Transmission of Sporadic Analog Samples over Wireless Channels

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***Abstract*—A low-latency, parameter modulation-estimation feedback protocol for wideband channels is introduced for both pure line-of-sight and more general fading channels with several degrees of freedom. One round of the protocol consists of a data phase and a control phase and uses non-coherent detection. The asymptotic optimality in energy efficiency of the protocol is analyzed and an upper bound on the distortion level is derived for two-rounds. The proposed scheme as well as known one-way schemes are compared with classical and very recent lower-bounds. Both the lower-bounds and performance evaluation of the feedback protocol are extended to a multi-channel fading model. The improvement of the feedback protocol over one-shot transmission is shown to be very significant on both line-of-sight and fading channels.** [1]

***Index Terms*—Joint source-channel coding, parameter modulation-estimation, non-coherent detection, distortion**

I. Introduction

In this work, we consider simple parameter modulation-estimation strategies applicable to future wireless sensor networks, where the sensor sporadically sends samples of analog information (temperature, magnetic field, current, speed, etc.) to a collecting node. The sensors can be seen as analog-to-digital converters which are distributed in space and use a wireless medium to relay their samples to the network. Such traffic is very low-rate, practically zero-rate, since in the majority of cases the sampling rate is very low, basically a few samples per second) and the available system bandwidth is very large (tens to hundreds of megahertz). The communication link from the samplers to the network often requires low-latency. The latter could arise for two reasons, either reactivity of an actuating element in the network or to minimize energy consumption in the sensing node itself by using discontinuous transmission and reception. Here the latency of the transmission is directly related to the ”on”-time of communication circuitry of the sensing node. This example captures the essense of *machine-type communications*, which refers to machines including sensors interconnected via cellular networks and exchanging information autonomously. It is widely believed that this sort of traffic will at least be shared with conventional voice and data communications on current and evolving cellular communication standards. Depending on the evolving usage scenarios, the amount of traffic produced by such low-rate devices could even vastly surpass that of conventional human communications. The purpose of this paper is to study modulation strategies for analog samples applicable to the wireless medium.

Imagine the simplest scenario of one sensor node tracking a slowly time-varying random sequence and sending its observations to a receiver over a wireless channel. The source is denoted by a random variable U of zero mean and variance $\sigma_u^2 = 1$, representing a single realization of the random sequence at a particular time t. The sensor should be seen as a tiny device with strict energy constraints. The communication channel between the sender and the receiver is an additive white Gaussian noise channel. An important question is how to efficiently encode the random variable U for transmission, and what performance can be achieved upon reconstruction as a function of the energy used to achieve this transmission. For this scenario, the slowly time-varying characteristic of the source has two main impacts on the way the coding problem should be addressed: firstly, the time between two observations is long, and the sensor should not wait for a sequence of observations to encode it. Therefore, the sensor will encode only one observation before sending it through the channel. Secondly, for each source realization the channel can potentially be used over many signal dimensions, for instance by encoding over a wide-bandwidth in the frequency-domain. This corresponds to the case for sensors connected directly to fourth-generation cellular networks. Hence, we can reasonably assume that there is no constraint on the dimensionality of the channel codebook. The latter

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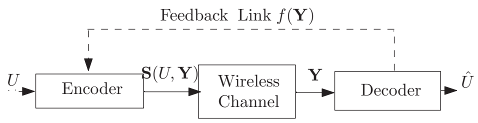

Fig. 1. Single-source Problem

amounts to saying that very low-rate codes should be used.

A. Addressed Problem and its Background

We consider a system as shown in Figure 1. An encoder maps one realization of a scalar source letter U into an N-dimensional vector $\sqrt{\mathcal{E}}\mathbf{S}(U,\mathbf{Y})$ which is transmitted across a wireless channel and received as an N-dimensional vector $\mathbf{Y}$. If the encoder makes use of feedback, we assume that this is causal, in the sense that the n^{th} dimension of the modulator is of the form $S_n(U,\mathbf{Y}) = g(U,Y_0,Y_1,\cdots Y_{n-1})$. We further assume that the transmitter is subject to a mean energy constraint $\mathbb{E}[||\mathbf{S}(U,\mathbf{Y})||^2] \leq 1$, so that the overall average energy is $\mathcal{E}$. The receiver is a mapping function which tries to construct an estimate $\widehat{U}$ of U given $\mathbf{Y}$. The fidelity criterion that we wish to minimize is the MSE distortion defined as $\mathcal{D} \triangleq \mathbb{E}[(U-\widehat{U})^2]$,

It is well-known that the linear encoder (i.e. $S_m = \sqrt{\mathcal{E}}U$) achieves the best performance under the mean energy constraint for the special case $N = 1$ [1]–[3] and normally-distributed U. In fact, a lower bound on the distortion over all possible encoders and decoders, both with and without a feedback link, was easily derived by Goblick in [1] using classical information theory, and given by

$$\mathcal{D}_g \geq e^{-2\mathcal{E}/N_0} \tag{1}$$

where $N_0/2$ is the channel noise variance per dimension. Note that, the form of (1) is adapted to a discrete-time complex Gaussian channel with noise variance $N_0/2$ to make the comparisons easier with lower-bounds to be introduced in Section V. Goblick's bound given above was derived through calculating the rate-distortion function and the channel capacity since for a particular source and a channel, the minimum mean-squared error in reconstructing the source using the channel output does not depend on the communication system. The author defined the channel capacity and the rate-distortion function in terms of the channel signal-to-noise ratio, more precisely minimum distortion in estimating the source message is obtained as a function of the channel signal-to-noise ratio which leads to the output signal-to-noise ratio in a continuous-time channel with limited bandwidth. Goblick also proposed a digital scheme shown in Figure 2 where a B bit uniform scalar quantizer is followed by 2^B-ary orthogonal modulation to transmit the source U using the energy $\mathcal{E}$. For this scheme, $\mathbf{S}(U,\mathbf{Y}) = \mathbf{S}_m$, where $m = 1,2,\cdots,2^B$ is the quantization index for the bin containing U. The performance loss with respect to (1) was heuristically argued to be on the order of 6-9 dB. At the end of the procedure the reconstruction error is the sum of the quantization error and a term proportional to the probability of error. In [4], [5] this reconstruction error is shown to be on the order of $e^{-\mathcal{E}/3N_0}$ both for coherent and non-coherent reception, which represents a 7.78 dB asymptotic energy gap with respect to (1).

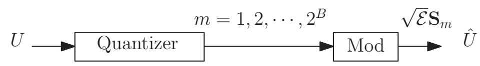

Fig. 2. Goblick's Digital Scheme

Several schemes can achieve $e^{-\mathcal{E}/3N_0}$ both with and without coherent detection and for both normally and uniformly distributed U. For instance, Wozencraft-Jacobs [6, pg:623-624] use analog pulse position modulation (PPM) with finite a finite bandwidth pulse where the delay of the pulse is proportional to the analog realization of U. The asymptotic performance of the distortion tends to $e^{-\mathcal{E}/3N_0}$ as the bandwidth goes to infinity [5]. A comparison in [5] with best-known joint medium-resolution source-channel codes [7] for high channel to source bandwidth ratios shows that simple hybrid yet separated joint-source channel techniques can outperform non-linear mappings. Such optimization for a different power constraint can be found in the literature for example in [8] and [9], where the authors try to bound the optimal number of quantization bits that minimizes distortion.

More recently, tighter lower-bounds for systems without feedback are derived by Merhav in [10], [11] for AWGN channels and discrete-memoryless channels, respectively. In [10], [12] the best-known lower bound for the reconstruction fidelity without feedback, with coherent detection and unlimited channel bandwidth behaves as $e^{-\mathcal{E}/2N_0}$ for uniformly-distributed U. The author achieves this lower-bound on the MSE through the threshold defined on the maximum exponential rate of error probability decay in estimating $|U-\hat{U}|$ rather than concentrating on the MSE as the performance criterion itself. In order to prove this threshold on the error probability of $|U-\hat{U}|$, the author adapts the well-known Ziv-Zakai bound [13] to the case with M hypotheses instead of 2 and the derivation proceeds as the Chazan-Zakai bound [14]. This narrows the asymptotic energy gap from Goblick's digital scheme to 1.79 dB. In a recent study [11], both upper and lower bounds for the best achievable exponential decay of $\mathbb{E}|\hat{U}-U|^m,\ m \geq 0$ are presented for a discrete memoryless channel.

In feedback systems, for example cellular networks, we could clearly imagine the use of reliable feedback from the down-link, with vanishing probability of error, i.e. perfect feedback. Some of the earliest work in analog transmission of low-bandwidth sources assumed feedback. Remarkably, stochastic control approaches [15], [16] can achieve, at least asymptotically, the lower bound on distortion in (1). This comes at the expense of delay, since, as in many adaptive systems, the feedback system must converge to minimize distortion. Moreover, these approaches are not directly applicable with non-coherent reception which would be required in practice to convey small amounts of information sporadically.

An example of a modern feedback-scheme for transmitting small amounts of sporadic information is the random-access procedure [17] in LTE systems, where a 6-bit message is conveyed using an orthogonal signal set occupying a large physical bandwidth (PRACH physical random access channel). The so-called *random-access response* contains the message hypothesized by the decoder, among other information, which serves either as an acknowledgment or an indication to retransmit. Although simplified, such a scheme was originally studied in [18] by Yamamoto which is an adaptation of the earlier work by Schalkwijk-Barron [19]. Note that, in these works the analysis with non-coherent detection is not provided. This type of transmission can exactly model any low-rate transmission strategy based on orthogonal modulation. For instance, to further put this in the context of the random-access procedure LTE systems, the $\mathbf{S}_m$ can represent the so-called *PRACH preamble* [20], where $m = 0, 1, \cdots, 63$, and conveys the 6-bit message (MSG1) described above. The preamble in LTE is a Zadoff-Chu roots-of-unity sequence which usually occupies $N = 839$ signaling dimensions for $B = 6$ information bits. Orthogonality over time-dispersive channels is guaranteed through up to 64 cyclic time-shifts of $\mathbf{S}_m$ coupled with the use of a cyclic extension. For very dispersive channels (i.e. with delay-spreads longer than the cyclic-shift between preambles), fewer than 64 (and hence longer) cyclic time-shifts can be used at the expense of using multiple preamble sequences which are quasi-orthogonal.

B. Contributions and Outline

The main contribution of this work is to analyze the use of a Yamamoto-style retransmission protocol [18] for the transmission of scalar quantized analog samples in terms of the energy-efficiency as a function of the reconstruction fidelity. It is shown that there is a very significant energy benefit coming from the use of feedback in comparison to one-shot transmission of the parameter as in Goblick's digital scheme [1]. The efficient use of such a protocol calls for joint optimization of the parameter quantization and modulation. It is important to note that in our scenario we are driven to assume unknown channels, i.e. non-coherent reception, in the formulation of the problem. Since the information content is very small, additional overhead for channel estimation is not warranted and thus, it is unreasonable to assume the channel state (i.e. channel amplitude and phase) be known to either the transmitter or receiver. The analysis is carried out for line-of-sight and non line-of-sight channels and we consider both cases of perfect and imperfect feedback. We furthermore provide new lower-bounds on the performance of such feedback-based schemes as well as numerical evaluation of recent bounds [10] for one-shot transmission. These bounds allow us to assess how close the proposed schemes are to fundamental limits.

In the upcoming section, we describe the channel models for the addressed problem. In Section III-A, we introduce a low-latency feedback protocol for a single source transmitting analog information over a non-coherent AWGN channel. In spirit, this is very similar to the first phase of the LTE random-access procedure described above. The analytical exponential behavior of the protocol with respect to the reconstruction error for estimating the source-message is observed and discussed subject to the energy used by the protocol. This is followed by the discussion regarding the effect of the feedback error on the distortion-energy trade-off made in Subsection III-B. We proceed with the analysis of a more general wireless channel model in Section IV. Additionally, for the case of one-shot transmission without feedback, in Subsection V-A we extend Merhav's recent lower-bounds derived in [10] for the problem addressed here, in order to provide the tightest numerical lower-bounds on performance. In Subsection V-B, we provide adaptations of Goblick's and Merhav's lower-bounds for the more general fading channel models. The numerical analysis results for a chosen configuration of the fading channel model are given in Section VI together with the results of the non-coherent AWGN channel. The numerical results are also contrasted with the best-known theoretical lower-bounds on the reconstruction fidelity. Comparisons between the two channel models are provided in Section VII for both single-round transmission without feedback in addition to the improvement achieved with two rounds of the novel protocol.

II. Channel Model

In this paper, we consider a general multi-channel wireless model [21, Section 12] where the channel amplitude and phase correspond to that of a multi-dimensional Ricean channel where the ratio of the non-line-of-sight component to the line-of-sight component is denoted as α. Let L represent the total number of statistically independent observations or diversity order of the transmitted signals and $L' \leq L$ denote the number of observations over which the average received energy is spread. To a first-order approximation, L' represents the number of coherence bandwidths and L/L' would represent the number of receive antennas. For example, $L = 4$, $L' = 2$ would correspond to a dual-antenna receiver with two coherence bandwidths. The output signal for this channel with a general N-dimensional channel input $\sqrt{\mathcal{E}}\mathbf{S}$ is given as

$$\mathbf{Y}'_l = \sqrt{\mathcal{E}/L'}\left(\sqrt{(1-\alpha)}\mathrm{e}^{j\Phi_l} + \sqrt{\alpha}h_l\right)\mathbf{S} + \mathbf{Z}_l, \quad (2)$$

for $l = 0, \cdots, L-1$ where $\mathrm{h} \sim N_{\mathbf{C}}(0,1)$ which have the desired statistics in both the frequency and time dimensions and α is a constant defined in the range $[0,1]$. The random phase sequence Φ is assumed to be i.i.d. with a uniform distribution defined on $[0, 2\pi)$. The N-dimensional vector noise sequence $\mathbf{Z}$ is complex, circularly symmetric with zero-mean and autocorrelation $N_0\mathbf{I}_{N\times N}$. Clearly, the channel model given above by (2) boils down to an AWGN channel for $\alpha = 0$ and $L' = 1$ with the N-dimensional channel observation given by

$$\mathbf{Y} = \sqrt{\mathcal{E}}\mathrm{e}^{j\Phi}\mathbf{S} + \mathbf{Z} \quad (3)$$

Throughout the paper, the general model given by (2) and the second model (3) will be referred as the fading channel and the AWGN channel, respectively. The following section will introduce a retransmission feedback protocol and provide an analytical upper bound on its performance for the AWGN channel. Due to its complexity the performance of the fading channel will be provided in Section IV only through numerical analysis.

III. Upper Bounds on the Performance of a Novel Feedback Protocol on Line-of-Sight Channels

A. Reliable feedback without energy cost

Let us consider now a Yamamoto-style protocol [18] applied to the transmission of isolated analog samples with non-coherent reception. In the analysis, we first focus on a simple non-coherent AWGN channel (3). The protocol consists of two phases, a data phase and a control phase which can be repeated up to two rounds. A pictorial representation of the protocol is shown in Figure 3. In our adaptation the two phases compose one round of the protocol. During the data phase, the source message is quantized, transmitted and its estimate is fed back from the receiver. The source denoted by U is uniformly distributed over $(-\sqrt{3}, \sqrt{3})$ which guarantees zero mean and unit variance. As in [1], a source sample is uniformly quantized to B bits

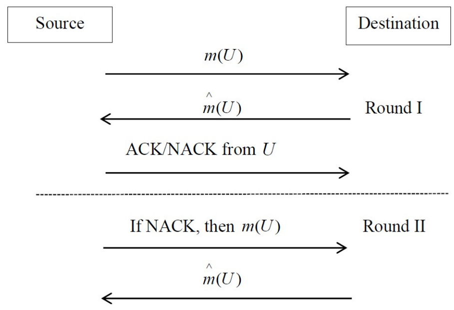

Fig. 3. Pictorial representation of the retransmission protocol.

is encoded into one of 2^B N-dimensional messages $\sqrt{\mathcal{E}}\mathbf{S}_m$, with $m = 1, 2, \cdots, 2^B$.

In this phase the channel observation (3) becomes

$$\mathbf{Y}_{D,i} = \sqrt{\mathcal{E}_{\mathrm{D},i}}\mathrm{e}^{j\Phi_i}\mathbf{S}_m + \mathbf{Z}_{D,i} \quad (4)$$

where the subscripts D and i represent the data phase and the i^{th} round of the protocol for $i = 1, 2$, respectively.

After the transmission of the source message, the receiver feeds $\hat{m}$ back to the encoder via the noiseless feedback link. We denote the corresponding error event by E_i. Square-law detection of the received signal produces the decision variables $U_{m'} = |\langle \mathbf{Y}_{D,1}, \mathbf{S}_{m'}\rangle|^2$ where $\langle .,. \rangle$ denotes the inner product. Let the projected noise on $\mathbf{S}_{m'}$ be denoted by $N_{m'} = \langle \mathbf{S}_{m'}, \mathbf{Z}_{D,1}\rangle$, a complex-valued zero-mean Gaussian random variable of variance N_0. Under the assumption that message m was transmitted, and given that $\mathbf{S}_{m'}$ is orthonormal with respect to $\mathbf{S}_m$ for $m' \neq m$, the first round decision variables are given as follows [21, sec.12-1-2].

$$U_{m'} = \begin{cases} |\sqrt{\mathcal{E}_{D,1}} + N_m|^2, & \text{for } m' = m \\ |N_{m'}|^2, & \text{for } m' \neq m \end{cases} \quad (5)$$

$|\sqrt{\mathcal{E}_{D,1}} + N_m|^2$ and $|N_{m'}|^2$ are random variables with non-central chi-square distribution (noncentrality parameter $\mathcal{E}_{D,1}$) and central chi-square distribution, respectively. According to (5), the receiver chooses $\hat{m} = \mathrm{argmax}_{\hat{m}'}\, U_{m'}$.

After the data phase, the encoder enters the control phase and informs the receiver whether or not its decision was correct via a signal $\sqrt{\mathcal{E}_{\mathrm{C},i}}\mathbf{S}_C$ of energy $\mathcal{E}_{\mathrm{C},i}$ if the decision is incorrect and $\mathbf{0}$ if the decision was correct where $\mathcal{E}_{\mathrm{C},i}$ denotes the energy of the control phase of the i^{th} round. $\mathbf{S}_C$ is an arbitary N-dimensional vector. During the control phase the receiver observes $\mathbf{Y}_{C,i} = \sqrt{\mathcal{E}_{\mathrm{C},i}}A\mathrm{e}^{j\Phi_i}\mathbf{S}_C + \mathbf{Z}_{C,i}$ where A takes the value 0 for an ACK and 1 for a NACK. Let $Y_{C,i} = \langle \mathbf{Y}_{C,i}, \mathbf{S}_C\rangle$ and assume a detector of the form $\hat{A} = \mathcal{I}\left(|Y_{C,i}|^2 > \lambda\mathcal{E}_{\mathrm{C},i}\right)$. Here $\mathcal{I}(\cdot)$ and λ represent the indicator function and a threshold to be optimized that is confined to an interval $[0,1)$, respectively. $E_{\mathrm{e}\to\mathrm{c},i}$ corresponds to an uncorrectable error since it acknowledges an error as correct decoding and $E_{\mathrm{c}\to\mathrm{e},i}$ represents a mis-detected acknowledged error declaring correct decoding as incorrect. If the receiver correctly decodes the control signal and it signals that the data phase was correct after the completion of the first round with probability $\Pr(E_1^c)(1-\Pr(E_{\mathrm{c}\to\mathrm{e},1}))$, the protocol halts, otherwise another identical round is initiated by the receiver. The retransmission probability, which is the probability of going on for a second round, is $\Pr(E_1)(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))$. This on-off signaling guarantees that with probability $\Pr(E_1^c)(1-\Pr(E_{\mathrm{c}\to\mathrm{e},1}))$ the transmitter will not expend more than $\mathcal{E}_{\mathrm{D},1}$ joules, which should be close to one. After each data phase, the receiver computes the ML or MAP message $\hat{m}_i(\mathbf{Y}_1,\cdots,\mathbf{Y}_i)$ based on all observations up to round i with error event E_i for $i=1,2$. The same control phase is repeated and the protocol is terminated after two rounds.

The error probability at the end of the second round is defined and consequently bounded by

$$\begin{aligned} P_{\mathrm{e}} = &\Pr(E_1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + \\ &\Pr(E_1)(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))\Pr(E_2|E_1) + \\ &(1-\Pr(E_1))\Pr(E_{\mathrm{c}\to\mathrm{e},1})\Pr(E_2|E_1^c) \\ &\overset{(a)}{\leq} \Pr(E_1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + \Pr(E_2). \end{aligned} \tag{6}$$

In step (a) the conclusive expression is obtained through bounding $\Pr(E_{\mathrm{c}\to\mathrm{e},1})$ and $(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))$ by 1. $Z_{C,i} = \langle \mathbf{S}_C, \mathbf{Z}_{C,i}\rangle$ is defined as a circularly-symmetric Gaussian zero-mean random variable with variance N_0. The probability of an uncorrectable error in round i, which is defined as $\Pr\left(|\sqrt{\mathcal{E}_{\mathrm{C},i}} + Z_{C,i}|^2 \leq \lambda\mathcal{E}_{\mathrm{c},i}\right)$, is obtained as

$$\Pr(E_{\mathrm{e}\to\mathrm{c},i}) = 1 - \mathrm{Q}_1\left(\sqrt{\frac{2\mathcal{E}_{\mathrm{C},i}}{N_0}}, \sqrt{\frac{2\lambda\mathcal{E}_{\mathrm{C},i}}{N_0}}\right), \tag{7}$$

where $\mathrm{Q}_1(\alpha,\beta)$ is the first-order Marcum-Q function. Furthermore, we have the recent bound on the $\mathrm{Q}_1(\alpha,\beta)$ for $\alpha > \beta$ from [22, eq:4] which is very useful for bounding (7) as

$$\Pr(E_{\mathrm{e}\to\mathrm{c},i}) \leq 1/2\exp\left(-\frac{(\sqrt{\lambda}-1)^2\mathcal{E}_{\mathrm{C},i}}{N_0}\right). \tag{8}$$

The probability of a mis-detected acknowledged error is obtained as

$$\begin{aligned} \Pr(E_{\mathrm{c}\to\mathrm{e},i}) &= \Pr\left(|Z_{C,i}|^2 > \lambda\mathcal{E}_{\mathrm{C},i}\right) \\ &= \mathrm{e}^{-\frac{\lambda\mathcal{E}_{\mathrm{C},i}}{N_0}}. \end{aligned} \tag{9}$$

The second round decision variables $U_{m'}^{(2)}$ can be obtained cumulatively through

$$U_{m'}^{(2)} = U_{m'} + |<\mathbf{Y}_{D,2}, \mathbf{S}_{m'}>|^2. \tag{10}$$

where $U_{m'}$ is given by (5). The receiver chooses $\hat{m} = \mathrm{argmax}_{\hat{m}'}\, U_{m'}^{(2)}$ over all possible sequences as in the first round. The probability of error for binary orthogonal signaling is defined in [21, eq:12.1-24] as

$$P_2(j) \leq \frac{1}{2^{2j-1}}\mathrm{e}^{-\gamma/2}\sum_{n=0}^{j-1} c_n \left(\frac{\gamma}{2}\right)^n \tag{11}$$

where $c_n = 1/n!\sum_{k=0}^{j-1-n}\binom{2j-1}{k}$ and γ represents the signal to noise ratio. The probability of making an error on a particular round j, $\Pr(E_j) \leq 2^B P_2(j)$ can be derived using (11) and given for the first and the second rounds by

$$\Pr(E_1) \leq 2^{B-1}\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}}, \tag{12}$$

$$\Pr(E_2) \leq 2^{B-3}\left(1 + 3\frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{N_0}\right)\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{2N_0}}. \tag{13}$$

Naturally, (12) and (13) are obtained using the first and the second round decision variables, respectively.

The reconstruction error of the source message is obtained by calculating the mean squared error distortion through $\mathcal{D} = \mathcal{D}_q(1-P_e) + \mathcal{D}_e P_e$ where $\mathcal{D}_q$ and $\mathcal{D}_e$ represent the quantization distortion and the MSE distortion for the case where an error was made, respectively. For a uniform source U on $(-\sqrt{3},\sqrt{3})$ and $B > 2$, the distortion for any round $\mathcal{N}$ is bounded by

$$\mathcal{D}(\mathcal{E}, N_0, \mathcal{N}, \lambda) \leq 2^{-2B}(1-P_e) + 2P_e. \tag{14}$$

A detailed derivation of the distortion terms in (14) is provided in Appendix VIII-A. The following subsections III-A1 and III-A2 respectively discuss the bound given above by (14) in the absence and presence of a feedback link in the system.

1) The performance of the protocol without feedback : For the case of $\mathcal{N} = 1$, i.e. the protocol terminates without retransmission, we obtain the bound on the reconstruction error in estimating the message of U as given in the following. The error probability defined by (6) consists of the probability of making an error in the first round $\Pr(E_1)$ solely, since there is no use of the control phase given that there will not be a second round to retransmit the message. Thus, through substitution of $P_e \leq 2^{B-1}\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}}$ into the distortion bound given by (14), we obtain the following bound

$$\mathcal{D}(\mathcal{E}, N_0, 1, \lambda) \leq \mathrm{e}^{-2B\ln 2} + \mathrm{e}^{B\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}}. \tag{15}$$

Through setting the two exponentials in (15) equal, it can be clearly seen that 2^{-B} is in the same order of $\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{6N_0}}$. In other words, the upper bound (15) is obtained as $\mathcal{D}(\mathcal{E}, N_0, 1, \lambda) \leq 2\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{3N_0}}$ for a single round.

2) The performance of the protocol exploiting feedback : At the end of the second round the resulting distortion is given by

$$\mathcal{D}(\mathcal{E}, N_0, 2, \lambda) \leq \mathrm{e}^{-2B\ln 2} + \mathrm{e}^{(B-1)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}}{2N_0} - \left(1+\lambda-2\sqrt{\lambda}\right)\frac{\mathcal{E}_{\mathrm{C},1}}{N_0}} + \left(1+3\frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{N_0}\right)\mathrm{e}^{(B-2)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{2N_0}} \quad (16)$$

through substituting (6) with (12), (13) and (8) for $i = 1$ into the distortion (14). By equating the three exponentials of (16) we have $\mathcal{E}_{\mathrm{C},1} = \frac{\mathcal{E}_{\mathrm{D},2}}{2(1+\lambda-2\sqrt{\lambda})}$. In order for $\Pr(E_1)$ to be exponentially bounded away from zero so that $\mathcal{E}$ can be made arbitrarily close to $\mathcal{E}_{\mathrm{D},1}$, we define $\mathcal{E}_{D,2} = (2-\mu)\mathcal{E}_{D,1}$ where μ is an arbitrary constant confined to $(0, 2)$. Finally, we obtain the final form of the bound on the distortion at the end of the second round as given by

$$\mathcal{D}(\mathcal{E}, N_0, 2, \lambda) \leq \mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}(1-\mu/3)}{N_0}}\left(3+3\frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{N_0}\right). \quad (17)$$

Note that, at the end of the second round, 2^{-B} is in the same order of $\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}(1-\mu/3)}{2N_0}}$.

Let $\mathcal{E}_{\mathrm{D},2}$ denote the required energy for retransmission, which is the energy to be used in the data phase of the second round. The average energy used by the protocol after two rounds is

$$\mathcal{E} = \mathcal{E}_{\mathrm{D},1} + \Pr(E_1)\mathcal{E}_{\mathrm{C},1} + (\Pr(E_1)(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1})) + (1-\Pr(E_1))\Pr(E_{\mathrm{c}\to\mathrm{e},1}))\mathcal{E}_{\mathrm{D},2}. \quad (18)$$

Clearly if $\Pr(E_1)$ is small, then the protocol achieves marginally more than $\mathcal{E}_{\mathrm{D},1}$ joules per source symbol. It is worth mentioning that (1) and the limiting expression in [15, eq.15] is achieved within a factor of 1/2 in the energy using only two rounds and, moreover, with non-coherent reception. Even though it is possible to obtain $\mathrm{e}^{-\frac{2\mathcal{E}_{\mathrm{D},1}}{N_0}}$ (i.e. twice better than the performance in (17)) by changing the relationship between the energies used in the different rounds, this causes the average energy used by the protocol to exceed the energy used in the data phase of the first round. As a result in this case, the proposed protocol could not achieve the error exponent in (1).

In Section III-B, we investigate the case when the feedback link from the decoder to the encoder is not perfect and discuss the effect of a possible error in feedback on the exponential behavior of the reconstruction error. Note that, for modeling systems where both the transmitter and receiver are subject to the constraints on energy usage, one would have to consider the energy consumption of the feedback link, and we also shed some light on this issue in Section III-B.

B. Unreliable feedback with and without energy cost

One might consider the case of an imperfect feedback link in the system described and analyzed above. Let m, $\hat{m}$ and $\hat{\hat{m}}$ denote the transmitted message, the messages decoded at the receiver and transmitter (after the feedback phase), respectively. Then $P_{\mathrm{fb},1}$ represents the following error probability $\Pr(\hat{\hat{m}} = m|\hat{m} \neq m)$ whereas $P_{\mathrm{fb},2} = \Pr(\hat{\hat{m}} \neq m|\hat{m} = m)$. The overall energy used by the protocol in this scenario becomes

$$\mathcal{E} = \mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{C},1}\Pr(E_1)(1-P_{\mathrm{fb},1}) + \mathcal{E}_{\mathrm{D},2}\left[\Pr(E_1)(1-P_{\mathrm{fb},1}) + (1-\Pr(E_1))P_{\mathrm{fb},2}\right] \quad (19)$$

whereas the error probability at the end of the second round yields as given on the top of the next page by (20). In step (a) of (20), $(1-P_{\mathrm{fb},1})$, $(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))$ and $\Pr(E_{\mathrm{c}\to\mathrm{e},1})$ are upper bounded by 1. Clearly, if we set $P_{\mathrm{fb},1} = P_{\mathrm{fb},2} = 0$ this case boils down to the perfect feedback scenario studied in the previous part, Section III-A and the expressions on average energy (19) and the error probability (20) yield (18) and (6), respectively. Now, we apply the modified error probability (20) to the overall distortion term (14). In order to obtain the same exponential behavior of $\mathrm{e}^{-\mathcal{E}_{\mathrm{D},1}/N_0}$ like in (17), $P_{\mathrm{fb},1}$ should be upper-bounded by the uncorrectable error $\frac{1}{2}\mathrm{e}^{-\frac{(\sqrt{\lambda}-1)^2\mathcal{E}_{C,1}}{N_0}}$ given earlier by (7). With respect to the energy consumption, we can say that in addition to the error probability in the first round, vanishing $P_{\mathrm{fb},2}$ guarantees the energy consumed by two rounds of the protocol to be upper bounded by the energy that is used by the data phase of the first round, i.e. $\mathcal{E}_{\mathrm{D},1}$.

In order to characterize the amount of the required energy for feedback, we consider an explicit scheme for feedback. The receiver uses waveform $\mathbf{S}_{\hat{m}}$ on the feedback link with energy $\mathcal{E}_{\mathrm{fb}}$ so that the received signal yields

$$\mathbf{Y}_{\mathrm{fb}} = \sqrt{\mathcal{E}_{\mathrm{fb}}}\mathrm{e}^{j\Phi}\mathbf{S}_{\hat{m}} + \mathbf{Z}. \quad (21)$$

In order to determine if message m was received correctly, the transmitter projects on waveform $\mathbf{S}_m$ and computes the statistic $U_{\mathrm{fb}} = |\langle\mathbf{Y}_{\mathrm{fb}}, \mathbf{S}_m\rangle|^2$ which is compared to a threshold $\lambda_{\mathrm{fb}}\mathcal{E}_{\mathrm{fb}}$ where $\lambda_{\mathrm{fb}} \in [0, 1)$. Then the feedback probability $P_{\mathrm{fb},1}$ is given by

$$P_{\mathrm{fb},1} = \Pr\left(|\langle\mathbf{Y}_{\mathrm{fb}}, \mathbf{S}_m\rangle|^2 \geq \lambda_{\mathrm{fb}}\mathcal{E}_{\mathrm{fb}}\right) = \mathrm{e}^{-\frac{\lambda_{\mathrm{fb}}\mathcal{E}_{\mathrm{fb}}}{N_0}}. \quad (22)$$

As a result, in order for $P_{\mathrm{fb},1}$ to be on the same exponential order as $\Pr(E_{\mathrm{e}\to\mathrm{c},1})$, we require that $\mathcal{E}_{\mathrm{fb}} = \frac{1-\mu/2}{\lambda_{\mathrm{fb}}}\mathcal{E}_{D,1}$ and that the energy used by the protocol approaches $\frac{\lambda_{\mathrm{fb}}+1-\mu/2}{\lambda_{\mathrm{fb}}}\mathcal{E}_{D,1}$.

The main conclusion is that when we account for the energy consumption required by the feedback link, it

$$
\begin{aligned}
P_{\mathrm{e}} &= \Pr(E_1)(1-P_{\mathrm{fb},1})\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + \Pr(E_1)P_{\mathrm{fb},1} + \Pr(E_1)(1-P_{\mathrm{fb},1})(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))\Pr(E_2|E_1) \\
&+ \left[(1-\Pr(E_1))(1-P_{\mathrm{fb},2})\Pr(E_{\mathrm{c}\to\mathrm{e},1}) + (1-\Pr(E_1))P_{\mathrm{fb},2}(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1})\right]\Pr(E_2|E_1^c) \\
&\overset{(a)}{\leq} \Pr(E_1)\left(\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + P_{\mathrm{fb},1}\right) + \Pr(E_2). \qquad (20)
\end{aligned}
$$

reduces the reconstruction fidelity in a non-negligible manner under a total energy constraint. In the primary application scenario considered here, namely energy-constrained sensors transmitting to cellular basestations, we believe that this does not pose a significant problem. Basestations are power constrained and not short-term energy constrained and if the aggregate downlink traffic dedicated to feedback for sensors is an order of magnitude less than other downlink services, this energy consumption is insigificant. If such schemes were to be used for transmission between energy-constrained devices, the benefits may be significantly reduced.

IV. More General Wireless Channels

We consider now the fading channel introduced in Section II by (2) and adapt this system to our retransmission feedback protocol proposed and analyzed in the previous section. In this fading channel model the output signal (2) in the data phase on channel l becomes

$$
\mathbf{Y'}_{D,i,l} = \sqrt{\mathcal{E}_{\mathrm{D},i}/L'}\left(\sqrt{(1-\alpha)}\mathrm{e}^{j\Phi_{i,l}} + \sqrt{\alpha}h_{i,l}\right)\mathbf{S}_m + \mathbf{Z}_{i,l}, \qquad (23)
$$

where $\mathrm{h}_i \sim N_{\mathbf{C}}(0,1)$. For this model, only the statistics of the mis-detected acknowledged error event is unchanged and is as given by (9). The probability of an uncorrectable error is given by (24) on the top of the next page. The error probabilities $\Pr(E_1)$ and $\Pr(E_2)$ corresponding to the first and second rounds, respectively are derived using an adaptation of [21, eq:12.1-22], which is given by (25) where j is the round index, I_n is the modified Bessel function of order n, $v = \frac{u}{2\mathcal{E}(N_0+\alpha\mathcal{E})}$ and $\gamma = \mathcal{E}/L'N_0$. u is the first decision variable with a non-central chi-square distribution having $2L$ degrees of freedom and non-centrality parameter $s^2 = \mathcal{E}(1-\alpha)$. Note that the probability $P_M(j)$ given by (25) reduces to [21, eq:12.1-22] for $\alpha = 0$. In the fading channel case, the protocol provides a more significant improvement when going from one to two rounds, due to the added diversity. Here it should be expected that the use of more than two rounds could be even more beneficial, unlike the AWGN case. The use of many rounds, however, will incur a non-coherent combining loss, despite the added diversity.

The upper bound on the reconstruction error given in Section III by (14) is adapted to the current model and by substituting (25) and (24), the following bound on the distortion at the end of the second round is obtained.

$$
\begin{aligned}
\mathcal{D}(\mathcal{E}, N_0, 2, \lambda) &\leq 2^{-2B}(1-P_e) + 2P_e \\
&\leq 2^{-2B} + 2\left[P_M(1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + P_M(2)\right] \qquad (26)
\end{aligned}
$$

In Section VI, we provide numerical evaluation results of the upper bound given above for different values of α for 0.5 and 0.1 since it is not possible to give an analytical result and discuss the improvement to be gained in two rounds through comparing (26) versus the distortion to be achieved in a single round without feedback, i.e. $\mathcal{D} \leq 2^{-2B} + 2P_M(1)$.

V. Lower Bounds on Distortion

In this part, we present lower bounds on the reconstruction error in estimating the source message that is transmitted over both types of channels (2) and (3) using information and estimation theoretic techniques.

A. AWGN Channel

The first set of bounds all rely on channel state knowledge at the receiving end which clearly is also a bound for the case where the channel phases are unknown. The simplest bound is Goblick's bound which in our case of a uniform random variable on $[-\sqrt{3}, \sqrt{3})$ is given by

$$
\mathcal{D}_G(\mathcal{E}, N_0) \geq \frac{6}{\pi e}\mathrm{e}^{-\frac{2\mathcal{E}}{N_0}}. \qquad (27)
$$

For the case of a single round without feedback we use the recent bounds from Merhav in [10] which are adaptations of the Ziv-Zakai lower-bound on mean-squared error for parameter modulation-estimation [13]. We consider only the case of zero-rate transmission in the context of [10] and adapt the results to the normalized uniform distribution considered here. We have the following bound on the distribution of distortion

$$
\Pr\left(|U-\hat{U}| > \frac{\sqrt{3}}{M}\right) \geq \frac{\sqrt{3}}{M}Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{M}{M-2}}\right). \qquad (28)
$$

$$\Pr(E_{\text{e}\to\text{c},i}) = \Pr\left(\sum_{l=0}^{L-1} |\sqrt{(1-\alpha)\mathcal{E}_{\text{C},i}/L'}\text{e}^{j\Phi_{i,l}} + \sqrt{\alpha\mathcal{E}_{\text{C},i}/L'}h_{i,l} + Z_{C,i,l}|^2 \le \lambda L\mathcal{E}_{\text{C},i/L'}\right)$$

$$= 1 - \text{Q}_L\left(\sqrt{\frac{2L(1-\alpha)\mathcal{E}_{\text{C},i}}{\alpha\mathcal{E}_{\text{C},i}+L'N_0}}, \sqrt{\frac{2L\lambda(1-\alpha)\mathcal{E}_{\text{C},i}}{\alpha\mathcal{E}_{\text{C},i}+L'N_0}}\right). \quad (24)$$

$$P_M(j) = 1 - \int_0^\infty \left(1 - e^{-v(1+\alpha\gamma)}\sum_{k=0}^{jL-1}\frac{(v(1+\alpha\gamma))^k}{k!}\right)^{M-1}$$

$$\left[v\left(\frac{1+\alpha\gamma}{\gamma(1-\alpha)}\right)\right]^{\frac{jL-1}{2}} e^{-v-\frac{\gamma(1-\alpha)}{(1+\alpha\gamma)}} I_{jL-1}\left(2\sqrt{\frac{v\gamma(1-\alpha)}{1+\alpha\gamma}}\right) \quad (25)$$

The right-hand side of (28) is the weakest version of Shannon's lower-bound on M-ary transmission over an AWGN channel [23, eq. 82]. Through the use of the Chebyshev inequality, this results in the following lower-bound on the distortion

$$\mathcal{D}_{M1}(\mathcal{E}, N_0) \ge \max_M \frac{3\sqrt{3}}{M^3} Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{M}{M-2}}\right). \quad (29)$$

A tighter version makes use of Shannon's best bound [23, eq. 81] and yields

$$\mathcal{D}_{M2}(\mathcal{E}, N_0) \ge \max_M \frac{6\sqrt{3}}{M^4}\sum_{n=2}^{M} Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{n}{n-1}}\right). \quad (30)$$

As suggested in [10, eq.23] an even tighter version based on [23, eq. 81] is derived using (31) as given on the top of the current page for any suitably large M. Numerical evaluation of the lower bounds introduced above and their comparison with the proposed transmission strategies are given in Section VI.

1) Relationships with classical conjectures on optimal signal sets: It is worth pointing out that certain classical and more recent results on the validity of conjectures on optimal signal sets are strongly related to the problem at hand and could provide tighter numerical lower-bounds on the reconstruction fidelity. In Merhav's bounding technique for the parameter modulation-estimation problem he relies on zero-rate lower-bounds on the probability of error (e.g. in [10, eq. 21]) in characterizing the tail-function of the estimation error at discrete values of its argument. For coherent detection on AWGN channels, it was long conjectured that the regular simplex was an optimal signal set for M-ary signaling in $M-1$ dimensions (i.e. without a bandwidth constraint). This was disproved by Steiner in [24] for the so-called *Strong Simplex Conjecture* which corresponds to the average energy constraint used here. The so-called *Weak Simplex Conjecture* is the classical conjecture [25] for equal-energy signaling which still has not been disproved and is valid for $M = 2, 3$. It is largely considered to be true for all M, and from a numerical perspective, was shown to be valid for $M \le 8$ in [26]. From a numerical perspective, the use of the constructive techniques in [26] for finding optimal signal sets could be used instead of Shannon's lower bound in (31). Although this will not provide an asymptotic difference, it could lead to tighter bounds for low signal-to-noise ratios. For the equal-energy case, it may be sufficient to use the error probability of the regular simplex in (31), at least if we limit the sum to $M \le 8$. Even if the Weak Simplex Conjecture is false, it is highly unlikely that any other signal set will provide a noticeable numerical difference in (31).

The equivalent equal-energy conjecture for non-coherent detection [27] also remains unproven. But it is reasonable for numerical purposes to use the error probability of orthogonal modulation with non-coherent detection as an approximate lower-bound. Using [27, eq. 28] instead of Shannon's lower bound in (31), we obtain

$$\mathcal{D}_{M4}(\mathcal{E}, M, N_0) \ge \frac{\sqrt{3}}{M^3}(5M+1)P_M + \sqrt{3}\sum_{i=2}^{M-1}\left(\frac{5i-4}{(i-1)^3} - \frac{5i+1}{i^3}\right)P_i \quad (32)$$

where

$$P_i = \sum_{n=1}^{i-1}(-1)^{n+1}\binom{i-1}{n}\frac{1}{n+1}\exp\left[-\frac{n}{n+1}\frac{\mathcal{E}}{N_0}\right] \quad (33)$$

which, strictly speaking, is only a true bound for equal-energy signaling and $M = 2$, subject to the validity of the classical conjecture. Note that (32) will have the same asymptotic behavior as (31).

2) Comments on variable-energy signaling: It is reasonable to expect that the use of variable-energy signaling (even orthogonal) can help close the 1.76dB asymptotic gap between (15) and (31) and the 3dB gap between (16) and (27). This is because with equal-energy signaling, erroneous decisions can lead to distortions at the peak or on the order of a bit with

$$
\begin{aligned}
\mathcal{D}_{M3}(\mathcal{E},M,N_0) &= 2\int_0^{2\sqrt{3}} d\Delta\cdot\Delta(2\sqrt{3}-(\left\lfloor 2\sqrt{3}/\Delta\right\rfloor-1)\Delta)\cdot\Pr\left(|U-\hat{U}|>\Delta\right)\\
&\geq\ 2\left(\int_0^{\sqrt{3}/M} d\Delta\cdot\Delta(2\sqrt{3}-(M-1)\Delta)\Pr\left(|U-\hat{U}|>\sqrt{3}/M\right)\right.\\
&+\ \left.\sum_{i=3}^{M}\int_{\sqrt{3}/i}^{\sqrt{3}/(i-1)} d\Delta\cdot\Delta(2\sqrt{3}-(i-1)\Delta)\Pr\left(|U-\hat{U}|>\sqrt{3}/(i-1)\right)\right)\\
&=\ \frac{\sqrt{3}}{M^4}(5M+1)\sum_{n=2}^{M}Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{n}{n-1}}\right)+\sqrt{3}\sum_{i=2}^{M-1}\left(\frac{5i-4}{(i-1)^4}-\frac{5i+1}{i^4}\right)\sum_{n=2}^{i}Q\left(\sqrt{\frac{\mathcal{E}}{N_0}\frac{n}{n-1}}\right). \quad (31)
\end{aligned}
$$

equal probability. A more judicious choice of energy distribution across the signal set would choose the energy difference between points according to their pairwise distortion. High distortion error events would then be less likely than low distortion error events.

B. Fading Channel

We consider the same two lower bounds on performance considered in the previous part. Merhav's bounding technique must be computed numerically in this case, as the upper bound (26) is presented in an analytical form. Merhav's results are applied to Rayleigh fading which is generalized here to channels with a line-of-sight component and more degrees-of-freedom. We also adapt the classical bound from Goblick [1] to a fading channel. Both of these techniques assume that the channel is known to the receiver and the distortion is averaged over all realizations of the random channel coefficients.

Merhav's bound (31) becomes

$$\overline{\mathcal{D}}_{M3}(\mathcal{E},M,N_0)\geq\mathbb{E}_a D_{M3}(a\mathcal{E},M,N_0) \quad (34)$$

where $a=\sum_{i=0}^{L-1}|\sqrt{1-\alpha}+\sqrt{\alpha}h_{i,l}|^2$ is a non-central chi-square distributed random variable with the non-centrality parameter $(1-\alpha)L$, $2L$ degrees of freedom and with the variance of the $2L$ underlying Gaussian random variables given by $\sigma^2=\alpha/2$. Its p.d.f. is given below.

$$
\begin{aligned}
f(a) &= \frac{1}{\alpha}\left(\frac{a}{(1-\alpha)L}\right)^{\frac{L-1}{2}}\exp\left(-\frac{a+(1-\alpha)L}{\alpha}\right)\\
I_{L-1}&\left(2\frac{\sqrt{a(1-\alpha)L}}{\alpha}\right). \quad (35)
\end{aligned}
$$

The behavior of the lower-bound (34) is presented numerically in the upcoming section.

The wireless adaptation of the Goblick bound (1) tries to capture the scenario considered in the achievable scheme (Section III), namely that a finite number of channel of realizations (or block-fading model) is exploited by the transmission strategy. To this end, we consider observations comprising N signaling dimensions split into R blocks of size N/R. Let $\mathbf{x}_k$ be the codeword in block k and constrain its energy as $\mathbb{E}||\mathbf{x}_k||^2\leq\mathcal{E}/R$. Each block witnesses an independent and identically distributed fading amplitude. We show in Appendix VIII-B that the distortion is bounded below by

$$
\begin{aligned}
\mathcal{D}\geq\frac{6}{\pi e}&\left(1+4\alpha\mathcal{E}/RN_0\right)^{-LR}\\
&\exp\left\{-\frac{2(1-\alpha)L\mathcal{E}/N_0}{1+4\alpha\mathcal{E}/RN_0}\right\}. \quad (36)
\end{aligned}
$$

VI. Numerical Evaluation

In this section, we provide numerical evaluation results for the bounds introduced in Sections III, IV and V. In Figure (4) we show the bound given by (14) for two rounds and different values of B from 6 to 14. The convex hull of these curves should be compared with the Goblick-bound given by (27) which is valid for systems with feedback. The curves labeled as the single-round scheme without feedback represent the upper bound (15). The convex hull of these curves should be compared with the Merhav bounds which are valid only without feedback. In Figure (4), the lower bounds given by equations (29), (30) and (31) are called as Merhav bound 1, 2 and 3, respectively. Firstly we see the significant effect of using the novel feedback protocol with respect to the reconstruction fidelity. The latter clearly provides an improvement in terms of distortion or approximately 3 dB in energy efficiency. We do not quite see the predicted 3dB gap (around 4.5 dB for 14-bits) in energy-efficiency with respect to the outer-bound with a known channel, even with a very high-resolution quantization level. Tighter bounding techniques for the case with feedback in addition to variable-energy schemes should therefore be considered for future work. The tightest of the Merhav bounds is clearly (31) but also does not quite predict the 1.7 dB asymptotic gap. Although not shown, numerical analysis also confirmed the asymptotic result given in Section III by (17) regarding the use of twice as much energy in the second round in comparison to the first.

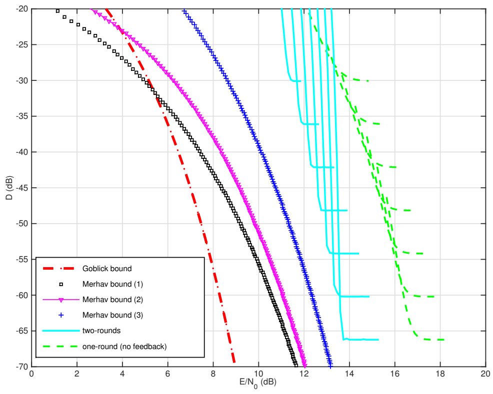

Fig. 4. Numerical evaluation of the upper and lower bounds on distortion for different values of B in an AWGN channel.

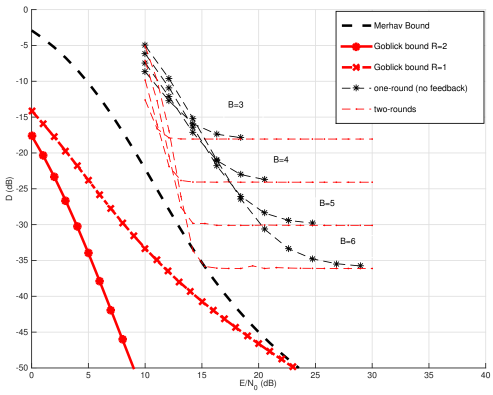

Fig. 5. Numerical evaluation of the distortion for B from 3 to 6 in a wireless channel for $\alpha = 0.1, L_p = 1, L = 1$

The upper-bound in (26) is depicted in Figures (5), (6) and (7) for several values of B for the cases of $\alpha = 0.1$ and $\alpha = 0.5$ and both high ($L = 4$) and low diversity orders ($L = 1$). In all cases we see a very significant effect ($\geq$10dB in energy-efficiency) in using a two-round feedback protocol compared to a one-shot transmission, and this still holds even in the case of a strong line-of-sight component ($\alpha = 0.1$). Both types of lower-bounds are looser in the case of the fading channels, and especially in the high-diversity case (Figure (7)). This can be attributed to the non-coherent combining loss which is not captured by the bounds which assume known channels. This motivates the search for better lower-bounds assuming unknown channels in their formulation.

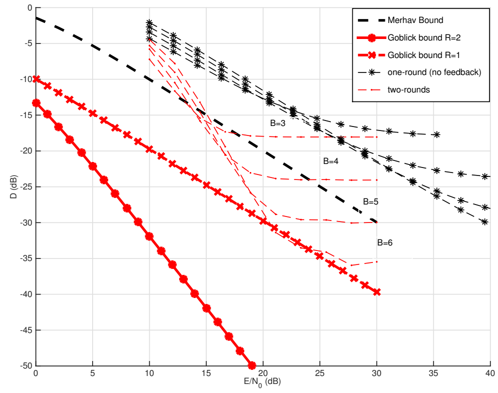

Fig. 6. Numerical evaluation of the distortion for B from 3 to 6 in a wireless channel for $\alpha = 0.5, L_p = 1, L = 1$

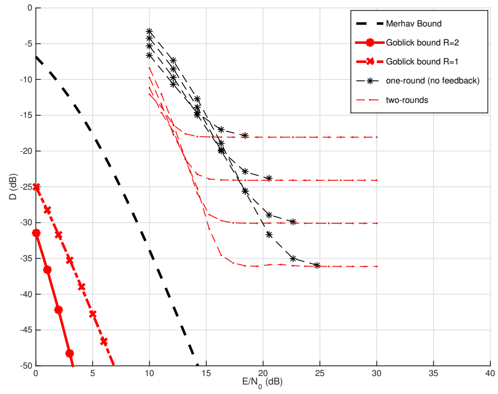

Fig. 7. Numerical evaluation of the derived bounds for B from 3 to 6 in a wireless channel for $\alpha = 0.5, L_p = 2, L = 4$

VII. Conclusion

We introduced a low-latency feedback protocol for the transmission of a single random variable over a wide-band channel and provided upper bounds on its performance with non-coherent detection on both pure line-of-sight and more general fading channels. The protocol and transmission strategy can be used for future energy-limited sensors making use of broadband cellular networks. We showed that the improvement over a one-shot transmission is on the order of 3-4 dB and asymptotically 4.7 dB. We have also included a discussion regarding the case of imperfect feedback and its effect on the trade-off between the required energy for the protocol and the reconstruction error in estimating the source message. We showed that in this case, if the energy consumption required by the feedback link is accounted, this reduces the reconstruction fidelity. Additionally, numerical evaluation of Merhav's recent lower-bounds for one-shot transmission are included and the tightest variant using his techniques is determined. Both the bounds and performance evaluation of the feedback protocol have been extended to a multi-channel fading model. The improvement of the feedback protocol over one-shot transmission is even more significant than in the line-of-sight case. We further suggest that tighter bounding techniques which rely on unknown channels should be found for the fading channel. Furthermore, schemes using variable-energy transmission should be considered to close the gap with the lower-bounds.

VIII. Appendix

A. Derivation of the Distortion $\mathcal{D}_e$

Let u be a realization of the random variable U and $\hat{u}$ denote its estimate. The distortion for the error case $\mathcal{D}_e = \mathbb{E}[(u-\hat{u})^2|\hat{u} \text{ in error}]$ is defined as follows

$$\begin{aligned}\mathbb{E}[(u-\hat{u})^2|\hat{u} \text{ in error}] &= \mathbb{E}[u^2|\hat{u} \text{ in error}] \\ &+ \mathbb{E}[\hat{u}^2|\hat{u} \text{ in error}] - 2\mathbb{E}[u\hat{u}|\hat{u} \text{ in error}]. \end{aligned} \tag{37}$$

The variance $\mathbb{E}[u^2]$ equals 1 by definition whereas $\mathbb{E}[\hat{u}^2|\hat{u} \text{ in error}]$ and $\mathbb{E}[u\hat{u}|\hat{u} \text{ in error}]$ are derived as follows, respectively.

$$\begin{aligned}\mathbb{E}[\hat{u}^2|\hat{u} \text{ in error}] &\overset{(a)}{=} \frac{1}{2^B-1}\sum_{i=0}^{2^{B-1}-1}\left(\frac{i\sqrt{3}}{2^{B-1}}+\frac{\sqrt{3}}{2^{B-2}}\right)^2 \\ &= \frac{3}{2^B-1}2^{-2B}\sum_{i=0}^{2^{B-1}-1}(2i+1)^2 \end{aligned} \tag{38}$$

Since U is distributed uniformly over $(-\sqrt{3},\sqrt{3})$, there are 2^B bins with a size of $2^{1-B}\sqrt{3}$ on each side of the origin. Conditioned on $\hat{u}$ being in error, there are as a total of both sides of the origin 2^B-1 bins left for $\hat{u}$ to fall in. The midpoint of the i^{th} bin and the distance between two adjacent bins is used in step (a) of (38) to obtain the variance of $\hat{u}$ for the error case.

$$\begin{aligned}\mathbb{E}[u\hat{u}|\hat{u} \text{ in error}] &= \int_{u>0} p(u)du \\ &(\mathbb{E}[\hat{u}u|\hat{u} \text{ in error}, U=u] + \mathbb{E}[\hat{u}u|\hat{u} \text{ in error}, U=-u]) \\ &= \int_{u>0}\frac{1}{2\sqrt{3}}du\,(\mathbb{E}[\hat{u}u|\hat{u} \text{ in error}, U=u] \\ &\qquad +\mathbb{E}[\hat{u}u|\hat{u} \text{ in error}, U=-u]) \\ &\overset{(b)}{=} \frac{1}{2\sqrt{3}}\sum_{i=0}^{2^{B-1}-1}\int_{u\in B_i}du \\ &\left(\frac{1}{2^B-1}\sum_{j \text{ s.t.} \hat{u}_j\notin B_i}u\hat{u}_j - \frac{1}{2^B-1}\sum_{j \text{ s.t.} -\hat{u}_j\notin B_i}u\hat{u}_j\right) \\ &= \frac{1}{2\sqrt{3}}\sum_{i=0}^{2^{B-1}-1}\int_{u\in B_i}du\left(\frac{1}{2^B-1}(-\hat{u}_i)-\frac{1}{2^B-1}\hat{u}_i\right) \\ &= -\frac{1}{\sqrt{3}(2^B-1)}\sum_{i=0}^{2^{B-1}-1}\hat{u}_i\int_{u\in B_i}duu \\ &= -\frac{1}{\sqrt{3}(2^B-1)}\sum_{i=0}^{2^{B-1}-1}\left(\frac{i\sqrt{3}}{2^{B-1}}+\frac{\sqrt{3}}{2^{B-2}}\right)\left[\frac{u^2}{2}\right]_{u\in B_i} \\ &= -\frac{1}{2\sqrt{3}(2^B-1)}\sum_{i=0}^{2^{B-1}-1}3\sqrt{3}2^{2-3B}(2i+1)^2 \\ &= -\frac{3(2^{1-3B})}{2^B-1}\sum_{i=0}^{2^{B-1}-1}(2i+1)^2 \end{aligned} \tag{39}$$

In step (b) of (39), B_i is the bin corresponding to $(i2^{1-B}\sqrt{3},(i+1)2^{1-B}\sqrt{3})$ and each sum reduces to a single term as $-\hat{u}_i$ and $\hat{u}_i$ in the following step. The distortion conditioned on $\hat{u}$ being in error (37) yields

$$\begin{aligned}\mathcal{D}_e &= 1+\frac{3}{2^B-1}2^{-2B}\sum_{i=0}^{2^{B-1}-1}(2i+1)^2 \\ &+\frac{3}{2^B-1}(2^{2-3B})\sum_{i=0}^{2^{B-1}-1}(2i+1)^2 \\ &= 1+\frac{3}{2^B-1}(2^{-2B}+2^{2-3B})\sum_{i=0}^{2^{B-1}-1}(2i+1)^2 \\ &\leq 2, \text{ for } B>2 \end{aligned} \tag{40}$$

Note that D_e decays as B increases. On the other hand, the quantization distortion $\mathcal{D}_q$ is simply the variance within a single bin which is 2^{-2B}.

B. Wireless Adaptation of the Goblick Bound

In order to derive a lower bound the distortion level of the wireless channel with feedback, we define the model $Y_{r,k} = \sqrt{h_r}X_{r,k} + Z_{r,k}$, $k = 1,\cdots,N/R$, $r = 1,\cdots,R$ where $Y_{r,k}$, $X_{r,k}$, h_r and $Z_{r,k}$ are the channel output, input, complex fadig amplitude and the noise terms, respectively. We start with two different expansions of the mutual information $I(U;\mathbf{Y}|\{\mathbf{H}_r=h_r, r=1,\cdots,R\})$ which are equated and given as follows.

$$\begin{aligned}I(U;\mathbf{Y}|\{\mathbf{H}_r=h_r\}) &= h(U|\{\mathbf{H}_r=h_r\}) \\ &- h(U-\hat{U}(\{\mathbf{H}_r=h_r\})|\mathbf{Y},\{\mathbf{H}_r=h_r\}) \\ &\overset{(a)}{=} h(U) - h(U-\hat{U}(\{\mathbf{H}_r=h_r\})|\mathbf{Y},\{\mathbf{H}_r=h_r\}, \\ &\quad \hat{U}(\{\mathbf{H}_r=h_r\})) \\ &\geq h(U) - h(U-\hat{U}(\{\mathbf{H}_r=h_r\})) \\ &\overset{(b)}{=} \frac{1}{2}\log 12 - \frac{1}{2}\log(2\pi e\mathcal{D}(h)) \\ &= \frac{1}{2}\log\left(\frac{6}{\pi e}\frac{1}{\mathcal{D}(h)}\right) \end{aligned} \tag{41}$$

where $\mathcal{D}(h)$ represents $\mathcal{D}(\{\mathbf{H}_r = h_r\})$. In step (a) given the independence between U and $\{\mathbf{H}_r = h_r\}$, the conditional entropy equals the entropy of the source. In step (b) the entropy of a uniform source defined on $(-\sqrt{3}, \sqrt{3})$ is substituted. For the second expansion we have

$$
\begin{aligned}
&I(U;\mathbf{Y}|\{\mathbf{H}_r = h_r\}) = h(\mathbf{Y}|\{\mathbf{H}_r = h_r\}) \\
&- h(\mathbf{Y}|U,\{\mathbf{H}_r = h_r\}) \\
&= \sum_{r=1}^{R}\sum_{k=1}^{N/R} h(Y_{r,k}|Y_r^{k-1}, Y_1^N, \cdots, Y_{r-1}^N, \{\mathbf{H}_r = h_r\}) \\
&- \sum_{r=1}^{R}\sum_{k=1}^{N/R} h(Y_{r,k}|Y_r^{k-1}, Y_1^N, \cdots, Y_{r-1}^N, U, \{\mathbf{H}_r = h_r\}) \\
&= \sum_{r=1}^{R}\sum_{k=1}^{N/R} h(Y_{r,k}|Y_r^{k-1}, Y_1^N, \cdots, Y_{r-1}^N, \{\mathbf{H}_r = h_r\}) \\
&- \sum_{r=1}^{R}\sum_{k=1}^{N/R} h(Y_{r,k}|Y_r^{k-1}, Y_1^N, \cdots, Y_{r-1}^N, U, \mathbf{X}, \{\mathbf{H}_r = h_r\}) \\
&= \sum_{r=1}^{R}\sum_{k=1}^{N/R} h(X_{r,k}\sqrt{h_r} + Z_{r,k}|Y_r^{k-1}, Y_1^N, \cdots, Y_{r-1}^N, \\
&\qquad \{\mathbf{H}_r = h_r\}) - \sum_{r=1}^{R}\sum_{k=1}^{N/R} h(Z_{r,k}) \\
&\le \sum_{r=1}^{R}\sum_{k=1}^{N/R} \log 2\pi e\left(N_0 + \mathcal{E}_{r,k}|h_r|^2\right) \\
&- \sum_{r=1}^{R}\sum_{k=1}^{N/R} \log 2\pi e N_0 \\
&\overset{(c)}{=} \sum_{r=1}^{R} \frac{N}{R}\log\left(1 + \frac{R}{N}\frac{\mathcal{E}_r|h_r|^2}{N_0}\right) \\
&\le \sum_{r=1}^{R} \frac{\mathcal{E}_r|h_r|^2}{N_0}.
\end{aligned}
\tag{42}
$$

In step (c) we used the following property $\log(1+x) \le x$. Equating the two expansions (41) and (42) yields

$$
\mathcal{D}(h) \ge \frac{6}{\pi e} e^{-2\sum_{r=1}^{R}\mathcal{E}_r|h_r|^2/N_0} \tag{43}
$$

which can be re-written as $\mathcal{D}(h) \ge \frac{6}{\pi e} e^{-2\frac{\mathcal{E}}{RN_0}\sum_{r=1}^{R}|h_r|^2}$ with $\mathcal{E}_r = \mathcal{E}/R \quad \forall r$. The average distortion $\mathcal{D} = \mathbb{E}[\mathcal{D}(h)]$ is therefore lower bounded by the moment generating function of $|h|^2$, as shown below by $M_{|h|^2}(t)$, with $t = -2\mathcal{E}/N_0$.

$$
\begin{aligned}
&M_{|h|^2}(t) = \\
&\frac{6}{\pi e}\prod_{r=1}^{R}\left(1 + 4\alpha\mathcal{E}/RN_0\right)^{-L}\exp\left\{-\frac{2(1-\alpha)L\mathcal{E}/RN_0}{1 + 4\alpha\mathcal{E}/RN_0}\right\}
\end{aligned}
\tag{44}
$$

The final form of the lower bound (43) is given in Section V-B by (36).

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