INTERFERENCE ALIGNMENT LIMITS FOR K-USER FREQUENCY-FLAT MIMO INTERFERENCE CHANNELS

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ABSTRACT

Starting from the zero-forcing constraints, some conditions to be satisfied in order to admit an interference alignment (IA) solution for any K-user frequency-flat MIMO interference channel are derived. Our conditions allow for analytical evaluation of the existence of IA solutions (or lack thereof) for a given K-user MIMO interference channel and degree of freedom allocation. We discuss interference alignment duality and the optimality of linear minimum mean squared error (LMMSE) interference reduction at the receiver for user rate maximization. Motivated by this, we suggest an algorithm for distributed interference alignment in a time division duplex (TDD) setting.

1. INTRODUCTION

The recent paper on interference alignment [1] (IA) that demonstrated the acheivability of a capacity prelog factor of $K/2$ in a K-user interference channel has resulted in a renewed interest in joint linear processing at transmitter and receiver with the aim of maximizing the capacity prelog (otherwise known as the degrees of freedom (DoF)) of a multiuser MIMO interference channel. IA exploits the availability of channel state information at the transmitter (CSIT) to compute appropriate beamforming matrices to be applied at each transmitter such that, at each receiver, all the interference is confined within a subspace of dimension complementary to the receiver's desired signal subspace dimension. Thus, simple zero-forcing (ZF) receivers are enough to separate the desired signal from the interferers. While it is known that interference alignment is the optimal (in the high SNR regime) scheme among approaches that use linear transmit/receive processing and treat interference as Gaussian noise, the existence of solutions in many cases is not known in general. In this paper we focus on one of these cases, namely, the frequency-flat MIMO interference channel.

2. SYSTEM MODEL

Fig. 1 depicts the K-user MIMO interference channel with K transmitter-receiver pairs. The k-th transmitter and the k-th receiver are equipped with M_k and N_k antennas respectively. The k-th transmitter generates interference at all $l \neq k$ receivers. Assuming the communication channel to be frequency-flat, the $\mathbb{C}^{N_k \times 1}$ received signal $\mathbf{y}_k$ at the k-th receiver, can be represented as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{H}_{kl}\mathbf{x}_l + \mathbf{n}_k \qquad (1)$$
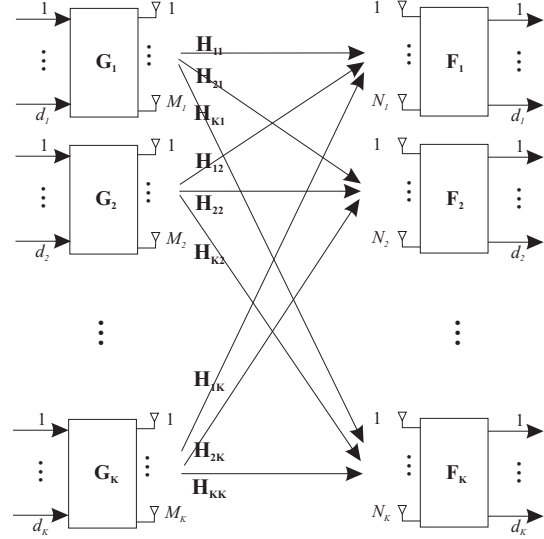

Figure 1: MIMO Interference Channel

where $\mathbf{H}_{kl} \in \mathbb{C}^{N_k \times M_l}$ represents the channel matrix between the l-th transmitter and k-th receiver, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector corresponding to the k-th transmitter, the $\mathbb{C}^{N_k \times 1}$ vector $\mathbf{n}_k$ represents the additive white Gaussian noise with zero mean and covariance matrix $\mathbf{R}_{n_k}$. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution without any deterministic relation between channel coefficients. We assume that each transmitter has complete knowledge of the channel matrices corresponding to its direct link and all the other cross-links. This limited set of assumptions is made only in the context of ZF interference alignment and in deriving the existence conditions in Sec. 3. It must be pointed out that all transmitters need to know all the channel matrices, transmitter power constraints and the receiver noise variances for sum rate (region) considerations.

Denoting the $\mathbb{C}^{M_k \times d_k}$ beamforming matrix of the k-th transmitter as $\mathbf{G}_k$, the transmitted signal vector $\mathbf{x}_k$ is given by $\mathbf{x}_k = \mathbf{G}_k\mathbf{s}_k$, where the $d_k \times 1$ vector $\mathbf{s}_k$ represents the transmitted symbols and d_k the number of independent streams transmitted to its receiver. We assume $\mathbf{s}_k$ to have a Gaussian distribution with zero mean and unit variance, $\mathcal{N}(0, \mathbf{I}_{d_k})$. At the k-th receiver, $\mathbf{F}_k \in \mathbb{C}^{d_k \times N_k}$, is applied to suppress interference and retrieve the d_k desired streams. Applying the interference suppressing filter $\mathbf{F}_k$ to $\mathbf{y}_k$ we obtain the follow-

ing $d_k \times 1$ vector $\mathbf{r}_k$

$$\mathbf{r}_k = \mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k \mathbf{s}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l \mathbf{s}_l + \mathbf{F}_k \mathbf{n}_k \qquad (2)$$

3. EXISTENCE CONDITIONS FOR INTERFERENCE ALIGNMENT SOLUTIONS

The objective of IA is to design aligning matrices to be applied at the transmitters such that, the interference caused by all transmitters at each non-intended receiver lie in a common *interference subspace*. Moreover, the interference subspace and the *desired signal subspace* of each receiver should be non-overlapping. If alignment is complete, simple zero forcing (ZF) can be applied to suppress the interference and extract the desired signal.[1] Since interference alignment is a condition for joint transmit-receive linear zero forcing, we need to consider the ZF conditions:

$$\mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l = \mathbf{0} \quad \forall l \neq k \qquad (3)$$

$$\mathrm{rank}(\mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \qquad (4)$$

Finally, the traditional single user MIMO constraint $d_k \leq \min(M_k, N_k)$ also needs to be satisfied for d_k streams to be able to pass over link k.

We now wish to translate the above equations into a set of conditions that need to be satisfied for a given K-user interference channel where each user pair has M_k transmit and N_k receive antennas so d_k interference-free streams are available for communication for the k-th user pair.

The purpose of the beamformer matrix $\mathbf{G}_k$ is to align the transmit signal of the k-th user to the interference subspace at all $l \neq k$ users while ensuring the rank of the equivalent channel matrix $\mathbf{F}_k \mathbf{H}_{kk} \mathbf{G}_k$ is d_k. This implies that the beamforming matrix is determined up to an arbitrary $d_k \times d_k$ square matrix. Since $\mathbf{G}_k$ has dimensions $M_k \times d_k$ the total number of variables in the beamforming matrix is reduced to $d_k(M_k - d_k)$. At receiver k the interference due to all other $(K - 1)$ transmitters are grouped into a $(N_k \times \sum_{l=1; l \neq k}^{K} d_l)$ matrix $\mathbf{H}_{INT}^{[k]} = [\mathbf{H}_{k1}\mathbf{G}_1, \ldots, \mathbf{H}_{k(k-1)}\mathbf{G}_{(k-1)}, \mathbf{H}_{k(k+1)}\mathbf{G}_{(k+1)}, \ldots, \mathbf{H}_{kK}\mathbf{G}_K]$ that spans the interference subspace at the k-th receiver.

The total signal-space dimension at receiver k is given by the total number of receive antennas N_k of which d_k interference-free signaling dimensions are to be reserved for the signal from the k-th transmitter. This is achieved when the interference from all other transmitters lies in an independent subspace whose dimension can be at most $N_k - d_k$. Thus the dimension of the subspace spanned by the matrix $\mathbf{H}_{INT}^{[k]}$ must satisfy

$$\mathrm{rank}(\mathbf{H}_{INT}^{[k]}) = r_k \leq N_k - d_k \qquad (5)$$

In order to satisfy this condition, we impose the following constraints on $\mathbf{H}_{INT}^{[k]}$

$$(N_k - r_k)(\sum_{\substack{l=1 \\ l \neq k}}^{K} d_l - r_k) \qquad (6)$$

[1]We say alignment is complete when there is no interference power in the desired signal subspace. However, it is possible that part of the signal lies in the interference space. Indeed, in such cases, the ZF receiver discards the part of the signal in the interference subspace.

r_k is maximized when the interference contribution of each interferer spans an independent subspace. Which leads us to the upperbound $r_k \leq \sum_{l=1; l \neq k}^{K} d_l$. However, accounting for the inequality in (5) we have

$$r_k = \min(d_{tot}, N_k) - d_k \qquad (7)$$

where $d_{tot} = \sum_{k=1}^{K} d_k$. Plugging this is in (6), the total number of constraints now become

$$(N_k + d_k - \min(d_{tot}, N_k))(d_{tot} - \min(d_{tot}, N_k)) \qquad (8)$$

In order to obtain the number of variables and constraints for the complete system, we sum up the results obtained above for each transmit-receive pair. Furthermore, since a solution for the interference alignment problem will exist only if the total number of variables are greater than or equal to the total number of constraints that need to be satisfied in the problem, we have

$$\sum_{k=1}^{K} d_k(M_k - d_k) \geq \sum_{k=1}^{K} d_k[d_{tot} - N_k]_+ \qquad (9)$$

where $[x]_+ = \max(0, x)$.

Remark: In most practical systems $d_{tot} \geq N_k$, in which case, the $[d_{tot} - N_k]_+$ on the RHS of (9) reduces to $(d_{tot} - N_k)$. Assuming that the number of receiving antennas is not greater than the total number of transmitted streams, $d_{tot} \geq N_k$, the total number of constraints in (8) becomes:

$$d_k(d_{tot} - N_k) \qquad (10)$$

The above equation only dictates the total number of transmit and receive antennas that should be available in the system and does not convey any information in terms of the actual distribution of antennas among users. Therefore, additional conditions arising from combinations of transmit and receive antennas and the number of streams need to be considered in order to decide if an interference alignment solution exists for the given K-user MIMO system. We do this by considering the following pairwise condition that takes into account the interference at receiver k due to transmitter l. It is known that this interference must lie in a subspace of proper dimension given by

$$\mathrm{rank}(\mathbf{H}_{kl}\mathbf{G}_l) = r_{kl} \quad \Rightarrow \quad r_{kl} \leq N_k - d_k \qquad (11)$$

In general $r_{kl} \in \{[d_l - [M_l - N_k]_+]_+, \ldots, d_l\}$. The rank is zero when $\mathbf{G}_l$ is perfectly aligned to the orthogonal complement of the space spanned by $\mathbf{H}_{kl}$ (which amounts to transmitter l suppressing all interference to receiver k) and is equal to d_l when none of streams from the l-th transmitter are aligned to the orthogonal complement. Denoting by (α_{kl}) the number of streams aligned to the orthogonal complement of $\mathbf{H}_{kl}$, we have $r_{kl} = d_l - \alpha_{kl}$ where α_{kl} satisfies the conditions: $0 \leq \alpha_{kl} \leq [M_l - N_k]_+$ and $\sum_{k=1; k \neq l}^{K} \alpha_{kl} \leq d_l$. Since l-th transmitter should align its streams at all $k \neq l$ receivers the condition (11) must be satisfied for all the receivers. So

$$[d_l + d_k - N_k]_+ \leq \alpha_{kl} \leq [M_l - N_k]_+ \quad \forall k \neq l. \qquad (12)$$

Finally, since the total number of streams that transmitter l can align must be bounded above by the rank of $\mathbf{G}_l$ we impose the inequality

$$\sum_{\substack{k=1 \\ k \neq l}}^{K} [d_l + d_k - N_k]_+ \leq d_l \tag{13}$$

We now have the set of conditions that need to be satisfied for a given MIMO interference channel with K users and M_k, N_k antennas to transmit d_k interference free streams which we enumerate here

$$\sum_{k=1}^{K} d_k(M_k - d_k) \geq \sum_{k=1}^{K} d_k[d_{tot} - N_k]_+ \tag{14}$$

$$[d_l + d_k - N_k]_+ \leq [M_l - N_k]_+ \quad \forall k \neq l \tag{15}$$

$$d_l \geq \sum_{\substack{k=1 \\ k \neq l}}^{K} [d_l + d_k - N_k]_+ \quad \forall l \tag{16}$$

$$d_k \leq \min(N_k, M_k) \quad \forall k \tag{17}$$

3.1 Alignment Duality

There are another set of conditions that need to be considered in order to complete the existence conditions. These conditions arise from the equations

$$\widetilde{\mathbf{F}}_l \widetilde{\mathbf{H}}_{lk} \widetilde{\mathbf{G}}_k = \mathbf{0} \quad \forall k \neq l \tag{18}$$

$$\text{rank}(\widetilde{\mathbf{F}}_k \widetilde{\mathbf{H}}_{kk} \widetilde{\mathbf{G}}_k) = d_k \quad \forall k \in \{1, 2, \ldots, K\} \tag{19}$$

which corresponds to the interference alignment constraints of the dual problem where all transmitters and receivers exchange roles. In other words, when $\widetilde{\mathbf{F}}_l = \mathbf{G}_l^H, \widetilde{\mathbf{G}}_k = \mathbf{F}_k^H, \widetilde{\mathbf{H}}_{lk} = \mathbf{H}_{kl}^H$ in (3) and (4). The dual problem of an interference channel is again an interference channel, involving the reciprocal channel. For the ZF case (interference alignment) the conditions (18)-(19) for the dual problem are obtained immediately by simply taking the transpose of (3)-(4) for the original problem [2]. If the ZF filters are replaced by MMSE receive filters that are the optimal interference suppressing filters (c.f Sec. 5) we conjecture a sum-rate duality for the K-user MIMO interference channel for an appropriate choice of receiver noise covariance matrices and transmit power constraints. This duality for the centralized approach is the motivation for the distributed TDD algorithm proposed in Sec. 5. In this section we focus on interference alignment duality and hence restrict ourselves to a ZF design. As a direct consequence of interference alignment duality, for an interference alignment solution to exist, the conditions in (14)-(16) should also be satisfied when the M_k and N_k are interchanged.

4. INTERFERENCE ALIGNMENT: SPECIFIC SCENARIOS

In the previous section, we derived the conditions that must be satisfied to admit an interference alignment solution in the most general case without any assumption on the system parameters. In this section we look at some interesting special cases and their implications on the interference alignment conditions (14)-(17).

4.1 Identical Stream and Antenna Configuration (ISAC)

Consider a K-user interference channel where each user pair has identical antenna configurations. Furthermore, it is required that user pairs have demanded the same number of interference free signaling dimensions. Simply put: $M_k = M, N_k = N, d_k = d \quad \forall k$. We call such a system an identical stream and antenna configuration (ISAC) system. For such a system, the conditions (14) through (17) take considerably simpler forms and can be expressed as

$$(M - d) \geq [Kd - N]_+ \tag{20}$$
$$[2d - N]_+ \leq [M - N]_+ \tag{21}$$
$$d \geq (K - 1)[2d - N]_+ \tag{22}$$
$$d \leq \min(N, M) \tag{23}$$

In the course of the work reported in this paper, the authors came across independent work in [3] where such a system is referred to as a symmetric case and denoted by $(M, N, d)^K$. Indeed, for $K \geq N$, the conditions in (20) through (23) coincide with the results [3]. However their conditions do not address the $K = 2$ case. We show this with an example of a 2 user ISAC system with $M = N = 3, d = 2$. For this case, the conditions derived in [3] suggest that $d = 2$ is feasible which is not true since the maximum DoF in the 2 user case with M antennas at each node is known to be a maximum of M [4]. The conditions derived in this paper address this and show that indeed $d = 2$ is infeasible. In addition, for the asymmetric cases (non ISAC cases) our conditions are more explicit and restrictive.

4.2 Interference Alignment For Real Signals

The key observation we make in this section is that by using real signal constellations in place of complex constellations, transmission over a complex channel of any given dimension can be interpreted as transmission over a real channel of double the original dimension (by treating the in-phase and quadrature components as separate channels). This doubling of dimensions provide additional flexibility in achieving the total DoF available in the network. We show this with a simple example of a 3 user ISAC MIMO interference channel where each transmitter and receiver has the same number of antennas. i.e., $M = N = 3$. From the results of the earlier section, we can show that the maximum interference-free streams available per-user obeys $d \leq 1.5$. Since the concept of transmitting 0.5 streams does not make any practical sense any interference alignment solution that exists for this system allows reception of a maximum of 1 interference-free stream per user. However, if the complex channel is considered to be composed of two real channels and if the transmitters and receivers use real signal constellations, the dimensions involved in the above problem are doubled and hence allows for a finer adjustment of d_k

Representing the $2M_k \times 2N_k$ *real* MIMO channel as

$$\hat{\mathbf{H}} = \begin{bmatrix} \text{Re}\{\mathbf{H}\} & -\text{Im}\{\mathbf{H}\} \\ \text{Im}\{\mathbf{H}\} & \text{Re}\{\mathbf{H}\} \end{bmatrix}$$

and using $\hat{\mathbf{x}}$ to represent the $2N_k \times 1$ real signal vector, the received signal at the k-th receiver in equation (1) can now be expressed as

$$\hat{\mathbf{y}}_k = \hat{\mathbf{H}}_{kk}\hat{\mathbf{x}}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \hat{\mathbf{H}}_{kl}\hat{\mathbf{x}}_l + \hat{\mathbf{n}}_k \tag{24}$$

In our example, each user is now capable of transmitting 3 real streams thereby exploiting fully all the available per-user DoF.

4.3 Linear Interference Suppression at One Side

For a fixed distribution of d_k among the K-users in the interference channel, (14) dictates the total number of transmit and receive antennas in the system. However, there exists a certain amount of flexibility in distributing these antennas among the transmitters and the receivers provided the new distribution M'_k and N'_k does not violate the conditions in (15) through (17). In this section we describe two particular cases where all the interference suppression is handled on one side of the communication link.

Let the K-users each transmit d_k streams. Apart from d_k antennas, all the antennas can be moved to the other side of the transmission link.

4.3.1 Complete Interference Suppression at the Receiver
If $M_k - d_k$ antennas are moved from the transmitter to the receiver, the total number of antennas in each link remains the same while we now have $M'_k = d_k$ transmit antennas and $N'_k = N_k + (M_k - d_k)$ receive antennas.

In such a system any kind of processing at the transmitter is ruled out. Thus the beamforming matrix reduces to the identity matrix. i.e., $\mathbf{G}_k = \mathbf{I}_{d_k}$. Interference suppression is now completely handled at the receiver using a zero forcing interference suppressor. For the k-th receiver we have

$$\mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l = 0 \quad \Rightarrow \mathbf{F}_k \mathbf{H}_{kl} = 0 \quad \forall l \neq k \qquad (25)$$

such that

$$\mathbf{F}_k^H = \mathbf{P}^{\perp}_{\mathbf{H}_{k\underline{k}}} \mathbf{H}_{kk} \qquad (26)$$

where $\mathbf{P}^{\perp}_{\mathbf{H}_{k\underline{k}}}$ is the orthogonal complement of the column space spanned by the matrix $\mathbf{H}_{k\underline{k}} = [\mathbf{H}_{k1}, \ldots, \mathbf{H}_{k(k-1)}, \mathbf{H}_{k(k+1)}, \ldots, \mathbf{H}_{kK}] \in \mathbb{C}^{N'_k \times \sum_{l \neq k} d_l}$ that in turn contains all channel matrices corresponding to the cross-links at receiver k ($\mathbf{H}_{k\underline{k}}$ represents the interference subspace spanned at the k-th receiver).

$$\mathbf{P}^{\perp}_{\mathbf{H}_{k\underline{k}}} = \mathbf{I}_{N'_k} - \mathbf{H}_{k\underline{k}}(\mathbf{H}_{k\underline{k}}^H \mathbf{H}_{k\underline{k}})^{-1} \mathbf{H}_{k\underline{k}}^H \qquad (27)$$

4.3.2 Complete Interference Suppression at the Transmitter
Alternatively, consider the situation where the transmitter completely suppresses all interference. We now move $N_k - d_k$ antennas from the receiver to the transmitter to obtain the following new distribution of antennas: $M'_k = M_k + (N_k - d_k)$ and $N'_k = d_k$.

In this case the matrix representing the receive filter reduces to an identity matrix ($\mathbf{F}_k = \mathbf{I}_{d_k}$) and the beamformer matrix satisfies the conditions

$$\mathbf{F}_k \mathbf{H}_{kl} \mathbf{G}_l = 0 \quad \Rightarrow \mathbf{H}_{kl} \mathbf{G}_l = 0 \quad \forall l \neq k \qquad (28)$$

The solution to this problem is similar to that of complete interference suppression at the receiver. The interference subspace spanned at the l-th receiver is described by the matrix $\mathbf{H}_{l\underline{l}} = [\mathbf{H}_{1l}^H, \ldots, \mathbf{H}_{(l-1)l}^H, \mathbf{H}_{(l+1)l}^H, \ldots, \mathbf{H}_{Kl}^H] \in \mathbb{C}^{M'_l \times \sum_{k \neq l} d_k}$. Hence the beamforming matrix at the l-th transmitter is given by the orthogonal complement of the subspace spanned by the matrix $\mathbf{H}_{l\underline{l}}$:

$$\mathbf{G}_l = \mathbf{P}^{\perp}_{\mathbf{H}_{l\underline{l}}} \mathbf{H}_{ll}^H$$

5. ITERATIVE ALGORITHM FOR INTERFERENCE ALIGNMENT

The sum rate duality for the centralized approach conjectured in Sec. 3.1 motivates us to propose a distributed algorithm in the TDD setting. We start with a brief discussion on the optimality of linear minimum mean squared error (LMMSE) interference suppressors (in terms of maximizing sum-rate in low and intermediate SNR regimes and maximizing achievable DoF in the high SNR regime) given the linear beamforming filters applied at the transmitters. In general, an interference alignment solution consists of beamformers applied at the transmitters that confine all the interference components in the N_k dimensional received signal $\mathbf{y}_k$ to an interference subspace of dimensions at most $N_k - d_k$ and interference suppressors applied at the receivers in order to extract the interference-free streams (represented by the vector $\mathbf{r}_k$) from the received signal where

$$\mathbf{y}_k = \mathbf{H}_{kk} \mathbf{x}_k + \mathbf{v}_k = \mathbf{H}_{kk} \mathbf{G}_k \mathbf{s}_k + \mathbf{v}_k \qquad (29)$$

$\mathbf{v}_k = \sum_{l=1; l \neq k}^{K} \mathbf{H}_{kl} \mathbf{x}_l + \mathbf{n}_k$ accounts for the total interference and noise contribution in $\mathbf{y}_k$ and $\mathbf{R}_{vv}$ represents its covariance matrix. Since the received signal has the same structure for all k we can drop the index k in order to simplify notations. The mutual information at each receiver can now be expressed as

$$I = \log \det(\mathbf{I} + \mathbf{R}_{vv}^{-1} \mathbf{H} \mathbf{G} \mathbf{Q} \mathbf{G}^H \mathbf{H}^H) \qquad (30)$$

where $\mathbf{Q}$ is the covariance matrix of the transmitted signal. As a consequence of applying an LMMSE interference suppressing filter $\mathbf{F} = \mathbf{Q} \mathbf{G}^H \mathbf{H}^H (\mathbf{R}_{vv} + \mathbf{H} \mathbf{G} \mathbf{Q} \mathbf{G}^H \mathbf{H}^H)^{-1}$ to $\mathbf{y}$, the mutual information at the output of the filter can be expressed as

$$I_{LMMSE} = \log \det(\mathbf{I} + (\mathbf{F} \mathbf{R}_{vv} \mathbf{F}^H)^{-1} \mathbf{F} \mathbf{H} \mathbf{G} \mathbf{Q} (\mathbf{F} \mathbf{H} \mathbf{G})^H) \quad (31)$$

Now, it is possible to show that the LHS of (31) is exactly the same as the LHS of (30). Which implies that, for a given linear beamforming filter applied at the transmitter, the LMMSE interference-suppressing filter applied at the receiver under the assumption that all interfering signals can be treated as Gaussian noise does not lose any information of the desired signal in the process of reducing the N_k dimensional $\mathbf{y}$ to a d_k dimensional vector $\mathbf{r}$. In other words, the linear MMSE interference suppressor filter is information lossless [5]. This together with the duality relationship between interference alignment on a given channel and its corresponding dual channel for the basis of our conjecture that the optimal transmit filter is the MMSE interference suppressing filter for a dual interference channel consisting of reciprocal channel matrices and certain transmit and noise covariance matrices. We note however that a related approach of per-stream MMSE processing in [6] and also in the max-SINR algorithm proposed in [2] that treats the $d_k - 1$ streams of each user as part of interference result in unnecessary constraints being imposed and thus lead to sub-optimal solutions.

5.1 Distributed TDD approach
We describe here a sub-optimal distributed approach to find the transmit and receive filters for the case of a TDD system assuming that each receiver has accurate knowledge of the channel matrix corresponding to its direct link and

the associated interference plus noise covariance matrix. We consider a K-user MIMO interference channel with arbitrary transmit and receive antenna configuration and a degree of freedom distribution such that an interference alignment solution exists.

The sketch of the algorithm is as follows:

Step 1: Compute the optimum MMSE interference suppressing filters $\mathbf{F}_k$ for at each receiver k for a fixed (initial) set of beamforming matrices $\mathbf{G}_k$ and the associated transmit signal covariances $\mathbf{Q}_k$.

Step 2: Use the MMSE filters computed in the previous step as beamformers in the dual interference channel along with an appropriate $\mathbf{Q}_k$.

Step 3: Compute the MMSE filters at each receiver in the dual link and apply this as the beamforming matrix for the original link (again taking into account the associated $\mathbf{Q}_k$ for this link).

Step: 4 Iterate between *Step 2:* and *Step 3:* till the beamforming matrices converge.

The following choices exist for $\mathbf{Q}_k$ matrices. Since the $\mathbf{F}_k^H$ of the original (dual) link is used to compute $\mathbf{G}_k$ for the dual (original) link. The normalized columns of $\mathbf{F}_k^H$ are be used to form $\mathbf{G}_k$ and $\mathbf{Q}_k = (P_k/d_k)\mathbf{I}_{d_k}$. Alternatively, $\mathbf{Q}_k$ might be determined by waterfilling (as in the case of single user MIMO with colored noise at the receiver).

6. NUMERICAL EXAMPLES

In this section we provide some numerical examples to validate the conditions derived in the paper.

Example 1: Consider a 2-user ISAC MIMO system with $M = 2, N = 4, d = 2$. Using the conditions for an ISAC system in Sec. 4.1, it can be verified that all the conditions are satisfied which implies that each receiver can indeed see 2 interference free signaling dimensions.

Example 2: In this example, we consider a 4-user MIMO interference channel where each user has $M = N = 5$ antennas and each requires $d = 2$ interference free dimensions at the receiver. Again, we are able to show that all the constraints are satisfied and we can therefore declare that this system admits an interference alignment solution.

Example 3: We now look at the 3 user case where the first and second user pair have 4 transmit and receive antennas, $(M_k = N_k = 4)$ $k \in \{1, 2\}$, and the third user pair has $M_3 = 6, N_3 = 2$ antennas. We set $d_k = 2$ $\forall k$. For this case, all the constraints in (14)-(17) are satisfied. The system should therefore have an interference alignment solution.

We compare our results against an iterative algorithm proposed in [7] that experimentally evaluates the possibility of an interference alignment solution for a given set of parameters (K, M_k, N_k, d_k). In all the above cases the algorithm was able to find an interference alignment solution.

Example 4: We now look at another 2-user ISAC MIMO system but this time with $M = N = 3, d = 2$. For this system, we see that (20) is not satisfied. Therefore we declare this system cannot achieve 2 degrees of freedom per user.

Example 5: The 3-user case that we now evaluate is characterized by $M_1 = 5, N_1 = 3, M_2 = 4, N_2 = 4, M_3 = 6, N_3 = 2, d_k = 2\forall k$. Here, (16) is no longer satisfied. Therefore there cannot be an interference alignment solution for this system.

As before, we compare the results of our analytical evaluation against the experimental results of [7] and find that indeed, the iterative algorithm is not able to find an interference alignment solution as was expected.

7. CONCLUDING REMARKS

We provide conditions to be satisfied for interference alignment solutions to exist for a given K-user frequency-flat MIMO interference channel. These conditions are used to analytically evaluate the existence of interference alignment solutions thereby circumventing the need for numerical simulations. We provide numerical examples where we compare the results of applying these conditions against an iterative algorithm proposed in [7] that experimentally checks for the existence of IA solutions. We conjecture a sum-rate duality for this inteference channel and propose an iterative algorithm to find the sum-rate maximizing

8. ACKNOWLEDGMENT

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