Channel Capacity for Linearly Precoded Multiuser MIMO for Discrete Constellations

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***Abstract*—This paper is based on the idea of exploiting the discrete constellation alphabets in linear precoding for the downlink of multiuser (MU) MIMO. We study the effect of discrete constellation inputs on the sum rate of different linear precoders while confining ourselves to the case of two single antenna users. We show that contrary to the case of Gaussian alphabets where altruistic solutions as interference cancellation (channel inversion - CI) and interference attenuation (regularized channel inversion - RCI) are the recommended strategies for linear precoding, it is beneficial not to attenuate or cancel the MU interference, if it is coming from discrete constellations. This interference belonging to finite sized constellations has a structure which can be effectively exploited in improving the error resilience at the users. Under such a scenario, it is better to use the degrees of freedom available at the transmitter to improve the desired signal strength at the users instead of utilizing them to nullify or attenuate the undesired signals (interferences) at the users. Therefore the egoistic linear solutions as matched filter (MF) based precoding bears the potential of enhanced sum rate and improved performance as compared to the altruistic linear solutions.**

I. Introduction

Spatial dimension surfacing from the usage of multiple antennas promises improved reliability, higher spectral efficiency and the spatial separation of users [1]. The discrete-time complex baseband multiple-input multiple-output (MIMO) channel (to be made precise in the subsequent sections) is defined by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \tag{1}$$

Despite its simplicity, the model is extremely rich and describes several situations of interest in data communications, depending on the constraints put on the transmitters and the receivers and on the assumptions about the channel matrix. If both transmitters and receivers are allowed to cooperate , (1) represents a single-user MIMO channel, arising in multiple-antenna wireless systems [2]. On the other hand, if both the transmitters and the receivers are not allowed to cooperate, (1) represents an interference channel [3], arising, for example, in peer-to-peer communication wireless networks. If only the receivers are allowed to cooperate and the transmitters are constrained to encode their signals independently, (1) represents a multiple-access channel [4] arising in the uplink of cellular communication. If only the transmitters are allowed to cooperate and the receivers are constrained to decode their signals independently, (1) represents a broadcast channel [5], arising in the downlink of a cellular system where the base station (BS) is equipped with an antenna array (downlink of multiuser-MU MIMO channel).

Amongst those, MIMO is particularly beneficial for precoding in the downlink of cellular system, where the spatial resources can be used to transmit data to multiple users simultaneously. Precoding can be termed as preprocessing at the transmitter using channel state information to pre-distort the signals in such a way that they either help in decoding at the receivers or they no longer interfere at the individual antennas of the receivers. The constraint that receivers are allowed or not allowed to cooperate leads to two different scenarios as precoding in single-user MIMO and precoding in multiuser (MU) MIMO downlink channel. Though the precoding techniques differ in these scenarios but they are based on the same underlying principle of coining independent parallel channels from the cross-coupled channels.

For transmission in single-user MIMO channel with the transmitter knowing the transfer coefficients between the antennas, optimal linear strategy for maximizing mutual information in the case of Gaussian assumption for alphabets is the conversion of cross-coupled matrix channels into parallel noninteracting channels by precoders and receive filters. Power is allocated on these parallel channels by classic waterfilling [1]. Although Gaussian inputs are optimum from a mutual information standpoint, they are too idealistic to be implemented in practical communication systems. The reason for this assumption is the convenience in mathematics to derive the elegant capacity formula [1]. For finite discrete inputs, optimal linear transmission strategy engulfs mercury-waterfilling [6] under the condition of noninteracting channels. The basic idea is not to allocate further power to a channel which is already close to saturation as the maximum mutual information of an M-ary constellation cannot exceed $\log_2 M$. However if the noninteracting condition is removed, then the optimal linear transmission strategy involves precoders that eventually result in cross-coupled effective channels [7] leading to joint detection at the receivers.

For transmission in MU MIMO Gaussian broadcast channel, optimal precoding involves a theoretical pre-interference cancellation technique known as dirty paper coding (DPC) [8]. Due to highly nonlinear nature of signal processing involved in DPC, its practical implementation is far from realizable. Moreover its optimality is constrained to idealistic Gaussian alphabets.

Linear precoding provides an alternative approach for transmission in MU MIMO Gaussian downlink channel, trading off a reduction in precoder complexity for suboptimal performance. Orthogonalization based schemes use channel inversion (CI) and block diagonalization (BD) to transform the MU downlink into parallel single user systems [9]. However, the performance of CI is degraded with a small number of users when the channel is ill-conditioned [10]. This is because inverting a poorly conditioned matrix unavoidably results in the reduction of effective channel gain which is more prominent in the case of low SNR. To overcome the drawbacks of CI, regularized channel inversion (RCI) precoding is proposed in [11] which adds a multiple of the identity matrix before channel inversion. In spite of introducing some crosstalk interference from other users, the RCI scheme can effectively increase the sum rate by alleviating the reduction in effective channel gain. Though optimum linear precoders [12] and optimum unitary linear precoders [13] for MU MIMO Gaussian broadcast channel have also been derived in the literature but the complexity associated with their calculation makes them less attractive for practical systems.

Gaussian being the worst case interference, the recommended precoding strategies for such inputs [14] [15] are pre-interference subtraction (DPC), interference cancellation (CI) and interference attenuation (RCI). These strategies therefore lead to simplified receiver structures for the users which is considered as the foremost advantage of precoding but are void of exploiting the interference structure in mitigating its effect. In the real world, inputs must be drawn from discrete constellations (often with very limited peak-to-average ratios) which may significantly depart from Gaussian idealization.

These interferences (discrete constellations) unlike Gaussian case have structures that can be exploited in the detection process and this is the main idea of this paper. We argue that the precoders may be designed to manage the interference in a way that this interference can be exploited in the detection process at the receivers. It is shown in [16] that when multiple antenna minimum mean square error (MMSE) receivers are employed by all users, and controlled interference cancellation based precoding is employed at the transmitter, then the lower bound on the maximum achievable sum rate is maximized leading to much higher throughput than that with CI precoder.

In this paper, we restrict ourselves to the case of linear precoders. We analyze the sum rate of different linear precoders under the constraint of finite size M-ary QAM alphabets (M is the number of points in the signal constellation) and show enhanced sum rate for the precoding schemes which do not suppress interference (matched filter-MF precoders and unitary precoders) as compared to the precoding schemes which are designed to attenuate or cancel the interference (CI and RCI precoders). We confine ourselves to the case of 2 single antenna users assuming that 2 users have been scheduled per time slot.

The definitions of some symbols and operators used in this paper are listed below.

$\mathbf{x}$: Boldface small symbol represents vector.

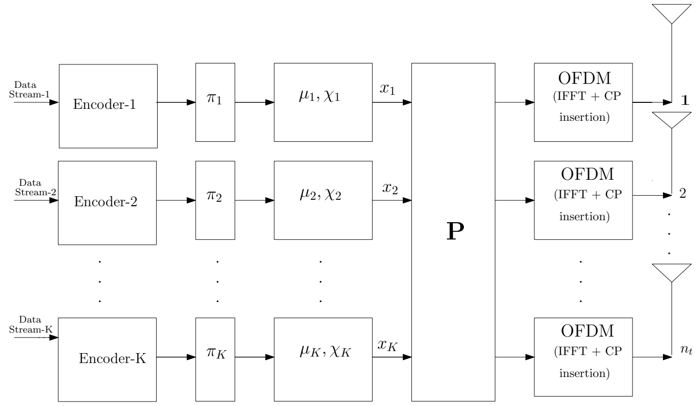

Fig. 1. Block diagram of the transmitter with n_t antennas. π_1 denotes the random interleaver, μ_1 the labeling map, χ_1 the signal set and x_1 the complex symbol for user-1.

$\mathbf{H}$: Boldface capital symbol represents a matrix.
E: Denotes expectation.
$\mathcal{CN}$: A complex circularly symmetric Gaussian random variable.
$\mathbf{I}_n$: $n \times n$ identity matrix
$\mathrm{tr}\,(\mathbf{H})$: Trace of matrix $\mathbf{H}$
$(.)_R$: Subscript R represents real part.
$(.)_I$: Subscript I represents imaginary part.
$|.|$: norm of scalar
$\|.\|$: norm of vector
$(.)^T$: Transpose
$(.)^*$: Conjugate
$(.)^\dagger$: Conjugate transpose
log: All logarithms are to the base 2.

This paper is organized in five sections. In section-II, we define the system model while section-III gives an overview of existing linear precoders. In section-IV, we derive the mutual information for linear precoders for discrete constellations while section-V concludes the paper.

II. System Model

Coherent with the next generation wireless systems as LTE [17] and IEEE 802.16m [18] which employ bit interleaved coded modulation (BICM) [19] with orthogonal frequency division multiplexing (OFDM) for downlink transmission, our system model is shown in fig.1. We consider the downlink of a wireless system with n_t transmit antennas at the transmitter while K users have one receive antenna each. We assume that one OFDM symbol has N subcarriers.

After encoding and interleaving, the output bits are mapped onto the tone $x_{k,n}$ using the signal map $\chi_k \subseteq \mathcal{C}$ with a Gray labeling map $\mu_k : \{0,1\}^{\log_2|\chi_k|} \rightarrow \chi_k$ where $k = 1, \cdots, K$ and n indicates the subcarrier. It is assumed that an appropriate length of cyclic prefix (CP) is used for each OFDM symbol. By doing so, OFDM converts downlink frequency selective channels into parallel flat fading channels denoted as $\mathbf{h}^\dagger_{k,n} \in \mathbb{C}^{n_t \times 1}$ where $\mathbb{C}^{n_t \times 1}$ denotes the n_t-dimensional complex space. We assume a spatially uncorrelated flat Rayleigh fading channel model so that the elements of $\mathbf{h}_{k,n}^{\dagger},\ (k=1,2\cdots,K)$ can be modeled as independent identically distributed (i.i.d) zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance of 0.5 per dimension. Each symbol for each tone is then multiplied by the corresponding precoding vector $\mathbf{p}_{k,n}$. Following precoding, OFDM is applied at each transmit antenna.

User scheduling is beyond the scope of this paper rather we just assume that basing on some criteria, two users are scheduled per time slot. Without loss of generality, we assume that the first 2 users are scheduled. Let the precoder matrix be $\mathbf{P}_n = [\mathbf{p}_{1,n}\ \mathbf{p}_{2,n}]$. Cascading IFFT at the transmitter and FFT at the user with CP extension, transmission at the n-th frequency tone for the first user can be expressed as::-

$$y_{1,n} = \mathbf{h}_{1,n}^{\dagger}\mathbf{p}_{1,n}x_{1,n} + \mathbf{h}_{1,n}^{\dagger}\mathbf{p}_{2,n}x_{2,n} + z_{1,n}, \quad n = 1,2,\cdots,N$$

where $y_{1,n}$ is the received symbol at user-1 and $z_{1,n}$ is ZMCSCG white noise of variance N_0. The complex symbols $x_{1,n}$ and $x_{2,n}$ are also assumed to be independent and of variances σ_1^2 and σ_2^2 respectively. The transmitter is subjected to an average power constraint $\mathbb{E}\left\|\mathbf{p}_{1,n}x_{1,n} + \mathbf{p}_{2,n}x_{2,n}\right\|^2 \leq P$. This power constraint may be met (long term power constraint) by designing the precoder matrix as $\mathbf{P}/\sqrt{\text{tr}\left(\mathbf{P}^{\dagger}\mathbf{P}\right)}$ and imposing the constraint that $\sigma_1^2 = \sigma_2^2 \leq P$. This may also be met (short term power constraint) by making $\mathbf{p}_{1,n}$ and $\mathbf{p}_{2,n}$ unit norm vectors and imposing the constraint $\sigma_1^2 + \sigma_2^2 \leq P$. We assume that the transmitter has perfect knowledge of channel state information of all users (perfect CSIT), and each user knows its own effective channel (scalar coefficient) and that of the other coscheduled user perfectly. This implies that user-1 has perfect knowledge of the coefficients $\mathbf{h}_{1,n}^{\dagger}\mathbf{p}_{1,n}$ and $\mathbf{h}_{1,n}^{\dagger}\mathbf{p}_{2,n}$. For channel estimation by the users, transmitter needs to transmit pilot symbols for the symbol intervals equal to the number of co-scheduled users (two). It would enable both the users not only to estimate their own coefficients but also the the coefficient of the other co-scheduled user. For notational convenience, we drop the frequency index for subsequent sections and rewrite the system equation as:-

$$\begin{aligned} y_1 &= \alpha_1 x_1 + \beta_2 x_2 + z_1 \\ y_2 &= \beta_1 x_1 + \alpha_2 x_2 + z_2 \end{aligned}$$

where α is the effective channel of the desired signal and β is the effective channel of the interferer.

III. Linear Precoders

In this section, we review some existing linear precdoing strategies. We denote the concatenation of the channels by $\mathbf{H}^{\dagger} = [\mathbf{h}_1\ \mathbf{h}_2]$ so $\mathbf{H}$ is the $2 \times n_t$ forward channel matrix with k-th row $\mathbf{h}_k^{\dagger}$ equal to the channel of the k-th user. Basing on this we can rewrite the system equation as

$$\mathbf{y} = \mathbf{HPx} + \mathbf{z} \tag{2}$$

where $\mathbf{x} = [x_1\ x_2]^T$ and $\mathbf{z} = [z_1\ z_2]^T$.

A. Maximum Ratio Transmission

The precoding vector is the matched filter (MF) [20] and is given as

$$\mathbf{p}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|} \tag{3}$$

As in this case the interference generated is completely ignored and the focus is on maximizing the useful signal received at the user, it is also referred as egoistic strategy in the literature.

B. CI Precoder

If enough antennas are available at the transmitter, nulling of the generated interference is possible and the optimal interference nulling precoding vector [20] is given by:

$$\mathbf{p}_k = \frac{\mathbf{\Pi}^{\perp,k}\mathbf{h}_k}{\|\mathbf{\Pi}^{\perp,k}\mathbf{h}_k\|} \tag{4}$$

where $\mathbf{\Pi}^{\perp,k}$ is the projection matrix on the null space of the channels being interfered. Alternatively the CI precoding matrix can be found by the Moore-Penrose pseudoinverse of $\mathbf{H}$ which is given as

$$\mathbf{W} = \mathbf{H}^{\dagger}\left(\mathbf{H}\mathbf{H}^{\dagger}\right)^{-1} \tag{5}$$

then the CI precdoing matrix $\mathbf{P}$ can be obtained from $\mathbf{W}$ by normalizing all of its columns. Hence each user will receive only the beam directed to it and no multi-user interference will be experienced. As the main focus is on trying to reduce the interference caused to others, this strategy is also referred in the literature as altruistic strategy. In case of ill-conditioned channel matrix, inversion will cause an excessive power penalty at the transmitter thereby degrading the performance.

C. RCI Precdoing

For rank-deficient channels, the performance of CI precoding can be improved by a regularization of the pseudo-inverse [11], which can be expressed as

$$\mathbf{P} = \mathbf{H}^{\dagger}\left(\mathbf{H}\mathbf{H}^{\dagger} + \frac{n_t N_0}{P}\mathbf{I}_2\right)^{-1} \tag{6}$$

where it is assumed that $\sigma_1^2 = \sigma_2^2 = P$. The precoder is normalized as $\mathbf{P}/\sqrt{\text{tr}\left(\mathbf{P}^{\dagger}\mathbf{P}\right)}$.

D. Unitary precoding (QR-QL decomposition)

Basing on the Gram Schmidt orthogonalization of $\mathbf{H}$, the channel matrix can be decomposed as

$$\mathbf{H}^{\dagger} = \mathbf{QR} \tag{7}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are unitary and upper triangular matrices respectively. Reversing the order of orthogonalization, we can decompose the channel matrix as

$$\mathbf{H}^{\dagger} = \mathbf{QL} \tag{8}$$

where $\mathbf{L}$ is the lower triangular matrix. The unitary matrix $\mathbf{Q}$ can be used as the precoding matrix, which in the two user case will lead to one interference free user while the other user will experience interference. Which of the two users is interference free depends on the $\mathbf{Q}$ matrix having been calculated by $\mathbf{QR}$ or $\mathbf{QL}$ decomposition of the channel matrix.

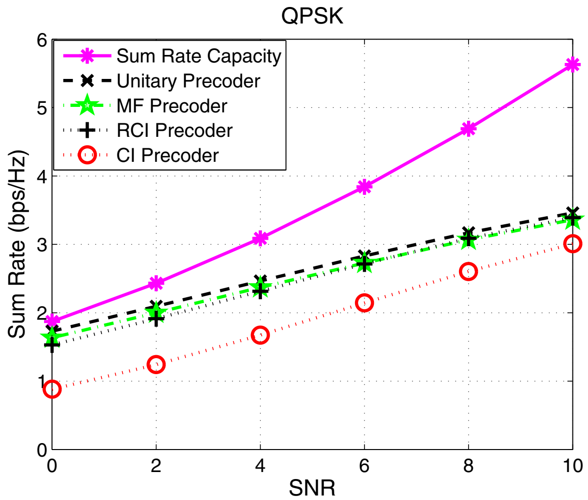

Fig. 2. Sum rate of downlink of 2 single antenna users with the transmitter having 2 antennas. Both users belong to QPSK constellations.

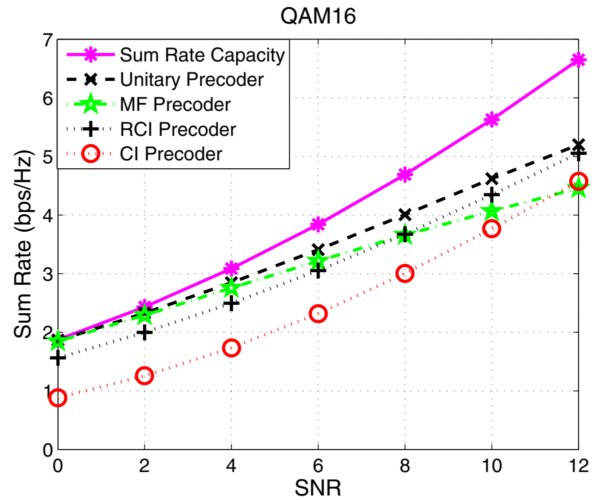

Fig. 3. Sum rate of downlink of 2 single antenna users with the transmitter having 2 antennas. Both users belong to QAM16 constellations.

IV. Channel Capacity Analysis

Sum rate of the downlink channel is given as

$$I(\mathbf{P}) = \mu I_1(\mathbf{P}) + (1-\mu) I_2(\mathbf{P}) \qquad 0 \leq \mu \leq 1 \tag{9}$$

where $I_1(\mathbf{P})$ and $I_2(\mathbf{P})$ are the mutual information of the first and second user respectively and μ is the parameter that defines the power distribution between the two users under the sum power constraint. The mutual information for the first user for finite size QAM constellation with $|\chi_1| = M_1$ takes the form as

$$\begin{aligned} I(Y; X_1|\alpha_1, \beta_2) &= \mathcal{H}(X_1|\alpha_1, \beta_2) - \mathcal{H}(X_1|Y_1, \alpha_1, \beta_2) \\ &= \log M_1 - \mathcal{H}(X_1|Y_1, \alpha_1, \beta_2) \end{aligned} \tag{10}$$

where $\mathcal{H}(.) = -E \log p(.)$ is the entropy function. The second term of eq. (10) is given as:-

$$\begin{aligned} &\mathcal{H}(X_1|Y_1, \alpha_1, \beta_2) \\ &= \sum_{x_1} \int_{y_1} \int_{\alpha_1} \int_{\beta_2} p(x_1, y_1, \alpha_1, \beta_2) \log \frac{1}{p(x_1|y_1, \alpha_1, \beta_2)} dy_1 d\alpha_1 d\beta_2 \\ &= \sum_{x_1} \int_{y_1} \int_{\alpha_1} \int_{\beta_2} p(x_1, y_1, \alpha_1, \beta_2) \log \frac{p(y_1, \alpha_1, \beta_2)}{p(x_1, y_1, \alpha_1, \beta_2)} dy_1 d\alpha_1 d\beta_2 \\ &= \sum_{x_1} \sum_{x_2} \int_{y_1} \int_{\alpha_1} \int_{\beta_2} p(x_1, x_2, y_1, \alpha_1, \beta_2) \\ &\qquad \times \log \frac{\sum_{x_1'} \sum_{x_2'} p\left(y_1|x_1', x_2', \alpha_1, \beta_2\right)}{\sum_{x_2'} p\left(y_1|x_1, x_2', \alpha_1, \beta_2\right)} dy_1 d\alpha_1 d\beta_2 \end{aligned} \tag{11}$$

For our purposes, it suffices to note that for each choice of x_1 and x_2, there are three sources of randomness as α_1, β_2 and noise. The above quantities can be easily approximated numerically using sampling (Monte-Carlo) methods with N_z realizations of noise and N_H realizations of the channel $\mathbf{H}$.

So the mutual information for the user-1 is given as

$$\begin{aligned} I(Y; X_1|\alpha_1, \beta_2) &= \log M_1 - \frac{1}{M_1 M_2 N_z N_H} \sum_{x_1} \sum_{x_2} \sum_{\mathbf{H}}^{N_H} \sum_{z_1}^{N_z} \\ &\times \log \frac{\sum_{x_1'} \sum_{x_2'} \exp\left[-\frac{1}{N_0}\left|y_1 - \alpha_1 x_1' - \beta_2 x_2'\right|^2\right]}{\sum_{x_2'} \exp\left[-\frac{1}{N_0}\left|y_1 - \alpha_1 x_1 - \beta_2 x_2'\right|^2\right]} \\ &= \log M_1 - \frac{1}{M_1 M_2 N_z N_H} \sum_{x_1} \sum_{x_2} \sum_{\mathbf{H}}^{N_H} \sum_{z_1}^{N_z} \\ &\times \log \frac{\sum_{x_1'} \sum_{x_2'} \exp\left[-\frac{1}{N_0}\left|\alpha_1 x_1 + \beta_2 x_2 + z_1 - \alpha_1 x_1' - \beta_2 x_2'\right|^2\right]}{\sum_{x_2'} \exp\left[-\frac{1}{N_0}\left|\beta_2 x_2 + z_1 - \beta_2 x_2'\right|^2\right]} \end{aligned} \tag{12}$$

Mutual information for the user-2 can be calculated in the similar manner.

Figs. 2, 3 and 4 show the sum rate of a broadcast channel with 2 transmit antennas and 2 single antenna users for QPSK, QAM16 and QAM64 alphabets respectively. SNR is the transmit SNR i.e. $\frac{\sigma_1^2 \|\mathbf{p}_1\|^2 + \sigma_2^2 \|\mathbf{p}_2\|^2}{N_0}$. Sum rate capacity (Gaussian broadcast channel) [8] along with the sum rate of unitary, MF, RCI and CI precoders are shown. Power distribution between the two streams i.e. the factor μ is optimized to maximize the sum rate for MF and CI precoders. For the case of unitary precoder, $\mathbf{Q}$ is selected from **QR** or **QL** decomposition depending on which decomposition leads to a higher sum rate.

The results show that in the low SNR regime, sum rates with the unitary and MF based precoders dominate those of CI and RCI precoders. It substantiates the argument that the precoding strategy of interference nulling or attenuation being devoid of exploiting interference structure will lead to degraded performance relative to other precoding strategies which allow interference to be propagated to the users. The

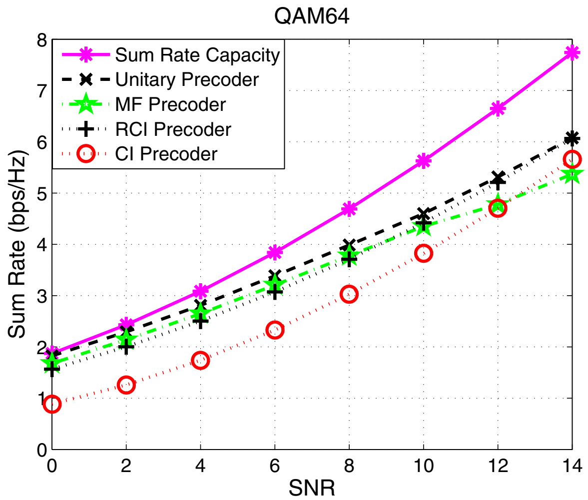

Fig. 4. Sum rate of downlink of 2 single antenna users with the transmitter having 2 antennas. Both users belong to QAM64 constellations.

degrees of freedom available at the transmitter need to be used in improving the desired signal strength instead of nulling or attenuating the interference strength. However at higher SNRs (not shown in the figures), RCI and CI precoders have higher sum rate than MF or unitary precoders.

An important point to underline is that interference eclipses the principle advantage of precoding and brings back the complexity to the receivers. In the presence of interference, MU detectors need to be used by the users. Users will also be required to estimate the effective channel of the interferers. However low complexity MU detectors [21] which can exploit the interference in mitigating its effect can replace the single user detectors. Users can also estimate the effective channels of the interferers once the BS transmits pilots for these users however the transmission of pilots by the BS for different users need to be orthogonal.

V. Conclusion

In this paper we have stressed that precoding in the downlink of MU MIMO can be used in 2 ways. It can be used to preprocess the signals at the transmitter so that they no longer interfere at the individual antennas of the receivers. This is the optimal strategy for Gaussian signals once the interference has no structure. However in real world where inputs belong to discrete constellations, precoding need to be used to preprocess the signals in such a way that they help in decoding at the receivers. A possible precoding strategy can be to enhance the desired signal strength (MF precoding) neglecting interference. The potential benefits lying in the interference structure then need to be exploited at the receiver.

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