Single Snapshot Joint Estimation of Angles and Times of Arrival: A 2D Matrix Pencil Approach

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***Abstract*—Two algorithms for the problem of joint angles and delays of arrival (JADE) of multiple paths are presented. The algorithms are based on a generalisation of the Matrix Pencil algorithm to the two dimensional case, i.e. 2D Matrix Pencils. Matrix pencil algorithms offer estimation of signal parameters, i.e. angles of arrival (AoA) or times of arrival (ToA), of multiple sources using a single snapshot. We focus on a scenario, where the OFDM symbol is transmitted in a rich multipath channel, which is the case of an indoor environment, and received through multiple antennas. The first algorithm seems more interesting than the second one, since it's motivated from an idea that most Wi-Fi systems use a large number of subcarriers compared to the number of antennas. Simulation results demonstrate the potential of the first algorithm and its performance as a function of Signal-to-Noise ratio (SNR).**

I. Introduction

Localisation has been one challenging topic over the past 60 years. In fact, many techniques have been developed in order to position a wireless emitter. The classical approach involves estimating the angle-of-arrival (AoA), received signal strength (RSS), time-of-arrival (ToA), etc.., of an emitter with respect to multiple base stations, so as to localise through triangulation or trilateration methods [1]. To estimate the signal parameters (ToA, AoA, etc..), the Maximum Likelihood (ML) technique was one of the first to be investigated [2]. However, it did not receive much attention due to the high computational load of the multivariate nonlinear minimisation problem involved, since it requires a $(pq + r)$-dimensional search, where p is the number of signal parameters of interest, q is the number of signals, and r represents additional parameters that are part of the model and have to be estimated jointly with signal parameters. These r additional parameters could be due to antenna calibration (see [3]) or synchronisation (see [4]).

As response to the highly nonlinear multidimensional ML problem, suboptimal techniques based on exploiting subspaces were developed, such as MUSIC [5]–[7] and ESPRIT [8]. Furthermore, these subspace algorithms are very efficient as compared to ML, but they perform poorly in case of a single snapshot. The spatial smoothing preprocessing technique [9] was discovered to overcome this issue.

The extension to joint estimation of signal parameters, i.e. JADE, was proposed in [10] to increase the number of resolvable signals using both space and time (or frequency) diversity. Therefore, extensions of classical subspace algorithms to the two dimensional (2D) case were derived so as to resolve more signals (see [11]–[14]). Even these 2D algorithms couldn't estimate the signal parameters of multiple sources using a single snapshot, unless a 2D preprocessing technique is applied such as [15].

The Matrix Pencil method for estimating signal parameters was first developed in [16]. The application of Matrix Pencils to the AoA problem was done in [17]. Furthermore, a similar methodology could be used for the ToA problem as in [18]. The extensions to the case of JADE are found in [19], [20]. In [19], the algorithm implemented uses the same technique as in [18] to estimate the ToAs of the multipath using 1D Matrix Pencils, then a beamforming step is done to estimate the AoAs per path. However, in [20], the authors implemented two algorithms. The first one is based on estimating the ToAs and AoAs using 2D Matrix Pencils and a matching technique was proposed to pair the ToAs and the AoAs. Their first algorithm is close to Algorithm 2 described herein, but we provide another pairing criterion. The second algorithm in [20] is based on 2D Unitary Matrix Pencils. In contrast, the first algorithm that we describe in this paper (Algorithm 1) is essentially different and is based on multiple stages of Least Squares (LS) fit to automatically pair the ToAs and AoAs. In addition, Algorithm 1, herein, has an advantage of resolving more paths than the algorithms described in [20] only when the number of subcarriers is much larger than the number of antennas. Furthermore, we present additional constraints that should be satisfied so that algorithms based on 2D Matrix Pencils estimate the signal parameters properly.

This paper is divided as follows. Section II presents the system model and problem statement. We explain how 2D Matrix Pencils could be used to estimate ToAs in Section III and AoAs in Section IV. The two algorithms that are based on 2D Matrix Pencils are described in Section V, followed by simulation results in Section VI. We conclude the paper in Section VII.

Notations: Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^{\mathrm{T}}$, $(.)^{*}$, and $(.)^{\mathrm{H}}$ represent the transpose, conjugate and the transpose-conjugate operators. The matrix $\mathbf{I}_{\mathrm{N}}$ is the identity matrix in $\mathbb{C}^{\mathrm{N\times N}}$. The operator $\|\mathbf{X}\|$ denotes the *Frobenius* norm of a square matrix $\mathbf{X}$. For any matrix $\mathbf{X} \in \mathbb{C}^{\mathrm{M\times N}}$, we use MATLAB notation to express entries or submatrices, i.e. $\mathbf{X}^{\langle i,j\rangle}$ is the $(i,j)^{\mathrm{th}}$ entry of $\mathbf{X}$ and $\forall j \geq i$ and $l \geq k$, the matrix $\mathbf{X}^{\langle i:j,k:l\rangle}$ is a submatrix formed by the the entries found between the i^{th} and j^{th} row and k^{th} and l^{th} column; also $\mathbf{X}^{\langle :,k:l\rangle} \triangleq \mathbf{X}^{\langle 1:M,k:l\rangle}$. The symbols $\otimes$ and $\boxtimes$ are the *Kronecker* and column-wise *Khatri-Rao* products, respectively. For $\mathbf{X} \in \mathbb{C}^{\mathrm{M\times N}}$, the matrix $\mathrm{ang}\{\mathbf{X}\} \in \mathbb{R}^{\mathrm{M\times N}}$ contains the phase of $\mathbf{X}^{\langle i,j\rangle}$ at its $(i,j)^{\mathrm{th}}$ entry. Also, for any integer N, the factorial of N is denoted as $N!$.

II. System Model

Consider an OFDM symbol composed of M subcarriers and centered at a carrier frequency f_c, impinging an array of N antennas via q multipath components, each arriving at different AoAs $\{\theta_i\}_{i=1}^{q}$ and ToAs $\{\tau_i\}_{i=1}^{q}$. In frequency domain, we could express the signal at the n^{th} antenna and m^{th} subcarrier as follows:

$$X_{m,n} = b_m \sum_{i=1}^{q} \gamma_i a_n(\theta_i) e^{-j2\pi m \triangle_f \tau_i} + N_{m,n} \tag{1}$$

where $T = \frac{1}{\triangle_f}$ is the OFDM symbol duration, $\triangle_f$ is the subcarrier spacing, b_m is the modulated symbol onto the m^{th} subcarrier,

$a_n(\theta)$ is the n^{th} antenna response to an incoming signal at angle θ. The form of $a_n(\theta)$ depends on the array geometry. γ_i is the complex coefficient of the i^{th} multipath component. The term $N_{m,n}$ is background noise on the m^{th} subcarrier at the n^{th} antenna.

We claim that the transmitted OFDM symbol is a preamble field of the Wi-Fi 802.11 frame, thus prior knowledge of the modulated symbols $\{b_m\}_{m=0}^{M-1}$ is a valid assumption. Therefore, we compensate for all such symbols (multiplying by $\frac{b_m^*}{|b_m|^2}$) and hence omit b_m from (1). Re-writing (1) in a compact matrix form, we have:

$$\mathbf{x} = \mathbf{H}\boldsymbol{\gamma} + \mathbf{n} \tag{2}$$

where $\mathbf{x}$ and $\mathbf{n}$ are $MN \times 1$ vectors

$$\mathbf{x} = [X_{1,1} \dots X_{1,N} \dots X_{M,1} \dots X_{M,N}]^{\mathrm{T}} \tag{3}$$

$$\mathbf{n} = [N_{1,1} \dots N_{1,N} \dots N_{M,1} \dots N_{M,N}]^{\mathrm{T}} \tag{4}$$

$\mathbf{H}$ is an $MN \times q$ matrix given as

$$\mathbf{H} = \mathbf{C}_{\mathrm{M}} \boxtimes \mathbf{A}_{\mathrm{N}} = [\mathbf{c}_{\mathrm{M}}(\tau_1) \otimes \mathbf{a}_{\mathrm{N}}(\theta_1) \dots \mathbf{c}_{\mathrm{M}}(\tau_q) \otimes \mathbf{a}_{\mathrm{N}}(\theta_q)] \tag{5}$$

where $\mathbf{C}_{\mathrm{M}}$ is an $M \times q$ matrix

$$\mathbf{C}_{\mathrm{M}} = [\mathbf{c}_{\mathrm{M}}(\tau_1) \dots \mathbf{c}_{\mathrm{M}}(\tau_q)] \tag{6a}$$

and $\mathbf{c}_{\mathrm{M}}(\tau)$ is an $M \times 1$ vector given as

$$\mathbf{c}_{\mathrm{M}}(\tau) = [z_\tau^{-\frac{M-1}{2}} \dots z_\tau^{\frac{M-1}{2}}]^{\mathrm{T}} \quad \text{with} \quad z_\tau = e^{-j2\pi\tau\triangle f} \tag{6b}$$

The matrix $\mathbf{A}_{\mathrm{N}}$ is an $N \times q$ matrix

$$\mathbf{A}_{\mathrm{N}} = [\mathbf{a}_{\mathrm{N}}(\theta_1) \dots \mathbf{a}_{\mathrm{N}}(\theta_q)] \tag{7a}$$

In what follows, we assume a Uniform Linear Array (ULA) setting and therefore $\mathbf{a}_{\mathrm{N}}(\theta)$ exhibits the following form

$$\mathbf{a}_{\mathrm{N}}(\theta) = [1, z_\theta, \dots, z_\theta^{N-1}]^{\mathrm{T}} \quad \text{with} \quad z_\theta = e^{-j2\pi\frac{d}{\lambda}\sin(\theta)} \tag{7b}$$

where d is the inter-element spacing and λ is the signal's wavelength. The $q \times 1$ vector $\boldsymbol{\gamma}$ is composed of the multipath coefficients

$$\boldsymbol{\gamma} = [\gamma_1 \dots \gamma_q]^{\mathrm{T}} \tag{8}$$

Note that we have made explicit the size of the matrices $\mathbf{C}_{\mathrm{M}}$ and $\mathbf{A}_{\mathrm{N}}$ as well as the vectors $\mathbf{c}_{\mathrm{M}}(\tau)$ and $\mathbf{a}_{\mathrm{N}}(\theta)$. Therefore, it should be understood that for any integer $K \geq 1$, the matrices $(\mathbf{C}_{\mathrm{K}}, \mathbf{A}_{\mathrm{K}}) \in \mathbb{C}^{\mathrm{K}\times\mathrm{q}}$ and are given as $\mathbf{C}_{\mathrm{K}} = [\mathbf{c}_{\mathrm{K}}(\tau_1) \dots \mathbf{c}_{\mathrm{K}}(\tau_q)]$ and $\mathbf{A}_{\mathrm{K}} = [\mathbf{a}_{\mathrm{K}}(\theta_1) \dots \mathbf{a}_{\mathrm{K}}(\theta_q)]$. Moreover, the vectors $\mathbf{c}_{\mathrm{K}}(\tau)$ and $\mathbf{a}_{\mathrm{K}}(\theta)$ are in $\mathbb{C}^{\mathrm{K}\times 1}$. The vector $\mathbf{n}$ is additive Gaussian noise of zero mean and variance $\sigma^2\mathbf{I}$, assumed to be white over space, and frequencies. Before stating the problem, we admit that the number of sources q is known a priori. The problem of estimating the number of sources is, in fact, a *detection* problem in signal processing. Techniques for estimating q are found in [21]–[23].

Any further assumptions will be mentioned. Now, we are ready to address our problem:

"Given one snapshot $\mathbf{x}$ *and the number of multipath components* q*, estimate the signal parameters* $\{(\theta_i, \tau_i)\}_{i=1}^{q}$*."*

III. ToA Estimation using 2D Matrix Pencil

A. Analytic Formulation

We start by forming a matrix from the data vector $\mathbf{x}$ given in equation (3). Let $\mathbf{X}$ be a $M_p \times K_M$ Hankel block matrix defined as follows

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_{K_M} \\ \mathbf{X}_2 & \mathbf{X}_3 & \cdots & \mathbf{X}_{K_M+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{M_p} & \mathbf{X}_{M_p+1} & \cdots & \mathbf{X}_M \end{pmatrix} \tag{9}$$

where $\mathbf{X}_i$ is an $N_p \times K_N$ Hankel matrix given by

$$\mathbf{X}_i = \begin{pmatrix} X_{i,1} & X_{i,2} & \cdots & X_{i,K_N} \\ X_{i,2} & X_{i,3} & \cdots & X_{i,K_N+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{i,N_p} & X_{i,N_p+1} & \cdots & X_{i,N} \end{pmatrix} \tag{10}$$

with

$$K_M = M - M_p + 1 \tag{11}$$

and

$$K_N = N - N_p + 1 \tag{12}$$

The matrix $\mathbf{X}$ could be written as

$$\mathbf{X} = \mathbf{L}\boldsymbol{\Gamma}\mathbf{R}^{\mathrm{T}} + \mathbf{N} \tag{13}$$

where $\mathbf{N}$ is a noise matrix with appropriate dimension, and $\mathbf{L}$ is an $M_pN_p \times q$ matrix expressed as

$$\mathbf{L} = \begin{pmatrix} \mathbf{A}_{\mathrm{N_p}} \\ \mathbf{A}_{\mathrm{N_p}}\mathbf{D}_\tau \\ \vdots \\ \mathbf{A}_{\mathrm{N_p}}\mathbf{D}_\tau^{M_p-1} \end{pmatrix} \tag{14}$$

and $\mathbf{R}$ is a $K_MK_N \times q$ matrix given by

$$\mathbf{R} = \begin{pmatrix} \mathbf{A}_{\mathrm{K_N}} \\ \mathbf{A}_{\mathrm{K_N}}\mathbf{D}_\tau \\ \vdots \\ \mathbf{A}_{\mathrm{K_N}}\mathbf{D}_\tau^{K_M-1} \end{pmatrix} \tag{15}$$

The matrices $\boldsymbol{\Gamma}$ and $\mathbf{D}_\tau$ are $q \times q$ diagonal matrices as

$$\boldsymbol{\Gamma} = \mathrm{diag}\ [\gamma_1 \dots \gamma_q] \tag{16}$$

and

$$\mathbf{D}_\tau = \mathrm{diag}\ [z_{\tau_1} \dots z_{\tau_q}] \tag{17}$$

Let $\mathbf{X}_l$ and $\mathbf{X}_r$ be two $N_pM_p \times K_N(K_M-1)$ matrices defined as

$$\mathbf{X}_l = \mathbf{X}^{\langle :,1:K_N(K_M-1)\rangle} \tag{18a}$$

$$\mathbf{X}_r = \mathbf{X}^{\langle :,(K_N+1):K_NK_M\rangle} \tag{18b}$$

In a noiseless case, it is easy to see that

$$\mathbf{X}_l = \mathbf{L}\boldsymbol{\Gamma}\mathbf{R}_o^{\mathrm{T}} \tag{19a}$$

and

$$\mathbf{X}_r = \mathbf{L}\boldsymbol{\Gamma}\mathbf{D}_\tau\mathbf{R}_o^{\mathrm{T}} \tag{19b}$$

where

$$\mathbf{R}_o = \mathbf{R}^{\langle 1:K_N(K_M-1),:\rangle} \tag{19c}$$

Consider the following matrix pencil

$$\mathbf{X}_r - \lambda\mathbf{X}_l = \mathbf{L}\boldsymbol{\Gamma}\big(\mathbf{D}_\tau - \lambda\mathbf{I}_{\mathrm{q}}\big)\mathbf{R}_o \tag{20}$$

Provided that the two matrices $\mathbf{L}$ and $\mathbf{R}_o$ are full column rank, i.e. the rank of both matrices is q, then the rank of the matrix pencil $\mathbf{X}_r - \lambda\mathbf{X}_l$ drops to $q-1$ at $\lambda = z_{\tau_i}$ for all $i = 1 \dots q$.

It is proved in [16] that if the singular value decomposition of $\mathbf{X}_l$ is $\mathbf{X}_l = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^{\mathrm{H}}$, then the q eigenvalues of the following matrix

$$\mathbf{T} = \bar{\boldsymbol{\Lambda}}^{-1}\bar{\mathbf{U}}^{\mathrm{H}}\mathbf{X}_r\bar{\mathbf{V}} \tag{21a}$$

where

$$\bar{\mathbf{U}} = \mathbf{U}^{\langle :,1:q\rangle} \tag{21b}$$

$$\bar{\boldsymbol{\Lambda}} = \boldsymbol{\Lambda}^{\langle 1:q,1:q\rangle} \tag{21c}$$

$$\bar{\mathbf{V}} = \mathbf{V}^{\langle :,1:q \rangle} \tag{21d}$$

are the values of λ that drop the rank of the matrix pencil $\mathbf{X}_r - \lambda \mathbf{X}_l$ to $q-1$. In other words, these q values of λ are called the generalised eigenvalues of the matrix pencil $(\mathbf{X}_r, \mathbf{X}_l)$. As a consequence, the q generalised eigenvalues of $(\mathbf{X}_r, \mathbf{X}_l)$ are estimates of $\{z_{\tau_i}\}_{i=1}^{q}$. We denote these estimates as $\{\hat{z}_{\tau_i}^{\text{MP}}\}_{i=1}^{q}$.

B. Conditions for ToA Estimates using 2D Matrix Pencil

Recall that under the assumption that both matrices $\mathbf{L}$ and $\mathbf{R}_o$ are full column rank, the generalised eigenvalues of the matrix pencil $(\mathbf{X}_r, \mathbf{X}_l)$ are estimates of $\{z_{\tau_i}\}_{i=1}^{q}$. Before deriving the conditions, we define the following:

Definition: Let P and Q be two integers defined as follows:

- Let q^τ be the number of distinct ToAs, i.e. $\tau^1, \ldots, \tau^{q^\tau}$; and let the following integers $P_1, \ldots, P_{q^\tau}$ denote their corresponding multiplicity.
  Note that $\sum_{i=1}^{q^\tau} P_i = q$. The maximum number of paths arriving at the same time but with different angles of arrival is $\max_i P_i = P$.
- Similarly, let q^θ be the number of distinct AoAs, i.e. $\theta^1, \ldots, \theta^{q^\theta}$; and let the following integers $Q_1, \ldots, Q_{q^\theta}$ denote their corresponding multiplicity.
  Note that $\sum_{i=1}^{q^\theta} Q_i = q$. The maximum number of paths arriving at same AoAs but with different ToAs is $\max_i Q_i = Q$.

It is straightforward to see that $\mathbf{L}$ and $\mathbf{R}_o$ have same structure, but different dimensions, i.e.

$$\mathbf{L} = \mathbf{C}_{\mathrm{M_p}} \boxtimes \mathbf{A}_{\mathrm{N_p}} \tag{22}$$

$$\mathbf{R}_o = \mathbf{C}_{\mathrm{K_M\text{-}1}} \boxtimes \mathbf{A}_{\mathrm{K_N}} \tag{23}$$

In order to proceed, we need the following theorem:

Theorem: *Let $\mathbf{H} \in \mathbb{C}^{MN \times q}$ be a matrix defined as in (5), i.e. $\mathbf{H} = \mathbf{C}_M \boxtimes \mathbf{A}_N$. The matrix $\mathbf{H}$ has full column rank if the following hold:*

- *Condition 1:* $q \leq MN$
- *Condition 2:* $P \leq N$
- *Condition 3:* $Q \leq M$

Proof: Omitted due to lack of space.

Using this theorem, it is easy to see that both matrices $\mathbf{L}$ and $\mathbf{R}_o$ are full column rank under the following conditions:

- **A.1**: $q \leq \min\{M_p N_p, K_N(K_M - 1)\}$
- **A.2**: $P \leq \min\{N_p, K_N\}$
- **A.3**: $Q \leq \min\{M_p, K_M - 1\}$

Therefore, if conditions **A.1** till **A.3** are satisfied, the ToAs could be estimated through the 2D Matrix Pencil technique described herein.

IV. AoA Estimation using 2D Matrix Pencil

A. Analytic Formulation

Let $\mathbf{Y}$ be a shuffled version of matrix $\mathbf{X}$, viz.

$$\mathbf{Y} = \mathbf{XP} \tag{24}$$

where $\mathbf{P}$ is a $K_M K_N \times K_M K_N$ permutation matrix defined as follows

$$\mathbf{P}^{\mathrm{T}} = \begin{pmatrix} \mathbf{e}(1) \\ \mathbf{e}(K_N+1) \\ \vdots \\ \mathbf{e}((K_M-1)K_N+1) \\ \mathbf{e}(2) \\ \mathbf{e}(K_N+2) \\ \vdots \\ \mathbf{e}((K_M-1)K_N+2) \\ \vdots \\ \mathbf{e}(K_N) \\ \mathbf{e}(2K_N) \\ \vdots \\ \mathbf{e}(K_M K_N) \end{pmatrix} \tag{25}$$

where $\mathbf{e}(i)$ is the i^{th} row of the identity matrix $\mathbf{I}_{K_M K_N}$. Now, as done in equation (18), form $\mathbf{Y}_l$ and $\mathbf{Y}_r$ by

$$\mathbf{Y}_l = \mathbf{Y}^{\langle :,1:K_M(K_N-1) \rangle} \tag{26a}$$

$$\mathbf{Y}_r = \mathbf{Y}^{\langle :,(K_M+1):K_N K_M \rangle} \tag{26b}$$

Using the same methodology as in equations (19), (20), and (21), one could obtain estimates of the AoAs, i.e. $\{\hat{z}_{\theta_i}^{\text{MP}}\}_{i=1}^{q}$. The conditions for proper estimation of AoAs using the 2D Matrix Pencil technique just described are similar to those in Section III-B and are given in the following subsection.

B. Conditions for AoA Estimates using 2D Matrix Pencil

The conditions for AoA estimation using 2D Matrix Pencil are the following:

- **B.1**: $q \leq \min\{M_p N_p, K_M(K_N - 1)\}$
- **B.2**: $P \leq \min\{N_p, K_N - 1\}$
- **B.3**: $Q \leq \min\{M_p, K_M\}$

V. Proposed Algorithms

In this section, we present two algorithms that allow joint estimation of the times and angles of arrival. The first algorithm is intended for systems where the number of subcarriers M is much larger than the number of antennas N, i.e. $M \gg N$. This is a reasonable assumption since most Wi-Fi technologies are equipped with 3 up to 8 antennas. Moreover, the number of subcarriers used in a Wi-Fi OFDM symbol varies between 64 and 512. Furthermore, the second algorithm could be used for any configuration, i.e. for any M and N. In addition, conditions for the two algorithms are provided.

A. Algorithm 1: ($M \gg N$)

Note that the parameters K_M and K_N (or equivalently M_p and N_p) are free in a noiseless case. However, in a noisy scenario, those parameters should be properly selected. For more details, the reader is referred to [16]. In any case, the parameters K_M and K_N parameters are jointly tuned so that conditions **A.1** till **A.3** (or **B.1** till **B.3**) are met, if the purpose is to estimate the ToAs (or AoAs) using 2D Matrix Pencil. If $M \gg N$, one could show that there exist integers K_M and K_N (or equivalently M_p and N_p) where conditions **A.1** till **A.3** are *less restrictive* than conditions **B.1** till **B.3**. In other words, if $M \gg N$, the 2D Matrix Pencil described herein allows estimation of more ToAs than AoAs. Therefore, we propose the following algorithm:

- *Step 1:* Given $\mathbf{x}$ and q, form $\mathbf{X}$ using equations (9) and (10).

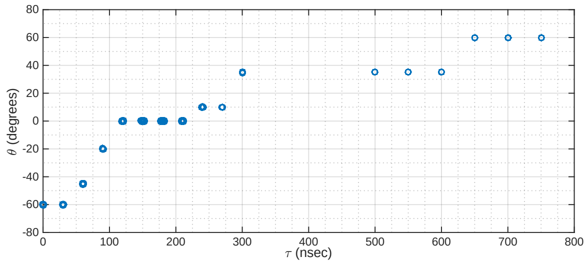

Fig. 1: Scatter plot of experiment 1 at SNR = 30 dB.

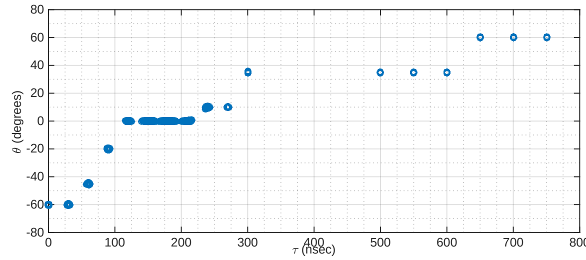

Fig. 2: Scatter plot of experiment 2 at SNR = 20 dB.

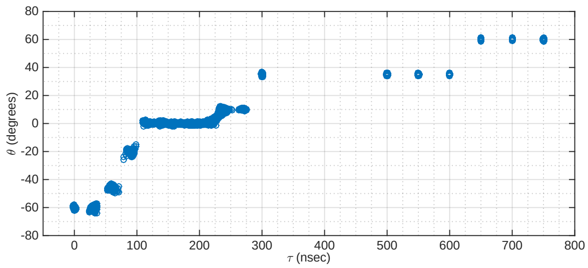

Fig. 3: Scatter plot of experiment 3 at SNR = 10 dB.

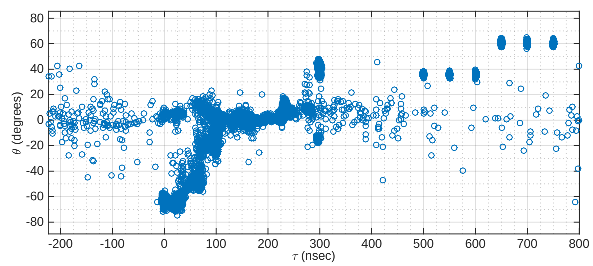

Fig. 4: Scatter plot of experiment 4 at SNR = 0 dB.

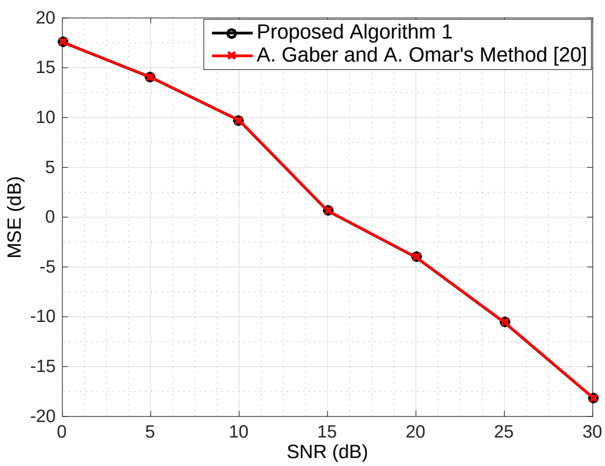

Fig. 5: MSE of ToAs vs. SNR of experiment 5.

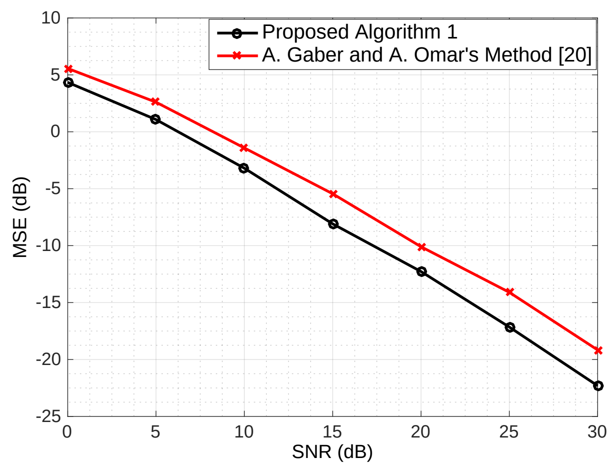

Fig. 6: MSE of AoAs vs. SNR of experiment 5.

- *Step 2:* Obtain $\{\hat{z}_{\tau_i}^{\text{MP}}\}_{i=1}^{q}$ using equations (18) till (21).
- *Step 3:* Estimate the ToAs of the q paths by the following relation:

$$\hat{\tau}_i^{\text{MP}} = -\frac{\text{ang}\{\hat{z}_{\tau_i}^{\text{MP}}\}}{2\pi \, \triangle_f} \tag{27}$$

- *Step 4:* Form an $N \times M$ matrix $\mathbf{Z}$ by using entries of the snapshot vector $\mathbf{x}$ as follows:

$$\mathbf{Z} = \begin{pmatrix} X_{1,1} & X_{2,1} & \cdots & X_{M,1} \\ X_{1,2} & X_{2,2} & \cdots & X_{M,2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1,N} & X_{2,N} & \cdots & X_{M,N} \end{pmatrix} \tag{28}$$

Note that $\mathbf{Z}$ is written as:

$$\mathbf{Z} = \mathbf{A}_{\text{N}}\mathbf{\Gamma}\mathbf{C}_{\text{M}}^{\text{T}} + \mathbf{W} \tag{29}$$

where $\mathbf{W}$ is the noise part. This step comprises in estimating the term $\mathbf{G} = \mathbf{A}_{\text{N}}\mathbf{\Gamma}$ using Least Squares (LS), i.e:

$$\hat{\mathbf{G}} = \arg\min_{\mathbf{G}} \|\mathbf{Z} - \mathbf{G}\mathbf{C}_{\text{M}}^{\text{T}}\|^2 \tag{30}$$

The solution of (30) is:

$$\hat{\mathbf{G}} = \mathbf{Z}\mathbf{C}_{\text{M}}^{\dagger} \tag{31}$$

where $\mathbf{C}_{\text{M}}^{\dagger}$ is the MoorePenrose pseudoinverse of $\mathbf{C}_{\text{M}}^{\text{T}}$ and is given by

$$\mathbf{C}_{\text{M}}^{\dagger} = \mathbf{C}_{\text{M}}^{*}\left(\mathbf{C}_{\text{M}}^{\text{T}}\mathbf{C}_{\text{M}}^{*}\right)^{-1} \tag{32}$$

Note that $\mathbf{C}_{\text{M}}^{\dagger}$ exists if and only if $q \leq M$ and all ToAs are distinct, i.e. $P = 1$. Finally, we use the 2D Matrix Pencil estimates of the ToAs obtained in *Step 3* to compute $\mathbf{C}_{\text{M}}^{\text{T}}$ in order to obtain the estimate of $\mathbf{G}$ using equation (32) then (31). In other words, $\mathbf{C}_{\text{M}}$ is obtained as

$$\mathbf{C}_{\text{M}} = [\mathbf{c}_{\text{M}}(\hat{\tau}_1^{\text{MP}}) \ldots \mathbf{c}_{\text{M}}(\hat{\tau}_q^{\text{MP}})] \tag{33}$$

- *Step 5:* Using $\hat{\mathbf{G}}$ from *Step 4*, we solve the following optimisation problem:

$$\begin{aligned} \hat{\mathbf{A}}_{\text{N}} &= \arg\min_{\mathbf{A}_{\text{N}}} \|\hat{\mathbf{G}} - \mathbf{A}_{\text{N}}\mathbf{\Gamma}\|^2 \\ &= \sum_{i=1}^{q} \arg\min_{\mathbf{a}_{\text{N}}(\theta_i)} \|\hat{\mathbf{G}}^{\langle :,i \rangle} - \gamma_i \mathbf{a}_{\text{N}}(\theta_i)\|^2 \end{aligned} \tag{34}$$

Note that the problem is decoupled in terms of $\mathbf{a}_{\mathrm{N}}(\theta_i)$ due to the diagonal structure of $\mathbf{\Gamma}$. The solution of the problem under a norm constraint, e.g. $\|\mathbf{a}_{\mathrm{N}}(\theta_i)\|^2 = N$ for $i = 1 \ldots q$, is

$$\hat{\mathbf{a}}_{\mathrm{N}}(\theta_i) = \frac{\hat{\mathbf{G}}^{\langle :,i\rangle}}{\|\hat{\mathbf{G}}^{\langle :,i\rangle}\|} \quad (35)$$

- *Step 6:* In the last step, we estimate the AoAs by using an LS fit, i.e.

$$\hat{\mathbf{e}}_i = \arg\min_{\mathbf{e}_i} \|\mathrm{ang}\{\hat{\mathbf{a}}_{\mathrm{N}}(\theta_i)\} - \mathbf{T}\mathbf{e}_i\|^2, \quad i = 1 \ldots q \quad (36)$$

where $\mathbf{T} \in \mathbb{C}^{\mathrm{N}\times 2}$ and is given by

$$\mathbf{T} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ N-1 & 1 \end{pmatrix} \quad (37)$$

and the solution is $\hat{\mathbf{e}}_i = [e_{i,1}, e_{i,2}]^{\mathrm{T}} = \mathbf{T}^{\dagger}\mathrm{ang}\{\hat{\mathbf{a}}_{\mathrm{N}}(\theta_i)\}$ with $\mathbf{T}^{\dagger} = (\mathbf{T}^{\mathrm{T}}\mathbf{T})^{-1}\mathbf{T}^{\mathrm{T}}$ and finally θ_i is estimated as follows

$$\hat{\theta}_i = -\sin^{-1}\Big(\frac{e_{i,1}\lambda}{2\pi d}\Big), \qquad i = 1 \ldots q \quad (38)$$

In short, Algorithm 1 is useful when $M \gg N$. Note that only the ToAs were estimated using the 2D Matrix Pencil technique in *Step 2*. Therefore, the conditions for Algorithm 1 are **A.1** till **A.3**, in addition to the condition of existance of a pseudoinverse of $\mathbf{C}_{\mathrm{M}}$ in *Step 4*. Combining all those conditions, we get the following:

- **C.1**: $q \leq \min\{M_pN_p, K_N(K_M-1), M\}$
- **C.2**: $P = 1$
- **C.3**: $Q \leq \min\{M_p, K_M - 1\}$

B. Algorithm 2

In this algorithm, both ToAs and AoAs are estimated using the 2D-Matrix Pencil technique, i.e.

- *Step 1* till *Step 3* are similar to Algorithm 1.
- *Step 4:* Form $\mathbf{Y} = \mathbf{X}\mathbf{P}$ where $\mathbf{P}$ is given in equation (25).
- *Step 5:* Obtain $\{\hat{z}_{\theta_i}^{\mathrm{MP}}\}_{i=1}^q$ using equations (26) and (21).
- *Step 6:* Estimate the AoAs of the q paths by the following relation:

$$\hat{\theta}_i^{\mathrm{MP}} = -\sin^{-1}\Big(\frac{\mathrm{ang}\{\hat{z}_{\theta_i}^{\mathrm{MP}}\}\lambda}{2\pi d}\Big) \quad (39)$$

Note that the ToAs and AoAs are estimated but are not matched; unlike Algorithm 1, where the matching happens naturally in *Step 5*. In other words, $\hat{\tau}_k^{\mathrm{MP}}$ and $\hat{\theta}_k^{\mathrm{MP}}$ are not necessarily the ToA and AoA of the k^{th} multipath. Therefore, a matching step is required to pair $\{\hat{\tau}_i^{\mathrm{MP}}\}_{i=1}^q$ with $\{\hat{\theta}_i^{\mathrm{MP}}\}_{i=1}^q$. Fixing the position of $\hat{\tau}_k^{\mathrm{MP}}$ at position k, there are $q!$ possible permutations of $\{\hat{\theta}_i^{\mathrm{MP}}\}_{i=1}^q$.

- *Step 7:* The matching criterion is based on evaluating the Maximum Likelihood (ML) cost function for joint angles and times of arrival estimation (see [24] for the JADE ML cost function) by fixing the positions of $\{\hat{\tau}_i^{\mathrm{MP}}\}_{i=1}^q$ and permuting $\{\hat{\theta}_i^{\mathrm{MP}}\}_{i=1}^q$ as done in the table **SubAlgorithm 1**.

Since the ToAs and AoAs are both estimated using 2D Matrix Pencil, Algorithm 2 needs conditions **A.1** till **A.3** and **B.1** till **B.3**, and therefore

SubAlgorithm 1: Step 7 of Algorithm 2

INITIALISE Step 7:
$\mathbf{C}_{\mathrm{M}} = [\mathbf{c}_{\mathrm{M}}(\hat{\tau}_1^{\mathrm{MP}}) \ldots \mathbf{c}_{\mathrm{M}}(\hat{\tau}_q^{\mathrm{MP}})]$
$\mathbf{A}_{\mathrm{N}} = [\mathbf{a}_{\mathrm{N}}(\hat{\theta}_1^{\mathrm{MP}}) \ldots \mathbf{a}_{\mathrm{N}}(\hat{\theta}_q^{\mathrm{MP}})]$
$\mathbf{\Upsilon_1} = \mathbf{I}_{\mathrm{q}}$
MAIN LOOP of Step 7:
for l = 1 *to* q! **do**
Step 7.1: $\mathbf{H} = \mathbf{C}_{\mathrm{M}} \boxtimes (\mathbf{A}_{\mathrm{N}}\mathbf{\Upsilon}_l)$
Step 7.2: $\mathscr{P}_{\mathbf{H}} = \mathbf{I}_{\mathrm{MN}} - \mathbf{H}(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}$
Step 7.3: $\mathbf{b}(l) = \big\|\mathscr{P}_{\mathbf{H}}\mathbf{x}\big\|^2$
Step 7.4: Choose another permutation matrix $\mathbf{\Upsilon}_{l+1}$
FIND BEST MATCH:
Step 7.5: Find $\hat{k} = \arg\max_k \mathbf{b}(k)$. This means that all columns of $\mathbf{C}_{\mathrm{M}}$ are matched to columns of $\mathbf{A}_{\mathrm{N}}\mathbf{\Upsilon}_{\hat{k}}$ according to the ML criterion in *Step 7.3*.

- **D.1**: $q \leq \min\{M_pN_p, K_N(K_M-1), K_M(K_N-1)\}$
- **D.2**: $P \leq \min\{N_p, K_N - 1\}$
- **D.3**: $Q \leq \min\{M_p, K_M - 1\}$

VI. Simulation Results

This section demonstrates the performance of Algorithm 1 as a function of SNR. The performance of Algorithm 2 was not provided due to lack of space.

In the first four experiments, the array was ULA with $N = 3$ antennas spaced half a wavelength apart. The transmitted OFDM symbol occupies 40 MHz of bandwidth, and uses $M = 64$ subcarriers with uniform spacing $\triangle_f =$ 0.625 MHz. The 2D Matrix Pencil parameters were $M_p = 30$ and $N_p = 2$. The number of multipath components were set to $q = 17$ paths. The ToAs and AoAs of each path are given as follows: The first 11 paths arrive with delays $\{\tau_k = 30(k-1)\ \mathrm{nsec}\}_{k=1}^{11}$ with corresponding AoAs as $\{\theta_k = -60°\}_{k=1}^{2}$, $\theta_3 = -45°$, $\theta_4 = -20°$, $\{\theta_k = 0°\}_{k=5}^{8}$, $\{\theta_k = 10°\}_{k=9}^{10}$, and $\theta_{11} = 35°$. The 6 other paths arrive with delays $\{\tau_k = 500 + 50(k-12)\ \mathrm{nsec}\}_{k=12}^{17}$ with corresponding AoAs as $\{\theta_k = 35°\}_{k=12}^{14}$ and $\{\theta_k = 60°\}_{k=15}^{17}$. Moreover, the multipath coefficients $\boldsymbol{\gamma}$ are randomly chosen. For each experiment, a different SNR was set and a scatter plot was depicted using 1000 Monte-Carlo simulations. Each Monte-Carlo simulation plots the ToA and AoA estimates using only one snapshot $\mathbf{x}$.

Note that the maximum number of paths arriving at the same time but with different AoAs is $P = 1$, and the maximum number of paths arriving with same AoAs but at different times is $Q = 4$. Moreover, one could easily verify that conditions **C.1** till **C.3** are satisfied and hence Algorithm 1 is applicable.

In the first experiment, i.e. Figure 1, the SNR was set to 30 dB, and we observe almost perfect estimation of all ToAs and AoAs since the variations of the estimates from their true values is negligible. The SNR was 20 dB in the second experiment (Figure 2) and we observe almost the same phenomena as the first experiment except for paths 5 till 8 where their AoAs were properly estimated but their ToAs tend to overlap. In the third experiment (Figure 3) where the SNR = 10 dB , paths 5 till 10 overlap and ToA/AoA estimates of all paths start to show more deviation from their true values. Finally, in the last experiment (Figure 4), the SNR was set to 0 dB and we could observe a clear degradation of the performance of Algorithm 1.

In the last experiment, i.e. experiment 5, we plot two Mean-Squared-Error (MSE) curves, one corresponding to the MSE of the Times-of-Arrival (Figure 5) and the other corresponding to the MSE of the Angles-of-Arrival (Figure 6). We compare with the first algorithm proposed by A. Gaber and A. Omar in Section III, [20]. The simulation parameters are the same as those in the first four experiments except for q which is now set to 3 sources. In addition, the ToAs are selected as follows: $\tau_1 = 0$ nsec, $\tau_2 = 25$ nsec, and $\tau_3 = 75$ nsec. Furthermore, the corresponding AoAs are chosen to be: $\theta_1 = 0^\circ$, $\theta_2 = 5^\circ$, and $\theta_3 = 10^\circ$. As expected, the MSE of the estimated ToAs using Algorithm 1 and the method in [20] is the same (See Figure 5) since *Steps 1* till 3 are similar in both algorithms, and therefore the ToA estimates are the same. However, the estimated AoAs are different, since both algorithms are essentially different. In particular, our proposed Algorithm 1 doesn't require ToA/AoA pairing, since this is automatically done in *Step 6*. Whereas, the method in [20] requires a matching criterion. This may explain why the proposed Algorithm 1 exhibits a lower MSE in AoAs than the one in [20] (See Figure 6),

VII. Conclusion

In short, we presented two algorithms based on 2D Matrix Pencils. These two algorithms allow joint estimation of times and angles of arrival of multiple paths using only one snapshot. Algorithm 1 resolves more sources than Algorithm 2 in the case where the number of subcarriers is much larger than the number of antennas, which is the case of most Wi-Fi systems. The performance of Algorithm 1 as a function of SNR was studied through simulations.

Acknowledgment

EURECOM's research is partially supported by its industrial members: ORANGE, BMW Group, SFR, ST Microelectronics, Symantec, SAP, Monaco Telecom, iABG. This work was also supported by RivieraWaves, a CEVA company, and a Cifre scholarship.

The authors wish to thank the anonymous reviewers for constructive comments which helped to improve the exposition.

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