# A Mutual Coupling Resilient Algorithm for Joint Angle and Delay Estimation

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***Abstract*—The problem of Joint Angle and Delay Estimation (JADE) in the presence of mutual coupling is addressed. The system consists of a Single Input Multiple Output (SIMO) link in an OFDM communication setting. This paper presents two resilient algorithms that could cope with mutual coupling. The first one is an extension of an existing algorithm in order to perform JADE, which allows resolvability of more signals. The second algorithm is an improvement of the first one, in the sense that it could handle more mutual coupling parameters. Simulation results show the difference between both algorithms. In addition, the algorithms are compared with the 2D-MUSIC method that performs JADE in the absence of mutual coupling, i.e. when a "coupling-free" model is assumed.**

***Index Terms*—Joint Estimation, Angle-of-Arrival, Time-of-Arrival, Resilient Mutual Coupling, OFDM**

## I. Introduction

Mutual coupling between antennas is a popular problem in array signal processing. This phenomenon arises when antennas are close to each other [1], and thus the current developed in an antenna element depends on its own excitation and on the contributions from adjacent antennas. As a consequence, an ideal model is no longer valid, and therefore the performance of the high resolution algorithms that perform Angle-of-Arrival (AoA) estimation, such as MUSIC [2], ESPRIT [3], etc., deteriorate significantly.

Methods that aim on solving the mutual coupling problem are sometimes referred to as calibration methods, which are of two types: Offline and Online. In an offline calibration approach, one estimates the mutual coupling parameters using known locations, such as the techniques in [4]–[6]. In contrast, online calibration consists of jointly estimating the coupling and AoA parameters. Herein, we focus on the latter.

In the literature, several techniques deal with the online calibration problem, such as those found in [7]–[17] and references within. The authors in [7] jointly estimate the coupling parameters and AoAs by alternating minimization steps between the former and the latter using the MUSIC cost function. In [8], the algorithm is iterative and the sources are assumed to be totally uncorrelated. Moreover, the method in [8] estimates the coupling parameters in order to utilize it in the MUSIC cost function. In [9], [10], the array elements are assumed to be partly calibrated. In other words, one has access to a few coupling parameters.Moreover, methods in [13]–[15] propose to use the *"middle sub-array"*. In particular, let $N$ be the number of antennas placed in a uniform and linear fashion, and $p$ be the number of coupling parameters. If $p \leq \frac{N}{2}$, then there exists a *"middle sub-array"*, of size $N - 2p + 2$, over which the effect of the mutual coupling is preserves the *Vandermonde* structure of the array response [13]–[15]. As a matter of fact, one could mathematically show that the array response on the *"middle sub-array"* is a known functional *Vandermonde* vector multiplied by an unknown scalar, and therefore high resolution techniques could be applied to estimate the AoAs. However, this is highly suboptimal because only an effective number of antennas, i.e. $N - 2p + 2$, are utilized for parameter estimation. To address this major loss of antennas, the authors in [17] suggest to add *"Guard"* antennas on the edges of the array, in particular $p - 1$ antennas on the left edge and the other $p - 1$ on the right edge. Therefore, the main array of size $N$ plays the role of a *"middle sub-array"*. The aforementioned arguement on the *"middle sub-array"* holds on the main array. Unfortunately, appending antennas is not possible in some applications, such as Wi-Fi.

Joint Angle and Delay Estimation, also known as JADE, was introduced in [26]. One motivation behind JADE is to increase the number of resolvable signals by transmitting known signals. As a result, this allows estimating signal parameters (AoA/ToA) of a number of signals beyond the number of antennas, thanks to temporal (or frequency, in the case of OFDM) diversity. We outline two contributions of this paper: The first one is an extension to an existing resilient mutual coupling algorithm in order to perform JADE in the presence of mutual coupling. Analogously to the motivation of JADE, this algorithm could handle more coupling parameters, as compared to AoA-only estimation. The second contribution is an improvement of the first algorithm. In particular, the first algorithm requires $p \leq \frac{N}{2}$ to function properly, whereas the second algorithm does not.

**Notations:** Upper-case and lower-case boldface letters denote matrices and vectors, respectively. $(.)^{\mathrm{T}}$, $(.)^*$, and $(.)^{\mathrm{H}}$ represent the transpose, conjugate, and transpose-conjugate operators. The matrix $\mathbf{I}$ is the identity matrix of suitable dimensions. The operator $\mathrm{E}\{\mathbf{X}\}$ returns the expectation of a random matrix $\mathbf{X}$. For any matrix $\mathbf{B}$, the operator $\|\mathbf{B}\|_2$ is the *Frobenius* norm.

## II. System Model

### A. Mathematical Formulation

Consider an OFDM symbol composed of $M$ subcarriers and centered at a carrier frequency $f_c$, impinging an array of $N$ antennas via $q$ sources, each arriving at different AoAs $\mathbf{\Theta} = \{\theta_1 \dots \theta_q\}$ and ToAs $\mathbf{T} = \{\tau_1 \dots \tau_q\}$. Note that the order in both sets should be respected. In frequency domain, we could express the signal at the $n^{th}$ antenna and $m^{th}$ subcarrier of the $l^{th}$ OFDM symbol as follows [19]:

$$X_{m,n}(l) = b_m \sum_{i=1}^{q} \gamma_i(l) \tilde{a}_n(\theta_i) e^{-j2\pi m \triangle_f \tau_i} + N_{m,n}(l) \qquad (1)$$

with $l = 1 \dots L$, $m = 1 \dots M$, and $n = 1 \dots N$. Moreover, $T = \frac{1}{\triangle_f}$ is the OFDM symbol duration, $\triangle_f$ is the subcarrier spacing, $b_m$ is the modulated symbol onto the $m^{th}$ subcarrier, $\tilde{a}_n(\theta)$ is the $n^{th}$ antenna response to an incoming signal at angle $\theta$. Moreover, $\gamma_i(l)$ is the complex coefficient of the $i^{th}$ source during the $l^{th}$ OFDM symbol. The term $N_{m,n}(l)$ is background noise.

We claim that the transmitted OFDM symbol is a preamble field of the Wi-Fi 802.11 frame, thus prior knowledge of the modulated symbols $\{b_m\}_{m=0}^{M-1}$ is a valid assumption. Therefore, we compensate for all such symbols (multiplying by $\frac{b_m^*}{|b_m|^2}$) and hence omit $b_m$ from (1). Re-writing (1) in a compact matrix form, we have:

$$\mathbf{x}(l) = \mathbf{H}\boldsymbol{\gamma}(l) + \mathbf{n}(l) \tag{2}$$

where $\mathbf{x}(l)$ and $\mathbf{n}(l)$ are $MN \times 1$ vectors

$$\mathbf{x}(l) = [X_{1,1}(l) \dots X_{1,N}(l) \dots X_{M,1}(l) \dots X_{M,N}(l)]^{\mathrm{T}} \tag{3}$$

The noise vector $\mathbf{n}(l)$ is defined in a similar manner. The vector $\boldsymbol{\gamma}(l) \in \mathbb{C}^{q\times 1}$ is given as

$$\boldsymbol{\gamma}(l) = [\gamma_1(l) \dots \gamma_q(l)]^{\mathrm{T}} \tag{4}$$

$\mathbf{H}$ is an $MN \times q$ matrix given as

$$\mathbf{H} = [\mathbf{c}(\tau_1) \otimes \tilde{\mathbf{a}}(\theta_1) \dots \mathbf{c}(\tau_q) \otimes \tilde{\mathbf{a}}(\theta_q)] \tag{5}$$

where

$$\mathbf{c}(\tau) = [1, z_\tau \dots z_\tau^{M-1}]^{\mathrm{T}} \quad \text{with} \quad z_\tau = e^{-j2\pi\tau\triangle f} \tag{6}$$

and $\tilde{\mathbf{a}}(\theta)$ is the response of a Uniform Linear Array (ULA) suffering from mutual coupling. Following [11], the response $\tilde{\mathbf{a}}(\theta)$ could be modelled as

$$\tilde{\mathbf{a}}(\theta) = \mathcal{T}_p(\mathbf{m})\mathbf{a}(\theta) \tag{7}$$

where $\mathcal{T}_p(\mathbf{m}) \in \mathbb{C}^{\mathrm{N}\times\mathrm{N}}$ is a banded symmetric Toeplitz matrix defined as follows

$$\mathcal{T}_p(\mathbf{m}) = \begin{bmatrix} 1 & m_1 & m_2 & \cdots & m_{p-1} & 0 & \cdots & 0 \\ m_1 & 1 & m_1 & \cdots & m_{p-2} & m_{p-1} & \cdots & 0 \\ \vdots & & & \ddots & \ddots & & & \vdots \\ 0 & \cdots & m_{p-1} & m_{p-2} & \cdots & m_1 & 1 & m_1 \\ 0 & \cdots & 0 & m_{p-1} & \cdots & m_2 & m_1 & 1 \end{bmatrix} \tag{8}$$

and

$$\mathbf{a}(\theta) = [1, z_\theta \dots z_\theta^{N-1}]^{\mathrm{T}} \quad \text{with} \quad z_\theta = e^{-j2\pi\frac{d}{\lambda}\sin(\theta)} \tag{9}$$

where $d$ is the inter-element spacing and $\lambda$ is the signal's wavelength. The model in equations (7) and (8) suggest that antennas that are placed at least $p$ inter-element spacings apart do not interfere, i.e. $m_i = 0$ for all $i \geq p$. This is due to the fact that the mutual coupling is inversely proportional to the distance between antennas.

### B. Assumptions and Problem Statement

Throughout the paper, we assume the following:

- **A1**: $\mathbf{H}$ is full column rank.
- **A2**: The complex coefficients $\boldsymbol{\gamma}(l)$ are fixed within a snapshot, and are uncorrelated over OFDM symbols.
- **A3**: The number of sources, i.e. $q$, is known.
- **A4**: The vector $\mathbf{n}(l)$ is additive Gaussian noise with zero mean and covariance $\sigma^2\mathbf{I}$ independent from the sources.

The conditions so that assumption **A1** is satisfied are given in [20]. Moreover, if assumption **A2** were not satisfied, i.e. when the sources are a result of multipath propagation, then one could preprocess OFDM symbols by applying Frequency Smoothing[1], which is a 1D version of [21]. For the sake of compactness, we shall assume that the sources are uncorrelated. Furthermore, algorithms exist for estimating the number of sources, such as Minimum Description Length [22], Modified MDL [23], Benjamin Hochberg procedure [24], and so forth. Therefore, assumption **A3** is reasonable. Any further assumptions will be mentioned. Now, we address our problem: *"Given $\{\mathbf{x}(l)\}_{l=1}^{L}$, $p$ and $q$, jointlyestimate the signal parameters $\boldsymbol{\Theta}$ and $\mathbf{T}$ in the presence of mutual coupling."*

[1]Note that one could apply the technique of spatial smoothing [18] over frequencies (i.e. frequency smoothing) since the VanderMonde structure is preserved over the vector $\mathbf{c}(\tau)$ even though mutual coupling is present. For simplicity and due to lack of space, we shall assume uncorrelation of sources.

## III. Resilient Mutual Coupling Algorithm

### A. Algorithm 1: Quadratic Program 1

We start by defining the covariance matrix of the received vector $\mathbf{x}(l)$, i.e.

$$\mathbf{R}_{xx} = \mathrm{E}\{\mathbf{x}(l)\mathbf{x}^{\mathrm{H}}(l)\} \tag{10}$$

In practical scenarios, this matrix is computed thru a sample average as follows $\hat{\mathbf{R}}_{xx} = \frac{1}{L}\mathbf{X}\mathbf{X}^{\mathrm{H}}$, where $\mathbf{X} = [\mathbf{x}(1) \dots \mathbf{x}(L)]$. Let $\hat{\lambda}_1 > \hat{\lambda}_2 > \ldots > \hat{\lambda}_{\mathrm{MN}}$ and $\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2 \dots \hat{\mathbf{u}}_{\mathrm{MN}}$ denote the eigenvalues and their corresponding eigenvectors of $\hat{\mathbf{R}}_{xx}$. Under assumptions **A1**, **A2** and **A4**, one could jointly estimate $\{(\theta_i, \tau_i)\}_{i=1}^{q}$ by evaluating the peaks of the following 2D-MUSIC cost function [25], [26]

$$(\hat{\theta}_i, \hat{\tau}_i) = \arg\max_{\theta,\tau} \frac{1}{\mathbf{h}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{h}(\theta,\tau)} \tag{11}$$

where $\mathbf{h}(\theta,\tau) = \mathbf{c}(\tau) \otimes \tilde{\mathbf{a}}(\theta)$ and $\hat{\mathbf{U}}_n = [\hat{\mathbf{u}}_{\mathrm{q+1}} \dots \hat{\mathbf{u}}_{\mathrm{MN}}]$ is called the "noise" subspace. The MUSIC cost function exploits the orthogonality between vectors in the signal and noise subspaces [25]. Unfortunately, MUSIC couldn't be directly applied since the functional form of $\tilde{\mathbf{a}}(\theta)$ is unknown. Before we proceed, we find the following useful:

**Definition 1:** For any vector $\mathbf{a} \in \mathbb{C}^{N\times 1}$ and matrix $\mathbf{A}_p \in \mathbb{C}^{N\times p}$. We say that "$\mathbf{a}$ *generates* $\mathbf{A}_p$", or $\mathbf{A}_p \triangleq \mathcal{G}_p(\mathbf{a})$, if

$$\mathbf{A}_p = \left[\ \mathbf{a} \ \middle|\ \mathbf{S}_1\mathbf{a} \ \middle|\ \dots \ \middle|\ \mathbf{S}_{p-1}\mathbf{a}\ \right] \tag{12}$$

where $\mathbf{S}_k \in \mathbb{C}^{N\times N}$ is an all-zero matrix except at the $k^{th}$ sub- and super-diagonals, which are set to 1.

**Theorem 1:** *(Commutativity of Symmetric Toeplitz Matrices) Let $\mathbf{m} = [m_0, m_1 \dots m_{p-1}]^T \in \mathbb{C}^{p\times 1}$ and $\mathbf{a} \in \mathbb{C}^{N\times 1}$. Define the corresponding matrix $\mathcal{T}_p(\mathbf{m})$ as in equation* (8)*. Then the following is true for any $1 \leq p \leq N$*

$$\mathcal{T}_p(\mathbf{m})\mathbf{a} = \mathbf{A}_p\mathbf{m} \tag{13}$$

*where $\mathbf{A}_p = \mathcal{G}_p(\mathbf{a})$.* (See **Definition 1**)

**Proof:** See [7], [27].

Using **Theorem 1**, we have that

$$\tilde{\mathbf{a}}(\theta) = \mathcal{T}_p(\mathbf{m})\mathbf{a}(\theta) = \mathbf{B}(\theta)\mathbf{m} \tag{14}$$

where $\mathbf{B}(\theta) = \mathcal{G}_p(\mathbf{a}(\theta))$. This theorem allows re-writing the denominator of the 2D-MUSIC cost function in equation (11) as follows

$$\begin{aligned} \mathbf{h}^{\mathrm{H}}(\theta,\tau)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}\mathbf{h}(\theta,\tau) &= \left\|\hat{\mathbf{U}}_n^{\mathrm{H}}\big(\mathbf{c}(\tau)\otimes\tilde{\mathbf{a}}(\theta)\big)\right\|_2^2 \\ &= \left\|\hat{\mathbf{U}}_n^{\mathrm{H}}\big(\mathbf{c}(\tau)\otimes\mathbf{B}(\theta)\big)\mathbf{m}\right\|_2^2 \end{aligned} \tag{15}$$

which is quadratic in $\mathbf{m}$. Now, let

$$\mathbf{S}(\theta,\tau) \triangleq \big(\mathbf{c}(\tau)\otimes\mathbf{B}(\theta)\big)^{\mathrm{H}}\hat{\mathbf{U}}_{\mathbf{n}}\hat{\mathbf{U}}_{\mathbf{n}}^{\mathrm{H}}\big(\mathbf{c}(\tau)\otimes\mathbf{B}(\theta)\big) \tag{16}$$

If we form the following Quadratic Program (QP)

$$\text{(QP1):} \begin{cases} \underset{\mathbf{t}\in\mathbb{C}^{p\times 1}}{\text{minimize}} & \mathbf{t}^{\mathrm{H}}\mathbf{S}(\theta,\tau)\mathbf{t} \\ \text{subject to} & \mathbf{e}_1^{\mathrm{T}}\mathbf{t} = 1 \end{cases} \tag{17}$$

then the solution is the cost function $f_1(\theta,\tau)$ [27], where its $q$ peaks are estimates of the signal parameters $\{(\theta_i,\tau_i)\}_{i=1}^{q}$:

$$f_1(\theta,\tau) = \mathbf{e}_1^{\mathrm{T}}\mathbf{S}^{\dagger}(\theta,\tau)\mathbf{e_1} \tag{18}$$

where the constraint in equation (17) is to exclude the trivial solution. Similar to [27], the identifiability conditions of $f_1(\theta,\tau)$ are:

- **B1:** $q+p \leq MN$
- **B2:** $p \leq \frac{N}{2}$

Condition **B2** is due to the following property:

**Property 1:** *For ULA type configurations, i.e.* $\mathbf{a}(\theta) = [1, z_\theta, \dots z_\theta^{N-1}]^T$ *with* $z_\theta = e^{-j2\pi\frac{d}{\lambda}sin(\theta)}$. *Define the following sets:*

$$\mathbf{\Theta}_+ = \{\theta = sin^{-1}(\frac{k\lambda}{Nd}), k = -\frac{N}{2}\dots\frac{N}{2}\} \tag{19a}$$

*and*

$$\mathbf{\Theta}_- = \{\theta = sin^{-1}(\frac{(k+\frac{1}{2})\lambda}{Nd}), k = -\frac{N}{2}\dots\frac{N}{2}\} \tag{19b}$$

*Also, let* $\mathbf{\Theta}_\pm = \{\mathbf{\Theta}_+ \cup \mathbf{\Theta}_-\}$. *The matrix* $\mathbf{B}(\theta) = \mathcal{G}_p(\mathbf{a}(\theta))$ *has the following characteristics:*

- *Iff* $p < \frac{N+2}{2}$, *the matrix* $\mathbf{B}(\theta)$ *is full column rank.*
- *When* $p \geq \frac{N+2}{2}$, *there are two cases*
  - *Iff* $N$ *is even and* $\theta \in \mathbf{\Theta}_+$, *then* $\text{rank}(\mathbf{B}(\theta)) = \frac{N}{2}$. *Furthermore, iff* $N$ *is even and* $\theta \in \mathbf{\Theta}_-$, *then* $\text{rank}(\mathbf{B}(\theta)) = \frac{N}{2}+1$. *Otherwise,* $\mathbf{B}(\theta)$ *is full column rank.*
  - *Iff* $N$ *is odd and* $\theta \in \mathbf{\Theta}_\pm$, *then* $\text{rank}(\mathbf{B}(\theta)) = \frac{N+1}{2}$. *Otherwise,* $\mathbf{B}(\theta)$ *is full column rank.*

**Proof:** See [28].

### *B. Algorithm 2: Quadratic Program 2*

When assumption **B2** is not satisfied, the function $f_1(\theta,\tau)$ is maximised at $\theta \in \mathbf{\Theta}_\pm$ for any $\tau$. This could be seen from the objective function of QP1 in (17): When $\theta \in \mathbf{\Theta}_\pm$, then according to **Property 1**, the matrix $\mathbf{B}(\theta)$ admits a null space, therefore the cost function is minimized. To circumvent this issue, we form a new Quadratic Program (QP2), which minimizes the same cost function, but under a different constraint, viz.

$$\text{(QP2):} \begin{cases} \underset{\mathbf{t}}{\text{minimize}} & \mathbf{t}^{\mathrm{H}}\mathbf{S}(\theta,\tau)\mathbf{t} \\ \text{subject to} & \mathbf{e}_1^{\mathrm{T}}\mathbf{B}(\theta)\mathbf{t} = 1 \end{cases} \tag{20}$$

The solution of QP2 is given in [28], which is the following

$$f_2(\theta,\tau) = \mathbf{a}_p^{\mathrm{T}}(\theta)\mathbf{S}^{\dagger}(\theta,\tau)\mathbf{a}_p^{*}(\theta) \tag{21}$$

where $\mathbf{a}_p(\theta)$ is a steering vector of a p-sized ULA array, namely $\mathbf{a}_p(\theta) = [1, z_\theta \dots z_\theta^{p-1}]^{\mathrm{T}}$. The ToAs and AoAs are obtained by selecting the $q$ peaks of the $f_2(\theta,\tau)$.

## IV. Analysis of $f_1(\theta,\tau)$ vs $f_2(\theta,\tau)$

In this section, we justify the identifiability conditions of $f_2(\theta,\tau)$, which are given as follows:

- **C1:** $q+p \leq MN$
- **C2:** $p < N$

Condition **C1** is common to $f_1(\theta,\tau)$ and $f_2(\theta,\tau)$. This is due to the fact that the projection matrix onto the noise subspace, i.e. $\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^{\mathrm{H}}$ is of rank $MN-q$. Furthermore, the dimension of matrix $\mathbf{S}(\theta,\tau)$ is $p \times p$. In order to preserve the rank of the noise noise subspace projector, we should have $p \leq MN-q$, provided that $\mathbf{B}(\theta)$ is full column rank (i.e. when **B2** is satisfied). This argument justifies the conditions of $f_2(\theta,\tau)$ when **B2** is true. In what follows, we argue the case when $\frac{N}{2} < p < N$.

For the sake of argument and compactness, let us assume true subspaces (i.e. high SNR or number of snapshots), so we shall replace $\hat{\mathbf{U}}_n$ by $\mathbf{U}_n$. The MUSIC criterion implies that

$$\Big\|\mathbf{U}_n^{\mathrm{H}}\big(\mathbf{c}(\tau_i)\otimes\mathbf{B}(\theta_i)\big)\mathbf{t}\Big\|_2^2 = 0, \qquad (\theta_i,\tau_i) \in (\mathbf{\Theta},\mathbf{T}) \text{ and } \mathbf{t} = \mathbf{m} \tag{22}$$

Equation (22) along with **Property 1** imply the following

- *Case 1*: The rank of $\mathbf{S}(\theta_i,\tau_i)$ is $p-1$, if $(\theta_i,\tau_i) \in (\mathbf{\Theta},\mathbf{T})$ and $\theta_i \notin \mathbf{\Theta}_\pm$. In addition, the only eigenvector that spans the nullspace of $\mathbf{S}(\theta_i,\tau_i)$ is $\mathbf{m}$. This is equivalent to equation (22).
- *Case 2*: The rank of $\mathbf{S}(\theta_i,\tau_i)$ is $p-\delta$ if $\theta_i \in \mathbf{\Theta}_\pm$ and $\theta_i \notin \mathbf{\Theta}$, $\forall\tau_i$, where $\delta$ is the dimension of the null space of $\mathbf{B}(\theta_i)$. This is a direct result of **Property 1** when $p \geq \frac{N+2}{2}$.
- *Case 3*: The rank of $\mathbf{S}(\theta_i,\tau_i)$ is $p-\delta-1$ if $(\theta_i,\tau_i) \in (\mathbf{\Theta},\mathbf{T})$ and $\theta_i \in \mathbf{\Theta}_\pm$. It has been proven in [28] that the vector $\mathbf{m}$ is linearly independent from the vectors that form the null space of $\mathbf{B}(\theta_i)$. In other words, $\mathcal{N}\big(\mathbf{S}(\theta_i,\tau_i)\big) = \mathcal{N}\big(\mathbf{B}(\theta_i)\big) \cup \{\mathbf{m}\}$
- *Case 4*: Otherwise, $\mathbf{S}(\theta_i,\tau_i)$ is full rank.

Let us discuss the first three cases. Using spectral decomposition, we can say

$$\mathbf{S}(\theta_i,\tau_i) = \mathbf{V}\mathbf{\Phi}\mathbf{V}^{\mathrm{H}} \tag{23}$$

where the $k^{th}$ column of $\mathbf{V}$ is the $k^{th}$ eigenvector[2] of $\mathbf{S}(\theta_i,\tau_i)$, denoted as $\mathbf{v}_k$ and its corresponding eigenvalue is the $k^{th}$ smallest eigenvalue found in the $k^{th}$ diagonal entry of $\mathbf{\Phi}$, denoted as $\nu_k$. We adopt "diagonal loading" for the pseudo inverse, namely

$$\mathbf{S}^{\dagger}(\theta_i,\tau_i) = \big(\mathbf{S}(\theta_i,\tau_i) + \epsilon\mathbf{I}\big)^{-1} = \mathbf{V}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}} \tag{24}$$

where $\mathbf{\Sigma} = (\mathbf{\Phi}+\epsilon\mathbf{I})^{-1}$. Consider the quadratic function $f(\theta,\tau) = \mathbf{y}^{\mathrm{H}}\mathbf{S}^{\dagger}(\theta_i,\tau_i)\mathbf{y}$. When $\mathbf{y} = \mathbf{e_1}$, then $f(\theta,\tau) = f_1(\theta,\tau)$ and when $\mathbf{y} = \mathbf{a}_p^*(\theta)$, then $f(\theta,\tau) = f_2(\theta,\tau)$. Furthermore, we can re-write $f(\theta,\tau)$ as follows

$$f(\theta,\tau) = \sum_{k=1}^{p}\frac{1}{\nu_k+\epsilon}\|\mathbf{y}^{\mathrm{H}}\mathbf{v}_k\|_2^2 \tag{25}$$

*Case 1* implies that $\nu_1 = 0$ while $\nu_k > 0$ for all $k \geq 2$. Therefore, $f(\theta_i,\tau_i) \backsim \frac{1}{\epsilon}\|\mathbf{y}^{\mathrm{H}}\mathbf{m}\|_2^2 \xrightarrow[\varepsilon\to 0]{} \infty$, provided that $\|\mathbf{y}^{\mathrm{H}}\mathbf{m}\|_2^2 \neq 0$. It is easy to see that the norm term is not zero if $\mathbf{y} = \mathbf{e_1}$, since the first element of $\mathbf{m}$ can not be zero. As for $\mathbf{y} = \mathbf{a}_p^*(\theta)$, the conditions on $\mathbf{m}$ so that $\|\mathbf{y}^{\mathrm{H}}\mathbf{m}\|_2^2 \neq 0$ should be found in [28]. We conclude that both functions $f_1(\theta,\tau)$ and $f_2(\theta,\tau)$ peak at the true values$(\theta_i,\tau_i)$ defined in *Case 1*.

*Case 2* implies that $\nu_1 = \dots = \nu_\delta = 0$, while $\nu_k > 0$ for all $k \geq \delta+1$. Therefore,

$$f(\theta_i,\tau_i) \backsim \frac{1}{\epsilon}\sum_{k=1}^{\delta}\|\mathbf{y}^{\mathrm{H}}\mathbf{v}_k\|_2^2 \tag{26}$$

When $\mathbf{y} = \mathbf{e_1}$, the norm term is not necessarily zero. Therefore, $f_1(\theta,\tau)$ peaks at values of $\theta_i \in \mathbf{\Theta}_\pm$ for any $\tau_i$. However, when $\mathbf{y} = \mathbf{a}_p^*(\theta_i)$, then the norm term is exactly zero because

$$f(\theta_i,\tau_i) \backsim \frac{1}{\epsilon}\sum_{k=1}^{\delta}\|\mathbf{a}_p^{\mathrm{T}}(\theta_i)\mathbf{v}_k\|_2^2 = \frac{1}{\epsilon}\sum_{k=1}^{\delta}\|\mathbf{e}_1^{\mathrm{T}}\mathbf{B}(\theta_i)\mathbf{v}_k\|_2^2 = 0 \tag{27}$$

since $\mathbf{v}_k$ span the null space of $\mathbf{B}(\theta_i)$. We conclude that $f_1(\theta,\tau)$ yields fake peaks as explained wherever $f_2(\theta,\tau)$ doesn't.

Using the same reasoning, both functions $f_1(\theta_i,\tau_i)$ and $f_2(\theta_i,\tau_i)$ peak at values given in *Case 3*.

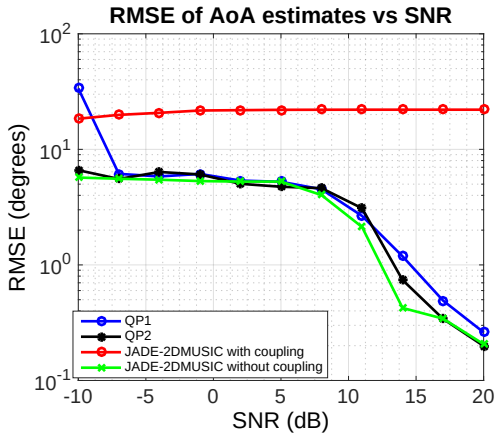


**Fig. 1:** RMSE of AoAs on a log-scale vs. SNR of the $1^{st}$ experiment.

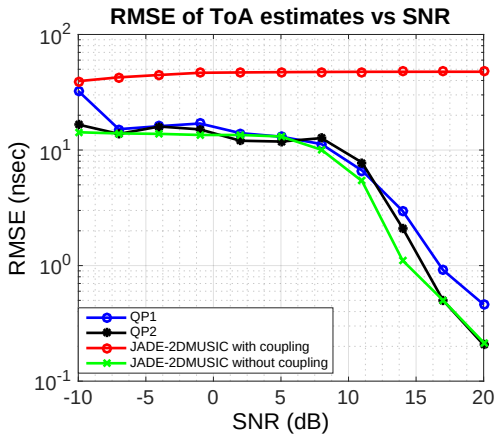


**Fig. 2:** RMSE of ToAs on a log-scale vs. SNR of the $1^{st}$ experiment.

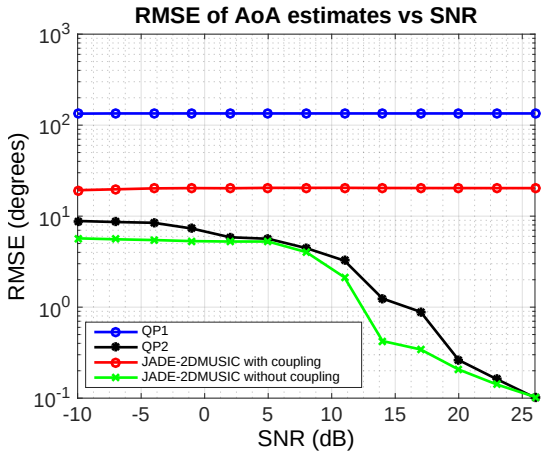


**Fig. 3:** RMSE of AoAs on a log-scale vs. SNR of the $2^{nd}$ experiment.

## V. Simulation Results

In this section, two experiments are conducted. In particular, we are interested in observing the Root-Mean-Squared-Error (RMSE) of ToAs and AoAs when condition **B2** is respected ($1^{st}$ experiment) and violated ($2^{nd}$ experiment). Note that **C2** is always respected. We fix the following parameters:

- $N = 5$ antennas and $M = 64$ subcarriers.
- $d = \frac{\lambda}{2}$ and $\triangle_f$= 312500 MHz (a total bandwidth of 20 MHz).
- $L = 100$ snapshots and 200 Monte-Carlo simulations.
- $q = 4$ multipaths, with:
  - $\mathbf{\Theta} = [-60°, -50°, -40°, -30°]$.
  - $\mathbf{T} = [10 \text{ nsec}, 35 \text{ nsec}, 50 \text{ nsec}, 75 \text{ nsec}]$.
  - $\boldsymbol{\gamma}(l)$ are chosen according to a Gaussian distribution.

In the $1^{st}$ experiment (Fig. 1 and Fig. 2), the number of coupling parameters is set to $p = 2$ (condition **B2** is respected), and are given as $\mathbf{m} = [1\ ,\ 0.343 + j0.3638]^{\mathrm{T}}$. Clearly, applying 2D-MUSIC to perform JADE in the presence of mutual coupling gives bad performance. Furthermore, at $-7$ dB $\leq$ SNR $\leq$ 10 dB, we can see that the performance of $f_1(\theta, \tau)$ and $f_2(\theta, \tau)$ is close to that of 2D-MUSIC, when applied to a "coupling-free" model. Moreover, we also observe that $f_2(\theta, \tau)$ coincides with the "coupling-free" model, when the SNR exceeds 17 dB. Also, there is a gain of almost $0.1°$ in terms of AoA (Fig. 1) and 0.3 nsec in terms of ToA (Fig. 2) when compared with $f_1(\theta, \tau)$.

In the $2^{nd}$ experiment (Fig. 3 and Fig. 4), the number of coupling parameters is set to $p = 4$ (condition **B2** is violated), and are given as $\mathbf{m} = [1\ ,\ 0.415 - j0.28\ ,\ -0.224 - j0.246\ ,\ 0.214 - j0.13]^{\mathrm{T}}$. It is obvious that the performance of $f_1(\theta, \tau)$ massively deteriorates. In addition, we see that $f_2(\theta, \tau)$ shows similar performance to 2D-MUSIC when applied to a "coupling-free" model when the SNR exceeds 25 dB.

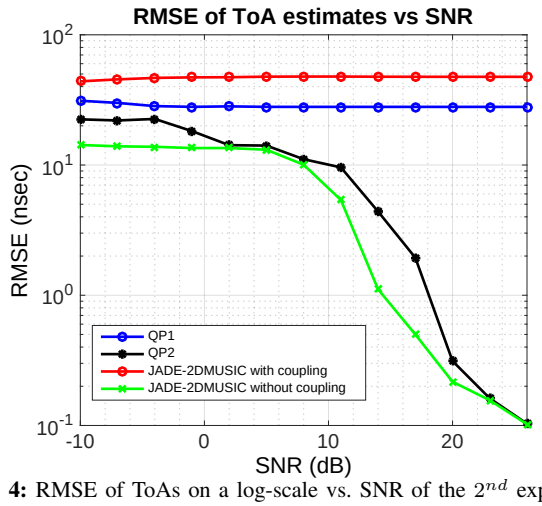


**Fig. 4:** RMSE of ToAs on a log-scale vs. SNR of the $2^{nd}$ experiment.

## VI. Conclusion

In this paper, we presented two contributions: The first one (Quadratic Program 1) is an extension to an existing resilient mutual coupling algorithm in order to perform Joint Angle and Delay Estimation (JADE). The second contribution (Quadratic Program 2) is an improvement of the first algorithm, in a sense that we could perform JADE when more antennas suffer from mutual coupling.


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[2]Indeed, $\mathbf{V}$ and $\mathbf{\Phi}$ are functions of $(\theta, \tau)$.

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