Two-Way Radio Networks with a Star Topology

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Abstract—**Achievable rates for two-way half-duplex relay channels are considered. This channel model is applicable to wireless networks with a star topology characterized by a traffic model where all traffic in the network traverses the relay due to the lack of a reliable direct link between the nodes. The coding and multiple-access strategies make use of network coding at the relay in addition to analog forwarding with interference cancellation at the receivers. An achievable rate region for discrete-memoryless networks is given as well as extensions for the additive white Gaussian network.**

I. Introduction

The two-tier network architecture shown in Figure 1 uses high-throughput radio links in a point-to-multipoint or *star* network topology to interconnect gateway nodes via a hub or relay. The gateway nodes provide large-range connectivity to dense networks of nodes. The networks around each gateway could be wired or wireless, or even comprise several tiers of P2MP topologies. Examples of star networks are satellite backhaul networks, high-altitude platforms (HAPS), maritime relay networks and future rapidly-deployable broadband wireless networks for civil protection. To a certain extent such topologies could be used for future cellular networks where relays are used to interconnect basestations (gateways).

This work considers two-way multiple-access strategies for star topologies which exhibit three primary characteristics:

1) Traffic patterns where data is primarily exchanged between the networks served by the relay and not relayed to other networks via secondary gateways on different links (radio or otherwise). This allows the relay and node to potentially use co-operative coding strategies for two-way multiple-access (e.g. interference cancellation at the physical layer, network coding at the MAC layer).
2) Radio equipment which uses some form of duplexing strategy to orthogonalize uplink (nodes to relay) and downlink (relay to nodes). For long-distance links frequency-division duplex (FDD) is typically used where relay and node transmit concurrently using different frequency carriers and potentially different channel bandwidths. For short-range links time-division duplex (TDD) is often chosen, where relay and node transmit using different time-intervals in a transmission frame.
3) Direct links between nodes served by the relay are non-existent. This implies that *all* traffic traverses the relay node.
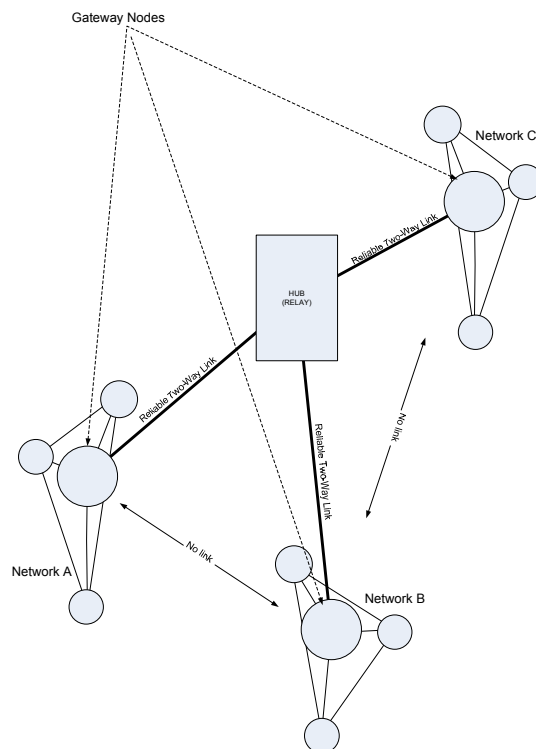

Fig. 1. Noisy Networks with a Single Relay Node

The paper is organized as follows: Section II compares two simple approaches using collaborative coding/multiple-access, namely analog relays with interference cancellation (modified Gaussian two-way channel) and purely digital relays which decode and reencode at relay and make use of network coding in the relay's encoder. Section III characterizes an achievable rate region for digital relays using a discrete-memoryless channel model. Section IV presents some conclusions and directions for ongoing work.

II. Analog vs. Digital Relays

Consider first the analog[1] relay for a network of two nodes. Here the relay does not attempt to decode the messages between the two nodes, but simply forwards them by normalizing

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[1]It is worth noting that it would still surely be implemented to a great degree with some form of digital signal processing

the noisy received signal and amplifying it to transmit at the desired power. The proportion of signaling dimensions allocated to the uplink is 1/2. Such analog forwarding for half-duplex two-way fading relay channels was considered recently in [1].

Aside from the duplexing component, this relay turns the channel into Shannon's Gaussian two-way channel [2] with a noise variance at the terminals dependent on the terminals' and relay's signal strengths and the relay's noise variance. The signal received by the relay is

$$y_{R,n} = \sqrt{2P_1}x_{1,n} + \sqrt{2P_2}x_{2,n} + z_{R,n}, n = 1, 2, \cdots, \frac{N}{2}, \quad (1)$$

where P_1 and P_2 are the received powers for each of the transmitted signals, x_1 and x_2, and $z_{R,n}$ is the noise at the relay which is assumed to be an i.i.d. Gaussian sequence with variance σ_R^2. The relay transmits the signal

$$x_{R,n} = \frac{y_{R,n}}{\sqrt{2P_1 + 2P_2 + \sigma_R^2}}, n = 1, 2, \cdots, \frac{N}{2}, \quad (2)$$

which is received as

$$y_{i,n} = \sqrt{2P_{R,i}}x_{R,n} + z_{i,n}, i = 1, 2, n = 1, 2, \cdots, \frac{N}{2}, \quad (3)$$

where $P_{R,i}$ is the received power at the ith node and $z_{i,n}$ is the noise at node i, and is also assumed to be an i.i.d. Gaussian sequence with variance σ_i^2.

Since the transmitted signal by user i is known at receiver i, it can strip it out from the received sequence leaving only the signal originating from the corresponding node. The interference-free signals at the two receivers are then

$$y'_{1,n} = \sqrt{\frac{2P_{R,1}P_2}{2P_1 + 2P_2 + \sigma_R^2}}(\sqrt{2P_2}x_{2,n} + z_{R,n}) + z_{1,n}, \quad (4)$$

$$y'_{2,n} = \sqrt{\frac{2P_{R,2}P_1}{2P_1 + 2P_2 + \sigma_R^2}}(\sqrt{2P_1}x_{1,n} + z_{R,n}) + z_{2,n}. \quad (5)$$

The achievable rates are therefore given by the single-user capacities

$$R_{12} = \frac{1}{4}\log_2\left(1 + \frac{2K_{12}P_1}{\sigma_1^2 + K_{12}\sigma_R^2}\right), \quad (6)$$

$$R_{21} = \frac{1}{4}\log_2\left(1 + \frac{2K_{21}P_2}{\sigma_1^2 + K_{21}\sigma_R^2}\right). \quad (7)$$

where $K_{12} = 2P_{R,2}/(2P_1 + 2P_2 + \sigma_R^2)$ and $K_{21} = 2P_{R,1}/(2P_1 + 2P_2 + \sigma_R^2)$. In the case of a symmetric system with a power factor K between relay and nodes (i.e. $P_1 = P_2 = P, P_{R,1} = P_{R,2} = KP$ and $\sigma_1^2 = \sigma_2^2 = \sigma_R^2 = \sigma^2$) we have

$$R_{12} = R_{21} = \frac{1}{4}\log_2\left(1 + \frac{4KP^2}{\sigma^2(\sigma^2 + 2(K+2)P)}\right) \quad (8)$$

$$\approx \frac{1}{4}\log_2\left(1 + \frac{2K}{K+2}\frac{P}{\sigma^2}\right), \frac{P}{\sigma^2} \gg 1. \quad (9)$$

Now consider the case of the digital relay which attempts to decode the two received signals and re-encode them for the downlink transmission. To exploit uplink/downlink signal
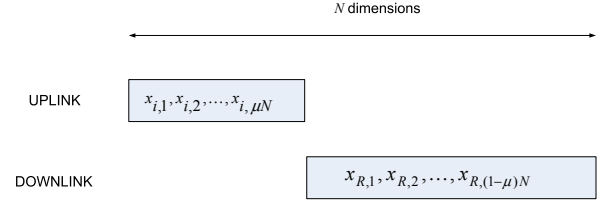

Fig. 2. UPLINK/DOWNLINK Duplexing

strength asymmetry, the proportion of signal-space dimensions is allowed to be different between the uplink and downlink transmissions. The proportion allocated to the uplink is denoted by $0 \leq \mu \leq 1$ as shown in Figure 2.

We will focus here on the symmetric user scenario and equal rates for the purpose of comparison with the analog relay. The general digital relay will be considered in the following sections. The uplink is a standard multiple-access channel and the equal-rate point is given by [6]

$$R_{\text{UL}} = \frac{\mu}{4}\log_2\left(1 + \frac{2}{\mu}\frac{P}{\sigma^2}\right). \quad (10)$$

On the downlink, we may use network coding [3] to combine the decoded information sequences (prior to channel coding) since the corresponding nodes know their own transmit signals. As a result, the full rate is achievable by both nodes on the downlink, and is given by

$$R_{\text{DL}} = \frac{1-\mu}{2}\log_2\left(1 + \left(\frac{K}{1-\mu}\right)\frac{P}{\sigma^2}\right) \quad (11)$$

The use of network coding for noisy two-way networks was recently considered in [4], [5] where cut-set outer bounds for the capacity region making use of network coding were found.

Since the code rates should be the same in both directions to guarantee a maximum end-to-end rate, we choose μ to guarantee this for the equal-rate point. For example, with $K = 1$, the uplink/downlink duplexing factor should be $\mu = 2/3$ which prioritizes the uplink. The resulting rate in this case is given by

$$R_{12} = R_{21} = \frac{1}{6}\log_2\left(1 + 3\frac{P}{\sigma^2}\right). \quad (12)$$

In comparison to the analog relay, we see that network coding gives a significant advantage in terms of signal-to-noise ratio (a factor of 3 for $K = 1$), and thus will outperform the analog relay for low spectral-efficiency systems. In contrast, however, the network coding approach suffers in throughput due to the

fact that successful decoding of both information sequences is required on the uplink and thus a multiplexing loss is incurred. For example, at an SNR of 20 dB and $K = 1$, the spectral-efficiency of the digital relay using network coding is 1.38 bit/dim, while that of the analog relay is 1.52 bit/dim. As the transmit power of the relay increases compared to the nodes(i.e. K), both tend to the same rate,.$25 \log_2(1 + 2P/\sigma^2)$, although very slowly in the case of the digital relay.

III. TWO-USER DISCRETE-MEMORYLESS SINGLE-RELAY NETWORKS

In this section we provide a more complete view of the achievable rates for the digital single-relay network with two communicating nodes. We begin with a discrete-memoryless channel model and then give an achievable rate region which makes use of a generalized form of network-coding for noisy channels.

A. Discrete-Memoryless Network Model

Consider the discrete-memoryless channel model as shown in Figure 3. Here each user generates an index $W_i, i = 1, 2$ for the codeword to be transmitted via the relay to the other user. Each encoder generates codewords comprising μN dimensions on the alphabets $\mathcal{X}_i, i = 1, 2$ for transmission during the uplink portion, where $0 \leq \mu \leq 1$ indicates the proportion of dimensions dedicated to the uplink (see Figure 2). The cardinality of the two codebooks are $2^{\mu N R_{12}}$ and $2^{\mu N R_{21}}$. The channel from the nodes to the relay is a classical multiple-access channel described by the transition probability on the received symbols $p(y_R|x_1, x_2)$.

The relay employs a multiuser receiver to decode the transmitted codeword indices yielding the estimates $\hat{W}_i, i = 1, 2$ at its output. Based on these indices, it then encodes the two indices using a codebook of dimensionality $(1 - \mu)N$ on the alphabet $\mathcal{X}_R$. The cardinality of the relay's codebook is at most $2^{\mu N(R_{12} + R_{21})}$. We assume that the codebooks at the nodes and relay are fixed. In the case of the relay-node this implies that the codebook cannot be a function of the received sequence $y_{R,n}$ and thus excludes, for example, the analog relay described in Section II.

The downlink channel is a classical broadcast channel except for the fact that the decoders have side information to exploit, namely the transmitted codeword indices that they themselves used during the uplink portion. For simplicity we assume that the two channel outputs are conditionally independent so that they can be separated into two transition probabilities $p(y_i|x_R), i = 1, 2$. Each node decodes the received sequence $y_{i,m}$ using the side information from its own transmission to yield the estimates $\hat{\hat{W}}_i, i = 1, 2$.

B. An Achievable Rate Region

In the following theorem we provide an achievable rate region for the above channel model.

Theorem: A set of achievable rates for the two-user single-relay network is given by the closure of the following set
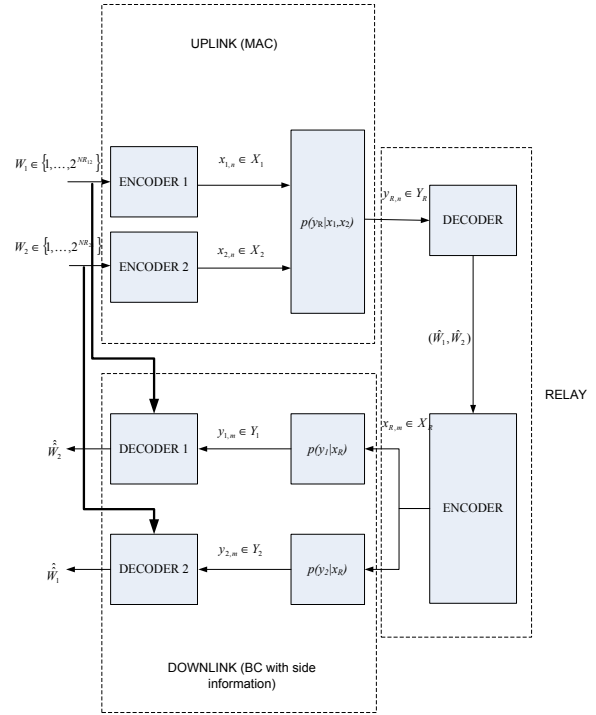

Fig. 3. Channel Model

of inequalities

$$R_{12} \leq \min\left(\mu I(X_1; Y_R|X_2), (1 - \mu)I(X_R; Y_2)\right)$$
$$R_{21} \leq \min\left(\mu I(X_2; Y_R|X_1), (1 - \mu)I(X_R; Y_1)\right)$$
$$R_{12} + R_{21} \leq \mu I(X_1, X_2; Y_R),$$
$$0 \leq \mu \leq 1 \qquad (13)$$

Proof: The probability of decoding error at each receiver assuming $W_1 = W_2 = 1$ is given by

$$P_{e,i} = \Pr(\hat{\hat{W}}_i \neq 0)$$
$$\leq \Pr(\hat{\hat{W}}_i \neq 0|\hat{W}_1 = \hat{W}_2 = 0) + \Pr((\hat{W}_1, \hat{W}_2) \neq (0, 0)) \qquad (14)$$

The second term in the sum in (14) is the probability of decoding error at the relay for the multiple-access channel on the uplink. As a result the following set of rates are achievable in the sense of vanishing $\Pr((\hat{W}_1, \hat{W}_2) \neq (0, 0))$ (see [6]),

$$R_{12} \leq \mu I(X_1; Y_R|X_2),$$
$$R_{21} \leq \mu I(X_2; Y_R|X_1),$$
$$R_{12} + R_{21} \leq \mu I(X_1, X_2; Y_R),$$
$$0 \leq \mu \leq 1 \qquad (15)$$

The first term in the sum in (14) is the probability of decoding error at the node *given* that the relay has correctly decoded both transmitted indices. To show the rates for which these probabilities vanish consider the following two-dimensional coding scheme between the relay and the two nodes, under the assumption that $R_{12} > R_{21}$:

1) generate $2^{NR_{12}}$ $(1-\mu)N$-dimensional codewords at random on the alphabet $\mathcal{X}_R$, $\mathbf{X}_R(i,j), i = 0,1,\cdots,2^{N(R_{12}-R_{21})}-1, j = 0,1,\cdots,2^{NR_{21}}-1$, generating each element i.i.d. according to $p(\mathbf{X}_R(i,j)) = \prod_{n=1}^{(1-\mu)N} p(x_{R,n})$.

2) Let $i \oplus j$ denote $(i+j) \mod 2^{NR_{21}}$. To send indices $i(1 \rightarrow 2)$ and j $(2 \rightarrow 1)$ choose $\mathbf{X}_R(\lfloor i2^{-NR_{21}} \rfloor, i \oplus j)$[**Network Coding**].

3) Let $A_{\epsilon,k}^{((1-\mu)N)}$ denote the set of jointly-typical $(\mathbf{x}_R(i,j), \mathbf{y}_k)$ sequences (see Chap. 8 [6]).

4) Receiver 1 has knowledge of i (side information due to its own transmission on the uplink) so it chooses the unique j' such that $(\lfloor i2^{-NR_{21}} \rfloor, j') \in A_{\epsilon,k}^{((1-\mu)N)}$. If none or more than one exist an error is declared. The decoded index is then $i \oplus j'$.

5) Receiver 2 chooses the unique (i',j') such that $(i',j') \in A_{\epsilon,2}^{((1-\mu)N)}$. If none or more than one exist an error is declared. It has knowledge of j (side information due to its own transmission on the uplink) so the decoded index is then $i'2^{NR_{21}} + j \oplus j'$

Define the conditional events $E_{ijk} = \left\{ (\mathbf{X}_R(i,j), \mathbf{y}_k) \in A_{\epsilon}^{((1-\mu)N)} | \hat{W}_1 = \hat{W}_2 = 0 \right\}$, then by the union of events bound and the joint asymptotic equipartition property (see Chap. 8 [6])

$$\Pr(\hat{\hat{W}}_i \neq 0 | \hat{W}_1 = \hat{W}_2 = 0) \leq \Pr(E_{01}^c) + \Pr(E_{002}^c) +$$
$$\sum_{j'=1}^{2^{NR_{21}}-1} \Pr(E_{0j'1}) +$$
$$\sum_{(i',j)' \neq (0,0)} \Pr(E_{i'j'2})$$
$$\leq 2\epsilon + 2^{-(1-\mu)N(I(\mathbf{X}_R;\mathbf{Y}_1) - \frac{R_{21}}{1-\mu} - 3\epsilon)} +$$
$$2^{-(1-\mu)N(I(\mathbf{X}_R;\mathbf{Y}_2) - \frac{R_{12}}{1-\mu} - 3\epsilon)}. \quad (16)$$

From this we see that in addition to (15),

$$R_{12} \leq (1-\mu)I(\mathbf{X}_R;\mathbf{Y}_2),$$
$$R_{21} \leq (1-\mu)I(\mathbf{X}_R;\mathbf{Y}_1),$$
$$0 \leq \mu \leq 1$$

must be satisfied, which when combined with (15) yields (13). The region is depicted in 4(a) and is characterized by three values of μ: μ_A is the value for which $\mu I(X_1;Y_R|X_2) = (1-\mu)I(X_R;Y_2)$, μ_B yields $\mu I(X_1,X_2;Y_R) = (1-\mu)(I(X_R;Y_2)+I(X_R;Y_1))$, and μ_C yields $\mu I(X_1;Y_R|X_2) = (1-\mu)I(X_R;Y_1)$.

IV. DISCUSSION

Consider again the Gaussian network as in section II. If now we combine the rate region found in the previous section
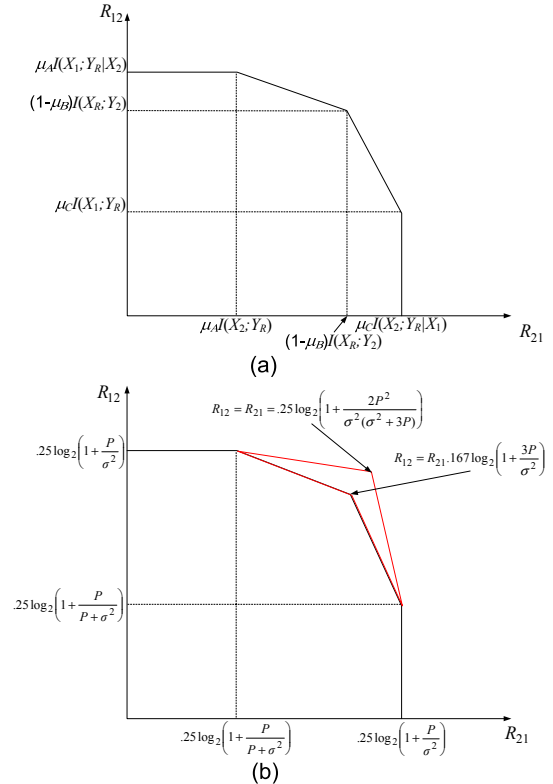

Fig. 4. Achievable Rate Regions. (a) : Discrete-Memoryless Network. (b) : Gaussian Network.

with that of the analog relay (after generalizing the result for continous alphabets), we see (as in Figure 4(b)) that the achievable region can be augmented around the equal-rate line by time-sharing between the extremities of the digital-relay and the analog-relay. This suggests that a more general form of relaying may be required to bring the individual information rates closer to the maximum point-to-point rates which are achievable through network coding on the downlink (i.e. to reduce the multiplexing-loss incurred on the uplink). In a similar vein, the encoding strategy at the nodes does not make use of any potentially available feedback paths to increase the decoding capability of the relay node. These are avenues for ongoing research on this problem.

REFERENCES

[1] B. Rankov, A. Wittneben, "Spectral Efficient Signaling for Half-Duplex Relay Channels," Asilomar Conference on Signals, Systems and Computers 2005, Pacific Grove, CA, Nov. 2005.

[2] C. Shannon, "Two-way Communication Channels", In Proc. 4th Berkeley Symp. Math. Stat. Prob., pages 611-644. University of California Press, Berkeley, 1961.

[3] R. Ahlswede, N. Cai, S.-Y.R. Li and R.W. Yeung, "Network Information Flow," IEEE Trans. on Information Theory, vol. 46, pp 1204-1216, 2000.

[4] G. Kramer, S. Savari, "On networks of two-way channels," DIMACS Workshop on Algegraic Coding Theory and Information Theory, Rutgers, 2003.

[5] G. Kramer, S.A. Savari, "Cut sets and information flow in networks of two-way channels"', IEEE Internation Symposium on Information Theory, Chicago, 2004.

[6] T. Cover and J.A. Thomas, "Elements of Information Theory", John Wiley and Sons, 1991.