Achievable Diversity-Multiplexing-Delay Tradeoff in Half-Duplex ARQ Relay Channels

Tarik Tabet * S. Dusad † Raymond Knopp ‡

Abstract— **In this paper, we present an efficient protocol for the delay-limited fading Automatic Retransmission reQuest single relay channel. The source is using an ARQ retransmission protocol to send data to the relay and the destination. When the relay is able to decode, both the relay and the source send the same data to the destination providing additional gains. The proposed protocol exploits two kinds of diversity: (i) space diversity available through the cooperative (relay) terminal, which retransmits the source's signals, (ii) ARQ diversity obtained by leveraging the retransmission delay to enhance the reliability. The performance characterization is in terms of the achievable diversity, multiplexing gain and delay tradeoff for a high signal-to-noise ratio (snr) regime.**

I. INTRODUCTION

The landmark paper of Gupta-Kumar [1] has driven interest in wireless sensor and ad hoc networks. The constraints on the size of the terminals in such ad hoc networks mitigates the presence of multiple antennas and full duplex transmissions. In such a scenario, distributed antennas can be used to provide a mean to combat fading with a similar flavor as that of space diversity. This kind of reliability obtained by the creation of virtual antennas is referred to as cooperative diversity because the terminals share their resources to get the information across to the destination. Such schemes have attracted significant attention recently, and a variety of cooperation protocols have been studied and analyzed in various papers like [2] [3] [4] [5].

Recently, the authors of [6] extended the Zheng-Tse formulation [7] and characterized the three dimensional diversity-multiplexing-delay tradeoff in MIMO ARQ channels. They established that delay can be exploited as a potential source for diversity. Thus, retransmission protocol is an appealing scheme to combat fading and its performance has been recently studied in decentralized ad hoc networks [8]. Inspired by [6], we propose a new scheme for transmission in relay channel utilizing the ARQ to increase the diversity gain. We look at the tradeoff in the high snr (Signal-to-Noise Ratio) regime and point out the gain achieved by the ARQ.

* LCM, School of Computer and Communication Sciences, Ecole Polytechnique Fédérale de Lausanne. Email: tarik.tabet@epfl.ch. T. Tabet is supported by the National Competence Center in Research on Mobile Information and Communication Systems (NCCR-MICS), a center supported by the Swiss National Science Foundation under grant number 5005-67322. .

† LTHC-LICOS, School of Computer and Communication Sciences, Ecole Polytechnique Fédérale de Lausanne. Email: sanket.dusad@epfl.ch.S. Dusad was supported in part by SNSF Grant #200021-105640/1

‡ Mobile Communications Department, Eurécom Institute, Sophia-Antipolis, France Email: raymond.knopp@eurecom.fr R. Knopp was supported by Eurécom and its industrial partners Hasler Stiftung, Swisscom, France Telecom, La Fondation Cegetel, Bouygues Telecom, Thales, ST Microelectronics, Hitachi Europe and Texas Instruments.

Following the setup in [3], the terminals are constrained to employ half-duplex transmission, i.e. they cannot transmit and receive simultaneously. The source and the relay are allowed to transmit in the same channel using cooperative protocols not relying on orthogonal subspaces. This is in contrast to [3], where the available bandwidth is divided into orthogonal channels allocated to the transmitting terminals. In the dynamic decode and forward scheme proposed in [4] the communication is across one block of fixed length l, where l is asymptotically large. In our setting the ARQ permits the use of communication over a variable number of blocks (henceforth referred to as number of rounds) of fixed length where the number of blocks used depend on the quality of the channel and are upper bounded by a fixed number L. If the destination is not able to decode at the end of these L blocks an outage is declared.

The outline of the paper is as follows. We introduce the channel model and the details of the algorithm in Section II. Section III contains a summary of the useful results and notations used in the rest of the paper. The actual achievable tradeoff for this protocol is analyzed and presented in Section IV for both long term and short term quasi-static channels. Section V proposes a power control scheme for ARQ relay protocol. Finally we summarize and present a few concluding remarks and future directions in Section VI.

II. SYSTEM MODEL AND SETTING

In this work, we consider communication over a relay network with one relay node (R) assisting the transmission of a source(S) destination(D) pair as described in Fig. 1. Each link has i.i.d. circularly symmetric complex Gaussian zero mean channel gain h_{sd}, h_{sr}, h_{rd}. Moreover we assume that each decoder has perfect knowledge of the channel gain. Perfect channel state information at the receivers implies that the S-R channel is known to the relay node, while the individual S-D R-D channels are known to the destination node. The channel state information (CSI) is assumed to be absent at the node which is transmitting. Moreover, perfect synchronization is assumed between nodes, which requires some form of distributed pilot signals in practice.

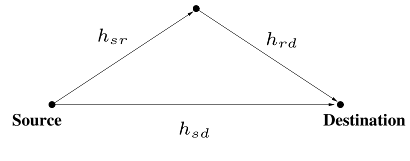

Fig. 1. System Model

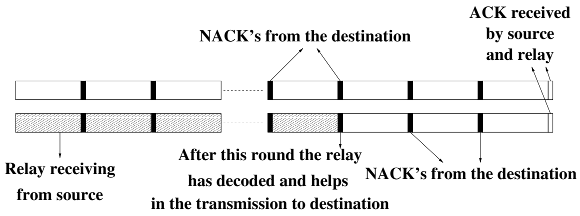

Fig. 2. Message as seen by the destination

We investigate two scenarios for the channel gains: 1) long-term static channel, where the fading is constant for all the channels over all retransmission (ARQ) rounds, and changes independently when the transmission of the current information message is stopped; 2) short-term static channel where the fading for all the channels is constant over each transmission round (or block) of the ARQ protocol and is an i.i.d process across successive rounds. The ARQ protocol considered in this work is a form of incremental redundancy as studied in [8] [9]. The transmission queue at the source is assumed to be infinite (not concerned by stability issues). The information message of b bits is encoded using a space-time code with code book $\mathcal{C} \subset \mathbb{C}^{2\times LN}$, where N is the number of channel uses taken to transmit one round and L is the maximum number of rounds that can be used to transmit the b information bits. We let $\mathcal{C}_l$ for $l = 1, \cdots, L$ denote the punctured space-time code of length lN obtained from $\mathcal{C}$ by deleting the last $(L-l)N$ columns of the space time code.

The protocol utilizes the ARQ as follows. The receiver feeds back a one bit success/failure indication to both the relay and the source. If the relay decodes before the destination then knowing the codebook $\mathcal{C}$ it begins transmitting the second row of the codebook $\mathcal{C}$ to the destination. Thus effectively it becomes a MISO channel increasing the diversity. If the destination decodes before the relay, it just sends the feedback to the source and relay and the source moves on to transmitting the next message. The source moves on to the next information message in the transmission queue either if L rounds have been exhausted for the message or if the destination sends success feedback. If successful decoding occurs at the l-th transmission, the effective coding rate for the current codeword is R/l bit/dim where $R = b/N$. In incremental redundancy, the receiver has memory of the past signals since it accumulates mutual information.

As defined above, the information message is encoded by a space-time encoder, and mapped in a sequence of L blocks, $\{\mathbf{x}_l \in \mathcal{C}^{2\times N} : l = 1, \ldots, L\}$, and the transmission is as in a MIMO system, where the columns of $\mathbf{x}_l = \begin{bmatrix} \mathbf{x}_{sd} & \mathbf{x}_{rd} \end{bmatrix}^T$ are transmitted in parallel by the source and the relay. Each symbol of the transmitted codeword has unit power constraint. Let us call $\mathcal{T}_r$ a random variable denoting the block in which the relay was able to decode the source information message. Then, the signal model of our channel is given by:

$$\mathbf{y}_l^d = \sqrt{\frac{snr}{2}}\mathbf{h}_l\mathbf{x}_l + \mathbf{n}_l^d \tag{1}$$

where l stands for the retransmission round, $\{\mathbf{y}_l^d \in \mathcal{C}^{1\times N}\}$ is the received signal block by the destination, and $\{\mathbf{n}_l^d \in \mathcal{C}^{1\times N}\}$ is the channel noise assumed to be temporally and spatially white with i.i.d entries $\sim \mathcal{N}\mathbb{C}(0,1)$. The channel of the l-th round is characterized by the matrix $\{\mathbf{h}_l \in \mathcal{C}^{1\times 2}\}$ as follows:

$$\mathbf{h}_l = \begin{cases} \begin{bmatrix} h_{sd} & 0 \end{bmatrix} & \text{if } l \in [1, \mathcal{T}_r] \\ \begin{bmatrix} h_{sd} & h_{rd} \end{bmatrix} & \text{if } l \in [\mathcal{T}_r + 1, L] \end{cases} \tag{2}$$

The received signal at the relay for $l = 1, \ldots, \mathcal{T}_r$ is given by:

$$\mathbf{y}_l^r = \sqrt{\frac{snr}{2}} h_{sr;l}\mathbf{x}_{sd;l} + \mathbf{n}_l^r \tag{3}$$

Note that as $N \to \infty$ using random coding arguments we can find codebooks which are good depending on the instant $\mathcal{T}_r$ at which the relay decodes. It can be shown that by taking the intersection over all the codebooks which are optimal for each $\mathcal{T}_r$ and using random coding arguments we can choose a codebook which is optimal irrespective of the instant $\mathcal{T}_r$ when the relay decodes.

III. Useful results and Notations

The symbol $\doteq$ will be used to denote the exponential quality, i.e. $f(snr) \doteq snr^b$ to denote:

$$\lim_{snr\to\infty} \frac{\log f(snr)}{\log snr} = b$$

The trade-off between diversity and multiplexing was formally defined and studied in the context of point-to-point coherent communications in [7]. A family of codes $C(snr)$ of block length T, with one code for each *snr* level, is said to have a diversity gain of d and spatial multiplexing gain of r if

$$r = \lim_{snr\to\infty} \frac{R(snr)}{\log\, snr} \quad , \quad d = -\lim_{snr\to\infty} \frac{\log\, P_e(snr)}{\log\, snr}$$

where $R(snr)$ is the rate of the code $C(snr)$ and $P_e(snr)$ is the average error probability.

We define the effective rate in a different manner as follows. Let $\mathcal{T}_d$ be a random variable denoting the stopping time of the transmission of the current message at the destination. Let E be the event that the mutual information at a particular decoder crosses the transmission rate R, i.e, $E_l = \{\sum_{i=1}^{l} I_i > R\}$ for $l = 1, \ldots, L-1$. Then, we have:

$$\begin{aligned} \Pr(\mathcal{T}_d = l) &= \Pr(\overline{E_{d,1}}, \ldots, \overline{E_{d,l-1}}, E_{d,l}) \\ &= \Pr(\overline{E_{d,1}}, \ldots, \overline{E_{d,l-1}}) - \Pr(\overline{E_{d,1}}, \ldots, \overline{E_{d,l}}) \\ &= \Pr(\overline{E_{l-1}}) - \Pr(\overline{E_l}) \end{aligned} \tag{4}$$

where we used the fact that the random sequence I_l is non-decreasing with probability 1, and $\overline{E_l} \subseteq \overline{E_m}$ for $l \leq m$ leading to $\Pr(\overline{E_1}, \ldots, \overline{E_l}) = \Pr(\overline{E_l})$. We have also $\Pr(\overline{E_0}) = 1$, and $\Pr(\mathcal{T}_d = L) = \Pr(\overline{E_{d,L-1}})$.

Let e_l denote the event that the destination makes an error in decoding at the end of the l^{th} round. Then it can be shown that,

$$\begin{aligned}P_e &= \sum_{l=1}^{L} \Pr\left(e_l, \mathcal{T}_d = l\right) \\ &\leq \sum_{l=1}^{L} \Pr\left(e_l, E_{d,l}\right) + \Pr\left(e_L, \overline{E_{d,L}}\right) \\ &= \sum_{l=1}^{L} \Pr\left(e_l, E_{d,l}\right) + \Pr\left(e_L | \overline{E_{d,L}}\right) \Pr\left(\overline{E_{d,L}}\right) \\ &\leq \sum_{l=1}^{L} \Pr\left(e_l, E_{d,l}\right) + \Pr\left(\overline{E_{d,L}}\right) \quad (5)\end{aligned}$$

Over here the destination can try to decode at the end of each round less than L. If it is able to decode it sends feedback to the source otherwise it waits for more rounds to allow the mutual information to accumulate. The above expression can be seen as,

$$P_e \leq \sum_{l=1}^{L} \Pr\left(\text{error at } l, \text{no outage at } l\right) + \Pr\left(\text{outage at } L\right) \quad (6)$$

It can be shown that in this expression only the second term dominates as the first one can be made arbitrarily small for sufficiently large N. Hence,

$$P_e \leq \Pr\left(\text{outage at } L\right) \leq \Pr\left(\text{outage at } l < L\right) \quad (7)$$

The effective multiplexing rate is then defined as,

$$\begin{aligned}r_e &= \lim_{snr \to \infty} \frac{R(snr)}{\left(\sum_{l=0}^{L-1} \Pr(\overline{E_l})\right) \log\, snr} \\ &= \lim_{snr \to \infty} \frac{R(snr)}{\left(1 + \sum_{l=1}^{L-1} \Pr(\overline{E_l})\right) \log\, snr} \\ &= \frac{r}{\left(1 + \sum_{l=1}^{L-1} \Pr(\overline{E_l})\right)} \quad (8)\end{aligned}$$

The channels are Rayleigh fading, i.e. $\gamma_i = |h_i|^2, i \in sd, sr, rd$ is exponentially distributed with unit mean. Defining $\mu_i = -\frac{\log\, \gamma_i}{\log\, snr}$ we note that μ_i is distributed as,

$$f_{\mu_i}(\mu) = \log(snr) snr^{-\mu} \exp(-snr^{-\mu}) \quad (9)$$

which, in the high snr gives:

$$f_{\mu_i}(\mu) \doteq \begin{cases} snr^{-\mu} & \text{for } \mu \geq 0 \\ 0 & \text{for } \mu < 0 \end{cases} \quad (10)$$

At high snr, we have $(1 + snr\gamma_i) \doteq snr^{(1-\mu_i)^+}$. Let us define $\mathcal{A}$ as the set describing the outage event. Then, for independent random variables $\underline{\mu} = [\mu_1, \ldots, \mu_n]$, the outage probability is given by:

$$P_{out} \doteq \int_{\mathcal{A}} f(\underline{\mu}) \mathrm{d}\underline{\mu} \doteq snr^{-d} \quad (11)$$

where:

$$d = \inf_{(\mu_1, \ldots, \mu_n) \in \mathcal{A}} \sum_{j=1}^{n} \mu_j \quad (12)$$

From this, it follows that:

$$\Pr(\log(1 + \gamma snr) < r \log(snr)) \doteq snr^{-(1-r)} \quad (13)$$

and the following results can be obtained:

$$\begin{aligned}&\Pr\left(\sum_{i=1}^{l} \log(1 + \gamma_i snr) < r \log(snr)\right) \\ &\doteq \Pr\left(\sum_{i=1}^{l} (1 - \mu_i)^+ < r\right) \\ &\doteq \int_{\mathcal{A}} snr^{-\sum_{i=1}^{l} \mu_i} \mathrm{d}\underline{\mu} \\ &\doteq snr^{-d} \quad (14)\end{aligned}$$

where the set $\mathcal{A} = \{\underline{\mu} : \sum_{i=1}^{l} (1 - \mu_i)^+ < r\}$ describes the outage event and d is given by:

$$d \doteq \inf_{\underline{\mu} \in \mathcal{A}} \sum_{i=1}^{l} \mu_i \doteq l\left(1 - \frac{r}{l}\right) \quad (15)$$

IV. Tradeoff Curves

In this section we derive the tradeoff curves for the case of the long term quasi-static and short term quasi-static channels. Since we are in the high snr regime we ignore the factor 2 and use $snr \doteq \frac{snr}{2}$ for the remaining sections of the paper. In this case the instantaneous average mutual informations for the j^{th} blocks are given by:

$$I_1^j = I^j(\mathbf{x}_{sd,j}; \mathbf{y}_j^d | h_{sd,j}) = \log(1 + snr\gamma_{sd,j}) \quad (16)$$

$$\begin{aligned}I_2^j &= I(\mathbf{x}_{sd,j}, \mathbf{x}_{rd,j}; \mathbf{y}_l^d | h_{sd,j}, h_{rd,j}) \quad (17) \\ &= \log(1 + snr(\gamma_{sd,j} + \gamma_{rd,j})) \\ &= \log(1 + snr^{(1-\mu_{sd,j})} + snr^{(1-\mu_{rd,j})}) \\ &\doteq \log(snr^{(1-\min(\mu_{sd,j}, \mu_{rd,j}))^+})\end{aligned}$$

$$I_3^j = I(\mathbf{x}_{sr,j}; \mathbf{y}_r^d | h_{sr,j}) = \log(1 + snr\gamma_{sr,j}) \quad (18)$$

A. Long term static channel

For a long term static channel the instantaneous average mutual informations do not vary from one round to another. Denote their common values as I_1, I_2 and I_3. At round l, the outage probability for this cooperative channel depends on the fact that the relay was able to decode the message from the source. From (4), we have:

$$\begin{aligned}\Pr(\mathcal{T}_r = k) &= \Pr((k-1)I_3 < r \log(snr)) \\ &\quad - \Pr(kI_3 < r \log(snr)) \\ &\doteq snr^{-(1-r/(k-1))} - snr^{-(1-r/k)} \quad (19)\end{aligned}$$

And the outage probability for the ARQ relay long-term static channel is,

$$\begin{aligned}
\Pr_{out}(l) &= \sum_{k=1}^{L} \Pr_{out|\mathcal{T}_r=k}(l)\Pr(\mathcal{T}_r = k) && (20)\\
&\doteq \sum_{k=1}^{l-1} \Pr(kI_1 + (l-k)I_2 < r\log(snr))\Pr(\mathcal{T}_r = k) \\
&\quad + \sum_{k=l}^{L} \Pr(lI_1 < r\log(snr))\Pr(\mathcal{T}_r = k) \\
&\doteq snr^{-d_{out}^{lt}(r,l)} && (21)
\end{aligned}$$

$$d_{out}^{lt}(r,l) = \begin{cases} (1-r) & \text{for } l = 1 \\ (1-\frac{r}{l}) + (1-\frac{r}{l-1}) & \text{for } l \neq 1,3 \\ 2-5r/6 & \text{for } l = 3, r < \frac{3}{4} \\ 5/2 - 3r/2 & \text{for } l = 3, r \geq \frac{3}{4} \end{cases} \quad (22)$$

We use the following result but omit the lengthy proof for space considerations,

$$\begin{aligned}
&\Pr(kI_1 + (l-k)I_2 < r\log(snr)) \\
&\quad \doteq \begin{cases} snr^{-2(1-r/l)} & \text{for } k \leq \lfloor l/2 \rfloor \\ snr^{-(l-r)/k} & \text{for } \lfloor l/2 \rfloor < k \leq l-1 \end{cases} \quad (23)
\end{aligned}$$

B. Short term static channel

Unlike in the case of long term static channel, the instantaneous mutual informations defined above vary from one block to the other. We have:

$$\begin{aligned}
\Pr(\mathcal{T}_r = k) &= \Pr\left(\sum_{i=1}^{k-1} I_3^i < r\log(snr)\right) \\
&\quad - \Pr\left(\sum_{i=1}^{k} I_3^i < r\log(snr)\right) \\
&\doteq snr^{-(k-1)(1-r/(k-1))} - snr^{-k(1-r/k)}
\end{aligned}$$

And the outage probability for the ARQ relay short-term static channel is:

$$\begin{aligned}
\Pr_{out}(l) &= \sum_{k=1}^{L} \Pr_{\text{out}|\mathcal{T}_r=k}(l)\Pr(\mathcal{T}_r = k) && (24)\\
&\doteq \sum_{k=1}^{l-1} \Pr\left(\sum_{i=1}^{k} I_1^i + \sum_{i=k+1}^{l} I_2^i < r\log(snr)\right)\Pr(\mathcal{T}_r = k) \\
&\quad + \sum_{k=l}^{L} \Pr\left(\sum_{i=1}^{l} I_1^i < r\log(snr)\right)\Pr(\mathcal{T}_r = k) \\
&\doteq snr^{-d_{out}^{st}(r,l)} && (25)
\end{aligned}$$

$$d_{out}^{st}(r,l) = \begin{cases} (1-r) & \text{for } l = 1 \\ l\left(1-\frac{r}{l}\right) + (l-1)\left(1-\frac{r}{(l-1)}\right) & \text{for } l \neq 1 \end{cases} \quad (26)$$

We again use the following result and omit the proof for space considerations,

$$\begin{aligned}
&\sum_{k=1}^{l-1} \Pr\left(\sum_{i=1}^{k} I_1^i + \sum_{i=k+1}^{l} I_2^i < r\log(snr)\right) \\
&\quad \doteq \begin{cases} snr^{-(2l-k)(1-r/l)} & \text{for } k \leq \lfloor l/2 \rfloor \\ snr^{-l(l-r)/k} & \text{for } \lfloor l/2 \rfloor < k \leq l-1 \end{cases} \quad (27)
\end{aligned}$$

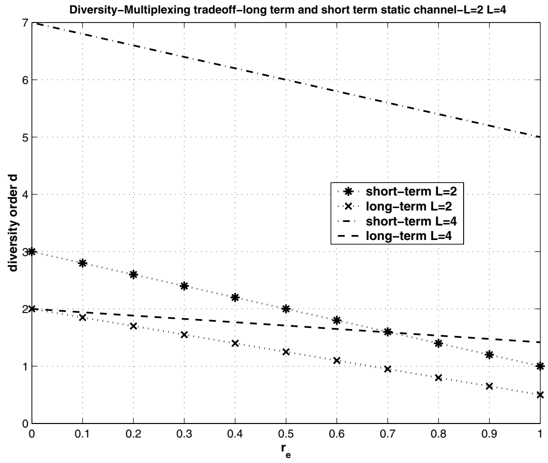

Fig. 3. The diversity-multiplexing tradeoff for different values of the maximum number of ARQ rounds for the short term and long term static channel.

Note that the way we have defined the effective rate earlier (8) and from the expressions above for both the short term and long term static channel, it follows that:

$$\begin{aligned}
r_e &= \frac{r}{\left(1 + \sum_{l=1}^{L-1} snr^{-d_{out}(r,l)}\right)} \\
\implies r_e &\doteq r && (28)
\end{aligned}$$

Hence the achievable diversity-multiplexing-delay tradeoff for the ARQ relay channel for the long term static and short term static relay channel is,

$$d_{out}^{lt}(r_e, L) = d_{out}^{lt}(r, L),\ d_{out}^{lt}(r_e, L) = d_{out}^{lt}(r, L) \quad (29)$$

The first interesting phenomena one can notice is that by increasing the value of the retransmission rounds, L, the diversity-multiplexing tradeoff curve for the long term static channel flattens out as in Fig. 3. Consider the tradeoff curve in (26) for the short term static channel. Since the channel fades independently to a new realization in each round, transmission in each new round gives additional diversity which explains the multiplicative L and $L-1$ factors in the diversity expression. Note that the factor is $(L-1)$ (both in the multiplication and the division) in the second term as the relay has to wait for at least one round before it can start transmitting to the destination. The reason this multiplicative factor does not show up in the case of the long term quasi-static channel is that the channel is constant over all ARQ rounds and there is no time diversity benefit. But still there is a gain in the diversity because of the relay to destination channel and because of the ARQ protocol (the factor r/l).

V. Power Control

We notice that $d_{out}^{lt}(0, l) = 2$ for all $l \neq 1$. Thus the long-term static channel is limiting the performance at low multiplexing-gains, which motivates the use of the power control.

Power control was recently applied to the cooperative relay channels. In [10], it was shown that by exploiting the channel state information at the transmitter and an adapted power control algorithm, the outage can be substantially lowered leading to an increase in diversity. In [11], the authors demonstrated that if the entire network state is used to determine the instantaneous transmitter power, only one bit feedback suffices to double the diversity order of the AF cooperative channel. Inspired by [6], and noticing that in long-term static channels the ARQ diversity is limited at low multiplexing gains, we construct a power control algorithm for this ARQ single relay channel. For simplicity, we consider a power control in which the relay is restricted to use a constant power in each round, but the source has the ability to vary its power to meet a long term average power constraint.

Let $P_l = snr^{p_l}$ be the power allocated per channel use for the l^{th} round. The power constraint for the long-term static channel is $\frac{\sum_{l=1}^{L} P_l \Pr_{out}(l-1)}{\sum_{l=0}^{L-1} \Pr_{out}(l)} \leq 1$ (where the denominator is the expected number of rounds needed for successful decoding at the receiver). It is straightforward to show that $P_l \leq \frac{L}{\Pr_{out}(l-1)}$ and $p_l \leq d_{out}(r, l-1)$ where $d_{out}(r, l-1)$ is the snr exponent of the l-th round outage probability for the ARQ relay channel. The power control policy is optimal when $P_l = snr^{d(r,l-1)}$, with $P_1 = 0$. Then, (16), (17), (18) become (for long-term static channels): $I_{1,pc}^{j} = \log(1+snr^{1-\mu_{sd}+d(r,j-1)})$, $I_{2,pc}^{j} \doteq \log(snr^{(1-\min(\mu_{sd}-d(r,j-1),\mu_{rd}))^{+}})$ and $I_{3,pc}^{j} = I(\mathbf{x}_{sr,j}; \mathbf{y}_r^d | h_{sr,j}) = \log(1 + snr^{1-\mu_{sr}+d(r,j-1)})$. We define for convenience,

$$q_k = \Pr(\sum_{i=1}^{k} I_{1,pc}^{i} + \sum_{i=k+1}^{l} I_{2,pc}^{i} < r\log(snr)) \quad (30)$$

$$\doteq \Pr\left(\sum_{i=1}^{k}(1-\mu_{sd}+d(r,k-1))^{+} + \sum_{i=k+1}^{l}(1-\min(\mu_{sd}-d(r,k-1);\mu_{rd}))^{+} < r\right)$$

The outage probability for the ARQ relay long-term static channel with power control is:

$$\Pr_{out}(l) \doteq \sum_{k=1}^{l-1} q_k \Pr(\mathcal{T}_r = k) + \sum_{k=l}^{L} \Pr\left(\sum_{i=1}^{l} I_{1,pc}^{i} < r\log(snr)\right)\Pr(\mathcal{T}_r = k) \quad (31)$$

Unfortunately, the diversity-multiplexing delay tradeoff is not derived analytically but computed through Monte-Carlo simulations. The diversity gain in a particular round, hence the power allocated can be numerically computed in a recursive manner. The diversity gain obtained using power control is significant compared to the constant power case especially at low multiplexing gains as shown in Fig. 4. Moreover, one can notice that the proposed power control is deterministic in the sense that it does not depend on the knowledge of the channel state but requires only the knowledge of the outage probabilities which can be estimated.

VI. Conclusion

From the graph in Fig. 3, Fig. 4, it can be seen that a significant gain in diversity is obtained by the proposed protocol.

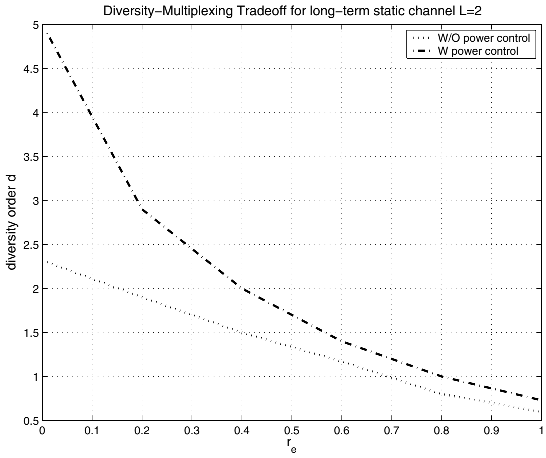

Fig. 4. The diversity-multiplexing tradeoff for $L = 2$ for the long term static channel with and without power control

This is also evident from the outage probability expressions for short term and long term static channels. We are investigating the impact of multiple antennas (in particular two antennas considering the practical implications) at the receiver (base station), where the source and the relay collaborate to reach the destination. We also propose to investigate the extension of these schemes to the case of multiple relays relaying the information for a single source destination pair. This protocol can then also be applied to ad-hoc TDMA wireless networks where in each slot all the remaining nodes in the network act as relays for a particular source destination pair.

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