Spectral efficiency of Cognitive Radio systems

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***Abstract*— In this contribution[1], we investigate the idea of using *cognitive radio* to reuse locally unused spectrum to increase the total system capacity. We consider a multiband/wideband system in which the primary and cognitive users wish to communicate to different receivers, subject to mutual interference and assume that each user knows only his channel and the unused spectrum through *perfect* sensing. Under this scheme, a cognitive radio will listen to the channel and, if sensed idle, will transmit during the voids. We impose the constraint that users successively transmit over available bands through proper water filling. Within this setting, we derive the total spectral efficiency of the cognitive radio system as well as the spectral efficiency gains and prove that we can improve the overall system spectral efficiency by considering cognitive communications in the system.**

I. Introduction

Observing that in some locations or at some times of day, 70% of the allocated spectrum may be sitting idle, the FCC has recently recommended [1] that significantly greater spectral efficiency could be realized by de-ploying wireless devices that can coexist with the licensed (primary) users, generating minimal interference while taking advantage of the available resources. The current approach for spectrum sharing is regulated so that wireless systems are assigned fixed spectrum allocations, operating frequencies and bandwidths, with constraints on power emission that limits their range. Therefore, most communications systems are designed in order to achieve the best possible spectrum efficiency within the assigned bandwidth using sophisticated modulation, coding, multiple antennas and other techniques. The most advanced systems are approaching Shannon's channel capacity limit [2], so further increase in capacity would require additional system bandwidth. On the other hand, the discrepancy between spectrum allocation and spectrum use suggests that this spectrum shortage could be overcome by allowing more flexible usage of a spectrum. Flexibility would mean that radios could find and adapt to any immediate local spectrum availability. A new class of radios that is able to reliably sense the spectral environment over a wide bandwidth, detect the presence/absence of legacy users (primary users) and use the spectrum only if the communication does not interfere with primary users is defined by the term *cognitive radio* [3]. Cognitive radios have been proposed as a mean to implement efficient reuse of the licensed spectrum. The key feature of cognitive radios is their ability to recognize their communication environment and *independently* adapt the parameters of their communication scheme to maximize the quality of service (QoS) for the secondary (unlicensed) users while minimizing the interference to the primary users. However, there are many challenges across all layers of a cognitive radio system design, from its application to its implementation [4].

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We consider an asynchronous TDD communication scenario in which the primary and cognitive users wish to communicate to different receivers, subject to mutual interference in a heterogeneous network where devices operates in a wideband/multiband context. However, contrary to the work addressed in [5], in this contribution we impose that only one user can simultaneously transmit over the same sub-band. We examine the total spectral efficiency of the cognitive radio system under the assumption of perfect sensing of the empty sub-bands and show that the overall system spectral efficiency can be considerably enhanced by considering cognitive communications with respect to the traditional system (without cognition). In particular, it is of major interest, in this context, to quantify the spectral efficiency gain as well as the maximum number of possible pairwise communications within this scenario in order to show the interest behind using cognitive radio terminals with respect to classical systems (without cognition). In fact, although cognitive radios have spurred great interest and excitement in industry, many of the fundamental theoretical questions on the limits of such technologies remain unanswered.
The rest of the paper is organized as follows: In section 2, we describe the cognitive radio protocol. Section 3 details the spectral efficiency analysis adopted throughout this paper when the number of sub-bands is limited. In section 4, we investigate the asymptotic performance of such a system in terms of spectral efficiency. Performance evaluation is provided in Section 5 and Section 6 concludes the paper.

II. Cognitive radio Protocol

The contribution of some recent studies has studied the spectral efficiency of cognitive systems with respect to classical approaches by allowing the cognitive users to transmit simultaneously with the primary users in the same frequency

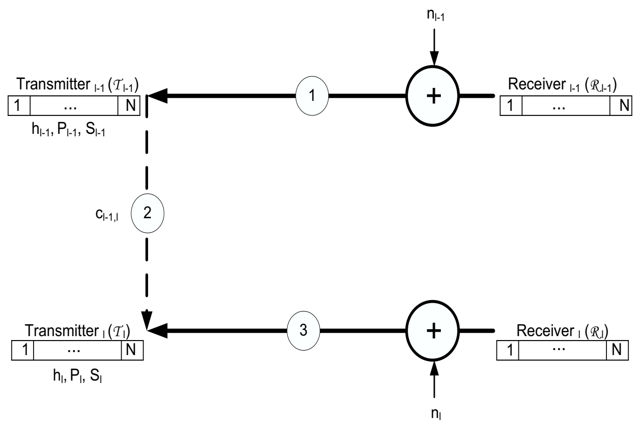

Fig. 1. The cognitive radio channel in a wideband/multiband context with N sub-bands.

band. In [6], the cognitive user is assumed to obtain an *a-priori* knowledge of the information that will be transmitted by the primary user. In [7], authors allow the primary and the secondary systems to *cooperate* and jointly design their encoder-decoder pairs. However, in practice, primary system should be unaware about the existence of the cognitive radio (unlicensed) system and operates according to the demands of the population of primary terminals. This implies that it is the role of cognitive radios to recognize their communication environment and adapt the parameters of their communication scheme to maximize the QoS for the secondary users.

Under the proposed protocol however, cognitive users listen to the wireless channel and determine, either in time or frequency, which part of the spectrum is unused. Then, they *successively* adapt their signal to fill detected voids in the spectrum domain. Each transmitter $\mathcal{T}_l$ for $l = 1, ..., L$ estimates the pilot sequence of the receiver $\mathfrak{R}_l$ in order to determine the channel gain h_l (see links (1) and (3) in fig. 1). Notice here that since we are in a TDD mode, when we estimate the channel in one way, we can also know it the other way. Thus, each user l is assumed to know only his proper channel gain h_l and the statistical properties of the other links.

Specifically, the primary user comes first in the system and estimates his channel gain. Then, cognitive users come after in an asynchronous way so that they will not transmit at the same moment. The second user who comes in the system randomly, for instance in a Poisson process manner, and estimates his channel link. Thus, within this setting, the primary user is assumed not to be aware of the cognitive users. Then, he communicates with his receiver in an ad-hoc manner while a set of cognitive radio transmitters that are able to reliably sense the spectral environment over a wide bandwidth, decide to communicate with theirs respective receivers only if the communication does not interfere with the primary user. Thus, under our opportunistic approach, a device transmits over a certain sub-band only when no other user does. Such an assumption is motivated by the fact that in an asynchronous context, the probability that two users decide to transmit at the same moment is negligible as the number of users is limited. The sensing algorithms for the cognitive users as well as the performance analysis of such an approach are proposed in [9]. Throughout the rest of the paper, we will adopt this framework to analyze the achievable performance of such a system in terms of spectral efficiency gains as well as the maximum number of possible pairwise communication within this scenario.

Moreover, in this work, we allocate transmit powers for each user in order to maximize his transmission rate over a total power budget constraint. In fact, when channel state information is made available at the transmitters, users know their own channel gains and thus they will adapt their transmission strategy relative to this knowledge. The corresponding optimum power allocation is the well-known *water filling* allocation [2] expressed by[2]:

$$P_l^i = \left(\frac{1}{\gamma_0} - \frac{N_0}{\left|h_l^i\right|^2} \right)^+ \tag{1}$$

Where γ_0 is the Lagrange's multiplier satisfying the average power constraint:

$$\frac{1}{N} \sum_{i=1}^{N} P_l^i = \overline{P}; \tag{2}$$

Without loss of generality, throughout the rest of the paper, we take $\overline{P} = 1$.

For clarity sake, let us take the following example with $N = 8$ sub-bands. As shown in figure 2, the primary user is always prioritized above cognitive users by enjoying the entire band while cognitive users adapt their signal to fill detected voids with respect to their order of priority. As a first step, the primary user maximizes his rate according to his channel process. As mentioned before in expression (1), only user with a channel gain h^i above a certain threshold equal to $\gamma_0.N_0$ transmits on the sub-band i (Ψ_2). User 2 senses the spectrum and decides to transmit only on sub-bands sensed idle. Thus, following his fading gains, user 2 adapts his signal to fill these voids in the spectrum domain in a complementary fashion (Ψ_3). Similarly, user 3 will sense the remaining sub-bands from user 1 and user 2 and decide to transmit during the remaining voids (Ψ_4).

III. Spectral efficiency analysis

Let us first define the set of the number of sub-bands sensed occupied by user l by:

$$\Psi_l = \left\{ i \in [1, N]; P_{l-1}^i \neq 0 \right\} \tag{3}$$

[2] $(x)^+ = \max(0, x)$.

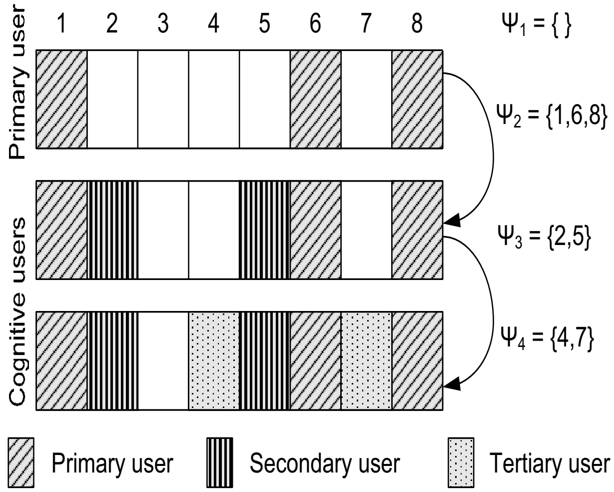

Fig. 2. One primary user and two cognitive users in a system with 8 sub-bands.

Where Ψ_l obeys to the following properties:

$$\begin{cases} \Psi_1 = \emptyset, \\ \bigcup_{l=1}^{L+1} \Psi_l \subseteq [1, N], \\ \bigcap_{l=1}^{L+1} \Psi_l = \emptyset \end{cases} \tag{4}$$

The capacity per band of user l given a number of sub-bands N is:

$$C_{l,N} = \frac{1}{card(\Omega_l)} \sum_{i \in \Omega_l} \log_2 \left(1 + \frac{P_l^i \mid h_l^i \mid^2}{N_0} \right) \tag{5}$$

Where Ω_l represents the set of the remaining idle sub-bands sensed by user l, namely:

$$\Omega_l = \left\{ i \in [1, N] \bigcap \overline{\bigcup_{k=1\ldots l} \Psi_k} \right\} \tag{6}$$

For a given number of sub-bands N, the optimal power allocation which maximizes the transmission rate of user l is the solution of the optimization problem:

$$\max_{P_l^1, \ldots, P_l^{card(\Omega_l)}} C_{l,N}, \qquad \text{for} \quad l \in [1, L]$$

subject to the average power constraint:

$$\begin{cases} \frac{1}{card(\Omega_l)} \sum_{i \in \Omega_l} P_l^i = 1, \\ P_l^i \geq 0, \end{cases} \tag{7}$$

The resulting optimal power control policy is derived in (1). Notice that the maximum number of users L allowed by such system must satisfy the condition that $card(\Omega_L) \neq 0$.

Let us now derive the spectral efficiency of such a system. The spectral efficiency per band of user l is given by:

$$\Phi_{l,N} = \frac{1}{N} . \sum_{i \in \Omega_l} \log_2 \left(1 + \frac{P_l^i \mid h_l^i \mid^2}{N_0} \right) \tag{8}$$

By multiplying and dividing (8) by $card(\Omega_l)$, we obtain[3]:

$$\Phi_{l,N} = \begin{cases} C_{1,N}, & \text{if} \quad l = 1 \\ \frac{card(\Omega_l)}{N} . C_{l-1,N}, & \text{for} \quad l \in [2, L] \end{cases} \tag{9}$$

As expected, when $l = 1$, the spectral efficiency without cognition is equal to the primary user capacity $C_{1,N}$. We define $\Delta_{l,N}$ as the band factor gain of user l for N sub-bands, namely:

$$\Delta_{l,N} \triangleq \frac{card(\Omega_l)}{N}, \qquad \text{for} \quad l \in [1, L] \tag{10}$$

In other words, the band factor gain represents the fraction of the band unoccupied at user l. The spectral efficiency per band of user l can therefore be expressed by:

$$\Phi_{l,N} = \begin{cases} \Delta_{1,N} . C_{1,N}, & \text{if} \quad l = 1 \\ \Delta_{l,N} . C_{l-1,N}, & \text{for} \quad l \in [2, L] \end{cases} \tag{11}$$

and the sum spectral efficiency of a system with N sub-bands per user is given by:

$$\Phi_{sum,N} = \sum_{l=1}^{L} \Phi_{l,N} \tag{12}$$

IV. Asymptotic Performance

Let us now study the achievable performance when devices operate in a wide-band context (i.e. $N \to \infty$). The instantaneous capacity of user l for a finite number of sub-bands in (5) becomes[4]:

$$C_{l,\infty} = \int_0^\infty \log_2 \left(1 + \frac{P_l(t).t}{N_0} \right) .e^{-t} dt, \quad \text{for} \quad l \in [1, L] \tag{13}$$

Where P_l is subject to the average constraint:

$$\int_0^\infty \left(\frac{1}{\gamma_0} - \frac{N_0}{t} \right)^+ .e^{-t} dt = 1 \tag{14}$$

and γ_0 is the Lagrange's multiplier satisfying[5]:

$$\frac{1}{\gamma_0} \int_{\gamma_0 . N_0}^{+\infty} e^{-t} dt - N_0 . E_i \left(\gamma_0 . N_0 \right) = 1 \tag{15}$$

[3]Notice that since the primary user enjoys the entire bandwidth, we have: $card(\Omega_1) = N$.

[4]All theoretical results as well as the methodology adopted in this paper can be translated immediately into results for any other probability distribution function of the channel model. In this way, the term e^{-t} in (13) will be replaced by the appropriate probability distribution function.

[5]$E_i(x)$ is the exponential integral function defined as: $E_i(x) = \int_x^{+\infty} \frac{e^{-t}}{t} dt$.

Moreover, the capacity of user l can be computed for $l = 1, ..., L$ as follows:

$$\begin{aligned} C_{l,\infty} &= \int_0^{\infty} \log_2\left(1 + \frac{P_l(t).t}{N_0}\right).e^{-t}dt \qquad (16) \\ &= \int_{\gamma_0 N_0}^{\infty} \log_2\left(1 + \frac{\left(\frac{1}{\gamma_0} - \frac{N_0}{t}\right).t}{N_0}\right).e^{-t}dt \\ &= \int_{\gamma_0 N_0}^{\infty} \log_2\left(\frac{t}{\gamma_0.N_0}\right).e^{-t}dt \\ &= \frac{1}{\ln(2)}.E_i\left(\gamma_0.N_0\right) \end{aligned}$$

In order to characterize the achievable performance of such system in terms of spectral efficiency, we define the capacity within the frequency bandwidth W, namely [8]:

$$C_{l,\infty}(W) = \frac{1}{W}\int_{\frac{-W}{2}}^{\frac{W}{2}} \log_2\left(1 + \frac{P_l(f).\left|H_l(f)\right|^2}{N_0}\right)df \qquad (17)$$

By identifying expression (13) with (17), we obtain a characterization of the frequency variation f as function of the channel gains t, namely:

$$f = -W.e^{-t} + \frac{W}{2}, \qquad l \in [1, L] \qquad (18)$$

Similar to our approach in the previous section, we define the band factor gain Δ_∞ as the fraction of the band sensed idle from user l to user $l+1$ over the total bandwidth W for an infinite number of sub-bands:

$$\Delta_\infty \triangleq \frac{\Delta f}{W}, \qquad (19)$$

Where Δf represents the frequency interval where the fading gain in (18) is below a certain threshold equal to $\gamma_0.N_0$. By deriving the appropriate vacant band Δf when $t \in [0, \gamma_0.N_0]$ in (18), we obtain:

$$\Delta_\infty = 1 - exp\left(-\gamma_0.N_0\right) \qquad (20)$$

Accordingly, the asymptotic spectral efficiency of user l is given by:

$$\Phi_{l,\infty} = \begin{cases} C_{1,\infty}, & \text{if} \quad l = 1 \\ \Delta_\infty.C_{l-1,\infty}, & \text{for} \quad l \in [2, L] \end{cases} \qquad (21)$$

Similar to the case where the number of sub-bands is fixed, when $l = 1$, the spectral efficiency without cognition is equal to the primary user capacity $C_{1,\infty}$. In particular, it is of major interest to quantify the spectral efficiency gain Δ_∞ in order to show the interest behind using cognitive radio terminals with respect to classical systems (without cognition). To do so, following the same procedure and going from user 2 to L, we obtain the expression of the asymptotic spectral efficiency as function of $C_{1,\infty}$:

$$\Phi_{l,\infty} = \Delta_\infty^{l-1}.C_{1,\infty}, \qquad \text{for} \quad l \in [1, L] \qquad (22)$$

The overall asymptotic sum spectral efficiency for a system with L users is therefore:

$$\begin{aligned} \Phi_{sum,\infty} &= \sum_{l=1}^{L} \Phi_{l,\infty} \qquad (23) \\ &= \sum_{k=0}^{L-1} \Delta_\infty^k C_{1,\infty} \\ &= \underbrace{\frac{1 - \Delta_\infty^L}{1 - \Delta_\infty}}_{\geq 1}.C_{1,\infty} \end{aligned}$$

Thus, the sum spectral efficiency obtained by considering cognitive communications is greater than or equal to the spectral efficiency without cognition $C_{1,\infty}$. Such a result justifies the increasing interest behind using cognitive radio terminals in future wireless communication systems since the sum spectral efficiency of such systems performs always better than classical communication systems (without cognition).
On the other hand, by substituting $C_{1,\infty}$ by its expression in (16), we obtain the final expression of the achievable sum spectral efficiency in such a system:

$$\Phi_{sum,\infty} = \frac{1}{\ln(2)}.\frac{1 - \Delta_\infty^L}{1 - \Delta_\infty}.E_i\left(\gamma_0.N_0\right) \qquad (24)$$

This result is very interesting as, by only knowing the statistics of the channel gains (through γ_0) and the SNR, one can derive the achievable spectral efficiency as well as the potential gain resulting from using cognitive radio.

V. Performance evaluation

In order to validate our approach in the previous Section, we compare theoretical expressions of the sum spectral efficiency in (24) to simulated expressions in (12). We model L i.i.d rayleigh channels (one for each user) and assume perfect sensing of the idle-sub-bands. Our numerical results in figure 3, show that the sum spectral efficiency in (12) matches expression (24) even for a low number of sub-bands N (from N = 32). Moreover, since the maximum number of users is not theoretically limited, we will consider only L that satisfies the condition that $card(\Omega_L) \neq 0$, otherwise, the L-th capacity would be negligible. Figure 4 characterizes the maximum number of users L as function of the received signal energy per information bit E_b/N_0 for different number of sub-bands N. As expected, we remark that the maximum number of users allowed to transmit increases with the number of sub-bands especially at low E_b/N_0 region. Furthermore, the maximum number of cognitive users ranges from 1 to 8. As an example, the proposed scheme allows up to 4 cognitive users to benefit from the licensed spectrum at 8 dB for $N = 2048$ sub-bands.

Figure 5 depicts the different configurations of the sum spectral efficiency gains for a system with 5 users and $N = 512$ sub-bands. It is clear that at low E_b/N_0 of interest,

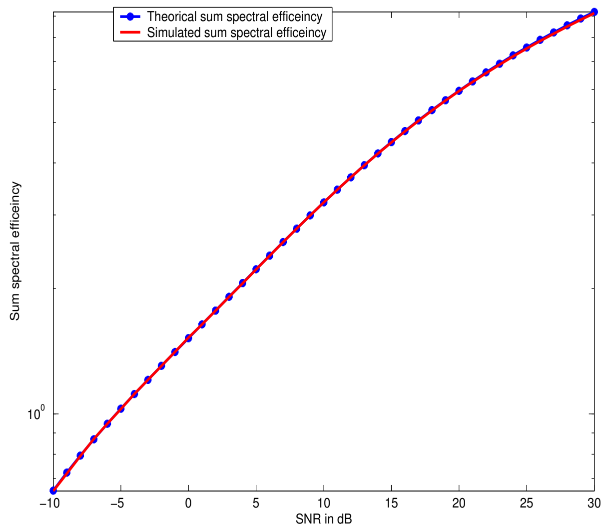

Fig. 3. Comparison between the theoretical expression of the sum spectral efficiency in (24) and the simulated one in (12) for $L = 5$ and N=32.

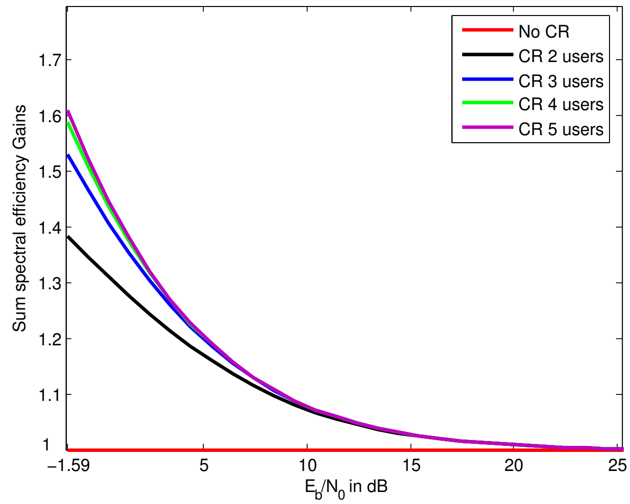

Fig. 5. Sum spectral efficiency gains of the system with 5 users.

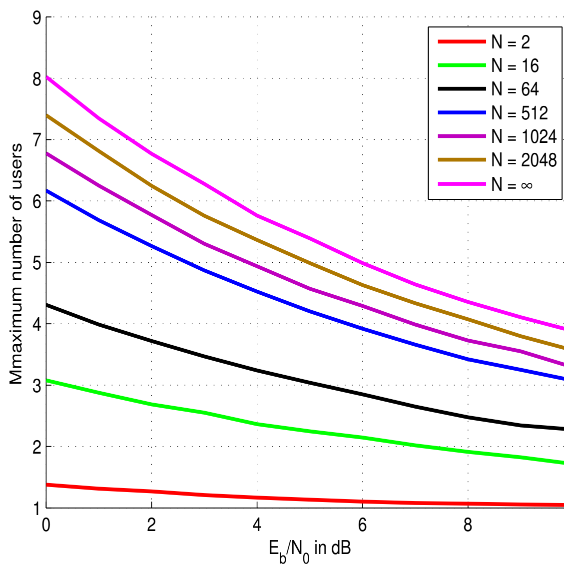

Fig. 4. The maximum number of users as function of the SNR.

the spectral efficiency is significantly increased with respect to the traditional system without cognition. The maximum spectral efficiency gain can not exceed the range of 60% for a configuration with 5 users. However, at high E_b/N_0 regime, the maximum sum capacity reaches $C_{1,\infty}$. This result is rather intuitive since that at high E_b/N_0 regime, the water-level $\frac{1}{\gamma_0}$ is becoming greater than the quantity $\frac{\sigma^2}{|h|^2}$ and more power is poured (see equation(1)). Notice here that as $E_b/N_0 \rightarrow \infty$, the band factor gain $\Delta_\infty \rightarrow 0$ and $\Phi_{sum,\infty} \rightarrow C_{1,\infty}$ as proved in equation (23).

VI. Conclusion

In this work, we analyzed a cognitive radio system where cognitive users benefit from the unused spectrum. For the first time, our study has quantified the achievable gain of using cognitive radio with respect to classical radio devices. In fact, we defined the band factor gain and obtained a characterization of the maximum achievable spectral efficiency as well as the maximum number of possible pairwise communications within such a scenario. We finally showed the fundamental principle of cognitive radios by proving that such systems always perform better than classical ones (without cognition). As a future work, it is of major interest to generalize the problem to unperfect feedback in order to characterize the sum capacity gain of such cognitive protocols with respect to the proposed scenario.

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