Advances in Multiuser MIMO Systems
(Tutorial Part II)
Emerging Topics in Multiuser MIMO Networks

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Outline

- General information
- Background on MIMO
- Essential results for MU-MIMO networks
- Living with partial channel knowledge
  - An important example: random opportunistic beamforming
- Multi-cell MU-MIMO: Key concepts and preliminary results
- Perspectives

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General information

This research-oriented talk aims at giving understanding over

- Fundamental paradigm change between MIMO and MU-MIMO
- Key features and advantages of MU-MIMO
- The issue of CSIT (channel state info at transmitter)
- Feedback reduction techniques
- Expanding MU-MIMO over a cellular network (Multi-cell MU-MIMO)

References

- Many references at the end of slides.
- Additional references: IEEE JSAC and EURASIP JSAP, special issues on MIMO communications with limited feedback
- D. Gesbert, M. Kountouris, R. Heath, C.B. Chae "From single user to multi user communications: shifting the MIMO paradigm", to appear in IEEE Signal Processing Magazine 2007. (available upon request to the authors)

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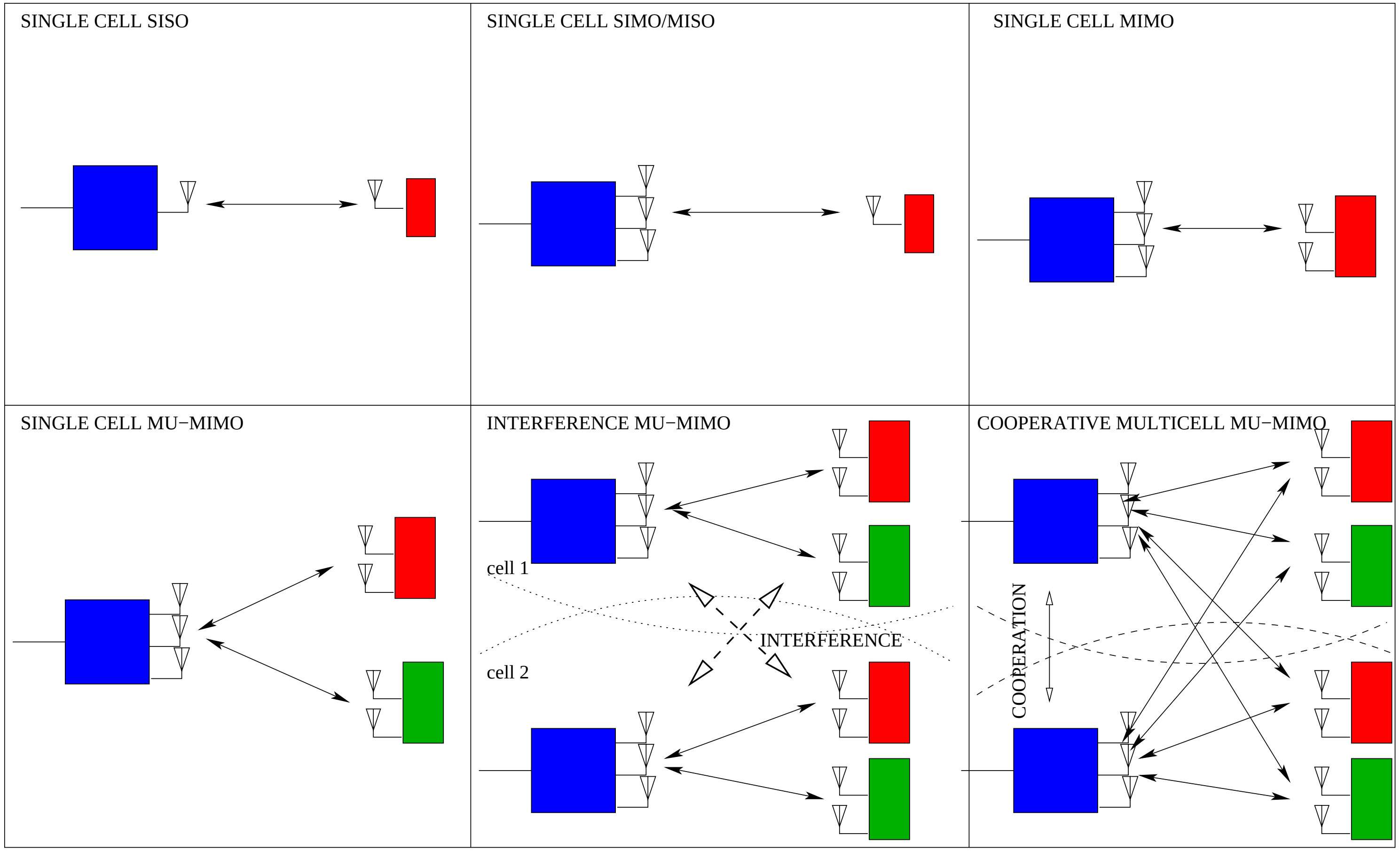
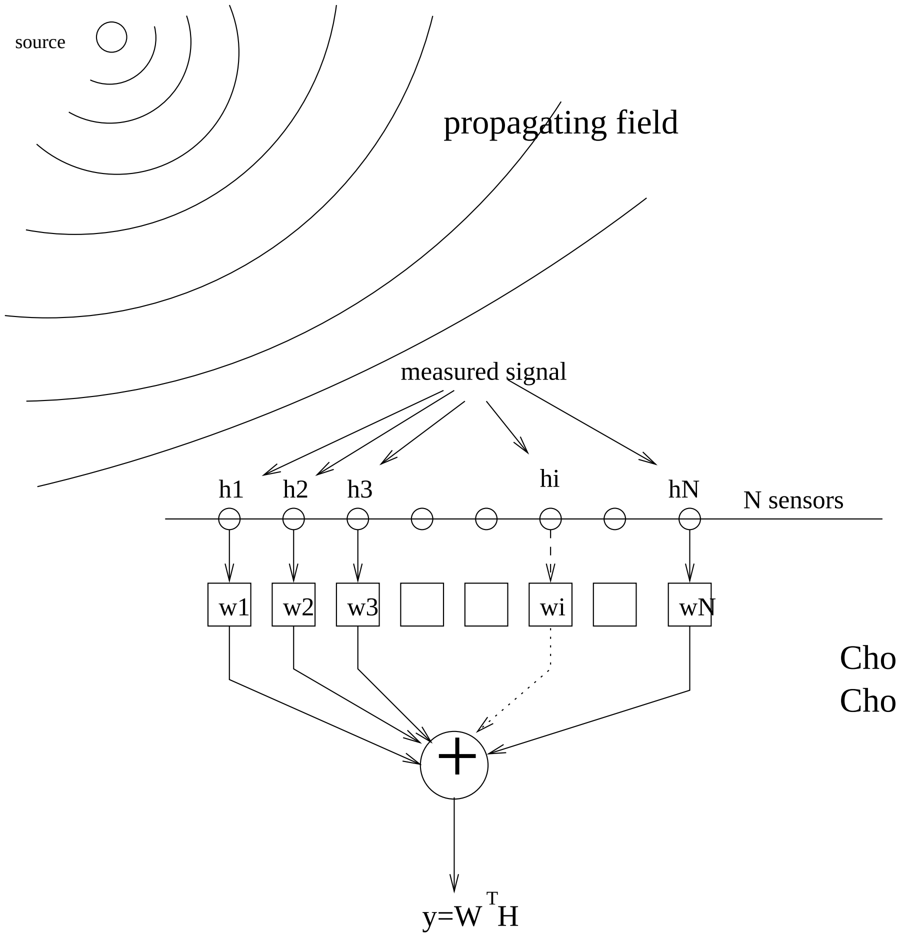
Background on MIMO

- MIMO configurations
- Basic principles of multiple antenna combining

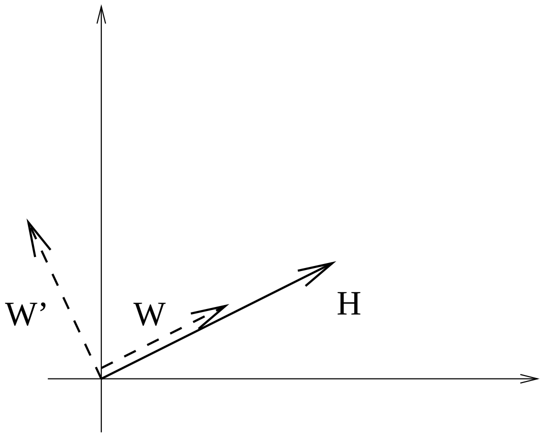
The MIMO Configurations



Multi-antenna combining



Vector space analogy (for two sensors)



Choosing W enhances the source (beamforming)
Choosing W' nulls the source out (interference nulling)

Basic algebra explains it all

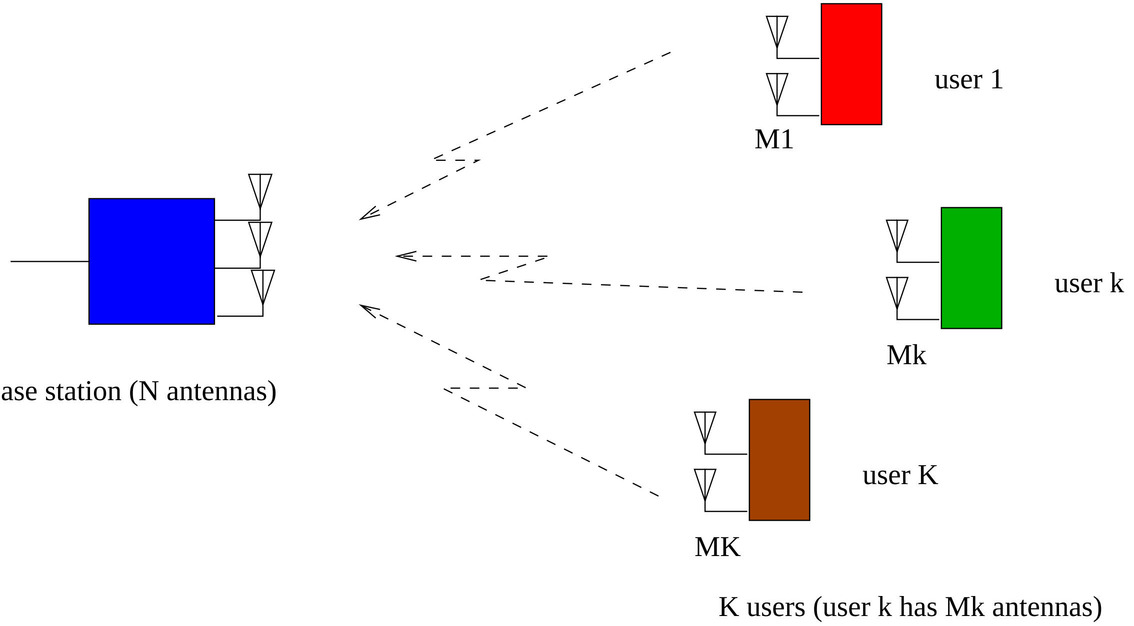
All key MIMO and MU-MIMO schemes (except diversity-oriented) can be interpreted from previous drawing:

- A N-antenna beamformer can amplify one source (no interference) by a factor N in the average SNR: Beamforming
- A N-antenna beamformer can extract one source and cancel out $N-1$ interferers simultaneously: Interference canceling
- Transmit beamforming realizes the same benefits/gains at receive beamforming if CSIT is given: Transmit beamforming and interference nulling
- All N sources can be simultaneously extracted (assuming the other $N-1$ are viewed as interferers) by beamformer superposition: Spatial multiplexing
- N sources can be assigned to N distinct users: MU-MIMO, SDMA
- Some of the N sources may belong to different cells: cooperative multicell MIMO

Notations for MU-MIMO networks

- Uplink
- Downlink
- Key differences

The uplink MU-MIMO channel



The uplink MU-MIMO channel: Notations

We have:

- Let's assume a group $K \leq N$ users are selected by the uplink scheduler
- Let the K users transmit to the base station.
- User k has M_k transmit antennas and peak power constraint P_k.
- User k transmits signal vector $\mathbf{X}_k$ with covariance $\mathbf{Q}_k$, Tr$(\mathbf{Q}_k) \leq P_k$
- Base has N receive antennas
- Channel between user k and base is matrix $\mathbf{H}_k^*$, of size $N \times M_k$.
- White noise with variance 1.

The uplink MU-MIMO signal model

Received signal model at the base:

$$\mathbf{y} = \mathbf{H}^{*}\mathbf{X} + \mathbf{n} \tag{1}$$

with global uplink channel matrix:

$$\mathbf{H}^{*} = [\mathbf{H}_1^{*}, \mathbf{H}_2^{*}, .., \mathbf{H}_K^{*}] \tag{2}$$

And global user transmit vector:

$$\mathbf{X} = \left[\mathbf{x}_1^T, \mathbf{x}_2^T, .., \mathbf{x}_K^T\right]^T \tag{3}$$

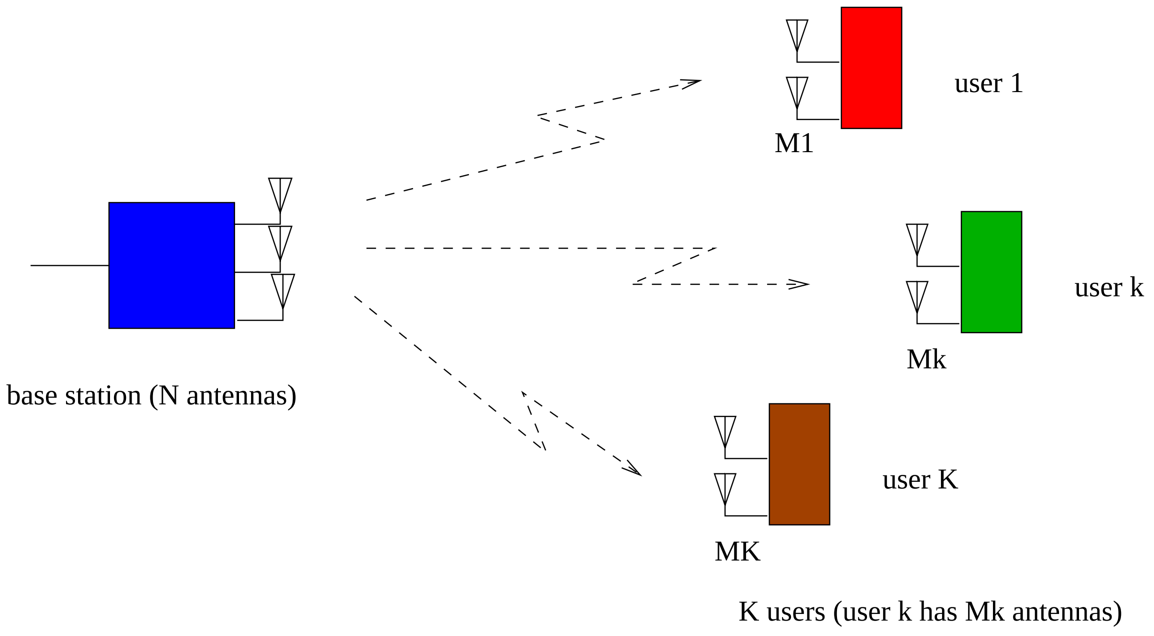
where vector $\mathbf{x}_k$ carries $m_k \leq M_k$ symbols per channel uses.

Focussing on the downlink MU-MIMO

Uplink vs. downlink MU-MIMO

- Several duality results exist:
  - duality of channel/signal models
  - Capacity region duality [Jindal, Goldsmith, Tse..]
  - MMSE beamforming duality [Shi, Shubert, Boche]
- Same multiplexing gain (limited by N typically) and diversity gains.
- Key difference: Downlink requires CSIT at the base for beamforming

The downlink MU-MIMO channel



The downlink MU-MIMO channel: Notations

We have:

- Let's assume a group of $K \leq N$ users are selected by the downlink scheduler
- Let the K users receive simultaneously from the base station.
- User k has M_k receive antennas.
- Base has N transmit antennas and peak power constraint P.
- Base transmits signal vector $\mathbf{X} = \sum_k \mathbf{X}_k$
- $\mathbf{X}_k$ is signal intended to user k, with covariance $\mathbf{Q}_k$.
- Power constraint ensured by $\sum_k \text{Tr}(\mathbf{Q}_k) \leq P$.
- Channel between user k and base is matrix $\mathbf{H}_k$, of size $M_k \times N$.
- White noise with variance 1.

The downlink MU-MIMO signal model

Received signal model at user k:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{X} + \mathbf{n}_k \text{ where } \mathbf{X} = \sum_k \mathbf{X}_k \tag{4}$$

usng the global downlink channel matrix:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \tag{5}$$

We have the global receive vector for all users:

$$\mathbf{y} = \left[\mathbf{y}_1^T, .., \mathbf{y}_K^T\right]^T = \mathbf{H}\sum_k \mathbf{X}_k + \mathbf{n} \tag{6}$$

where $\mathbf{X}_k$ is the signal vector designed to reach user k.

Fundamental CSIT/performance trade-off

There exists an interesting trade-off between

(i) the capacity performance

(ii) the number of antennas at the users

(iii) the need for CSIT.

- Capacity scales with $\min(K, N)$ provided the base has CSIT.
- In the absence of CSIT, user multiplexing is generally not possible: The base does not know in which "direction" to form beams!
- This is contrast with single user MIMO where CSIT is not necessary to get multiplexing gain.
- One case where multiplexing gain is restored is when at least $M_k = \min(N, K)$ antennas are installed at each user (exercise!)

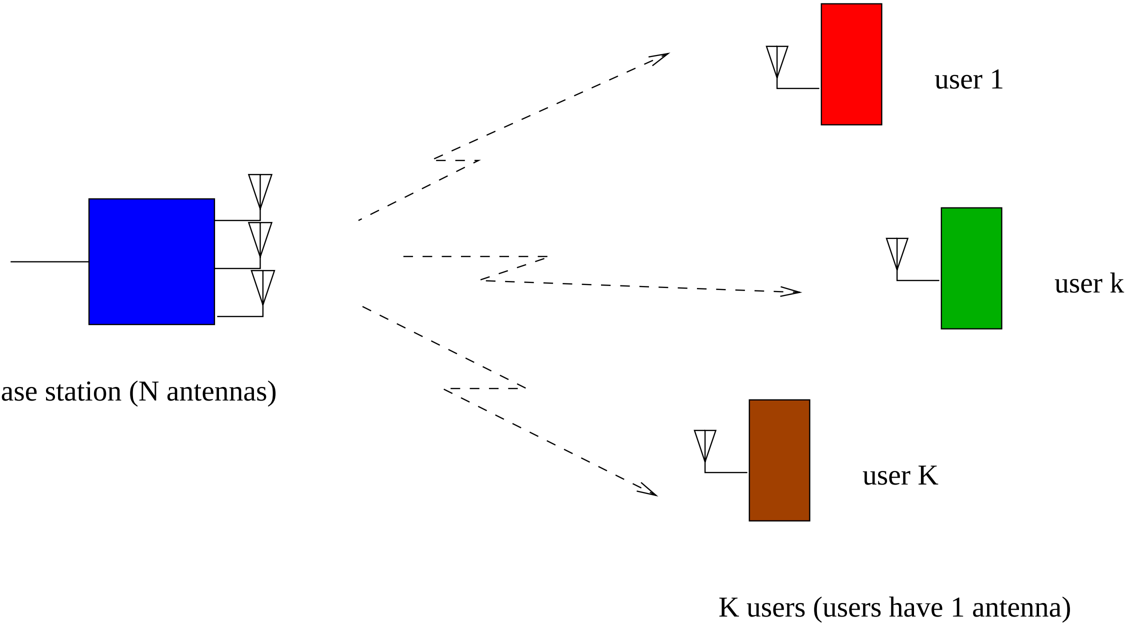
Linear multi-user MIMO downlink

The complexity/performence trade-off:

- Linear solutions favored for their reduced complexity
- Do not generally attain capacity bounds
- However achieve the optimial capacity *scaling* when nb of users is large [Yoo, Goldsmith et al]

Multi-user MIMO: The downlink with $M_k = 1$

We consider single antenna users ($M_k = 1$)



Signal model for MU-MIMO downlink beamforming

The base transmits signal vector $\mathbf{X} = \mathbf{W}\sqrt{\mathbf{Q}}\mathbf{s}$ where

- $\mathbf{W}$ is the $N \times K$ downlink beamformer and
- $\mathbf{s} = (s_1, .., s_K)^T$ contains the symbols.
- $\mathbf{Q} = \text{diag}(q_1, .., q_K)$ is the power allocation matrix.

The received signal at all users becomes:

$$\mathbf{y} = \mathbf{H}\mathbf{W}\sqrt{\mathbf{Q}}\mathbf{s} + \mathbf{n} \tag{7}$$

Essential results for MU-MIMO networks

- Single user vs. multiuser MIMO
- Performance limits of MU-MIMO
  - The role of CSIT

Single user vs. multi-user MIMO (I)

Multi-user MIMO makes certain things difficult:

- Dealing with users of unequal channel conditions (fairness issues).
- Mixing antenna filtering and scheduling problems into a harder problem.
- Multiple users can't cooperate as well as multiple antennas on a single device.
- Leads to multiple (rather than single) power constraints.
- Makes CSIT a stringent requirement (at least for downlink).

Single user vs. multi-user MIMO (II)

But provides a lot of advantages:

- Provides multi-user diversity (less reliance on antenna diversity).
- Provides decorrelation of spatial signatures.
- Allows for user- (in addition to stream-) multiplexing.
- Low rank channels no longer a problem but an advantage.
- Mitigates the need for multiple antennas at mobile (see later).

Single user vs. multi-user MIMO (III)

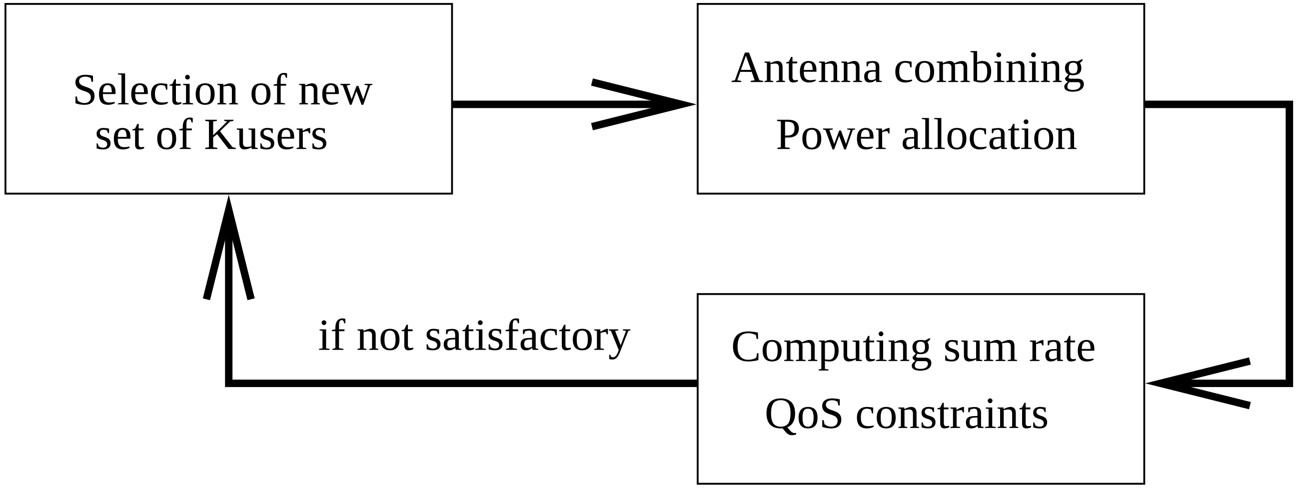
MU-MIMO makes cross-layer design essential:

- Admission control
- Multi-antenna combining (for MIMO case)
- Power control
- User scheduling

Optimal SDMA user scheduling

MU-MIMO scheduling provides the multiuser diversity gain extended to SDMA.

Assume we wish to select K out of a total of U system users.



SDMA user scheduling

Practical scheduling rules are akin to single-user mode (see slides by T. Ohtsuki)

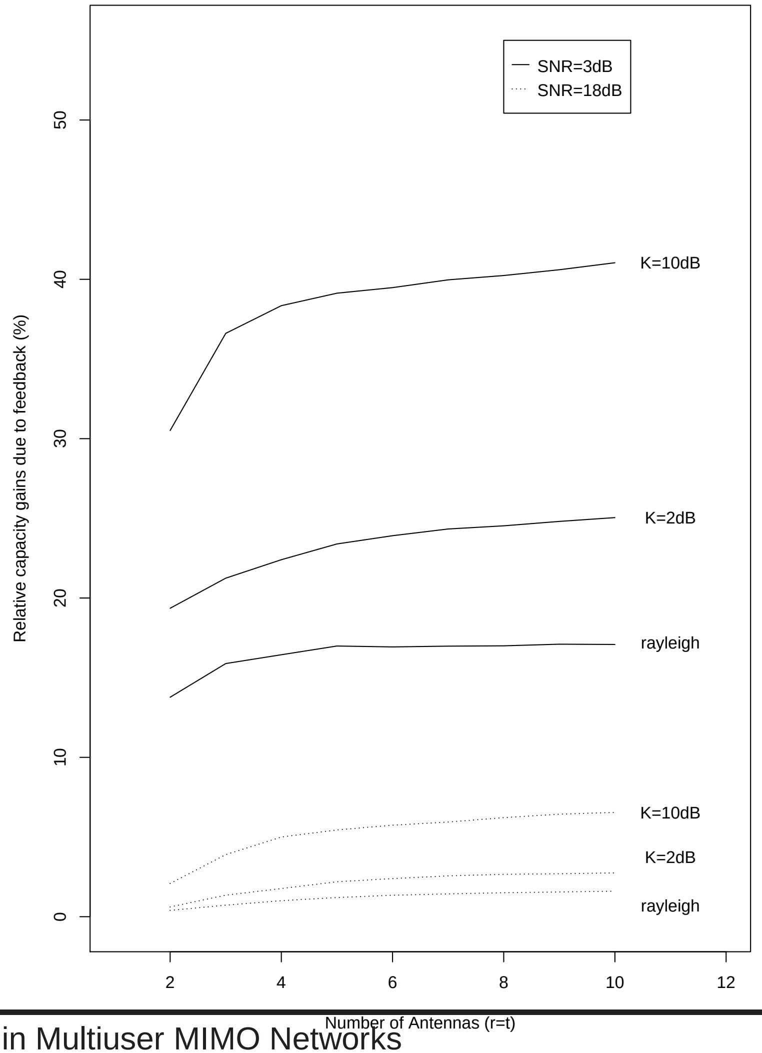
- Max rate scheduler (optimum but unfair)
- SDMA-based PFS scheduler
- Weighted delay-based scheduler
- Round-robin (fair)

Some system issues

- Channel aware scheduling transforms the fading statistics as seen by upper layer
- Gives less reliance in PHY-layer diversity (e.g. STC)
- Allows for compact antenna spacing at BTS, mobile.
- Multiple antennas at mobile only give a bonus (extra SNR, allow for feedback reduction)

On the role of CSIT in MU-MIMO

Relative capacity gain with CSIT (SU-MIMO case)



Role of CSIT in MU-MIMO

Role of CSIT in downlink evidenced by capacity scaling analysis.

With CSIT, it is found that [Hassibi05], with $M_k = M \; \forall k$:

$$\lim_{U \to \infty} \frac{E(R_{DPC})}{N \log \log(MU)} = 1 \tag{8}$$

where R_{DPC} is the sum rate achieved by dirty paper coding (optimal scheme).

Interpretation:

- CIST allows for transmit beamforming.
- With large U, the base can select and spatially multiplex the N best users out of U with negligible interference loss.
- Mobile antenna provide extra M diversity factor
- Multiplexing gain is not limited by single-antenna mobiles!

Role of CSIT in MU-MIMO (II)

Without CSIT, it is found that:

$$\lim_{U\to\infty} \frac{E(R_{DPC})}{\min(M,N)\log \mathsf{SNR}} = 1 \tag{9}$$

Interpretations:

- In the absence of CSIT, multiuser diversity gain vanishes
- multiplexing gain is limited to $\min(M,N)$.
- multiplexing gain vanishes if mobile are equiped with single antenna.

Acquiring CSI

- Receive side (easy): Channel estimated from training sequence
- Transmit side (hard):
  - TDD system: Base recycle uplink channel estimate. Quality depends on ”ping-pong time” and Doppler.
  - FDD systems: Exploit a dedicated feedback channel with quantizing

Acquiring CSIT with feedback

A numerical example:

- 4x2 MIMO-OFDM complex channel with 512 OFDM tones.
- 100 Hz Doppler (vehicular application).
- Channel estimation approx 10 times faster than Doppler.
- 8 bits quantizing per real-vaued coefficients.
- Total feeback load: 8x512x16*1000= 65.5 Mb/s per user !!!

→ Feedback reduction techniques are critical

→ Fortunately, a little information yields large gains!

Living with incomplete channel knowledge...

Feedback reduction techniques

A panorama:

1. Efficient quantizing (Lloyd-max, Grassmanian,..) [Love, Heath, et al.]
2. Quantizing the leading channel eigen directions (rather than the channel)
3. Eliminating users from feedback pool using *Selective Multiuser Diversity* (SMUD)
4. Dimension reduction (includes concept of random beamforming!)
5. Exploiting redundance (temporal, frequency) to reduce feedback close to *rate of innovation*
6. Exploiting spatial statistics
7. Using hybrid direction/gain information
8. Splitting feedback between scheduling and beamforming tasks
9. More??

Let us now investigate approaches 3, 4, 5, 6, 7, 8 in greater detail.

Selective Multiuser diversity

Principles:

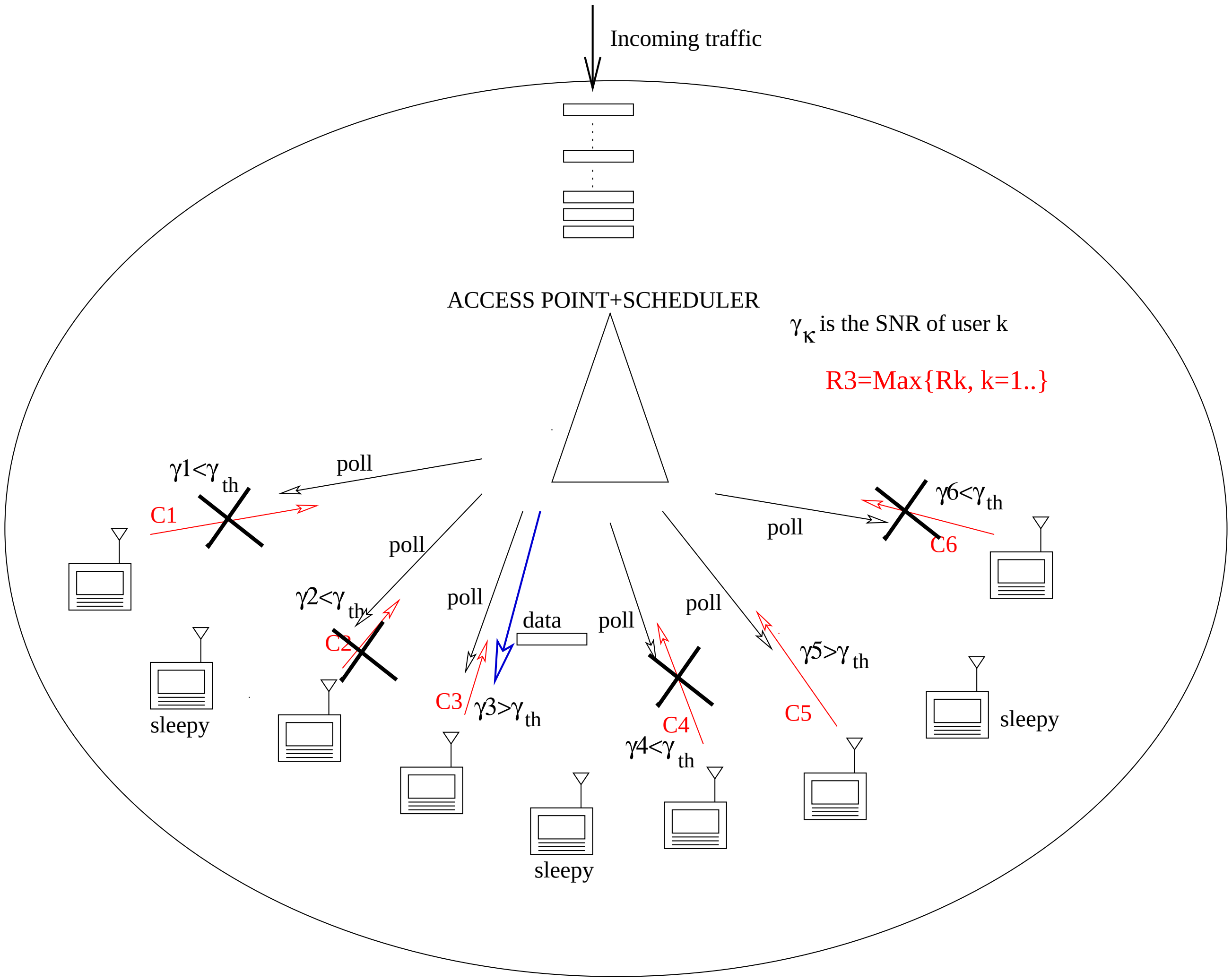
⇒ Proposed in IEEE ICC2004 [Gesbert et al.]

⇒ Selective MUD (SMUD) exploits idea that **scheduled user is bound to have a "good" channel**.

⇒ By **thresholding channel quality**, one can reduce feedback dramatically

⇒ SMUD can be analyzed/optimized in closed form (SISO case, ICC 2004).

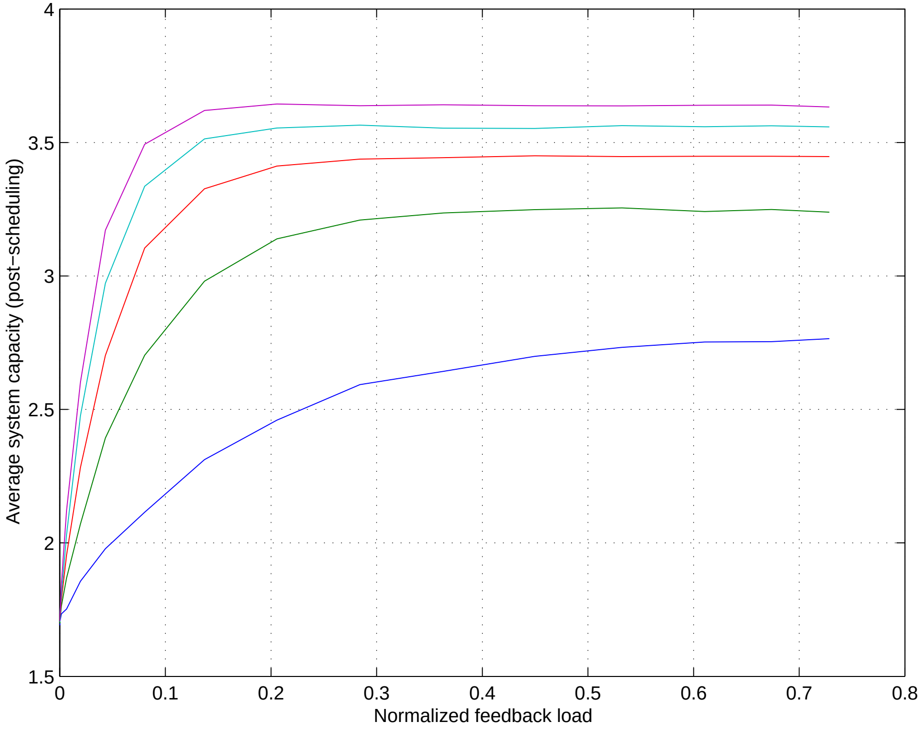
Selective multi-user diversity scheduling



Capacity loss vs. Feedback reduction

We compare SMUD+PFS with full feedback MUD+PFS (SISO case)

- t_c (PFS time constant) is 500 slots. Average SNR is 5 dB. Number of users is $4, 10, 16, 22, 28$ (bottom to top).



Dimension reduction techniques

Key idea: Mapping the $M \times N$ scalar channel dimensions of CSI down to a smaller number p.

- Projection of the channel matrix/vector onto one or more basis vectors known to the Tx and Rx.
- Once the projection is carried out, user k feeds back a metric $\xi_k = f(\mathbf{H})$ which is typically related to the square magnitude of the projected signal.
- Important example: projection onto a *unitary* precoder known by both BS and user.

Projection on a unitary precoder (I)

- Let $M_k = 1$, the BS designs an arbitrary unitary precoder $\mathbf{Q}_{M \times p}$. $p \leq M$.
- Each terminal identifies the projection of its vector channel onto the precoder and reports the SINR on the best precoding column:

$$\xi_k = \max_{1 \leq i \leq p} \frac{|\mathbf{h}_k^H \mathbf{q}_i|^2}{\sigma^2 + \sum_{j \neq i} |\mathbf{h}_k^H \mathbf{q}_j|^2} \tag{10}$$

  where $\mathbf{q}_i$ denotes the i-th column of $\mathbf{Q}$.

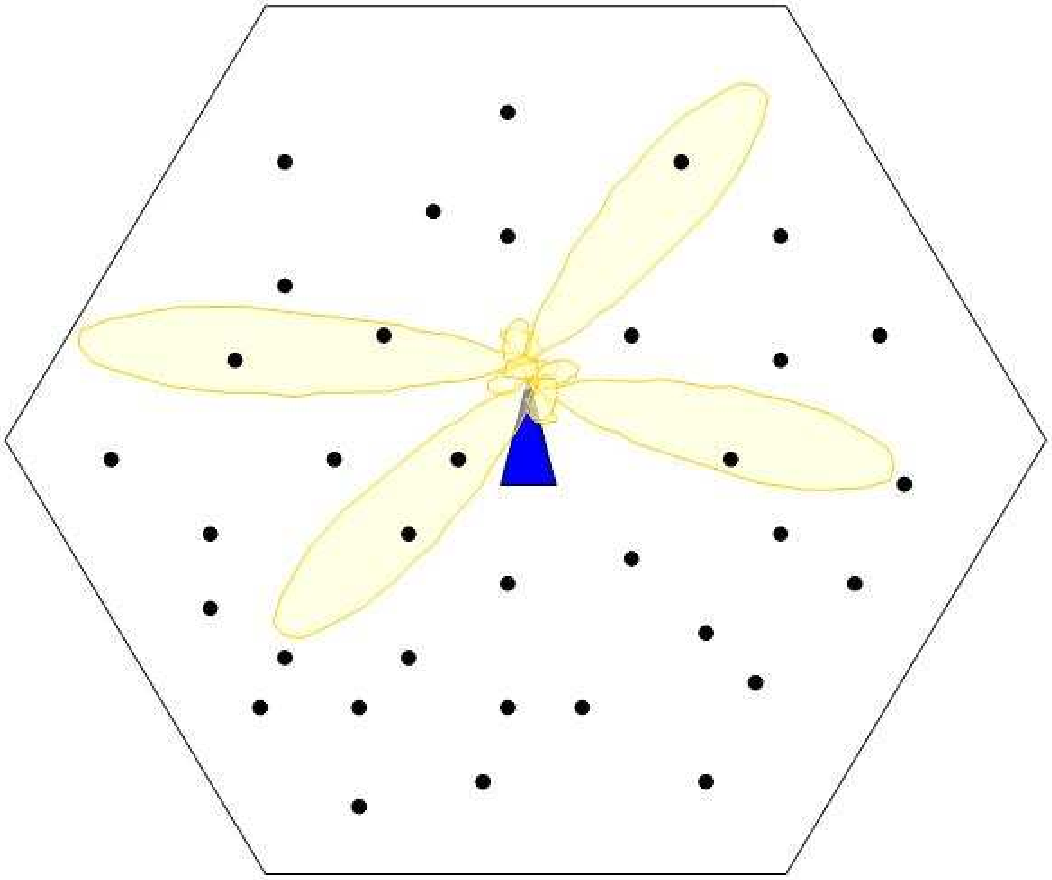
- The scheduling algorithm opportunistically assigns to each beamformer $\mathbf{q}_i$ the user which has selected it and has reported the highest SINR.

Projection on a unitary precoder (II)

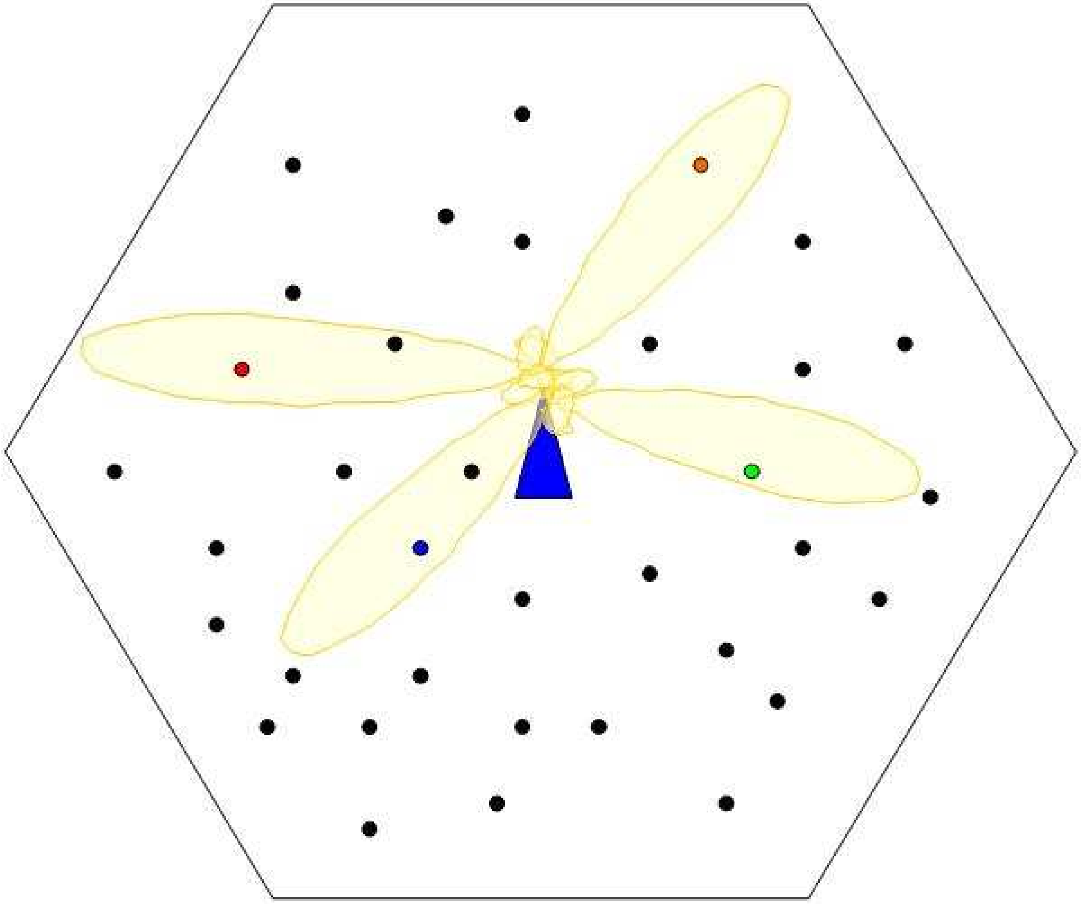
Some interesting particular cases:

- $p = M$, $\mathbf{Q}$ is fixed and equal to identity. This yields the per-antenna SDMA scheduler.
  - This scheduler is optimal with large number of users but unfair in low Doppler scenarios.
- $p = 1$, $\mathbf{Q}$ is random, unit-norm. This yields opportunistic beamforming [Viswanath et al.'02].
- $p = M$, $\mathbf{Q}$ is random, unitary. This yields opportunistic multiuser beamforming [Sharif,Hassibi'05].
- In both cases, randomization restores fairness on shorter horizons.

Opportunistic multi-user beamforming (I)



Opportunistic multi-user beamforming (II)



Opportunistic multi-user beamforming: Performance

- Each user reports the SINR observed on his *preferred* beam.
- Sum rate performance (in the large number of users case):

$$SR \approx E\left\{\sum_{m=1}^{N} \log_2(1 + \max_{1 \leq k \leq U} SINR_{k,m})\right\} \tag{11}$$

Opportunistic BF performance

For very large number of users:

- The sum rate converges to sum rate obtained under optimal unitary precoder with CSIT.
- The scaling laws (with nb of users) of rate under unitary and optimal precoder are identical ($N \log \log U$)
- Threfore opportunistic multiuser beamforming is asymptotically optimal in the number of users U.

For low number of users ("sparse network"):

- Random beams do not reach users precisely
- Severe degradation

This problem can be fixed by monitoring matching between users and beams and adjusting beam power accordingly.

Beam power control (I)

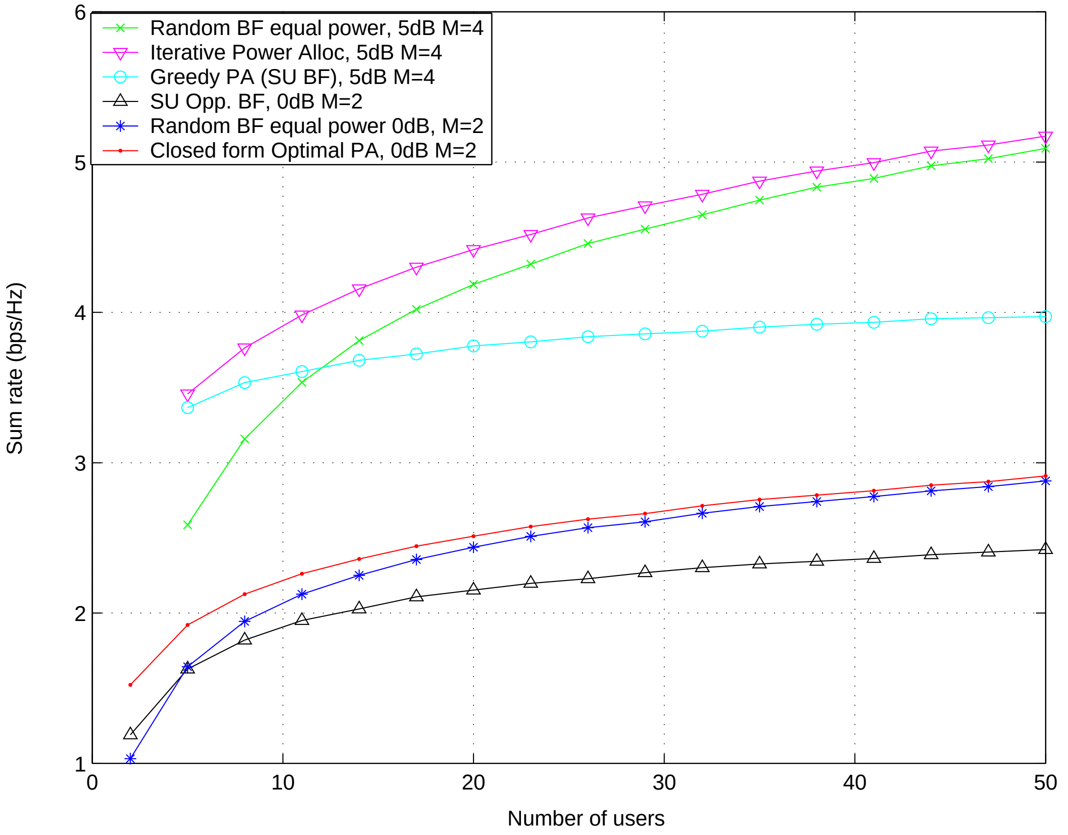
- Denote $\mathcal{S}$ the set of selected users and $\mathbf{p}$ the beam power vector.
- BS knows $g_{km} = \left|\mathbf{h}_k^H \mathbf{q}_m\right|^2$ for $k \in \mathcal{S}, m = 1, \ldots, N$.
- The sum-rate optimal beam power allocation [Kountouris et al 05]:

$$\max_{\mathbf{p}} \sum_{k \in \mathcal{S}} \log \left(1 + \frac{P_m g_{km}}{1 + \sum_{j \neq m} P_j g_{kj}} \right)$$

$$\text{subject to} \sum_{i=1}^{N} P_i = P$$

- Closed-form solution for $M = 2$ antennas (optimal).
- Iterative WF-like algorithm for $M > 2$ (optimality is not guaranteed).

Beam power control (II)

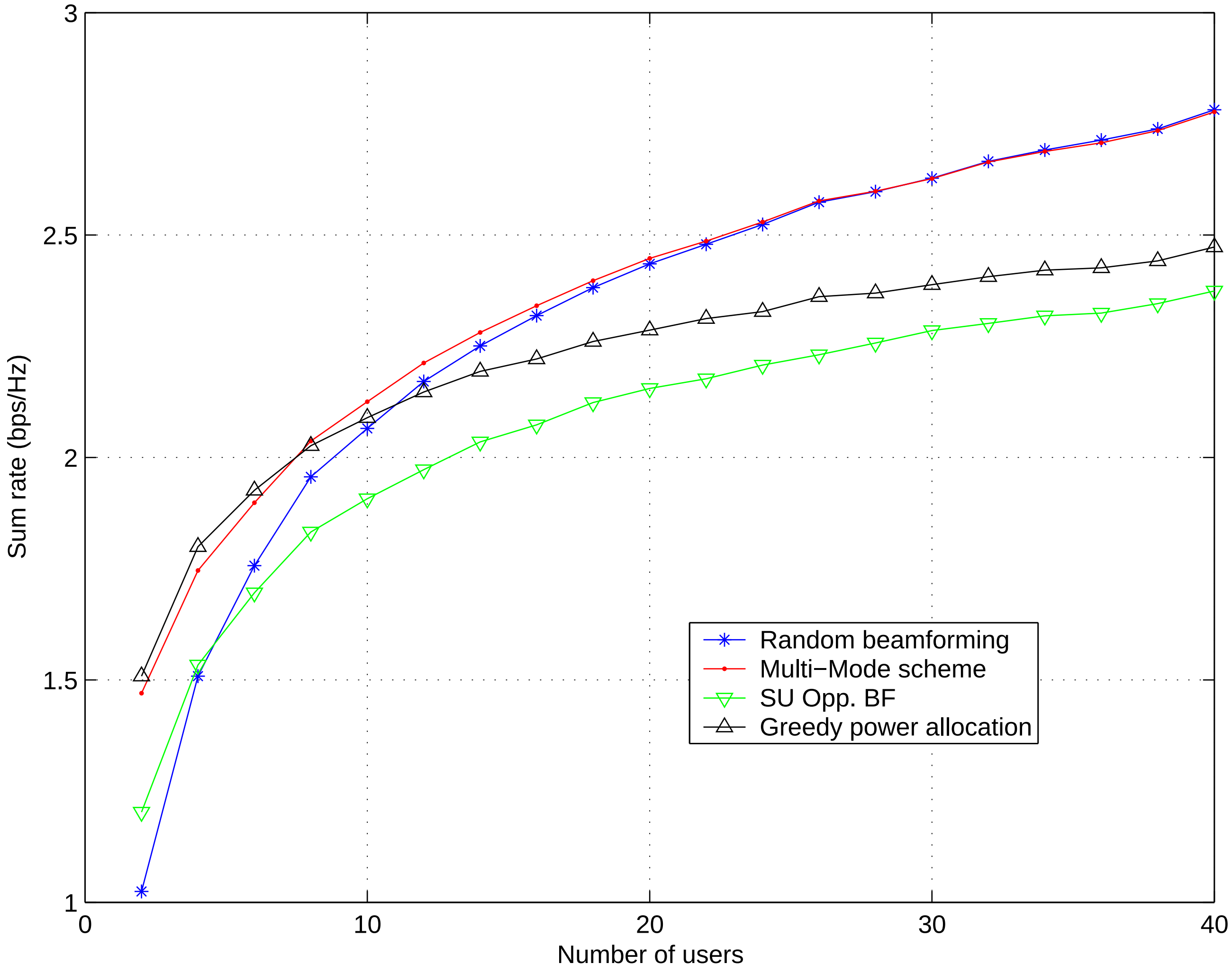


Performance of Beam Power Allocation vs. the number of users for $N = 2, 4$ Tx antennas.

On-off beam selection

- Turn off the worst beams → reducing inter-user interference.
- Decision based on comparing SINR on each beam with a threshold.
- Power on unallocated beams is reported to active beams.
- Gives discrete transition between TDMA and SDMA.

Performance with beam power control



Sum rate vs. number of users for $N = 2$ Tx antennas and $SNR = 0$ dB

Exploiting temporal redundance (I)

- Random opportunistic beamforming can be made robust to sparsity thanks to redundance.
- Temporal redundance exists for slow varying channel scenarios.
- Feedback aggregation concept: information derived from low-rate feedback channel can be cumulated over time to approach the performance of full CSIT scenario.
- Idea: use successive refinement of random beams (single user [Avidor et al 2004], multiuser [Kountouris et al. 2005])

Memory based opportunistic beamformer

First phase ('best' unitary matrix selection)

Initialize Set with random BF matrices Q_i, with sum rate $SR(Q_i)$

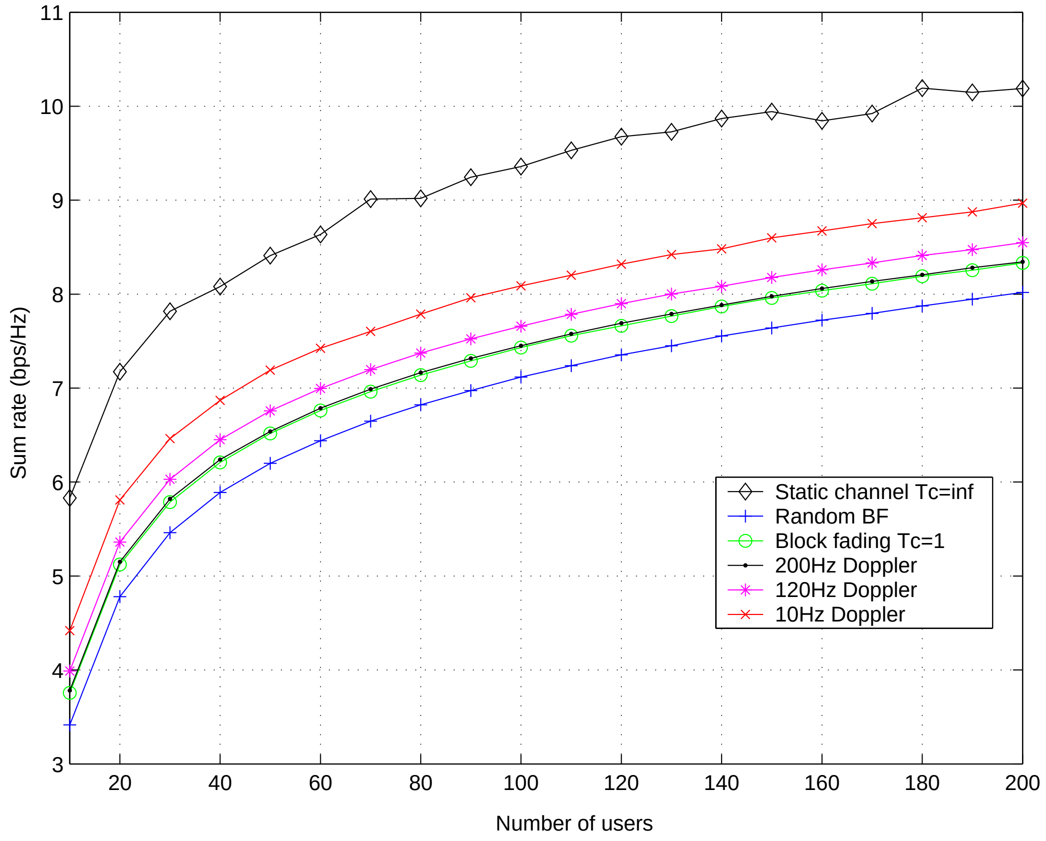
At each time slot t,

- Generate a new random Q_{rand},with sum rate $SR(Q_{rand})$
- Select from the Set of 'preferred' matrices, Q_i*, such that $i* = arg\max_{Q_i} SR(Q_i)$
- Calculate $SR(Q_{i*})$ given updated channel
- If $(SR(Q_{i*}) > SR(Q_{rand}))$ use Q_{i*} , else use Q_{rand}

Second phase (update of the Set)

- Update $SR(Q_{i*})$ value of the set
- If $(SR(Q_{rand}) > SR(Q_{imin}))$, replace Q_{imin} by Q_{rand}, where Q_{imin} is matrix with minimum sum rate ($i_{min} = arg\min_{Q_i} SR(Q_i)$)

Exploiting temporal redundance: Performance



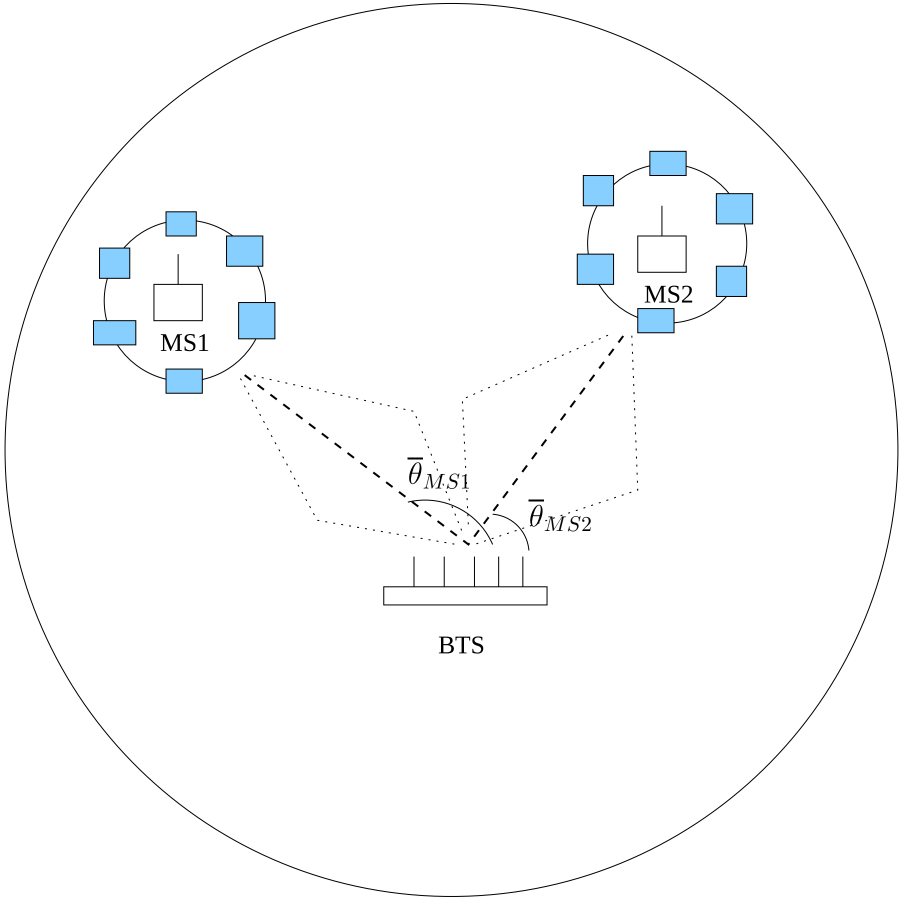
Exploiting spatial structure

- Spatial channel statistics reveal a great deal of information on the *macroscopic* nature of the channel:
  - multipath's mean AoA
  - angular spread
- Spatial statistics have a long coherence time compared with that of fading.
- Several forms of statistical CSI are reciprocal (second-order correlation matrix, power of Ricean component, etc.) → no additional feedback required.
- Second-order statistical information $\mathbf{R}_k = \mathrm{E}\left[\mathbf{h}_k^H \mathbf{h}_k\right]$ can be used to infer knowledge on users' average spatial separability.

Previous work: [Hammarwall et al. 06, Kountouris et al 06]

Interpretation spatial statistics

The BTS should schedule users which are *likely* to be away from each other *statistically*.



Using spatial statistical feedback in MU-MIMO downlink

- Consider a correlated Rayleigh MISO channel $\mathbf{h}_k \sim \mathcal{CN}(0, \mathbf{R}_k)$, where $\mathbf{R}_k \in \mathbb{C}^{N \times N}$ is the transmit covariance matrix (known to BS).
- *Objective*: How to combine long-term CSIT with instantaneous scalar CSIT in order to exploit Multiuser Diversity ?
- Instantaneous CSIT given by:

$$\gamma_k = \|\mathbf{h}_k \mathbf{Q}_k\|^2 \tag{12}$$

  where $\mathbf{Q}_k \in \mathbb{C}^{N \times L}$ is a training matrix containing L orthonormal vectors $\{\mathbf{q}_{ki}\}_{i=1}^{L}$.

- Key idea: Conditioned on short-term CSIT γ_k, derive a coarse channel estimate.

ML estimation framework

- We estimate a coarsely the channel by maximizes the likelihood of $\mathbf{h}_k$ under the scalar constraint $\gamma_k = |\mathbf{h}_k \mathbf{q}_k|^2$ $(L = 1)$.
- The solution to the optimization problem

$$\begin{aligned} \max_{\mathbf{h}_k} \;& \mathbf{h}_k \mathbf{R}_k \mathbf{h}_k^H \\ s.t. \;& |\mathbf{h}_k \mathbf{q}_k|^2 = \gamma_k \end{aligned} \tag{13}$$

is given by [Kountouris et al, Eusipco'06]

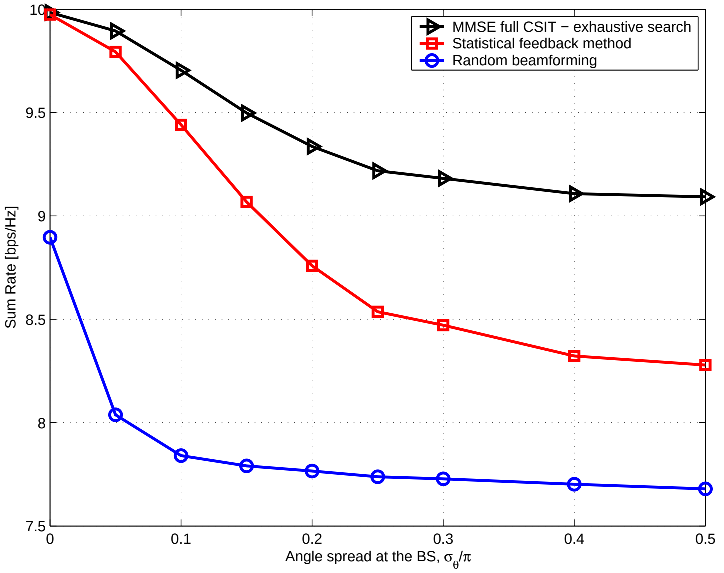
$$\widehat{\mathbf{h}}_k = \arg\max_{\mathbf{h}_k} \frac{\mathbf{h}_k \mathbf{R}_k \mathbf{h}_k^H}{\mathbf{h}_k (\mathbf{q}_k \mathbf{q}_k^H) \mathbf{h}_k^H} \tag{14}$$

which corresponds to the (dominant) generalized eigenvector associated with the largest positive generalized eigenvalue of the Hermitian matrix pair $(\mathbf{R}_k, \mathbf{q}_k \mathbf{q}_k^H)$.

What to do with a coarse $\hat{\mathrm{h}}$ estimate ?

- $\hat{\mathbf{h}}$ can be used to schedule and form beams to selected users (risky)
- $\hat{\mathbf{h}}$ can be used to form a scheduling metric at BTS, but not form the beams (robust)

We evaluate the second approach..



ML estimation framework - approach 2

Sum rate vs. angle spread σ_θ at the base station ($N = 2$, SNR = 10 dB and $K = 50$

ML estimation framework - approach 2

Conclusions:

- Performance close to that of full CSIT when angular spread per user is small enough (less 30°.
- ideal for wide area networks in suburban environment.
- Robustness to the case of wide angular spread (worst case performance is that of random beamforming).

Hybrid direction/gain feedback metrics

Consider that the feedback channel is divided into 2 types of information:

- Channel Direction Information (CDI)
- Channel Quality Information (CQI)

CDI Finite Rate Feedback Model

- Quantization codebook known to both the k-th Rx and Tx:
- $\mathcal{V}_k = \{\mathbf{v}_{k1}, \mathbf{v}_{k2}, \ldots, \mathbf{v}_{kN}\}$ containing 2^B unit norm vectors
- the k-th mobile sends index (using B bits) for following vector:

$$\hat{\mathbf{h}}_k = \mathbf{v}_{kn} = \arg \max_{\mathbf{v}_{ki} \in \mathcal{V}_k} |\bar{\mathbf{h}}_k^H \mathbf{v}_{ki}|^2 = \arg \max_{\mathbf{v}_{ki} \in \mathcal{V}_k} \cos^2(\angle(\bar{\mathbf{h}}_k, \mathbf{v}_{ki})) \qquad (15)$$

where $\bar{\mathbf{h}}_k = \mathbf{h}_k / \|\mathbf{h}_k\|$.

CQI under Zero Forcing beamforming (1/2)

- Let $\mathcal{S}$, be a group of $|\mathcal{S}| = K \leq N$ users selected for transmission.
- The signal model is given by

$$\mathbf{y}(\mathcal{S}) = \mathbf{H}(\mathcal{S})\mathbf{W}(\mathcal{S})\mathcal{P}\mathbf{s}(\mathcal{S}) + \mathbf{n} \tag{16}$$

  where $\mathbf{H}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathrm{s}(\mathcal{S})$ are the concatenated channel vectors, beamforming vectors, uncorrelated data symbols.

- Assuming ZF beamforming on the quantized channel directions:

$$\mathbf{W}(\mathcal{S}) = \hat{\mathbf{H}}(\mathcal{S})^H(\hat{\mathbf{H}}(\mathcal{S})\hat{\mathbf{H}}(\mathcal{S})^H)^{-1}\mathbf{\Lambda} \tag{17}$$

CQI under Zero Forcing beamforming (2/2)

- The SINR at the k-th receiver is

$$SINR_k = \frac{P_k |\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{j \in \mathcal{S}-\{k\}} P_j |\mathbf{h}_k^H \mathbf{w}_j|^2 + 1} \tag{18}$$

  where $\sum_{i \in \mathcal{S}} P_i = P$ (power constraint)

- Sum rate is measured by:

$$\mathcal{R}_k = \mathbb{E}\left\{ \sum_{k \in \mathcal{S}} \log\left(1 + SINR_k\right) \right\} \tag{19}$$

- Key problem: How can the user report the SINR? without knowing the beamformer? it can't..

Using an upper bound of SINR as CQI

- Let $\phi_k = \angle(\hat{\mathbf{h}}_k, \bar{\mathbf{h}}_k)$ be the angle between the normalized channel vector and the quantized channel direction.
- Each user feeds back the following scalar metric [Jindal 06, Kountouris 06]

$$\xi_k^{UB} = \frac{P \left\|\mathbf{h}_k\right\|^2 \cos^2 \phi_k}{P \left\|\mathbf{h}_k\right\|^2 \sin^2 \phi_k + M} \tag{20}$$

- This gives information on the channel gain as well as the CDI quantization error ($\sin^2 \phi_k$).
- Can be interpreted as an upper bound (UB) on the received SINR$_k$ (under equal power allocation)
- Exact for orthogonal user sets (valid for case with many users)

Splitting the feedback

Key ideas:

- MU-MIMO schemes can be decomposed into scheduling and beamforming stages
- Both stages require CSIT
- Scheduling requires CSIT from $U >> N$ users, but can live with coarse estimates.
- Beamforming to selected users requires CSIT from $< N$ users, but CSIT must be precise.

$\Rightarrow$ Why not split the feedback load over the two stages? [Zakhour et al. PIMRC 07]

Feedback split model

Let $0 \leq \alpha \leq 1$ be the split factor:

- Let B_{total} denote the total number of bits available for feedback
- $B_1 = \alpha B_{\text{total}}$ bits dedicated to the scheduling
- $B_2 = (1-\alpha) B_{\text{total}}$ bits dedicated to beamforming matrix design
- A user selected in second phase refines his initial B_1/U-bit feedback with B_2/N-bit feedback
- Achievable distorsion at each stage:

$$
\begin{aligned}
\sigma_{e_1}^2 &= 2^{-b_1/N} = 2^{-\alpha B_{total}/(U \times N)} && (21)\\
\sigma_{e_2}^2 &= 2^{-(b_1+b_2)/N} = 2^{-\frac{B_{total}}{N}\left(\frac{\alpha}{U}+\frac{1-\alpha}{N}\right)}, && (22)
\end{aligned}
$$

Feedback split optimization

The optimal α is that which maximizes the average sum rate

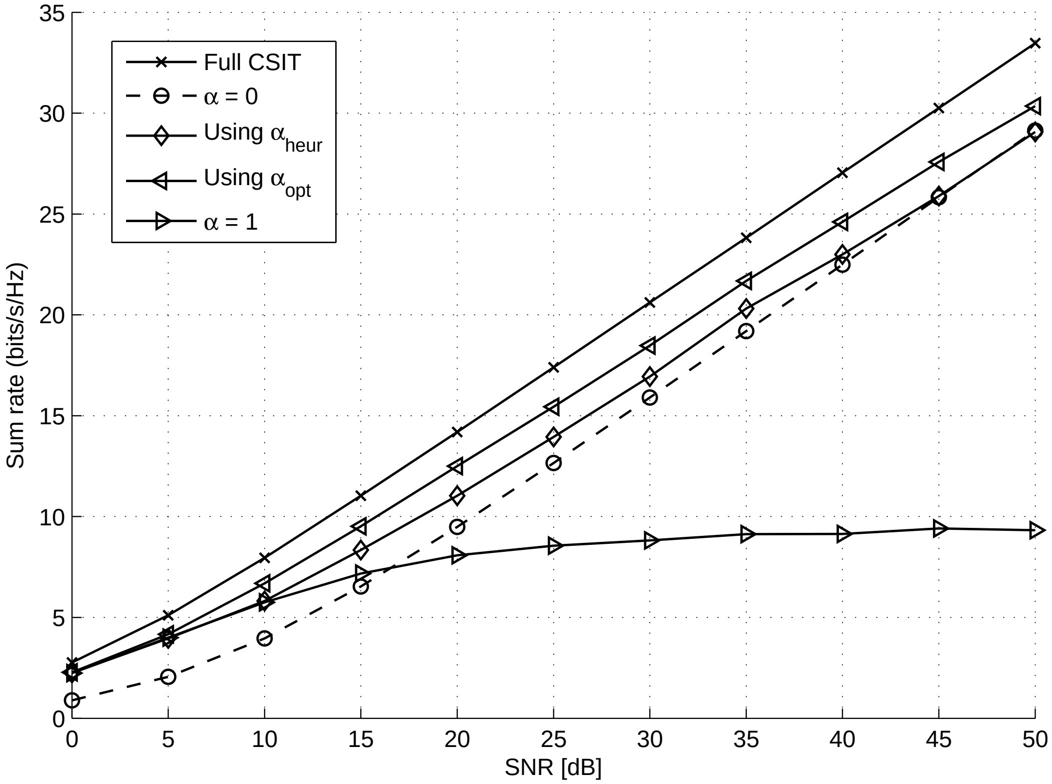
Lemma: α_{opt} is approximated by the following solution [Zakhour PIMRC07]:

$$\text{PL} = \frac{1-\sigma_{e_1}^2}{1+P\sigma_{e_2}^2} + \frac{\sigma_{e_1}^2 - \sigma_{e_2}^2}{\log U(1+P\sigma_{e_2}^2)} \tag{23}$$

$$\alpha_{opt} \approx \arg\max_{\alpha\in[0,1]} \text{PL} \tag{24}$$

Sume rate performance

Sum rate for $N = 2$ base antennas, $U = 30$ single-antenna users, $B_{total} = 120$ bits



Multicell MU-MIMO: An introduction

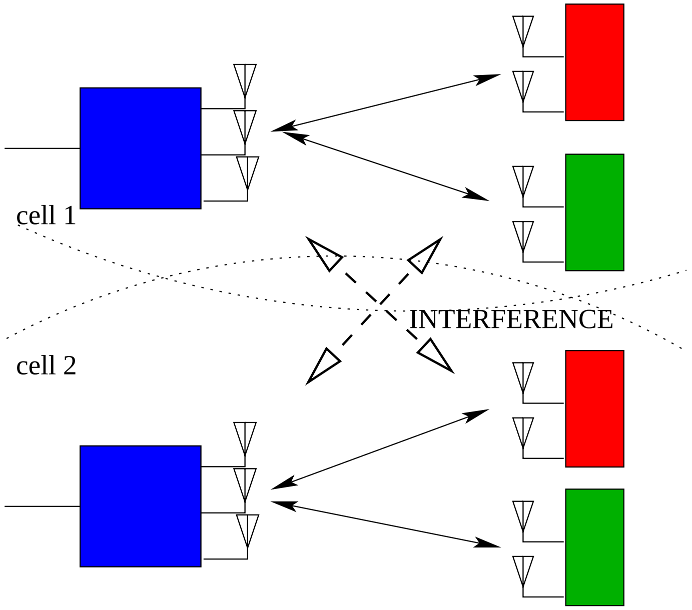
- The network contains several multiuser MIMO links, sharing the same resource.
- Neighboring coverage regions overlap each other.

Two key approaches are possible

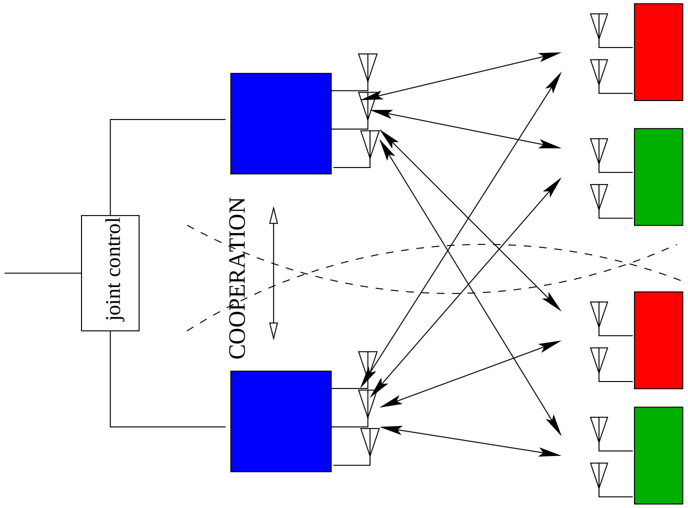
- MIMO links can be competing or cooperating.
- Cooperation is infrastructure based
- Unlike mobile relaying, infrastructure cooperation works with standard user devices
- No spectral efficiency consumed in BTS relaying (BTS are connected via high speed optical fibers)
- Cooperation can still however be limited by partial channel knowledge

Multicell (competing) MU-MIMO

Competing links are undermined by co-channel interference

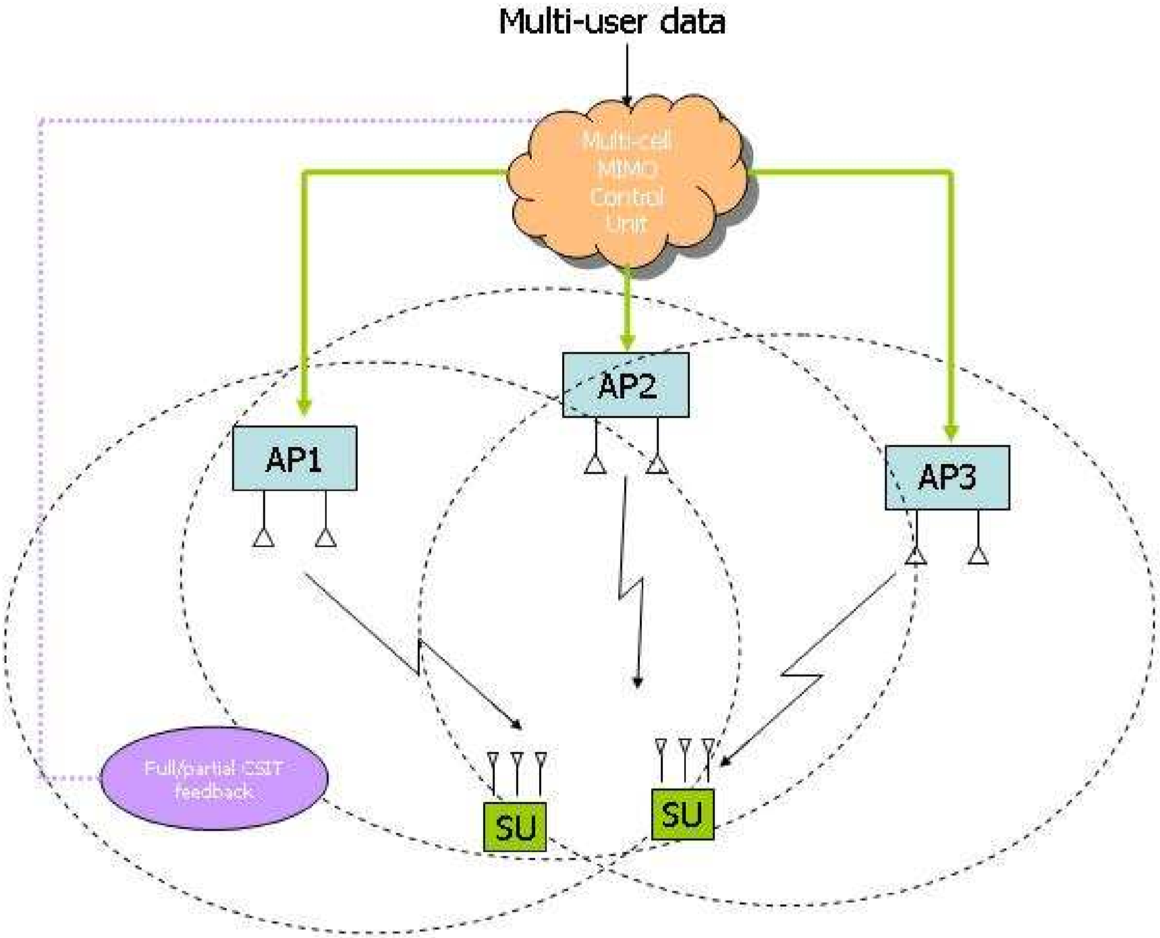


Multicell (cooperative) MU-MIMO



- Information theoretic results recently obtained [Shamai et al]
- But does Multi-cell MIMO really help users with severe interference (cell-edge)?

Cooperative signaling for cellular downlink



Cooperative MIMO strategies

Several possible approaches:

1. 1 We are after diversity: distributed space time coding
2. 2 We are after boosting data rates: multicell multiplexing (assuming scheduling takes care of diversity)

We go for multiplexing..

- Same algorithms as single-cell MU-MIMO (DPC,linear beamforming, etc.)
- Main challenges: Per-base power constraints and inter-cell signaling overhead.

A first example

Cooperative spatial multiplexing with two bases

Notations:

- Two bases with N antennas each.
- Two mobile users with single antenna.
- Network wishes to transmit $[s_0, s_1]$ (one symbol s_i per user i) via *both bases*.
- Symbols are uncorrelated.
- Each base has peak power constraint P_i.
- Channel from base i to all users is $\mathbf{H}_i \in \mathbb{C}^{2 \times N}$.

Transmit-receive signal model

Base i transmits $N \times 1$ signal vector formed by:

$$\boldsymbol{x}_i = \boldsymbol{A}_i \boldsymbol{s}. \tag{25}$$

where $\boldsymbol{A}_i \in \mathbb{C}^{N \times 2}$ is such that

$$\mathrm{Tr}\left\{\boldsymbol{A}_i \boldsymbol{A}_i\right\} = P_i. \tag{26}$$

The received signal vector, $\boldsymbol{y} = [y_0, y_1]^T$, $\boldsymbol{y} \in \mathbb{C}^{2 \times 1}$, is then given as

$$\boldsymbol{y} = \boldsymbol{H}_0 \boldsymbol{A}_0 \boldsymbol{s} + \boldsymbol{H}_1 \boldsymbol{A}_1 \boldsymbol{s} + \boldsymbol{v}. \tag{27}$$

Problem: obtain optimal transmit filters under CSIT and power constraint

Cooperative spatial multiplexing with full CSIT

We use the MMSE criterion:

$$\arg\min_{\boldsymbol{A}_0,\boldsymbol{A}_1} \text{MSE} = \mathbb{E}_{\boldsymbol{s},\boldsymbol{v}}\left[\|\boldsymbol{y}-\boldsymbol{s}\|^2\right] \tag{28}$$

under constraint:

$$\operatorname{Tr}\left\{\boldsymbol{A}_0\boldsymbol{A}_0^H\right\} = P_0. \tag{29}$$

$$\operatorname{Tr}\left\{\boldsymbol{A}_1\boldsymbol{A}_1^H\right\} = P_1. \tag{30}$$

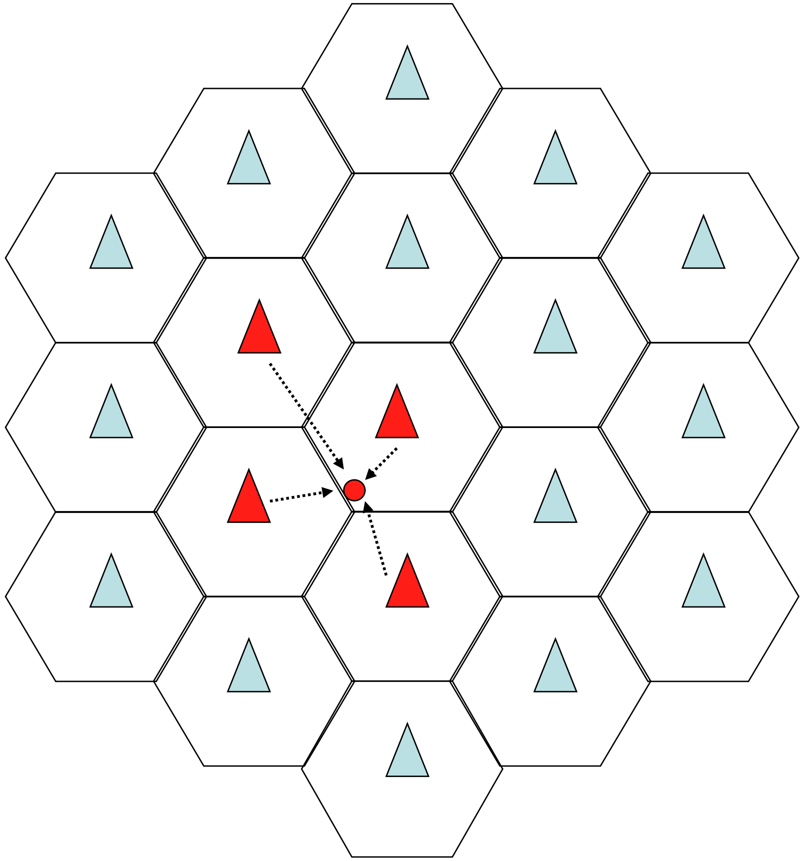
Cooperative multiplexing with full CSIT (2)

The optimal filters are given by the equation:

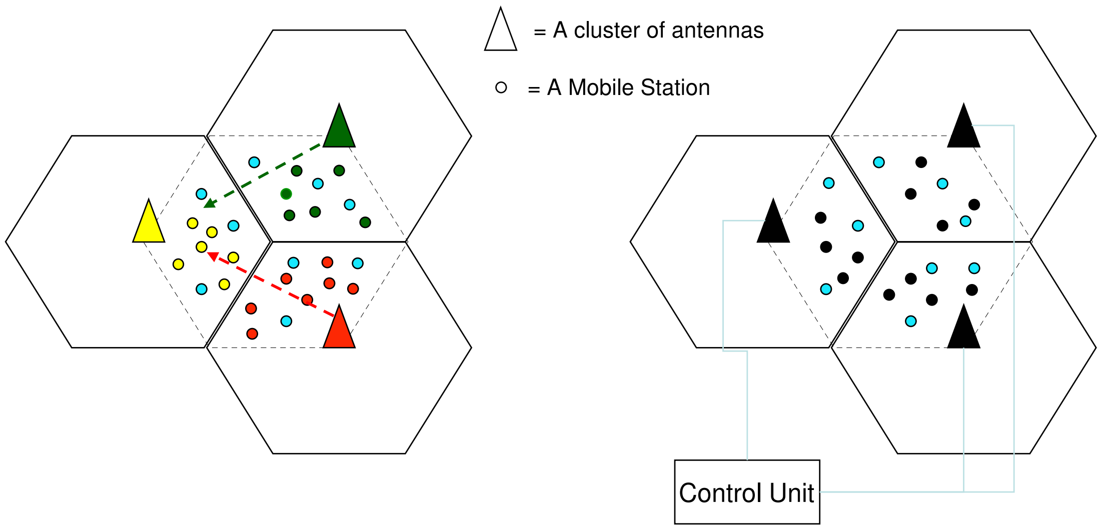
$$\begin{bmatrix} \boldsymbol{H}_0^H \boldsymbol{H}_0 + \mu_0 \boldsymbol{I}_N & \boldsymbol{H}_0^H \boldsymbol{H}_1 \\ \boldsymbol{H}_1^H \boldsymbol{H}_0 & \boldsymbol{H}_1^H \boldsymbol{H}_1 + \mu_1 \boldsymbol{I}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_0 \\ \boldsymbol{A}_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_0^H \\ \boldsymbol{H}_1^H \end{bmatrix}, \tag{31}$$

where μ_0 and μ_1 must be chosen such that the power constraints are satisfied.

Evaluation in a cellular network context

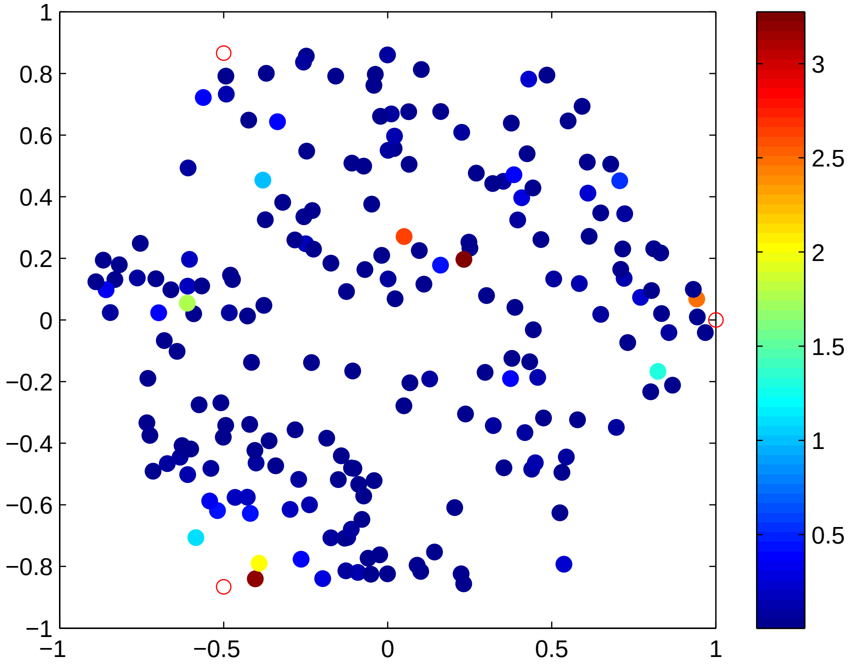


Zooming on cooperative subnetwork



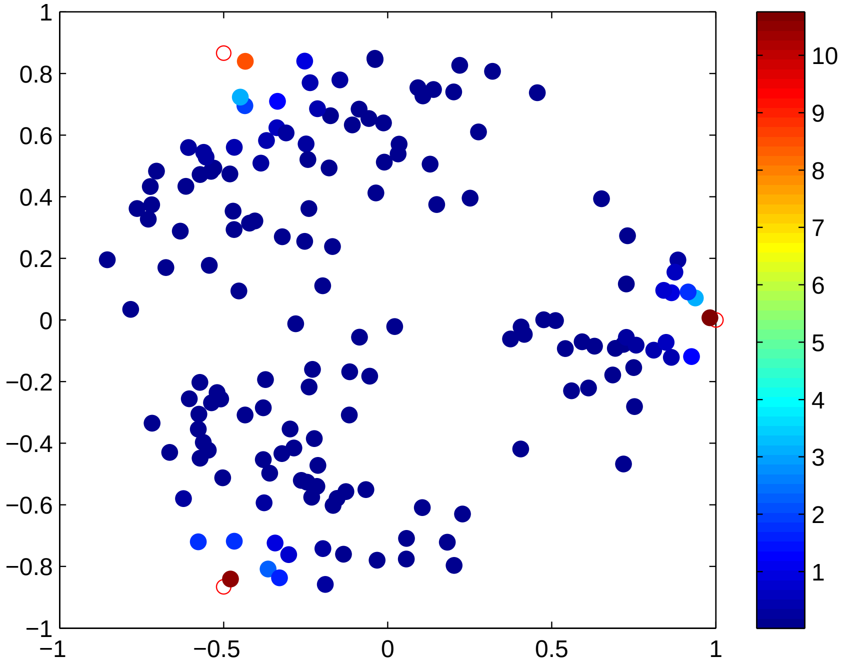
Evaluation with max rate (greedy) scheduling

Single-cell processing (no cooperation) - 2 antennas per BTS



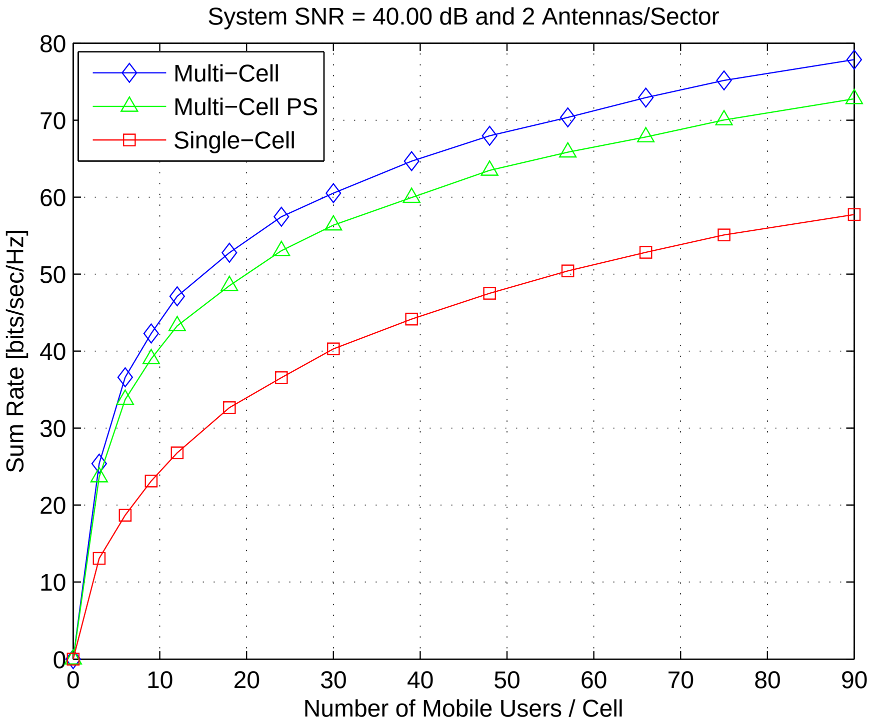
Evaluation with max rate (greedy) scheduling

Multi-cell processing (cooperation)



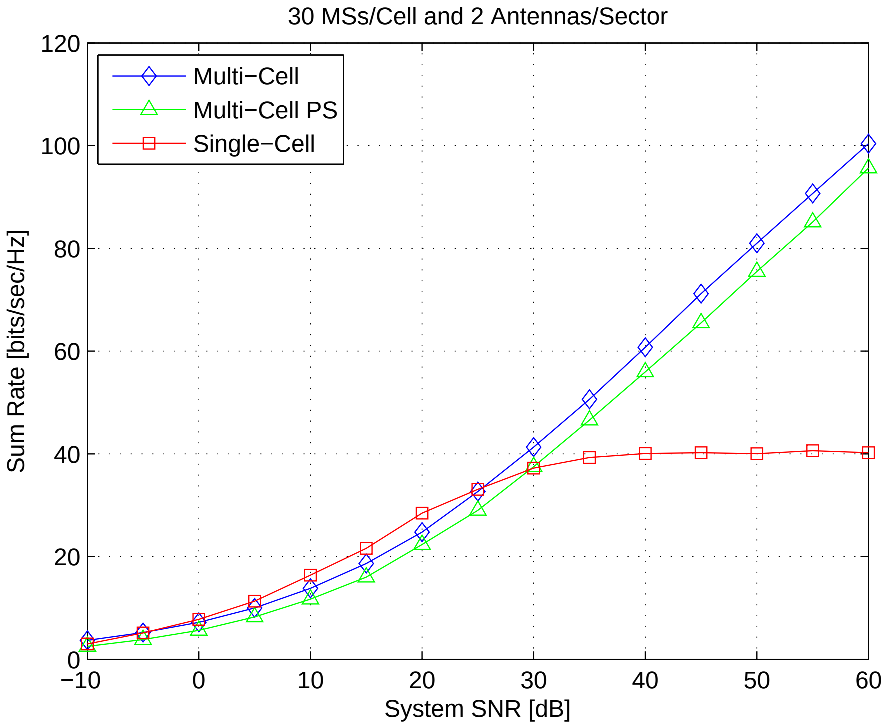
Evaluation with max rate (greedy) scheduling

Sum rate performance vs. number of users (SNR=40)



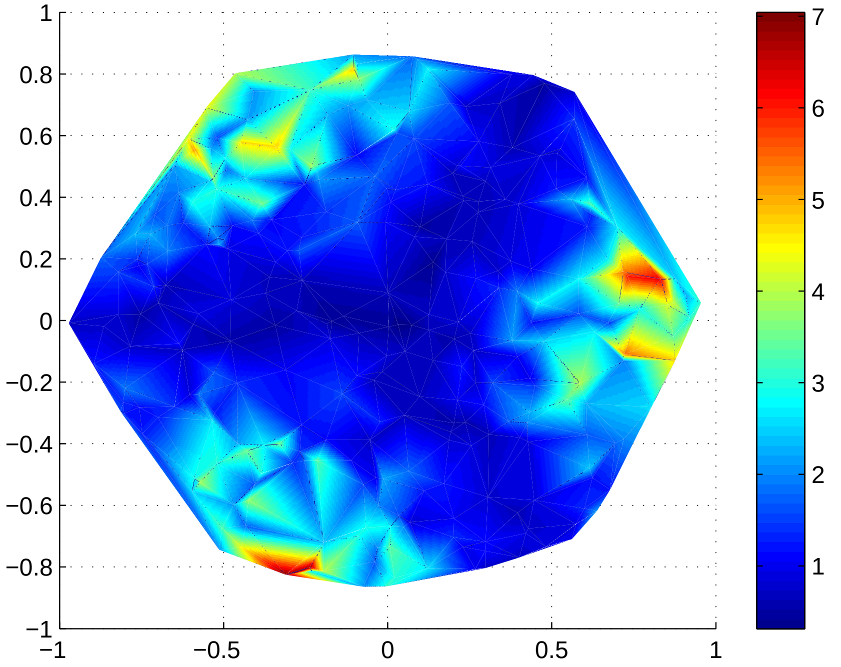
Evaluation with max rate (greedy) scheduling

Sum rate performance vs. SNR (30 users per cell)



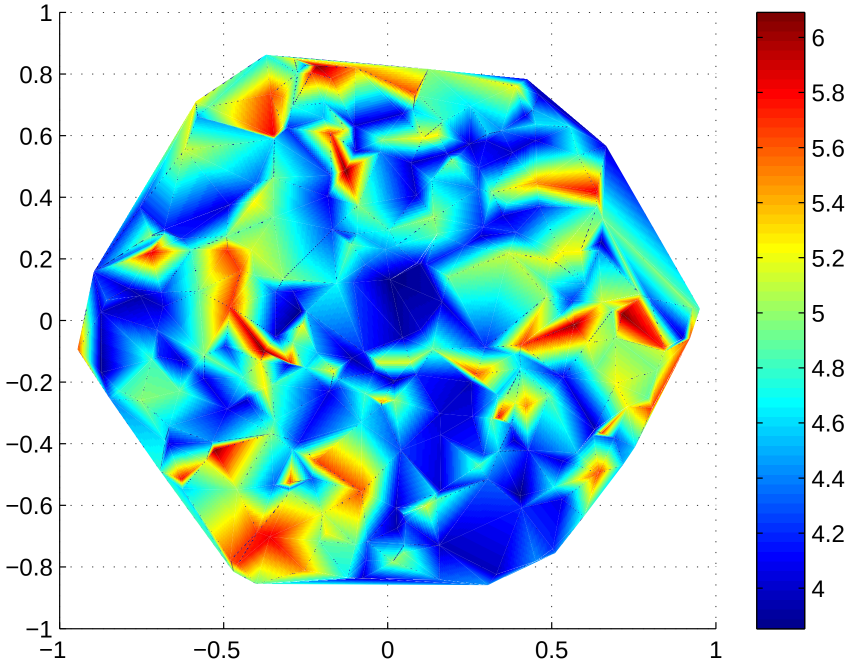
Evaluation with round robin scheduling

Rate performance without cooperation (single cell processing)



Evaluation with round robin scheduling

Rate performance with cooperation (multi cell processing)



Multi-cell multiplexing: Conclusions

- Significant rate gains
- Does not give significant advantage for edge-of-cell users, unless hard fairness is enforced.
- Easy to implement for small subnets (2 cells)
- More than 2 cells cooperating may be difficult due to inter-cell CSI overhead
- Distributed solutions preferred to get scalability.

Multicell MU-MIMO: distributed implementations?

Distributed multicell MU-MIMO

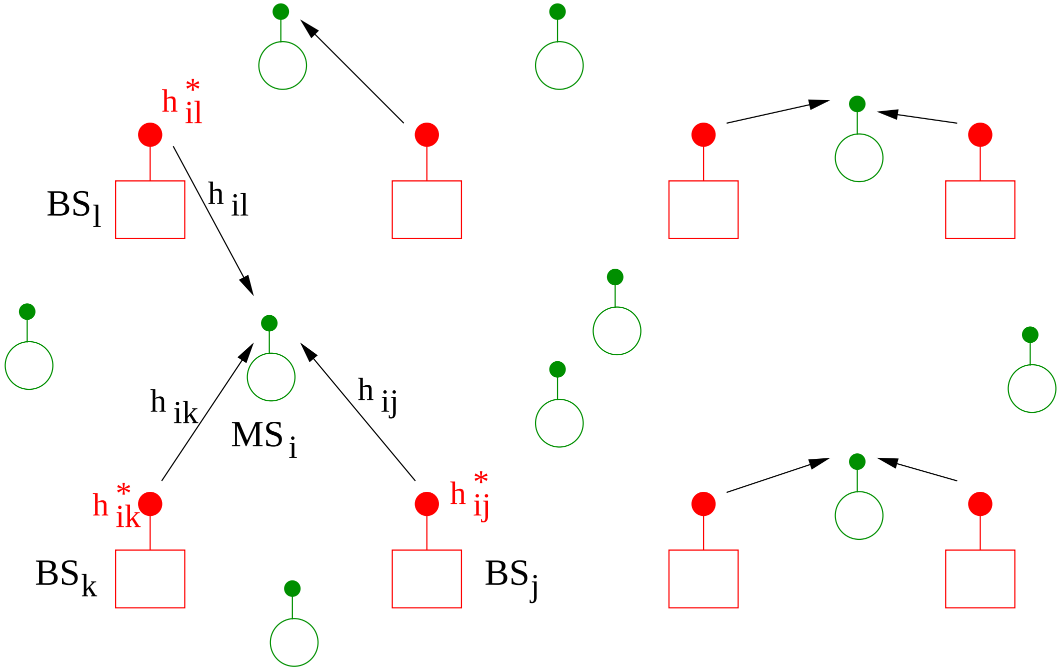
Idea 1: The optimization of transmission in cell n is done based on locally available *instantaneous* information, and external *statistical* information.

Idea 2: Maximum ratio combining lends itself naturally to distributed implementation.

Proposed scenario [Skjevling 07]:

- BTS performs distributed MRC-based beamforming based on local instantaneous channel phase compensation
- Selects exactly one user in the network
- One user may be selected by several bases
- BTS selects best user based on conditional expected system capacity, using statistics of non-local channels.

Multi-cell Maximum-Ratio-Combining



Network seen as a graph

N base stations (BS), U mobile stations.

Definition: a scheduling graph, given by the $U{\times}N$-sized matrix $\boldsymbol{G} = [\boldsymbol{g}_1\boldsymbol{g}_2\ \ldots\ \boldsymbol{g}_N]$, $\boldsymbol{g}_j = [g_{1j}\ g_{2j}\ \ldots\ g_{Uj}]^T$.

The set of *feasible* scheduling graphs $\mathcal{S}_{\boldsymbol{G}}$ include all $\boldsymbol{G}$ for which each column contains a single non-zero element.

$$\mathcal{S}_{\boldsymbol{G}} = \{\boldsymbol{G}\ :\ \boldsymbol{g}_j \in \boldsymbol{e}_i,\ i \in \{1,2,\ldots,U\},\ j \in \{1,2,\ldots,N\}\},$$

such that

$$g_{ij} = \begin{cases} 1 & \text{if BS}_j \text{ transmits to MS}_i\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Maximizing the sum capacity

The network sum capacity is

$$C(\boldsymbol{G},\boldsymbol{H}) = \sum_{i=1}^{U} \log_2\Big(1 + \mathsf{SINR}_i(\boldsymbol{G},\boldsymbol{H})\Big),$$

where the SINR of user i is

$$\mathsf{SINR}_i(\boldsymbol{G},\boldsymbol{H}) = \frac{\left(\sqrt{P}\sum_{j=1}^{N} g_{ij}|h_{ij}|\right)^2 \sigma_s^2}{\sqrt{P}\sum_{k=1}^{U}\left|\sum_{j=1}^{N} h_{ij}g_{kj}h_{kj}^*/|h_{kj}|\right|^2 \sigma_s^2 + \sigma_n^2}.$$

Distributed B/F but **centralized** scheduling:

$$\boldsymbol{G}_{\mathsf{pref}} = \arg\max_{\boldsymbol{G}\in\mathcal{S}_{\boldsymbol{G}}} \{C(\boldsymbol{G},\boldsymbol{H})\}$$

Distributed scheduling

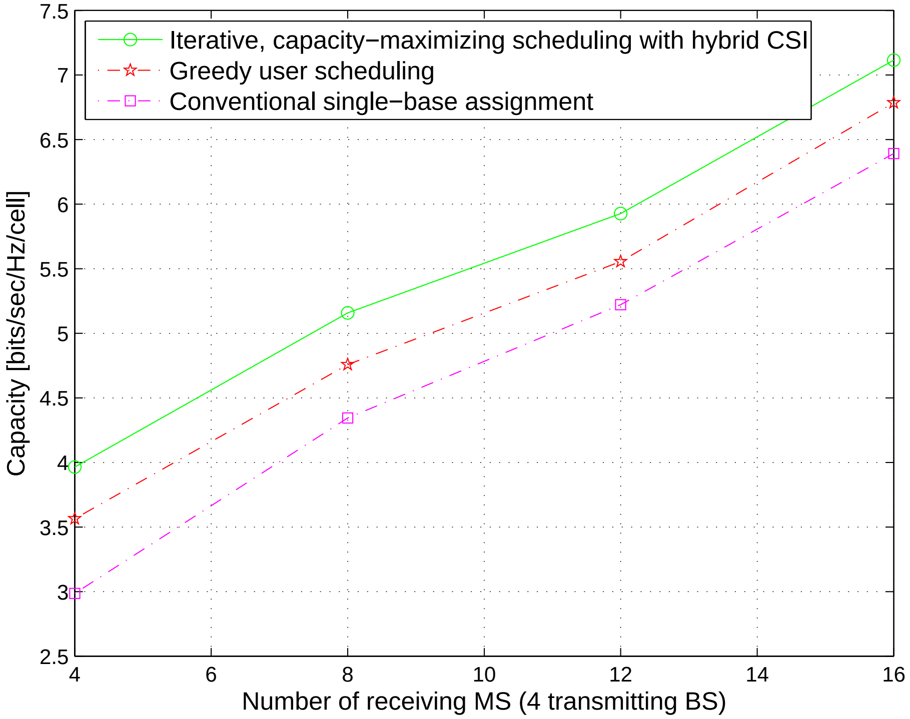
Start from an initial graph $\boldsymbol{G}$.

Next, in a given order, each BS$_j$ updates its corresponding vector in the scheduling matrix:

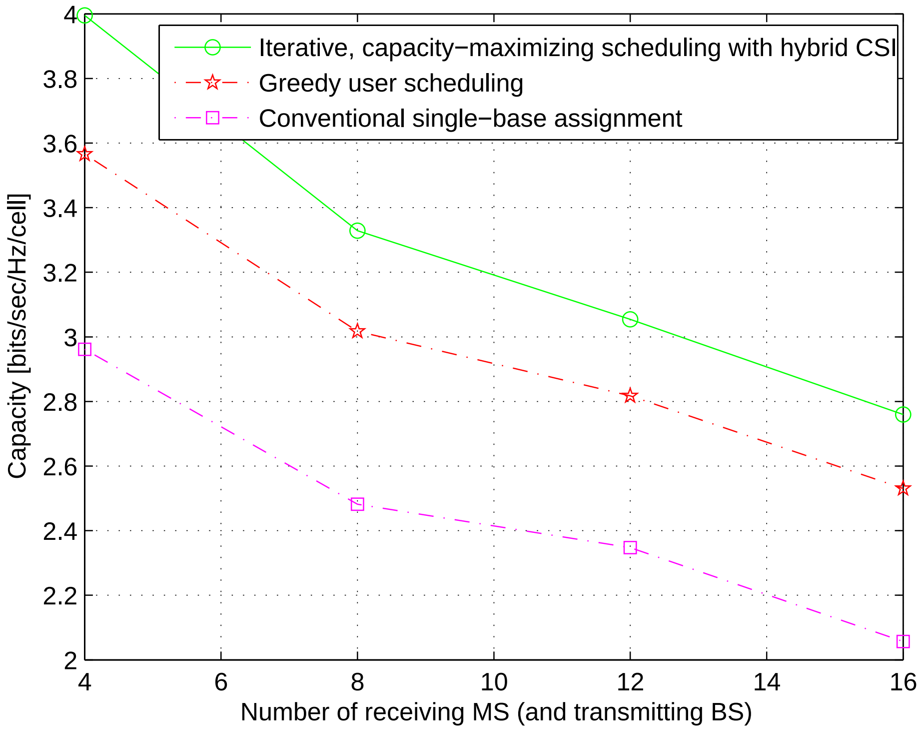
$$(\boldsymbol{g}_j)_{\text{pref}} = \arg\max_{\boldsymbol{g}_j \in \boldsymbol{e}_i} \mathbb{E}_{\boldsymbol{h}_l}\left\{C(\boldsymbol{G}, \boldsymbol{H})\right\},$$

where $\mathbb{E}_{\boldsymbol{h}_l}$, $l \in \{1, 2, \ldots, N\} \setminus j$, reflects that BS$_j$ only has local, instantaneous channel state information.

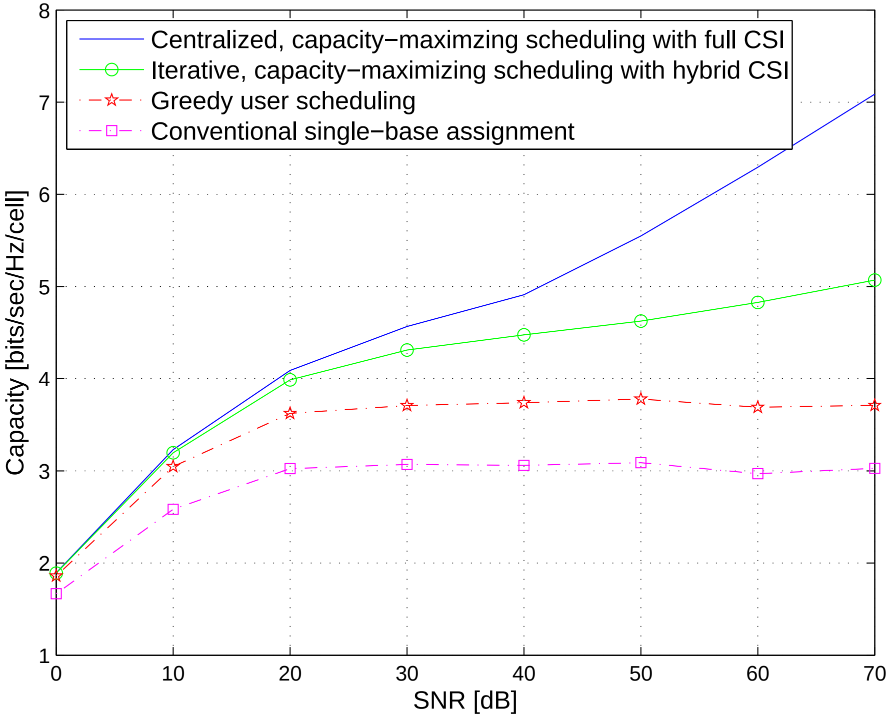
Performance vs. number of mobiles



Performance vs. number of mobiles and cells



Performance vs. SNR



Outline

- General information
- Background on MIMO
- Essential results for MU-MIMO networks
- Living with partial channel knowledge
  - An important example: random opportunistic beamforming
- Multi-cell MU-MIMO: Key concepts and preliminary results
- Perspectives

Perspectives

- Advantages of MU-MIMO over SU-MIMO are substantial.
- Main challenges are extra complexity (computation, cross-layer protocols), extra overhead signaling
- Many techniques for feedback reduction. Optimal approach still open.
- Design of robust schemes (to feedback errors) important.
- Random beamforming works well if associated with additional limited CSI in case of low nb of users
- Multi-cell MU-MIMO gives additional gains at cost of inter-cell signaling overhead
- Design of distributed schemes for multi-cell pretty much open.

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