INTERFERENCE ALIGNMENT IN THE PARTIALLY CONNECTED K-USER MIMO INTERFERENCE CHANNEL

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ABSTRACT

We consider interference alignment in the partially connected K-user MIMO interference channel (IC). Conversely to the fully-connected case, we show that interference alignment can be achievable for an arbitrary number of users K in the network, while the per-user signaling dimension remains fixed, provided that the number of interference links per user is bounded. For this class of channels, which we denote by L-interfering K-user MIMO IC, we provide a criterion applicable to symmetric systems for the system of IA equations to be proper, according to the framework introduced earlier by Yetis et al. Properness is a necessary condition for IA to be feasible. Interestingly, this criterion is independent from the number of users K. Furthermore, we propose an iterative algorithm to solve the alignment problem for this class of channels.

I. INTRODUCTION

Interference Alignment (IA) over the K-user interference channel (IC) was introduced in [1]. The method, based only on linear precoding at the transmitters and zero-forcing at the receivers, enables the simple removal of interference through zero-forcing filtering, thanks to the alignment of all interfering signals in the same subspace from the point of view of each receiver. IA was shown in [1] to achieve almost surely a sum-rate multiplexing gain of $\frac{K}{2}$ per time, frequency and antenna dimension. In comparison, independent operation of K *isolated* point-to-point links would incur a sum-rate multiplexing gain of K per dimension. This indicates that IA allows virtually interference-free communications, at the cost of halving the multiplexing gains with respect to what the users could achieve over isolated point-to-point links.

In the K-user Gaussian MIMO IC, under mild assumptions on the distribution of the channel coefficients, the existence with probability 1 of a solution to the IA problem depends only on the dimensions of the problem (number of users K and number of antennas at each node). The existence of an IA solution was considered in [2], where the notion of a proper system of equation provided a necessary condition on the size of the channel matrices for the almost sure (a.s.) existence of a solution. This criterion was later shown in [3] and [4] to be sufficient as well under certain conditions. In particular, properness has been shown to ensure a.s. feasibility in the case of square channels for $K > 3$ in [3], and for symmetric systems when all users seek to achieve the same DoF, and this number evenly divides the number of antennas at all nodes [4].

An iterative algorithm was introduced in [5] to find numerically the precoding matrices achieving IA. Closed-form solutions are available for certain particular cases (e.g. when all nodes have $N = K - 1$ antennas, see [6]). An extension of IA to the case where the interference-free subspace at the receiver is strictly larger than the dimension of the signal to decode (enabling receive diversity) was introduced in [7].

One important consequence of the achievability results from [2], [7] is that, in the case of the K-user MIMO IC with a fixed number of antennas at each node, IA is only achievable among a finite number of users K, because of the finite number of degrees of freedom offered by the spatial dimension at the transmitter and at the receiver. There exists a few approaches to allow for a scaling of IA across a growing number of links or cells: (i) let the number of signaling dimensions (antennas, or time in the case of the compound channel) grow arbitrarily large with the number of cells, (ii) apply a power control scheme, which effectively reduces the number of active nodes to preserve the feasibility of IA over the network, or (iii) assume a network-wise channel model exhibiting a limited connectivity between the various nodes.

Although interesting from an analytical point of view, approach (i) is not realistic in practice. Approach (ii) is more practical and was considered e.g. in [8], nevertheless, power control results in deactivating certain links and goes at the expense of the total multiplexing gain of the network.

This paper considers the situation (iii) in which natural attenuation effects (path loss, fading) cause the (at least partial) loss of connectivity between certain receivers and interfering transmitters. This scenario arises for instance in the cellular network context where the distance between non-neighboring cells causes a strong attenuation on far-away interference signals. On-off models are typically considered in order to approximate this situation, in which each receiver is assumed to be receiving non-zero interference from a bounded set of transmitters only. This model is referred to as *partially connected IC* in the following.

The partially connected MIMO IC has been considered in [9], where an achievable scheme is proposed for certain problem dimensions. We revisit this channel model, and show that under mild conditions on the connectivity of the interference links in the model, IA can be feasible among an arbitrary number of users while keeping the signaling dimension bounded. Specifically, our key contributions are as follows:

- We extend the properness criterion of Yetis et al. [2], to the case of the symmetric L-interferer K-user MIMO IC (see Definition 1 below), which is a special scenario of partially connected K-user MIMO IC.
- We show that 1 degree of freedom (DoF) per user is achievable through alignment among K users having each 2 transmit and 4 receive antennas, and where each user has 4 interferers, irrespective of K. This means that the total DoF scales unbounded with K. Note also that this represents exactly one half of the achievable interference-free DoF. This results is a concrete example of a network where the number of links can scale unbounded while interference can be ideally cancelled from IA despite the finite number of antennas.
- We provide an algorithm for how to compute precoding matrices and receiver beamforming vectors that achieves good performance, and, at convergence, realizes IA over the network.

II. THE L-INTERFERING K-USER MIMO IC

Let us introduce the L-interfering K-user MIMO IC. This model is a special case of the partially connected K-user MIMO IC model, where each user receives interference only from a limited subset of the K transmitters, and where each transmitter interferes with a limited subset of receivers (see Fig. 1). Formally, for any $k \in \{1,\ldots,K\}$, let $\mathrm{I}(k) \subset \{1,\ldots,k-1,k+1,\ldots,K\}$ denote the set of transmitters which interfere with receiver k. We also let $\mathrm{I}^{-1}(l) = \{k / l \in I(k)\}$ for $l \in \{1,\ldots,K\}$, i.e. $\mathrm{I}^{-1}(l)$ is the set of receivers which are affected by interference from transmitter l. In the example pictured in Fig. 1, $\mathrm{I}(1) = \{2\}$ while $\mathrm{I}^{-1}(1) = \{2,3\}$. For simplicity, we assume that all channels have the same dimension, i.e. all transmitters and receivers are equipped respectively with N and M antennas.

In the partially connected K-user MIMO IC, the M-dimensional signal at receiver k is therefore

$$\mathbf{y}^{(k)} = \mathbf{H}^{(k,k)}\mathbf{x}^{(k)} + \sum_{l\in \mathrm{I}(k)} \mathbf{H}^{(k,l)}\mathbf{x}^{(l)}, \qquad (1)$$

where $\mathbf{x}_k^{(l)}$ denotes the N-dimensional vector signal at transmitter l, and $\mathbf{H}^{(k,l)}$ is a $M \times N$ matrix representing the channel between transmitter l and receiver k.

We now introduce the model used in the remainder of this work:

Definition 1 (L-interfering K-user MIMO IC): The partially connected K-user MIMO IC described above is *L-interfering* for some $L < K$, iff

$$\forall k \in \{1,\ldots,K\}, \quad |\mathrm{I}(k)| \le L \quad \text{and} \quad \left|\mathrm{I}^{-1}(k)\right| \le L, \qquad (2)$$

where $|\cdot|$ denotes the cardinality operator.

II-A Interference Alignment

We are concerned with the feasibility of IA over the L-interfering K-user MIMO IC introduced above. We define alignment as follows: assume that each user is transmitting a signal of rank $D < N$, and let $\mathbf{V}^{(l)}$ denote the $N \times D$ precoding matrix at transmitter l. We can write $\mathbf{x}^{(l)} = \mathbf{V}^{(l)}\mathbf{s}^{(l)}$ where $\mathbf{s}^{(l)}$ is a vector containing D transmitted symbols.

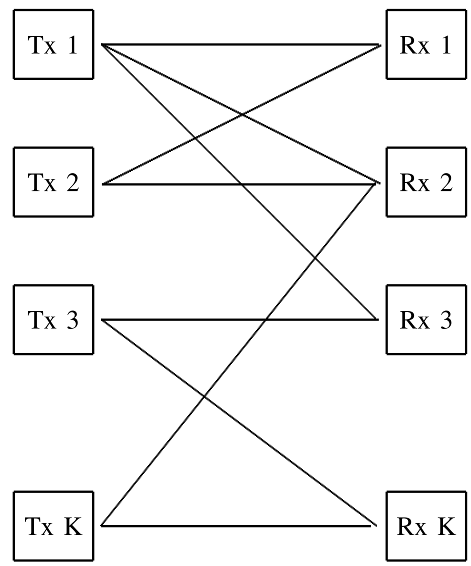

Fig. 1. Example connectivity graph of the L-interfering K-user IC. Lines represent channels with non-zero coefficients.

We wish all interference at receiver k to be restricted to a subspace of dimension $M - D'$, where $D' < M$ denotes the dimension of the interference-free subspace[1]. This is equivalent to finding a $M \times D'$ projection matrix $\mathbf{U}^{(k)}$ which suppresses all interference, i.e. such that ${\mathbf{U}^{(k)}}^H \sum_{l\in \mathrm{I}(k)} \mathbf{H}^{(k,l)}\mathbf{V}^{(l)}\mathbf{s}^{(l)} = 0$. Since this must be true for all values of $\mathbf{s}^{(l)}$, this yields the following definition of IA:

Definition 2: IA with parameters (D,D') is achieved in the L-interfering K-user MIMO IC iff there exist full column-rank $M \times D'$ matrices $\mathbf{U}^{(k)}$, $k = 1,\ldots,K$, and $N \times D$ matrices $\mathbf{V}^{(l)}$, $l = 1,\ldots,K$, s.t.

$$\forall k \in \{1,\ldots,K\},\ \forall l \in \mathrm{I}(k),\ {\mathbf{U}^{(k)}}^H \mathbf{H}^{(k,l)}\mathbf{V}^{(l)} = 0. \qquad (3)$$

II-B Achievability of IA over the L-Interfering IC

We now investigate the achievability of IA over the considered channel using the framework introduced in [2]. The method is based on the comparison of the number of variables and constraints in the system of IA equations (3). Let us first paraphrase the definition of a *proper* system of equations from [2]:

Definition 3 (Proper system): A system of equations is proper iff, for any subset of its equations, the number of variables involved is at least as large as the number of equations.

As outlined in [2], in a proper system, and in the absence of particular structure in the channel coefficients, IA is achievable almost surely (a.s.) at least for the case $D =$

[1] Note that this is a generalization of the original definition of IA [1] in which $D' = D$. Clearly, the DoF achievable per user under this scheme can not be higher than $\min(D', D)$. However, in cases where DoF maximization is not the sole objective, choosing $D' > D$ might be desirable [7]. The achievable DoF is considered in more detail in Section II-C.

$D'=1$ (see the remarks in [2, Section VII]). For $D>1$ or $D'>1$, having a proper system of IA equations is sufficient to guarantee the achievability of IA under certain conditions – see the recent results on sufficiency of this condition in [4] and [3]). It is therefore important to characterize whether a considered system is proper in order to assess the achievability of IA. For the fully connected K-user interference channel treated in [2], it has been shown that the amount of equations involved in (3) is the limiting factor for the system to be proper. Indeed, in the fully connected MIMO IC, $|\mathrm{I}(k)| = K-1\ \forall k$, and the number of equations in (3) scales quadratically with K. Conversely, in the case of the L-interfering K-user MIMO IC, the constraint in (2) ensures that the number of scalar equations involved in (3) scales linearly with K. Intuitively, this means than the feasibility conditions for IA will be relaxed in the L-interfering K-user MIMO IC, in particular for large K.

We next give formal feasibility conditions for the system of IA equations (3) to be proper.

Theorem 1: The system of equations (3) is proper if

$$D(N-D)+D'(M-D')-LDD' \geq 0. \tag{4}$$

Furthermore, (4) is also a necessary condition if $\forall k \in \{1,\ldots,K\}, \quad |\mathrm{I}(k)| = L$.

Proof: We first prove that (4) is necessary in the case $|\mathrm{I}(k)| = L\ \forall k$. For this, we consider the total number of equations and variables involved in (3). The number of distinct tuples (k,l) involved in (3) is trivially KL, and each one of the matrix equalities contains $D' \times D$ scalar equations. This yields a total of $N_e = KLDD'$ equations. The total number of variables N_v in the $\mathbf{V}^{(l)}$ and $\mathbf{U}^{(k)}$ matrices must be counted while paying attention to the fact that multiple parameterizations of the same choice of a subspace are possible, and must be counted only once. As shown in [2], each $\mathbf{V}^{(l)}$ must be counted as $D(N-D)$ variables, while each $\mathbf{U}^{(k)}$ represents $D'(M-D')$ variables. Therefore, we have $N_v = KD(N-D)+KD'(M-D')$. Notice now that if (4) is not fulfilled, we have immediately that $N_v < N_e$, i.e. the system is not proper. Therefore, (4) is necessary.

We now prove the sufficient part. Due to the lack of symmetry in the interfering connections as defined by $I(k)$, comparing N_v and N_e is not sufficient to guarantee that the system is proper. Therefore, we have to check that the inequality between number of equations and variables is verified for all possible subsets of equations. Let us introduce some formalism. Let $\mathrm{S} = \{(d',k,l,d) \in \{1,\ldots,D'\} \times \{1,\ldots,K\} \times\{1,\ldots,K\} \times \{1,\ldots,D\} \text{ s.t. } l \in \mathrm{I}(k)\}$. Each tuple in S corresponds to one scalar IA equation from eq. (3). Let $\mathrm{A} \subset S$ an arbitrary subset of S. Let N_v^{A} denote the number of variables involved in any of the equations designated by A, and $N_e^{\mathrm{A}} = |\mathrm{A}|$ the number of those equations. We need to prove that $N_v^{\mathrm{A}} \geq N_e^{\mathrm{A}}$.

We need the following definitions:

$$\overline{\mathrm{K}} = \left\{k \text{ s.t. } \exists(d',l,d) \text{ s.t.} (d',k,l,d) \in A\right\}, \tag{5}$$
$$\overline{\mathrm{L}} = \left\{l \text{ s.t. } \exists(d',k,d) \text{ s.t.} (d',k,l,d) \in A\right\}, \tag{6}$$
$$\overline{\mathrm{D}}(l) = \left\{d \text{ s.t. } \exists(d',k) \text{ s.t.} (d',k,l,d) \in A\right\}, \tag{7}$$
$$\overline{\mathrm{D}'}(k) = \left\{d' \text{ s.t. } \exists(l,d) \text{ s.t.} (d',k,l,d) \in A\right\}, \tag{8}$$
$$\overline{\mathrm{KL}} = \left\{(k,l) \text{ s.t. } \exists(d',d) \text{ s.t.} (d',k,l,d) \in A\right\}. \tag{9}$$

Intuitively, $\overline{\mathrm{K}}$ is the set of indices c which appear in at least one tuple in A; $\overline{\mathrm{D}}(l)$ is the set of indices d which appear in at least one tuple in A together with l; etc.

Using these definitions, the number of variables involved in the beamformer at transmitter l is $\left|\overline{\mathrm{D}}(l)\right|\left(N-\left|\overline{\mathrm{D}}(l)\right|\right)$, while the number of variables involved in the projection filter at receiver k is $\left|\overline{\mathrm{D}'}(k)\right|\left(M-\left|\overline{\mathrm{D}'}(k)\right|\right)$. We have therefore

$$\begin{aligned} N_v^{\mathrm{A}} &= \sum_{k\in\overline{\mathrm{K}}} \left|\overline{\mathrm{D}'}(k)\right|\left(M-\left|\overline{\mathrm{D}'}(k)\right|\right) \\ &\quad + \sum_{l\in\overline{\mathrm{L}}} \left|\overline{\mathrm{D}}(l)\right|\left(N-\left|\overline{\mathrm{D}}(l)\right|\right) \qquad (10) \\ &\geq \sum_{k\in\overline{\mathrm{K}}} \left|\overline{\mathrm{D}'}(k)\right|(M-D') + \sum_{l\in\overline{\mathrm{L}}} \left|\overline{\mathrm{D}}(l)\right|(N-D) \qquad (11) \end{aligned}$$

since the cardinalities of $\overline{\mathrm{D}}(l)$ and $\overline{\mathrm{D}'}(k)$ are upper bounded respectively by D and D' by definition of those sets.

Let us now fix k and l, and consider the tuples (d',k,l,d) that appear in A. Clearly there are at most $\left|\overline{\mathrm{D}}(l)\right|\left|\overline{\mathrm{D}'}(k)\right|$ such tuples. Therefore, summing over all possible (k,l),

$$|\mathrm{A}| \leq \sum_{(k,l)\in\overline{\mathrm{KL}}} \left|\overline{\mathrm{D}}(l)\right|\left|\overline{\mathrm{D}'}(k)\right|. \tag{12}$$

Since $\left|\overline{\mathrm{D}'}(k)\right| \leq D'\ \forall k$, we have

$$\begin{aligned} |\mathrm{A}| &\leq \sum_{(k,l)\in\overline{\mathrm{KL}}} \left|\overline{\mathrm{D}}(l)\right| D' \qquad (13) \\ &\leq \sum_{l\in\overline{\mathrm{L}}} \left|\mathrm{I}^{-1}(l)\right|\left|\overline{\mathrm{D}}(l)\right| D' \qquad (14) \\ &\leq \sum_{l\in\overline{\mathrm{L}}} L\left|\overline{\mathrm{D}}(l)\right| D' \qquad (15) \end{aligned}$$

where (14) stems from the fact that $(k,l) \in \overline{\mathrm{KL}}$ implies $k \in \mathrm{I}^{-1}(l)$, and that $\mathrm{I}^{-1}(l)$ has at least as many elements as its restriction to those appearing in A. (15) stems directly from Definition 1. Starting again from eq. (12), and bounding $\left|\overline{\mathrm{D}}(l)\right|$ instead of $\left|\overline{\mathrm{D}'}(k)\right|$, we obtain symmetrically

$$|\mathrm{A}| \leq \sum_{k\in\overline{\mathrm{K}}} LD\left|\overline{\mathrm{D}'}(k)\right|. \tag{16}$$

Combining (11), (15) and (16) yields

$$N_v^{\mathrm{A}} \geq |\mathrm{A}|\left(\frac{M-D'}{LD} + \frac{N-D}{LD'}\right). \tag{17}$$

Finally, we note that the condition (4) ensures that the second term in the right-hand side of (17) is greater or equal to 1, yielding $N_v^{\mathrm{A}} \geq |\mathrm{A}| = N_e^{\mathrm{A}}$.

□

Remark 1: Note that condition (4) is *independent of K*. This means that we can potentially let the number of users K grow unbounded, and achieve IA all the while keeping the number of antennas at each node fixed. This is in sharp contrast with previous feasibility results obtained for IA, where at least one of the signaling dimensions had to grow unbounded with K. Note that this property stems directly from our assumption on the limited number of outgoing interfering links at the transmitters, and incoming interfering links at the receivers.

II-C DoF analysis of IA on the L-interfering MIMO-IC

Using the IA transmission scheme outlined in Section II-A, i.e. after transmission along the beamforming vectors $\mathbf{V}^{(l)}$, and zero-forcing of the interference using $\mathbf{U}^{(k)}$, the channel available to user k is ${\mathbf{U}^{(k)}}^H\mathbf{H}^{(k,k)}\mathbf{V}^{(k)}$. The DoF achievable by this user is equal to the rank of this matrix. Furthermore, under mild assumptions on the channel fading statistics (e.g. if all channel coefficients are selected independently from a continuous distribution such as the Gaussian one – see [5]), this matrix can be shown to be full rank a.s. Therefore, each user achieves $\min(D,D')$ DoF. Taking $D'=D$, eq. (4) yields $D \leq \frac{M+N}{L+2}$. Furthermore, since $D \leq N$ and $D' \leq M$, the DoF per user achievable using IA is $\min\left(M,N,\frac{M+N}{L+2}\right)$, independently of K. In comparison, the DoF per user achievable in the interference-free case would be $\min(M,N)$.

Note that this indicates that if either $M \geq (L+1)N$ or $N \geq (L+1)M$, the interference-free DoF is achievable. However, this is trivial since this can be achieved using a simple scheme whereby the L+1 interferers (or interferees) are zero-forced.

Cases where $\frac{M+N}{L+2} < \min(M,N)$ are less trivial but potentially more interesting: for instance, for $N=2$ transmit and $M=4$ receive antennas per user, and $L=4$ interferers per users, IA with $D=1$ DoF per user is achievable *regardless of the number K of users*. In that case, the 1 DoF per user achieved by IA represents half of the interference-free DoF ($\min(M,N)=2$), whereas without alignment, interference would occupy the whole receive space since $DL=M$.

III. SIMULATIONS

In order to experimentally verify the feasibility of IA as outlined in Section II-A, we introduce now an algorithm to compute the matrices $\mathbf{V}^{(k)}$ and $\mathbf{U}^{(k)}$ for the L-Interfering K-user MIMO IC . The following algorithm is adapted from the one in [5]:

Algorithm 1 Iterative computation of the IA solutions for the L-Interfering K-user MIMO IC

initialize the $\mathbf{V}^{(k)}$ by truncating independent Haar-distributed matrices to D columns.
repeat
for $k \in \{1,\ldots,K\}$ **do**

$$\mathbf{U}^{(k)} \leftarrow \mathrm{EVmin}_{D'}\left(\sum_{l\in \mathrm{I}(k)} \mathbf{H}^{(k,l)}\mathbf{V}^{(l)}{\mathbf{V}^{(l)}}^H{\mathbf{H}^{(k,l)}}^H\right)$$

end for
for $l \in \{1,\ldots,K\}$ **do**

$$\mathbf{V}^{(l)} \leftarrow \mathrm{EVmin}_{D}\left(\sum_{k\in \mathrm{I}^{-1}(l)} {\mathbf{H}^{(k,l)}}^H\mathbf{U}^{(k)}{\mathbf{U}^{(k)}}^H\mathbf{H}^{(k,l)}\right)$$

end for
until $\max_{k\in\{1,\ldots,K\}} \sum_{l\in \mathrm{I}(k)} \left|\left|{\mathbf{U}^{(k)}}^H\mathbf{H}^{(k,l)}\mathbf{V}^{(l)}\right|\right|^2 \leq \varepsilon.$

The $\mathrm{EVmin}_n(\cdot)$ operator denotes the selection of the eigenvectors associated with the n eigenvalues of lowest magnitude. ε is a small value below which the interference leakage is deemed negligible. The difference with the algorithm in [5, Section V] lies in the sets over which the sums are taken in the above algorithm.

Remark 2: Note that for any $k \in \{1,\ldots,K\}$, Algorithm 1 only requires channel and precoding/interference suppression matrix information for the users in $\mathrm{I}(k)$ and $\mathrm{I}^{-1}(k)$. The algorithm is naturally distributed, and global channel state information knowledge is not required at any point of the network. Note also that, for the same reason, its global complexity per iteration scales linearly with K, i.e. the complexity per user and per iteration of the main loop remains constant, regardless of the size of the network.

We verified that the algorithm converges reliably to an IA solution, provided that it is known to exist. When no solution exists, the algorithm converges to a fixed point which does not achieve the target maximum interference leakage ε. Figure 2 shows a histogram of the convergence times (in terms of the number of iterations of the outer loop to reach $\varepsilon = 10^{-3}$, with Gaussian i.i.d. channel coefficients with unit power) of the algorithm for the case of the L-Interfering K-user MIMO IC with $K=15$ users with 4×2 channel matrices, $|I(k)| = |I^{-1}(k)| = 4\ \forall k$, and $D=D'=1$. According to Theorem 1 this system is proper, and therefore IA is feasible a.s. according to [2]. Indeed, Algorithm 1 converges reliably to an IA solution. The mean convergence time is 794 iterations.

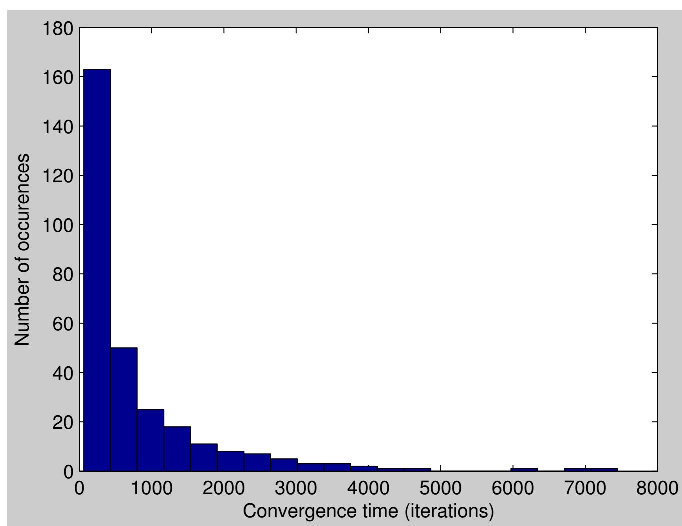

Fig. 2. Convergence times for IA over the 15 user L-Interfering MIMO IC with 4×2 antennas per user, histogram over 300 channel realizations.

IV. CONCLUSION

We have considered IA in a particular case of the K-user MIMO partially connected interference channel (IC), namely the L-interferer K-user MIMO IC. We have shown that, under a mild condition on scaling on the number of interference links in the network, interference alignment can be achievable among an arbitrary, potentially infinite number of users, while the per-user signaling dimension (i.e.

the dimension of the MIMO channels) remains bounded. We provided a necessary and sufficient criterion for the system of IA equations to be proper, as well as an iterative algorithm capable of solving the alignment problem when it is feasible.

V. ACKNOWLEDGEMENTS

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