EXACT SPHERE CONSTRAINED MAXIMUM LIKELIHOOD DETECTION OF PARAMETERS FOR CDMA

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ABSTRACT

We describe a method for Maximum Likelihood (ML) parameter estimation corrupted by additive white Gaussian noise. The ML cost function is maximized over the constraint that the detected data vector lie on the sphere. The results are compared with MMSE and with [2]. Simulations results shows superior performance of our method comparing to both the methods.

1. INTRODUCTION

In CDMA system all resources are in principle available to all users simultaneously. The users are distinguished from each other by user specific signature sequences, modulating the transmitted data sysmbols using direct sequence spread spectrum techniques. In the past many iterative techniques have been considered. Talwar et al [1] proposed iterative least square with enumeration (ILSE), which solves the problem by estimating the channel by short training sequence or from previous estimate and find data sequence over all possible data in the finite alphabet (FA). They also proposed iterative least square with projection (ILSP), which also initially estimates the channel with the same method as for ILSE and treats the problem as continous optimization problem and projects the result onto the closest discrete alphabet. In [3], a constrained Maximum Likelihood problem was considered with the data vector to lie with in hyper cube and called it as Box constrained ML. Similarly, they also proposed problem of maximizing likelihood function over sphere i.e. confine the solution vector to lie within the sphere and project the solution vector on the sphere. Infact, in the sphere constrained problem the solution vector lies on the sphere and not in the interior of the constraining sphere (as is done in [2]). The other problem with their method is that small error in the solution vector can cause large error when projected on to sphere (provided the solution vector is well inside the sphere). In this paper we constrain the solution vector to lie on the sphere and estimate data vector.

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2. SIGNAL MODEL

In this section, discrete-time base band uplink signal model for CDMA communication system is described. We consider asynchronous CDMA with single path channels. The signal is corrupted by the presence of the additive white Gaussian noise (AWGN) with zero mean and variance $\frac{N_o}{2} = \sigma^2$. The number of users in the system are assumed to be K. The processing gain, $N = T_d/T_c$, where T_d is symbol duration and T_c is the chip duration. The users transmit binary information symbol stream $d_k(n) \in \{-1, 1\}$, $n = 0, 1, ..., L-1$ is symbol interval index and L is the length of the data block. $s_k(n) = (s_k(nN+1).......s_k(n+1)N)^T$ with $s_k(i) \in (-1/\sqrt{N}, 1/\sqrt{N})$ is the spreading code of the user k to modulate n^{th} bit. The received base band signal can be written as

$$r = \sum_{i=0}^{L-1} \sum_{k=1}^{K} d_k(i) \begin{pmatrix} 0_{iN+\tau_k} \\ s_k(i) \\ 0_{(L-i)N-\tau_k-1} \end{pmatrix} + n \quad (1)$$

The convenient matrix notation is given by

$$r = Sd + n \quad (2)$$

where data symbol vector is given by $d = (d_1(0), d_2(0), ..., d_K(L-1))^T = (d_1, d_2, ..., d_{LK})^T$. S is the matrix of transmitted waveforms with the column j expressed as

$$s_j = \begin{pmatrix} 0_{iN+\tau_k} \\ s_k(i) \\ 0_{(L-i)N-\tau_k-1} \end{pmatrix} \quad (3)$$

A minimal set of sufficient statistics of dimension LK is obtained through correlation, matched to the received signal. This also ensures the maximization of the SNR, i.e,

$$y = S^T r = S^T Sd + S^T n = Rd + z \quad (4)$$

where R is the correlation matrix and z is zero mean Gaussian vector with covariance $\sigma^2 R$.

3. SPHERE CONSTRAINED ML

Given the set of data $y \in R^{LK}$, our goal is to find parameters that miximize the $log P(y|\theta)$ or minimize the negative of it. The negative loglikelihood function of y is given by

$$l(d) = d^T R d - 2y^T d \tag{5}$$

The sphere constrained ML problem for the asynchronous CDMA is then described as

$$d = arg_{min_d} d^T R d - 2y^T d \tag{6}$$

subject to $d^T d = LK$. The Lagrangian function associated with the above problem can be written as

$$L(d, \lambda) = d^T R d - 2y^T d + \lambda(d^d - LK) \tag{7}$$

To calculate the stationary points, we differentiate $L(d, \lambda)$ with respect to d and λ. The solution of the above problem is given by

$$d = (R + \lambda I)^{-1} y \tag{8}$$

Now the problem is to find the Lagrange multiplier. Using the quadratic constraint we can write

$$y^T (R + \lambda I)^{-2} y = LK \tag{9}$$

We proceed by computing computing the eigenvector decomposition oh matrix R.

$$f(\lambda) = y^T (U \Sigma U^T + \lambda I)^{-2} - LK \tag{10}$$

Which can further be written as

$$f(\lambda) = \sum_i \frac{z_i^2}{(\lambda + \sigma_i)^2} - LK = 0 \tag{11}$$

where $z_i = (U^T y)_i$, σ_i are the eigenvalues of R with $\sigma_1 < \ldots < \sigma_n$, and U are the eigenvectors of R. $f(\lambda)$ can be solved iteratively using Newton-Raphson method to finf its zero. We choose λ such that $R + \lambda I$ is positive definite. This selection of λ forces the $f(\lambda)$ to be convex. Now the problem is to find λ for which $f(\lambda)$ is zero. We find the bounds for λ in order to restrict our search to find the zero of $f(\lambda)$. The bounds are given by

$$\lambda \leq \frac{||y||}{LK} - \sigma_1 \tag{12}$$

and

$$\lambda \geq \frac{||y||}{LK} - \sigma_n \tag{13}$$

4. SIMULATIONS

In this section we investigate BER performance based on the simulations. The codes were selected at random and we considered a lightly loaded case with six number of users. The processing gain was kept 32. We simulated for BER for our case. It is clear from the figure that our receiver performs better than MMSE and the receiver proposed in [2] (they have the same performance). The MMSE receiver on average constrain the data vector to lie within sphere [2]. In [2] the authors considered the constraint that the data vector lie within sphere, which is a loose constraint comparing to that of ours. Also we compared this algorithm with the one proposed by the authors [6], which was approximation to the exact sphere constraint. It is clear from the figure that our algorithm outperforms the approximate constraint sphere receiver.

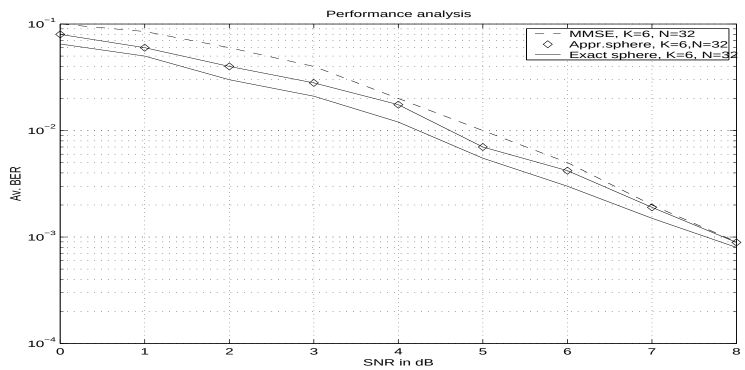

Fig. 1. Av. BER of six users vs $SNR(dB)$.

5. REFERENCES

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