Joint delay and angle of arrival estimation using EM algorithm for multipath signals arriving at antenna array

Ejaz Khan
Mobile Communications Department
Institut Eurécom*
B.P. 193, 06904 Sophia–Antipolis CEDEX, France
Tel: +33 4 93 00 26 76, Fax: +33 4 93 00 26 27
E-mail: firstname.lastname@eurecom.fr

November 22, 2002

Abstract

We propose a novel approach to estimate the angles of arrival (AOA) and delays of the multipath signals from digitally modulated sources arriving at an antenna array. Our method uses a collection of estimates of a space-time vector channel. The method avoids computationally expensive optimization search by using Expectation Maximization (EM) algorithm. The useful behavior of the proposed algorithm is verified by simulations.

1 Introduction

Source localization is an issue of interest in wireless communications. In this system, mobiles emit signals that arrive at the base station via multiple paths with different angle of arrivals and delays, which are in turn used for source (mobile) localization. This paper focuses on jointly estimating AOAs and relative time delays of multipath propagation signals emnating from a single source and received by antenna array. Parametric joint angle/delay estimation has been investigated i.e. see [1],[2] and [3]. In [2] the authors jointly estimates both parameters using MUSIC type algorithm, which though suboptimal, but involves only two dimentional search. In [3], the authors used ESPRIT type algorithm to estimate jointly the parameters. Wireless communications channel can be characterized by slow varying parameters and fast varying parameters. The

*Eurecom's research is partially supported by its industrial partners: Ascom, Cégétel, France Télécom, Hitachi, IBM France, Motorola, Swisscom, Texas Instruments, and Thomson CSF

amplitudes and the relative phase of each path are fast varying and is subject to (Rayleigh) fading. The stationarity of the fading depends on the speed of the mobile. If the mobile is moving fast enough then we can consider fading to be stationary within single time slot and are uncorrelated among slots. In contrast, AOAs and delays does not change over many slots even if mobile is at relatively high speed. Typically, a mobile moving at 30 m/s changes angular position only by $0.1°$ with respect to the base station 3 Kms away. Our method exploits the stationarity of angles and delays, as well as the independence of fading over many time slots. We assume that AOAs and time delays do not change over multiple channel estimates because they are slow varying parameters. We also assume that antenna array response has known structure.

2 Data Model

Consider a single usere case which transmit signal via multiple paths. The received base band signal at an M-element antenna array at time t in the n_{th} interval is $y^n(t) = [y^1(t).......y^M(t)]_T$ which can be written as

$$y^n(t) = \sum_l S_l^n h^n(t - lT) + n^n(t) \tag{1}$$

where T is symbol period and $n^n(t)$ is additive white Gaussian noise. S_l^n is digital transmitted sequence. $h^n(t)$ is the channel. The channel can be modeled as

$$h^n(t) = \sum_i^Q a(\theta_i)\beta_i(n)g(t - \tau_i) \tag{2}$$

where Q are the total number of paths. Each path is parametrized by a DOA θ_i, time delay τ_i and complex path attenuation (fading) $\beta_i(n)$. The array response to the path from direction θ_i is $a(\theta_i)$ and $g(t)$ is known pulse shape (modulation waveform). After sampling at a rate P times the symbol rate. A block formulation is obtained by taking N snapshots, yielding

$$Y^n = H^n S^n + N^n, n = 1, 2...., S \tag{3}$$

The first step is to estimate the channel impulse response from the user to the antenna array. This can be accomplished by using training sequence or using some blind method. Let $\widehat{H^n} = X(n)$ be the estimate of the true channel H^n. We have

$$X(n) = \widehat{H^n} = H^n + V^n \tag{4}$$

where V^n is the estimation noise matrix. H^n can be written as

$$H^n = A(\theta)diag(\beta(n))G^T(\tau) \tag{5}$$

where $A(\theta) = [a(\theta_1)......a(\theta_Q)]$, $\beta(n) = [\beta_1(n).....\beta_Q(n)]$ and $G^T(\tau) = [g^T(\tau_1)......g^T(\tau_Q)]$. Applying vectorization operator to eq(4) and using eq(5) and writing the resulting equation in matrix form as

$$x(n) = U(\theta, \tau)\beta(n) + v(n), n = 1, 2...., N \tag{6}$$

where

$$U(\theta,\tau) = G(\tau) \circ A(\theta) = [u(\theta_1, \tau_1 ......u(\theta_Q, \tau_Q)] \tag{7}$$

$u(\theta,\tau)$ is called space time matrix. It is assumed that the structure of the space time matrix U is known. It is also assumed that path fadings are normally distributed with zero mean. Estimation noise V is white Gaussian. To be able to identify (θ,τ) using eq(6), we need $U(\theta,\tau)$ to be strictly tall and full column rank.

3 EM based JADE

Expectation Maximization (EM) [5], is an iterative approach to Maximum Likelihood (ML) estimation. Each iteration is composed of two steps: an Expectation (E) step and a Maximization (M) step. The aim is to maximize the log likelihood $l(\psi; D)$, where ψ are the unknown parameters of the model and D are the data. Suppose that this optimization problem would be simplified by the knowledge of the additional variable χ, known as missing or hidden data. The set $D_c = D \cup \chi$ is refered to as the complete data set (in the same context D is refered to as incomplete data set). Correspondingly, the loglikelihood function $l_c(\psi; D_c)$ is refered to as complete data likelihood. χ is chosen such that the function $l_c(\psi; D_c)$ would be easily maximized if χ were known. However, since χ is not obsevable, l_c is a random variable and cannot be maximized directly. Thus, the EM algorithm relies on integrating over the distribution of χ, with the auxiliary function $Q(\psi, \hat{\psi}) = E_\chi[l_c(\psi; D_c|D, \hat{\psi}]$, which is the expected value of the complete data likelihood, given the observed data D and the parameter $\hat{\psi}$ computed at the previous iteration. Intuitively, computing Q corresponds to filling the missing data using the knowledge of the observed data and previous parameters. The auxiliary function is deterministic and can be maximized. An EM algorithm iterates the following two steps, for k=1,2,...., until local or global maximum of the likelihood is found.

Expectation: Compute

$$Q(\psi; \psi^{(k)}) = E_\chi[l_c(\psi; D_c|D, \psi^{(k)}] \tag{8}$$

Maximization: Update the parameters as

$$\psi^{(k+1)} = arg_{max_\psi} Q(\psi; \psi^{(k)}), \tag{9}$$

In some cases, it is difficult to analytically maximize $Q(\psi; \psi^{(k)})$, as required by the M-step of the above algorithm, and we are only able to compute a new value $\psi^{(k+1)}$ that produces an increase of Q at each iteration. In this case we have so called Generalized EM (GEM) algorithm. Given the model of eq(6), we are now ready to define the complete data set. Let $z = (x(n), \beta(n))$ be complete data, where $x(n)$ is estimated channel and $\beta(n)$ (hidden data), is fading coefficients. We are now in position to start the derivation of the E-step of the EM algorithm.

the pdf of z as function of u can be written as product between the likelihood and priori distribution of β, i.e.,

$$f(z;u) = f(x(n),\beta(n);u) = f(x(n)|\beta(n),u)f(\beta(n);u) \tag{10}$$

which can be further written as (after omitting $f(\beta(n);u)$ because it does not depend on the parameter u

$$f(z;u) = K_1 exp(\frac{1}{2\sigma^2}(x(n)-u\beta(n))^H(x(n)-u\beta(n))) \tag{11}$$

having multiple channel estimation over slots i.e., $x(1), x(2)....., x(N)$, we have E-step given by

$$Q(u;u^{(k)}) = E(\sum_{n=1}^{N} log f(z;u|x(n);u^{(k)})) \tag{12}$$

which can be further expanded as

$$Q(u;u^{(k)}) = \sum_{n=1}^{N} E((x(n)^H x(n) - x^H u\beta(n) - \beta(n)^H u^H x + \beta(n)^H u^H u\beta(n))|x(n));u^{(k)}) \tag{13}$$

the above equation can be written as after omitting constant term

$$Q(u;u^{(k)}) = \sum_{n=1}^{N}(-x(n)^H u\widehat{\beta(n)} - \widehat{\beta(n)}^H u^H x(n) + Trace(u^H u\widehat{\beta(n)\beta(n)^H}) \tag{14}$$

where $\widehat{\beta(n)} = E[\beta(n)|x(n);u^{(k)}]$, and

$$\widehat{\beta(n)\beta(n)^H} = E(\beta(n\beta(n)^H|x(n);u^{(k)}) \tag{15}$$

where E is expectation operator.The conditional mean will also be Gaussian. The conditional mean and second order moment is given by

$$\widehat{\beta(n)} = R_{bb}u^H x(n) \tag{16}$$

where

$$R_{bb}^{-1} = u^H u + R^{-1}_{\beta(n)\beta(n)} \tag{17}$$

where

$$R_{\beta(n)\beta(n)} = exp(-\beta(n)R_{\beta(n)\beta(n)}\beta(n)) \tag{18}$$

where covariance matrix, $R_{\beta(n)\beta(n)}$ is assumed to be known and

$$\widehat{\beta(n)\beta(n)^H} = R_{bb} + \widehat{\beta(n)}\widehat{\beta(n)^H} \tag{19}$$

The maximization step (updated parameter values) is given by equation the following equations to zero to find the stationary points i.e.,

$$\frac{\partial Q}{\partial \tau}|_\theta = \frac{\partial Q}{\partial u}(\frac{\partial u}{\partial \tau})|_\theta = 0 \tag{20}$$

and

$$\frac{\partial Q}{\partial \theta}|_{\tau} = \frac{\partial Q}{\partial u}(\frac{\partial u}{\partial \theta})_{\tau} = 0 \tag{21}$$

where

$$\frac{\partial Q}{\partial u} = \sum_{n=1}^{N}(2\widehat{\beta(n)}^{H} x(n) + 2\widehat{\beta(n)\beta(n)}u \tag{22}$$

These two equations can be solved iteratively to get updated value of the parameters to be estimated. The above fixed point equations are nonlinear and hence there could be local minima or saddle points. That solution should be chosen which gives maximum value of the likelihood function.

References

[1] M.C. Vanderveen, C.B. Papadias and A. Paulraj, "Joint Angle and Delay estimation(JADE) for Multipath Signals Arriving at an Antenna Array", IEEE Communications Letters, vol.1, Jan. 1997

[2] M.C. Vanderveen, C.B. Papadias and A. Paulraj, "Joint Angle and Delay estimation(JADE) for Multipath environment", in Proc. 30th Asilomar Conf. Circuit,Syst.,Comp., Nov. 1996.

[3] M.C. Vanderveen, A. J. Van der Veen and A. Paulraj, "Estimation of multipath parameters in wireless communications", IEEE Trans. on Signal Processing, Vol. 46, March 1998.

[4] M. Wax and A. Leshem, "Joint estimation of time delays and directions of arrival of multiple reflections of a known signal," in Proc. IEEE ICASSP, May, 1996.

[5] G. McLachlan and T. Krishnan, *The EM algorithm and extension.* John Wiley and sons Inc, 1997.