

DIRECTION OF ARRIVAL ESTIMATION USING QUADRATIC TIME FREQUENCY DISTRIBUTIONS

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ABSTRACT

The paper introduces a novel time-frequency linearly constrained minimum variance (LCMV), also known as Capon method, for the direction of arrival (DOA) estimation of nonstationary signals impinging on a multisensor array receiver. The results are compared with the conventional Capon method of DOA estimation technique. In Capon method, the weights are chosen to minimize the weighted array power output subject to the unity gain constraint in the desired look direction. Time-frequency distributions localize the signal power in the time-frequency domain and as such enhance effective Signal to noise ratio (SNR), leading to improved DOA estimates. Time-frequency distributions result in signal separation in the time-frequency domain and hence fewer signals can be selected for processing. Therefore, the interference signals are eliminated by selecting proper time-frequency regions. This paper focuses on the class of frequency modulated (FM) signals because of their clear representation in time-frequency plane.

1. INTRODUCTION

The problem of signal parameter estimation in the sensor array processing has received much interest for several years. Many algorithms have been proposed for the estimation of DOA of the signals impinging on the array of sensors [1]. Classical DOA estimation problem, however, requires that the number of source signals impinging on an antenna array is less than the number of sensors in the array. One of the non-parametric method used to estimate the DOAs is Capon method. The conventional Capon method is based on the estimates of the data covariance matrix. In our approach, the evaluation of quadratic time-frequency distributions of the data snapshots across the array yields spatial time frequency distributions (STFD). These distributions are most appropriate to handle non-stationary sources. STFD localize the signal energy while spreading the noise in the entire

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time-frequency plane and thus enhance SNR. Our method (time-frequency Capon method) is shown to have superior performance than conventional Capon method for DOA estimation. This superior performance is attributed to the following reasons:

- 1) Increase in SNR
- 2) the localization of signals in time-frequency domain permits to select fewer signals than those incident on the array and hence the space time frequency distribution matrices can be constructed by taking into account fewer signals which can be used in place of data covariance matrix in Capon method.

The paper is organized as follows: In section 2, the signal model is presented and conventional Capon method is discussed. Section 3 is devoted to spatial time frequency distributions and the Capon method based on these distributions. In section 4 simulations are presented, and the results are discussed in section 5.

2. SIGNAL MODEL

Consider an array of m sensors. The output from all the sensors can be written as

$$y(t) = A(\theta)s(t) + n(t) \quad (1)$$

where $m \times n$ spatial matrix $A(\theta) = [a(\theta_1)a(\theta_2) \cdots a(\theta_n)]$ represents the mixing matrix or the steering matrix. The functional form of this matrix is assumed to be known. $a(\theta_i) \in C^m$, i.e., the steering vectors corresponds to the angle of arrival θ_i . We shall refer to $a(\cdot)$ as the array manifold. $y(t) \in C^m$ and $s(t)$ is given by the following equation

$$s(t) = [s_1(t)s_2(t) \cdots s_n(t)]^T \quad (2)$$

is the n -vector containing the signals, $(\cdot)^T$ is transpose operator and $n(t)$ is complex zero mean white Gaussian noise with variance σ^2 . We assume that the sources are in the far field of the array and that the sources and the array lie in the same plane. We also assume that the array is calibrated, i.e., we know the positions, gains, and phases for all

antenna elements, for all possible frequencies and all possible DOAs. In this paper we focus on frequency modulated signals. These signals are modeled as

$$s(t) = [S_1 e^{j\phi_1(t)} S_2 e^{j\phi_2(t)} \dots S_n e^{j\phi_n(t)}]^T \quad (3)$$

where S_i and $\phi_i(t)$ are the fixed amplitudes and the time-varying phase of the i^{th} source signal respectively. The instantaneous frequency of the i^{th} source is given by $f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$. The DOA problem can now be stated as follows. Given N measurements of the output of the array and given the model in equation (1), determine the DOAs of n sources. In other words, given $\{y(t)\}_{t=1}^N \in C^m$ and the mapping $A(\theta)$, we wish to determine the parameter θ . The DOA will be estimated in a non-parametric fashion. The main attribute of the non-parametric method is that it does not make any assumption on the covariance structure of the data $y(t)$. As such, it does not require the number of signals and noise covariance matrix to be specified. Only knowledge of the functional form of the array is assumed to be known. We consider FM signals because these signals are characterized by instantaneous frequency and they have clear time-frequency distributions which facilitates in separating region of interest in the time-frequency domain. The region of interest contains signal of interest and hence space time frequency distribution matrices are constructed by considering only the region of interest. In the following lines we will briefly describe Capon method and regularized form of Capon method.

2.1. Capon method

The Capon filter minimizes the following criteria:

$$\min_w (w^H R_{yy} w) \quad (4)$$

subject to

$$w^H a(\theta_i) = 1 \quad (5)$$

i.e., minimize the output power, $E|y(t)|^2$ but pass undistorted the signal from the desired direction, θ_i , E is the expectation operator. In Capon method, the criteria is data dependent. It also takes into account the spatial distribution of the signal energy which is described by the covariance matrix R_{yy} . The goal is to steer the beam to the desired direction θ_i while attenuating as much as possible all other signals that impinge on the array from the directions other than θ_i . The solution to the above optimization problem is

$$w = \frac{R_{yy}^{-1} a(\theta_i)}{a(\theta_i)^H R_{yy}^{-1} a(\theta_i)} \quad (6)$$

and the corresponding output energy is

$$E|y(t)|^2 = \frac{1}{a(\theta_i)^H R_{yy}^{-1} a(\theta_i)} \quad (7)$$

$(\cdot)^H$ is Hermitian transpose. The minimum value of equation (7) for a value of θ_i is the DOA of the i^{th} source. Similarly the DOAs for other sources can be estimated in the same way.

2.2. Regularized Capon method

In this method the idea is to minimize the sum of weighted array output power plus a penalty term, proportional to the square of the norm of the weight vector, subject to the unity gain constraint in the desired signal direction.

$$\min_w (w^H R_{yy} w + \eta w^H w) \quad (8)$$

subject to

$$w^H a(\theta_i) = 1 \quad (9)$$

The parameter η penalizes large values of w . The solution of the above problem is

$$w = \frac{(R_{yy} + \eta I)^{-1} a(\theta_i)}{a(\theta_i)^H (R_{yy} + \eta I)^{-1} a(\theta_i)} \quad (10)$$

3. SPATIAL TIME FREQUENCY DISTRIBUTIONS

The STFDs based on quadratic (Cohens class) time-frequency distributions was introduced in [3]. We will discuss two STFDs methods, i.e., method based on pseudo Wigner-Ville distribution and the method based on Wigner-Hough transform. First of all, we will consider the pseudo Wigner-Ville distributions (PWVD). The discrete form of PWVD of a signal $y(t)$, using rectangular window of length L , is given by

$$C_{yy}(t, f) = \sum_{\tau=-(L-1)/2}^{(L-1)/2} y(t + \tau) y(t - \tau)^H e^{-j4\pi f \tau} \quad (11)$$

where $(\cdot)^H$ denotes the Hermitian transpose. Taking separately, i.e., for each antenna element, PWVD of the received signal will give PWVD at each antenna element. We consider FM signals, these have clear (well separated) PWV representation. It is shown in [3] that if we select the time frequency points along the time-frequency signature or the instantaneous frequency of the i^{th} FM signal, the SNR is improved by factor of L , i.e., the selected window size. The PWVD of each FM source has a constant value over the observation period, provided that we leave out the rising and falling power distributions at both ends of the data record. For convenience analysis, we select those $N - L + 1$ time-frequency points of constant distribution value for each source signal. The averaged space time frequency distribution matrices over the time-frequency signature of the signal of interest is given by

$$\hat{C}_{yy} = \frac{1}{N - L + 1} \sum_{i=1}^{N-L+1} C_{yy}(t_i, f_i(t_i)) \quad (12)$$

where $f_i(t_i)$ is the instantaneous frequency of the signal of interest at i^{th} time sample. In the above equation we construct the space time frequency distribution matrix by taking into account the region of interest in the time-frequency plane i.e., signal of interest region plus noise in that region. Hence \hat{C}_{yy} will almost be free of the interfering signals (neglecting interference terms of the distribution). By replacing the covariance matrix in the Capon method with the above space time frequency distribution matrix, we obtain time-frequency Capon method. More precisely, we have

$$w = \frac{\hat{C}_{yy}^{-1} a(\theta_i)}{a(\theta_i)^H \hat{C}_{yy}^{-1} a(\theta_i)} \quad (13)$$

and the corresponding output energy is

$$E|y(t)|^2 = \frac{1}{a(\theta_i)^H \hat{C}_{yy}^{-1} a(\theta_i)} \quad (14)$$

The estimate of θ_i is

$$\hat{\theta}_i = \underset{\theta_i}{\operatorname{argmin}}_i (a(\theta_i)^H \hat{C}_{yy}^{-1} a(\theta_i))^{-1} \quad (15)$$

The advantage of the time-frequency Capon method over conventional Capon method are follows:

- 1) Better DOA estimates
- 2) Conventional Capon method will completely fail if the two or more sources have the same DOA. On the other hand time-frequency Capon method will resolve the DOA problem successfully by constructing space time frequency distribution matrix for each source separately.

3.1. Chirp detection/parameter estimation using Wigner-Hough transform

The parameters of the chirp can also be estimated by Wigner-Hough transform [4]. The discrete Wigner-Hough transform (WHT) of a sequence $x(n)$, $n = 0, 1, \dots, N-1$ (for N even) is defined as

$$\begin{aligned} WH_x(f, g) = & \sum_{n=0}^{\frac{N}{2}-1} \sum_{k=-n}^n x(n+k)x(n-k)^H e^{-j4\pi k(f+gn)} \\ & + \sum_{n=\frac{N}{2}}^{N-1} \sum_{k=-(N-1-n)}^{N-1-n} x(n+k)x(n-k)^H e^{-j4\pi k(f+gn)} \quad (16) \end{aligned}$$

The WHT of the chirp signal

$$s(n) = A e^{j(\phi_0 + 2\pi f_0 n + \pi g_0 n^2)} \quad (17)$$

assumes its maximum absolute value in the point of coordinated (f_0, g_0) where it is equal to $\frac{N^2 A^2}{2}$ [4]. This means that the detection and estimation of chirp signals embedded in noise can be recast as search for the peaks in the domain (f, g) . There is one advantage of using WHT, is that, there are only two peaks present in the final domain, since the cross terms are cancelled by the integration operated by the HT.

4. SIMULATIONS

We consider a six-element linear array with half wavelength interelement spacing, and two chirp signals arrive at this array. We consider a simple case of FM signals such that their time-frequency signatures do not overlap. The start and the end frequency of the first signal, $s_1(t)$ are $f_{1s} = 0.3$ and $f_{1e} = 0.5$ respectively and those for the second signal $s_2(t)$ are $f_{2s} = 0$ and $f_{2e} = 0.4$ respectively. The DOAs of the two signals are $\theta_1 = 5^\circ$ and $\theta_2 = -5^\circ$ respectively. Both Capon method and time-frequency Capon method gives satisfactory results upto SNR of 5 dB. Figure 1 shows the mixture of two noiseless chirp signals. Figure 2 and figure 3 shows Wigner-Ville and Pseudo Wigner-Ville distribution of the two chirp signals respectively. In these figures we can see clearly the interference terms. Figure 4 shows Smoothed Pseudo Wigner-Ville distribution, it is clear from the figure the suppression of interference terms but this is achieved at the expense of signal resolution in the time-frequency plane. In figure 5, the Wigner-Hough transform is shown. The coordinates of the two peaks are the estimates of the parameters of two chirp signals. Now we fix SNR to be -5dB. The window length is chosen to be, $L = 129$ and the number of samples across the array, $N = 256$. For eight independent trials for DOA estimation (see figure 6 and figure 7), it is evident that the Capon method based on space time frequency distribution matrix out performs conventional Capon method. Now we consider two chirp signals arriving at the array from the same direction with DOA equal to $\theta = 5^\circ$. Conventional Capon method fails to estimate DOAs. But the same problem can be solved successfully by time-frequency Capon method as shown in figure 8.

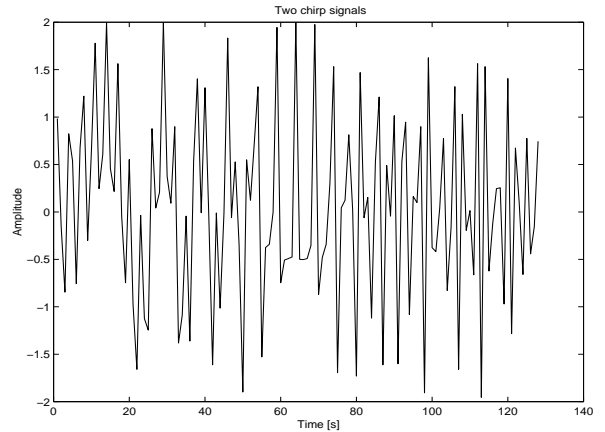


Fig. 1. Mixture of two chirp signals.

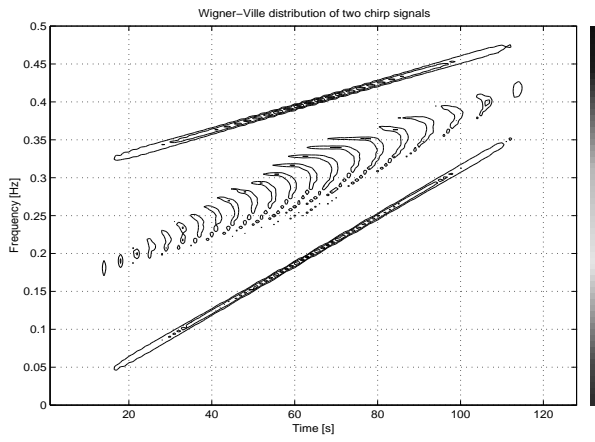


Fig. 2. Wigner-Ville distribution of mixture of two chirp signals.

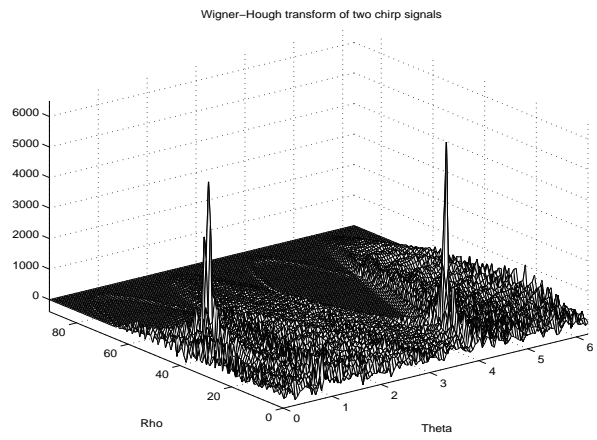


Fig. 5. Wigner-Hough transform of mixture of two chirp signals.

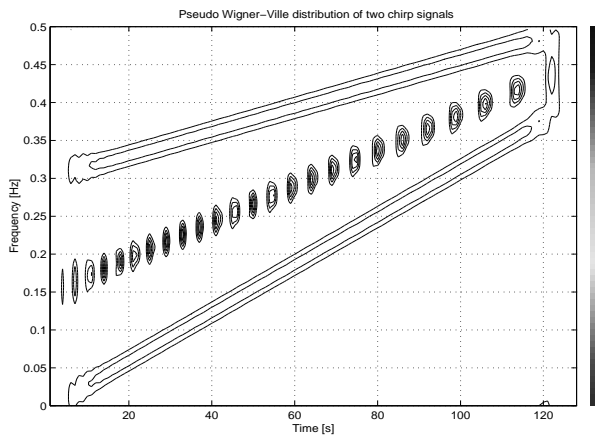


Fig. 3. Pseudo Wigner-Ville distribution of mixture of two chirp signals.

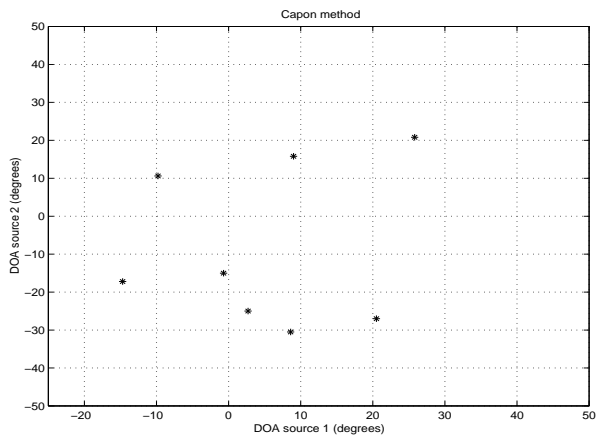


Fig. 6. Capon method for DOA estimation.

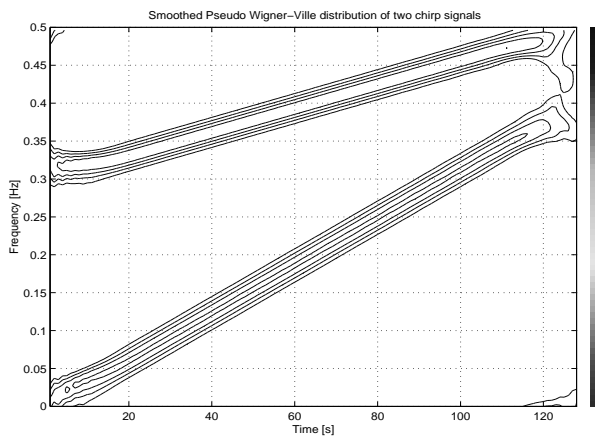


Fig. 4. Smoothed Pseudo Wigner-Ville distribution of mixture of two chirp signals.

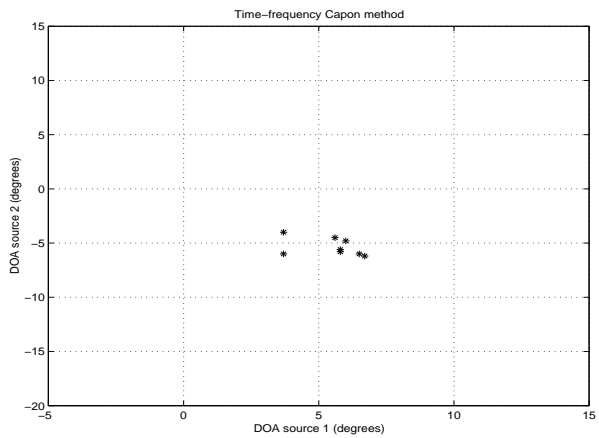


Fig. 7. Time-frequency Capon method for DOA estimation.

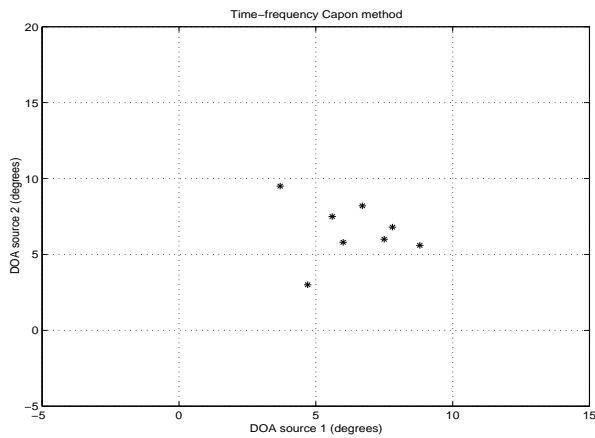


Fig. 8. Time-frequency Capon method for DOA estimation.

5. CONCLUSIONS

In this paper we presented nonparametric DOA estimation using time-frequency distributions. The results are compared with the Capon method for DOA estimation. Better results are obtained because the covariance matrix in the Capon method is replaced with the space time frequency distribution matrix. We considered the case of two chirp signals. These signals have clear time-frequency signatures. Using time-frequency distributions we enhance the SNR of signals and we can construct space time frequency distribution matrices by selecting only region of interest, i.e., in order to calculate the DOA of source 1, we select source 1 region of interest in the time-frequency plane. In this manner we eliminate interference term which is due to the second source and the space time frequency distribution matrix can be constructed only from the signal of interest, the same procedure applies in estimating the DOA of the second source. In this way better estimates for DOAs are obtained by using space time frequency distribution matrices in place of covariance matrix in Capon method.

6. REFERENCES

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