BLIND ITERATIVE RECEIVER FOR MULTIUSER SPACE-TIME CODING SYSTEMS

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ABSTRACT

We consider a space time coding system. We propose to detect symbols of the each user and estimate the channel iteratively. The channel gets estimated blindly via Expectation Maximization (EM) algorithm by formulating the problem as Gaussian mixture model (GMM). The estimated channel is then used to detect the symbols for each user, which is also done in an iterative fashion, i.e., user-wise detection. We consider using finite alphabet for MAI, to simplify and to reduce the complexity of the resulting EM algorithm, we consider the introduction of Mean field methods for the approximation of the posterior MAI symbol probabilities. Simulations shows very useful behaviour of the proposed receiver.

1. INTRODUCTION

Deploying multiple antennas at both the base station and the mobile stations increase the capacity of wireless channels. The recently developed space-time coding (STC) techniques [3] integrate the methods of transmitter diversity and channel coding, and provide significant capacity gains over the traditional communication sytems in fading wireless channels.

Recently, iterative processing has attracted vast attention due to its successful applications in many areas of coding and signal processing.

In this paper we iteratively detect symbols of the each user and estimate the channel. The channel gets estimated blindly via Expectation Maximization (EM) algorithm by formulating the problem as Gaussian mixture model. The estimated channel is than used to detect the symbols for each user, which is also done in iterative fashion, i.e., user-wise detection. We consider exploiting the finite alphabet for the MAI symbols, leading to significant MAI reduction capability. To simplify and to reduce the complexity of the resulting EM algorithm, we consider the introduction of Mean field methods for the approximation of the posterior MAI symbol probabilities. The paper is organized as follows: In section 2, we define the signal model. Section 3 describes Gaussian mixture model based estimation of the channel and the effect of dimensionality reduction on Gaussian mixture problem. In section 4, we describe the detection procedure for our problem. In section 5 conclusions are drawn.

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2. SIGNAL MODEL

We consider the Space-time block coding (STBC) system with K users. Each user is equiped with N transmit antennas. The base station has M receiving antennas. The kth user's STBC is defined by a $(P \times N)$ code matrix G_k, where P denotes the number of time slots for transmitting an STBC codeword or the temporal transmitter diversity order. A STBC encoder takes as input the code vector d_k, and transmits each row of symbols in G_k at P consecutive time slots. At each time slot, the symbols contained in an N-dimensional row vector of G_k are transmitted through N transmitter antennas simultaneously. For two antennas system the code matrix is given by

$$\mathbf{G} = \begin{pmatrix} d(1) & d(2) \\ -d^*(2) & d^*(1) \end{pmatrix} \tag{1}$$

where $(.)^*$ denotes the transpose and $d(i) \in \{1, -1\}$. We consider flat fading channel between each transmitter-receiver pair. The coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i to the receive antenna j at time t. The path gains $\alpha_{i,j}$ are modeled as samples of independent complex Gaussian random variables with mean zero and variance 1. This is equivalent to the assumption that signals transmitted from different antennas undergo independent Rayleigh fades. It is also assumed that the fading gain remains constant over the entire signal frame and vary from one frame to another (quasi-static fading).

The model for our problem is given by [2]

$$\underbrace{\begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^M \end{pmatrix}}_{x=MP\times 1} = \underbrace{[H_1 H_2 \cdots H_K]}_{H=MP\times NK} \cdot \underbrace{\begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_K \end{pmatrix}}_{NK\times 1} + \underbrace{\begin{pmatrix} n^1 \\ n^2 \\ \vdots \\ n^M \end{pmatrix}}_{MP\times 1} \quad (2)$$

In the above equation, $x^m = [x^m(1), x^m(2)\cdots x^m(P)]^*$, $m = 1, 2\cdots M$, consist of the received signal from time slots 1 to P, at the mth receiver antenna. H_k denotes the channel response of the user k. $d_k = [d_k(1)d_k(2)\cdots d_k(N)]^*$ is the code vector of the kth user, with $d_k(i) \in \{1, -1\}$; and $n^m = [n^m(1)n^m(2)\cdots n^m(P)]^H$ is the additive noise vector at the mth receiver antenna.$(.)^*$ denotes transpose operator.
For single user, the Alamouti scheme, (two transmit antennas), STBC is given by

$$\begin{pmatrix} x^m(1) \\ x^m(2) \end{pmatrix} = \begin{pmatrix} d(1) & d(2) \\ -d^*(2) & d^*(1) \end{pmatrix} \begin{pmatrix} \alpha_{m,1} \\ \alpha_{m,2} \end{pmatrix} + \begin{pmatrix} n^m(1) \\ n^m(2) \end{pmatrix}$$

It can be further written as

$$\underbrace{\begin{pmatrix} x^m(1) \\ x^m(2)^H \end{pmatrix}}_{x^m} = \underbrace{\begin{pmatrix} \alpha_{m,1} & \alpha_{m,2} \\ \alpha^H_{m,2} & -\alpha^H_{m,1} \end{pmatrix}}_{H_1^m} \underbrace{\begin{pmatrix} d(1) \\ d(2) \end{pmatrix}}_{d_1} + \underbrace{\begin{pmatrix} n^m(1) \\ n^m(2)^H \end{pmatrix}}_{n^m} \quad (3)$$

where $(.)^H$ is the Hermitian transpose. From the above equation we can see the analogy between multiuser STBC signal model and synchronous CDMA signal model [5,6]. By stacking all x^m, we get the following equation for two transmit antenna system.

$$\underbrace{\begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^M \end{pmatrix}}_{x=2M\times 1} = \underbrace{\begin{pmatrix} H_1^1 \\ H_1^2 \\ \vdots \\ H_1^M \end{pmatrix}}_{H_1=2M\times 2} \underbrace{\begin{pmatrix} d(1) \\ d(2) \end{pmatrix}}_{d_1=2\times 1} + \underbrace{\begin{pmatrix} n^1 \\ n^2 \\ \vdots \\ n^M \end{pmatrix}}_{n=2M\times 1} \quad (4)$$

3. GAUSSIAN MIXTURE MODEL BASED CHANNEL ESTIMATION

The Gaussian mixture model was considered for the synchronous CDMA system in [5,6]. Due to analogy between synchronous CDMA and Space time multiuser system, we can use the method used for the synchronous CDMA to the problem at hand. In ML estimation problem we have density function $P(x|\theta)$ that is governed by the set of parameters θ (e.g. P might be set of Gaussians and θ could be the means and covariances). The data is of size T, supposedly drawn from this distribution, i.e., $X = [x_1, \ldots\ldots x_T]$. That is, we assume that these data vectors are independent identically distributed (i.i.d) with distribution P. Therefore the resulting density for the samples is

$$p(X|\theta) = \prod_{j=1}^{T} P(x_j|\theta) = L(\theta|X).$$

This function $L(\theta|X)$ is called the likelihood of the parameters given the data, or just the likelihood function. In the ML problem, our goal is to find θ that maximizes L. That is, we wish to find θ^* where

$$\theta^* = \arg\max_{\theta} L(\theta|X). \quad (5)$$

Assuming that the channel output, i.e., x can be approximated by Gaussian distributions, i.e., $P(x|\theta)$ can be modeled as MP-dimensional mixture of Gaussians. We can write

$$P(x|\theta) = \sum_{j=1}^{W} \alpha_j P(x|m_j, \Sigma_j), \quad (6)$$

where $W = 2^{NK}$ and

$$P(x|m_j, \Sigma_j) = \frac{1}{(2\pi)^{(MP/2)}|\Sigma_j|^{1/2}} \exp\left(-\frac{1}{2}(x - m_j)^H \Sigma_j^{-1}(x - m_j)\right), \quad (7)$$

$\alpha_j \geq 0$, and $\sum_{j=1}^{W} \alpha_j = 1$. The parameter vector θ consists of mixing proportions α_j, the means vectors m_j, and the covariance matrices Σ_j. Given W and given T independent, i.i.d samples $\{x_t\}_1^T$, we obtain the following likelihood

$$l(\theta) = \sum_{t=1}^{T} \log \sum_{j=1}^{W} \alpha_j P(x_t|m_j, \Sigma_j) \quad (8)$$

which is difficult to optimize because it contains logarithm of a sum. If we consider X as incomplete, since we do not know which index j, within the mixture probability density function resulted for a specific output. The complete "data set" in this case is, $[x_1, \ldots, x_T, i_1, \ldots, i_T]$, where i_n denotes the component of the pdf from which x_n is drawn. Using complete data set we can optimize our problem using EM algorithm.

The update for means is given by the following equation.

$$m_j^{(k+1)} = \frac{\sum_{t=1}^{T} h_j^{(k)}(t) x_t}{\sum_{t=1}^{T} h_j^{(k)}(t)}, \quad (9)$$

where the posteriori probabilities $h_j^{(k)}(t)$ is defined as follows:

$$h_j^{(k)}(t) = \frac{\alpha_j^{(k)} P\left(x_t|m_j^{(k)}, \Sigma_j^{(k)}\right)}{\sum_{i=1}^{W} \alpha_i^{(k)} P\left(x_t|m_j^{(k)}, \Sigma_j^{(k)}\right)}. \quad (10)$$

The mixing proportions (α_j) and Covariance matrices in our case are of constant values and are given by 2^{-KN} and $\sigma^2 I$ respectively.

The EM algorithm for GMM works as follows, first posteriori probabilities are calculated using initial estimates of means. The posteriori probabilities tells us the likelihood that a point belongs to each of the separate component densities. These posteriori estimates are used to find the update means of the mixture. These two steps are repeated until convergence. The convergence of the EM algorithm to a solution and the number of iterations depends on the tolerance, the initial parameters, the data set, etc. After convergence of the algorithm the estimate of H is given by

$$\hat{H} = \sum_{j=1}^{W} m_j b_j^* \left(\sum_{j=1}^{W} b_j b_j^* \right)^{-1}. \quad (11)$$

where $\{b_j\}_1^W$, is the set of all possible transmitted symbol sequences.

3.1. Effect of dimensionality reduction on the Gaussian mixture problem

In the following lines we will consider the effect of random projections (to reduce the dimensionality of the problem) on the Gaussian mixture problem.

Consider d identically distributed independent (i.i.d) random variables which are components of the mean vector μ. The goal is to calculate the probability distribution of the sum of the squares of the difference of the two mean vectors X and Y, each of which is a vector with d components which are i.i.d random variables, i.e., $X = (X_1, X_2, \cdots X_d)$ and $Y = (Y_1, Y_2, \cdots Y_d)$. Let Z be the components of the difference of the means, i.e., $Z = Y - X = (Z_1, Z_2, \cdots Z_d)$. The probability density function of Z is follows

$$R(Z) = \int_{\infty}^{\infty} P(Z+X)U(X)dX \quad (12)$$

The joint probability density function, $J(Z_1, Z_2, \cdots Z_d)$ is

$$J(Z_1, Z_2, \cdots Z_d) = R(Z_1)R(Z_2)\cdots R(Z_d) \quad (13)$$

Considering Gaussian probability density function for each component, the joint probability density function for means is:

$$J(Z_1, Z_2, \cdots Z_d) = \frac{1}{(2\pi 2\rho^2)^{\frac{d}{2}}} exp(-\sum_{i=1}^{d} \frac{Z_i^2}{4\rho^2}) \quad (14)$$

where we have assumed that the Gaussian is zero mean with variance ρ^2. This density function is spherically symmetric hence it cal be written in the form of

$$\int\int\cdots\int f(r)dX^d = \int_0^{\infty} f(r) S_d r^{d-1} dr \quad (15)$$

where S_d denotes the surface area of d-dimsional sphere. Let $z_i = \frac{Z_i}{\sqrt{2}\rho}$. The criterion for the separation of two Gaussian is that the distance between two means is greater than twice the standard deviation. We are interested to find the probability that

$$(\sum_{i=1}^{d} Z_i^2)^{\frac{1}{2}} < 2\rho \quad (16)$$

i.e.,

$$(\sum_{i=1}^{d} z_i^2)^{\frac{1}{2}} < \frac{\sqrt{2}\sigma}{\rho} \quad (17)$$

The solution to this problem takes the form of the standard distribution of the sum of the squares of d normally distributed random variables, $P(\chi^2|d)$. χ^2 distribution is one of the classic distribution of statistics. We can calculate the probability that

$$(\sum_{i=1}^{d} z_i^2)^{\frac{1}{2}} < \sqrt{\chi^2} \quad (18)$$

as

$$S(d) = P(\chi^2|d) = \int_0^{\sqrt{\chi^2}} (\frac{1}{2\pi})^{\frac{d}{2}} e^{\frac{-r^2}{2}} S_d r^{d-1} dr \quad (19)$$

which can be written as

$$S(d) = \frac{1}{2^{\frac{d}{2}}\Gamma(\frac{d}{2})} \int_0^{\chi^2} t^{\frac{d-2}{2}} e^{\frac{-t}{2}} dt \quad (20)$$

where $\Gamma(.)$ is Gamma function. After bit of algebra, (not shown due to lack of space), the minimum dimension for which the probability that the separation between two means is less than 2σ given a small number ϵ, is given by

$$P(\frac{2\sigma^2}{\rho^2}|q_{min}) = \frac{(\sigma/\rho)^{q/2}}{2^{q/2}\Gamma(q/2+1)} \quad (21)$$

Fig (3) shows that when the data is projected into a space of smaller dimension, $q < d$, the probability of overlap between the means increases rapidly as q decreases and the EM algorithm fails to converge to true means when there is a substantial amount of overlap between Gaussians, leading to wrong channel estimates.

4. USER-WISE SYMBOL ESTIMATION USING DISCRETE MAI PRIOR

The discrete time received signal is given by equation (2), i.e., $x = Hd + n$. This equation can be further written as $x = H[d_1^* d_r^*]^*$. Where $(.)^*$ denotes the transpose operator, d_1, is the information bits of user 1 and d_r are the information bits of the rest of the users. Without loss of generality, we detect symbols for user 1 first and user K in the last, i.e., in the ascending order of the users.

Given the above model we are now ready to define complete data set. We choose complete data set as $y = \{x, d_r\}$. The derivation of the algorithm is as follows: The pdf of the complete data set can be written as

$$f(x, d_r; H, d_1) = f(x|H, d)f(d_r; H, d_1) \qquad (22)$$

where $f(x|H, d)$ and $f(d_r; H, d_1)$ is given by

$$f(x|H, d) = K_1 exp(\frac{-1}{\sigma^2}(x - Hd)^H (x - Hd)) \qquad (23)$$

where K_1 is constant not depending on parameters to be estimated. Having the above equations we are now ready to evaluate the E-step of the algorithm. Since we are conditioning on the received data, we take expectations with respect to d_r (interfering users' symbols).

$$Q(d_1; d_1^{(k)}) = E\{log f(x, d_r; H, d_1|x; d_1^{(k)})\} \qquad (24)$$

where $(.)^k$ is the iteration index. In the above equation, we will use the estimated value of the channel which is estimated by GMM approach and has fixed value. Evaluating the expectations and dropping the terms that do not depend on the parameters the above equation can be written as

$$Q(d_1; d_1^{(k)}) =$$
$$E\{(x - H_1 d_1 - H_r d_r)^H (x - H_1 d_1 - H_r d_r)|x; d_1^{(k)}\} \qquad (25)$$

where $H = [H_1|H_r]$. The above equation can be further written as

$$Q(d_1; d_1^{(k)}) =$$
$$E\{(x - D_1 h_1 - D_r h_r)^H (x - D_1 h_1 - D_r h_r)|x; d_1^{(k)}\} \qquad (26)$$

where $D_1 = d_1 \otimes I$ and $h_1 = vec(H_1)$ and I is identity matrix. Similarly, we can define $D_r = d_r \otimes I$ and $h_r = vec(H_r)$. We have used the property that $vec(ABC) = (C^T \otimes A)vec(B)$. The symbols, d_1, can be detected by maximizing the above expectation equation over finite alphabet. From the above equation it is clear that

$$\hat{d}_r = E\{d_r|x; H, d_1^{(k)}\}. \qquad (27)$$

As the expectation equation depends on the conditional mean of d_r, therefore now the problem is to find expressions for conditional mean of d_r.

The conditional mean for d_r is given by

$$\hat{d}_r = E\{d_r|x; H, d_1^{(k)}\} = \sum_{d_r} d_r f(d_r|x; H, d_1^{(k)}) \qquad (28)$$

From now for the sake of simplicity we will omit the EM iteration index, i.e., k. In order to calculate the conditional mean we have to evaluate the above expression, which is summation of all interfering users' symbols multiplied by their corresponding pdfs, which is computationally very expensive. Mean field (MF) methods [1], provide tractable approximations for the computation of high dimensional sums and integrals in the probabalistic models. By neglecting certain dependencies between the random variables, a closed set of equations for the expected values of these variables are derived which often can be solved in a time that grows polynomially in the number of variables [1, chapter.2]. The MF approximation is obtained by taking the approximating family of probability distribution by all product distribution, i.e., $Q(d_r) = \Pi_j Q_j(d_{rj})$. We now choose a distribution which is close to the true distribution, i.e., $f(d_r|x; H, d_1)$. The parameter of the distribution is chosen so as to minimize Kullback-Leibler (KL) distance, i.e.,

$$KL(Q||f(d_r|x; H, d_1) = \sum_{d_r} Q(d_r) ln \frac{Q(d_r)}{f(d_r|x; H, d_1)} \qquad (29)$$

where $Q(d_r) = \Pi_{j=1}^{(K-1)N} Q_j(d_{rj})$ and $d_{rj} \in \{-1, 1\}$.

$$f(d_r|x; H, d_1) = \frac{f(x|H, d)}{\sum_{d_r} f(x|H, d)} = \frac{exp(-H(d))}{Z} \qquad (30)$$

where Z is independent of d_r and $f(x|H, d)$ has Gaussian distribution. After some simplification $H(d)$ can be written as

$$H(d) = \frac{1}{\sigma^2}(-x^H Hd - d^H H^H x + d^H H^H Hd) \qquad (31)$$

The above equation has the form

$$H(d) = \sum_{i,j} d_{ri} J_{ij} d_{rj} - 2 \sum_i d_{ri} \theta_i + C \qquad (32)$$

where C is a term independent of d_r, $J_{ij} = \frac{1}{\sigma^2}(H^H H)_{ij}$ and $\theta_i = real(\frac{1}{\sigma^2}(H^H x)_i)$, is the i^{th} element of the vector $H^H x$. The KL distance between Q and $f(d_r|x; H, d_1)$ can be written as

$$KL(Q||f(d_r|x; H, d_1) = lnZ + V[Q] - S[Q] \qquad (33)$$

where

$$S[Q] = -\sum_{d_r} Q(d_r) ln Q(d_r) \quad (34)$$

is the entropy and

$$V[Q] = \sum_{d_r} Q(d_r) H(d) \quad (35)$$

is the variational energy. The most general form of probability distribution for our problem is

$$Q_j(d_{rj}; m_j) = \frac{1 + d_{rj} m_j}{2} \quad (36)$$

where m_j is the variational parameter which corresponds to the mean, i.e., $m_j = E\{d_{rj}\}$. The entropy can be written as

$$S[Q] = -\sum_i \frac{1+m_i}{2} ln \frac{1+m_i}{2} + \frac{1-m_i}{2} ln \frac{1-m_i}{2} \quad (37)$$

and similarly variational energy can be written as

$$V[Q] = \sum_{i,j} J_{ij} m_i m_j - 2 \sum_i m_i \theta_i \quad (38)$$

In order to evaluate m_i we have to minimize the variational free energy, i.e.,

$$F[Q] = V[Q] - S[Q] \quad (39)$$

Differentiating this equation with respect to $m_i^{'} s$ gives nonlinear fixed point equations, i.e.

$$m_i = tanh(-\sum_j J_{ij} m_j + \beta_i), i = 1, 2 \cdots (K-1)N \quad (40)$$

In the matrix form we can write the above equation as

$$\mathbf{m} = \mathbf{tanh}(-\mathbf{Jm} + \beta) \quad (41)$$

where $\beta_i = 2\theta_i$.

4.1. Linear response theory

In approximating the posteriori probability $f(d_r|x; H, d_1)$, the correlations were neglected, when $Q(d_r)$ is chosen to factorize, i.e.,

$$E_{exact}\{d_{ri} d_{ri}\} \simeq E_Q\{d_{ri} d_{ri}\} = E_Q\{d_{ri}\} E_Q\{d_{ri}\}, \quad (42)$$

where $E_Q\{.\}$ stands for expectation with respect to distribution Q. A correction to the estimate is found by differentiating the following equation

$$E\{d_{ri}\} = Z^{-1} \sum_d d_{ri} e^{H(d)} \quad (43)$$

with respect to β_i to obtain linear response relation [4], i.e.,

$$\frac{\partial E\{d_{ri}\}}{\partial \beta_j} = E\{d_{ri} d_{rj}\} - E\{d_{ri}\} E\{d_{rj}\}. \quad (44)$$

The above relation is exact when expectation is taken according to exact probability distribution. However, if $E\{d_{ri}\}$ is reasonably well approximated with the mean field method, we can get the right hand side of the above equation by differentiating the left side of the equation with respect to β_j. In this way, we can improve the covariance and hence the second moment of the interfering users' bits which will result in improved symbol detection of the UoI as compared to Naive Mean Field Theory (NMFT). NMFT does not take into account correlations between random variables. This improvement is gained at the expense of very little additional complexity.

The huge computational task (exponential complexity) of exact averages over $f(d_r|x; H, d_1)$ has been replaced by solving the above set of $(K-1)N$ nonlinear equations, which often can be done in time that grows only polynomially. As the above equation is nonlinear there may be local minima or saddle points. In order to avoid it, the solution must be compared by their value of variational free energy $F[Q]$.

The overall algorithm works as follows: First, user 1 symbols are detected from the above procedure. Then the contribution of that user is subtracted from the received signal to get more clean signal. Then the same procedure is repeated for the other users. We can also vary the detection procedure by detecting user-wise bit by bit, resulting in minimizing the complexity of the algorithm because the expectation equation will be maximized for only one symbol (not the whole block of the user). For the later case, the algorithm will work as follows: First of all, bit number one will be detected by maximizing the Expectation equation with respect to that bit. Secondly, the contribution of this detected bit will be subtracted from the received signal. Then we estimate the second information symbol. We continue in this fashion until the last symbol of the user to be detected is estimated. By doing so the bit detection will improve because at each step the Intersymbol Interference (ISI) caused by the detected bit is subtracted from the received signal. The improvement will result provided that the bits are correctly detected and this also will improve detection for the rest of the symbols because at each step more clean signal will be processed.

5. CONCLUSIONS AND SIMULATIONS

In this paper, we proposed channel estimation and symbol detection for the Space-time block coded multiuser system. The channel is estimated blindly by formulating the STBC systems as Gaussian mixture model. In this proposed receiver a discrete prior is assumed on the interfering users' bits. In this case, the complexity of computing the posteriori probabilities grows exponentially in the number of interfering users times the symbols per user. We derived low complexity method to circumvent this problem. The exact posteriori probabilities are replaced by the approximate separable distributions. The distributions are calculated by MFT (variational approach). In simulations we consider the four user case. We consider two transmit and two receive antennas case. Fig (1) shows the mean square error (MSE) of the channel estimation error. Fig (2) gives the performance in terms of BER for the proposed receiver. Very close performance to the exact ML is obtained using Linear response theory (LRT).

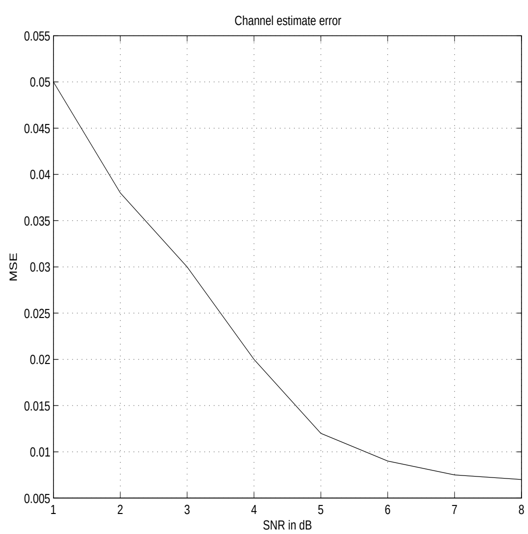

Fig. 1. Channel estimate error vs $SNR(dB)$.

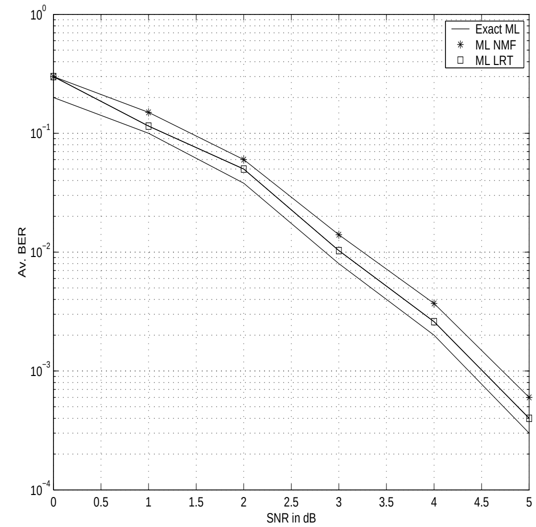

Fig. 2. Av. BER of K=4, N=2, M=2 vs $SNR(dB)$.

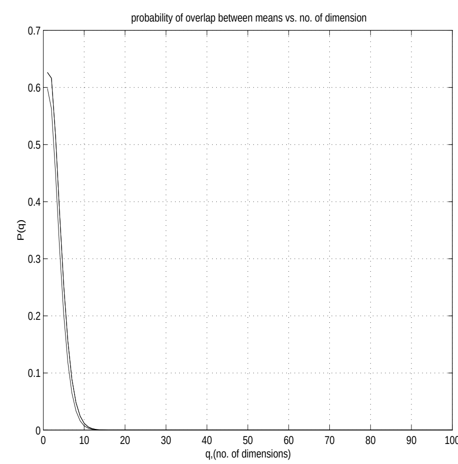

Fig. 3.

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