Low complexity semi-blind channel estimation with a Pseudo Random Postfix OFDM modulator

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***Abstract*— This contribution[1] details a new OFDM modulator based on the use of a Pseudo Random Postfix (PRP-OFDM) and low complexity equalization architectures. The main advantage of this new modulation scheme is the ability to estimate and track the channel variations blindly using order one statistics of the received signal. This scheme is thus very well suited in presence of large Doppler spreads where channel tracking becomes essential. Moreover, the proposal of various equalization structures derived from the zero padded transmission schemes, allows implementations ranging from low-complexity/medium performance to increased-complexity/high performance.**

I. Introduction

Nowadays, Orthogonal Frequency Division Multiplexing (OFDM) seems the preferred modulation scheme for modern broadband communication systems. Indeed, the OFDM inherent robustness to multi-path propagation and its appealing low complexity equalization receiver makes it suitable either for high speed modems over twisted pair (digital subscriber lines xDSL), terrestrial digital broadcasting (Digital Audio and Video Broadcasting: DAB, DVB) and 5GHz Wireless Local Area Networks (WLAN: IEEE802.11a and ETSI BRAN HIPERLAN/2) [2]–[5].

All these systems are based on a traditional Cyclic Prefix OFDM (CP-OFDM) modulation scheme. The role of the cyclic prefix is to turn the linear convolution into a set of parallel attenuations in the discrete frequency domain. Recent contributions have proposed an alternative: replacing this time domain redundancy by null samples leads to the so called Zero Padded OFDM (ZP-OFDM) [6]–[9]. This solution relying on a larger FFT demodulator, has the merit to guarantee symbol recovery irrespective of channel null locations in absence of noise when the channel is known (coherent modulations are assumed).

Channel coefficients estimation is usually performed using known training sequences periodically transmitted (e.g. at the start of each frame), implicitly assuming that the channel does not vary between two training sequences. Thus in order to enhance the mobility of wireless systems and cope with the Doppler effects, reference sequences have to be repeated more often resulting in a significant loss of useful bitrate. An alternative is to track the channel variations by refining the channel coefficients blindly using the training sequences as initializations for the estimator.

Semi-blind equalization algorithms based on second order statistics have already been proposed for the CP-OFDM and ZP-OFDM modulators [8]–[10].

In this contribution we introduce a new OFDM modulator that capitalizes on the advantages of ZP-OFDM. It is proposed to replace the null samples inserted between all OFDM modulated blocks by a known vector weighted by a pseudo random scalar sequence: the Pseudo Random Postfix OFDM (PRP-OFDM). This way, unlike for the former OFDM modulators, the receiver can exploit an additional information: the prior knowledge of a part of the transmitted block [11]. This paper explains how to build on this knowledge and perform an extremely low complexity order one semi-blind channel estimation and tracking. Moreover, several PRP-OFDM equalization architectures derived from the zero padded transmission scheme, are proposed allowing implementations ranging from low-complexity/medium performance to increased-complexity/high performance.

Note that a similar idea has been proposed in the single carrier context in [12], but the exploitation of the training data is not detailed and no efficient equalization scheme is presented. Moreover spectrum wise, it is important to avoid the insertion of the same training sequence at each block otherwise this generates peaks in the transmitted signal spectrum: the pseudo-random sequence weighting used in this contribution deals efficiently with this issue.

This paper is organized as follows. Section II introduces the notations and presents the new PRP-OFDM modulator. Then a blind channel estimation method is presented section III exploiting only the portion of the received vector corresponding to the postfix location. Section IV details the receiver including several equalization schemes (Zero Forcing, ZF and Minimum Mean Square Error, MMSE) and decoding strategies in presence of bit interleaved convolutional coded modulation. Some considerations for designing a suitable postfix are discussed section V from a spectral and envelope point of view. Finally, simulation results in the context of 5GHz IEEE802.11a and ETSI BRAN HIPERLAN/2 illustrate the behavior of the proposed scheme compared to the standardized CP-OFDM systems in section VI.

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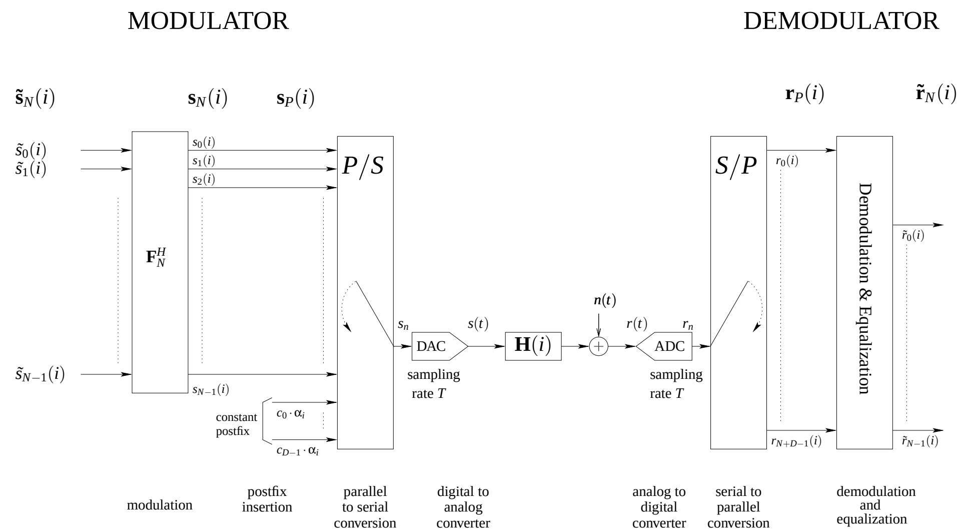

Fig. 1. Discrete model of the PRP-OFDM modulator.

II. Notations and PRP-OFDM modulator

Figure 1 depicts the baseband discrete-time block equivalent model of a N carrier PRP-OFDM system. The ith $N \times 1$ input digital vector [2] $\tilde{\mathbf{s}}_N(i)$ is first modulated by the IFFT matrix $\mathbf{F}_N^H = \frac{1}{\sqrt{N}}\left(W_N^{ij}\right)^H, 0 \le i < N, 0 \le j < N$ and $W_N = e^{-j\frac{2\pi}{N}}$. Then, a deterministic postfix vector $\mathbf{c}_D = (c_0, \ldots, c_{D-1})^T$ weighted by a pseudo random value $\alpha(i) \in \mathbb{C}$ is appended to the IFFT outputs $\mathbf{s}_N(i)$. With $P = N + D$, the corresponding $P \times 1$ transmitted vector is $\mathbf{s}_P(i) = \mathbf{F}_{\mathrm{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P$, where

$$\mathbf{F}_{\mathrm{ZP}}^H = \left[\begin{array}{c} \mathbf{I}_N \\ \hline \mathbf{0}_{D,N} \end{array}\right]_{P\times N} \mathbf{F}_N^H \quad \text{and} \quad \mathbf{c}_P = \left(\mathbf{0}_{1,N}\ \mathbf{c}_D^T\right)^T$$

The samples of $\mathbf{s}_P(i)$ are then sent sequentially through the channel modeled here as a Lth-order FIR $H(z) = \sum_{n=0}^{L-1} h_n z^{-n}$ of impulse response $(h_0, \cdots, h_{L-1})$. The OFDM system is designed such that the postfix duration exceeds the channel memory $L \le D$.

Let $\mathbf{H}_{\mathrm{ISI}}(P)$ and $\mathbf{H}_{\mathrm{IBI}}(P)$ be respectively the Toeplitz inferior and superior triangular matrices of first column: $[h_0, h_1, \cdots, h_{L-1}, 0, \rightarrow, 0]^T$ and first row $[0, \rightarrow, 0, h_{L-1}, \cdots, h_1]$. As already explained in [13], the channel convolution can be modeled by $\mathbf{r}_P(i) = \mathbf{H}_{\mathrm{ISI}}\mathbf{s}_P(i) + \mathbf{H}_{\mathrm{IBI}}\mathbf{s}_P(i-1) + \mathbf{n}_P(i)$. $\mathbf{H}_{\mathrm{ISI}}(P)$ and $\mathbf{H}_{\mathrm{IBI}}(P)$ represent respectively the intra and inter block interference. Since $\mathbf{s}_P(i) = \mathbf{F}_{\mathrm{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P$, we have as illustrated by figure 2:

$$\mathbf{r}_P(i) = (\mathbf{H}_{\mathrm{ISI}} + \beta_i \mathbf{H}_{\mathrm{IBI}})\mathbf{s}_P(i) + \mathbf{n}_P(i)$$

where $\beta_i = \frac{\alpha(i-1)}{\alpha(i)}$ and $\mathbf{n}_P(i)$ is the ith AWGN vector of variance σ_n^2. Note that $\mathbf{H}_{\beta_i} = (\mathbf{H}_{\mathrm{ISI}} + \beta_i \mathbf{H}_{\mathrm{IBI}})$ is pseudo circulant: i.e. a circular matrix whose $(D-1)\times(D-1)$ upper triangular part is weighted by β_i.

[2] Lower (upper) boldface symbols will be used for column vectors (matrices) sometimes with subscripts N or P emphasizing their sizes (for square matrices only); tilde will denote frequency domain quantities; argument i will be used to index blocks of symbols; ^{H} (^{T}) will denote Hermitian (Transpose).

The expression of the received block is thus:

$$\begin{aligned} \mathbf{r}_P(i) &= \mathbf{H}_{\beta_i}\left(\mathbf{F}_{\mathrm{ZP}}^H \tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{c}_P\right) + \mathbf{n}_P(i) \qquad (1) \\ &= \mathbf{H}_{\beta_i}\begin{pmatrix} \mathbf{F}_N^H \tilde{\mathbf{s}}_N(i) \\ \alpha(i)\mathbf{c}_D \end{pmatrix} + \mathbf{n}_P(i) \end{aligned}$$

Please note that equation (1) is quite generic and captures also the CP and ZP modulation schemes. Indeed ZP-OFDM corresponds to $\alpha(i) = 0$ and CP-OFDM is achieved for $\alpha(i) = 0$, $\beta_i = 1 \forall i$ and $\mathbf{F}_{\mathrm{ZP}}^H$ is replaced by $\mathbf{F}_{\mathrm{CP}}^H$, where

$$\mathbf{F}_{\mathrm{CP}}^H = \left[\begin{array}{c|c} \mathbf{0}_{D,N-D} & \mathbf{I}_D \\ \hline \multicolumn{2}{c}{\mathbf{I}_N} \end{array}\right]_{P\times N} \mathbf{F}_N^H.$$

III. An inherent order one semi-blind channel estimation

As mentioned in the introduction, PRP-OFDM allows an order one and low-complexity channel estimation. For explanation sake let assume that the Channel Impulse Response (CIR) is static.

Define $\mathbf{H}_{\mathrm{CIR}}(D) = \mathbf{H}_{\mathrm{ISI}}(D) + \mathbf{H}_{\mathrm{IBI}}(D)$ as the $D \times D$ circulant channel matrix of first row $row_0(\mathbf{H}_D) = [h_0, 0, \rightarrow, 0, h_{L-1}, \cdots, h_1]$. Note that $\mathbf{H}_{\mathrm{ISI}}(D)$ and $\mathbf{H}_{\mathrm{IBI}}(D)$ contain respectively the lower and upper triangular parts of $\mathbf{H}_{\mathrm{CIR}}(D)$.

Denoting by $\mathbf{s}_N(i) = [s_0(i), \cdots, s_{N-1}(i)]^T$, splitting this vector in 2 parts: $\mathbf{s}_{N,0}(i) = [s_0(i), \cdots, s_{D-1}(i)]^T$, $\mathbf{s}_{N,1}(i) = [s_{N-D}(i), \cdots, s_{N-1}(i)]^T$, and performing the same operations for the noise vector: $\mathbf{n}_P(i) = [n_0(i), \cdots, n_{P-1}(i)]^T$, $\mathbf{n}_{D,0}(i) = [n_0(i), \cdots, n_{D-1}(i)]^T$, $\mathbf{n}_{D,1}(i) = [n_{P-D}(i), \cdots, n_{P-1}(i)]^T$, the received vector $\mathbf{r}_P(i)$ can be expressed as:

$$\mathbf{r}_P(i) = \begin{pmatrix} \mathbf{H}_{\mathrm{ISI}}(D)\mathbf{s}_{N,0}(i) + \alpha(i-1)\mathbf{H}_{\mathrm{IBI}}(D)\mathbf{c}_D + \mathbf{n}_{D,0} \\ \vdots \\ \mathbf{H}_{\mathrm{IBI}}(D)\mathbf{s}_{N,1}(i) + \alpha(i)\mathbf{H}_{\mathrm{ISI}}(D)\mathbf{c}_D + \mathbf{n}_{D,1} \end{pmatrix}.$$

As usual the transmitted time domain signal $\mathbf{s}_N(i)$ is zero-mean. Thus the first D samples $\mathbf{r}_{P,0}(i)$ of $\mathbf{r}_P(i)$ and its last D samples $\mathbf{r}_{P,1}(i)$ can be exploited very easily to find back the channel matrices relying on the deterministic nature of the postfix as follows:

$$\hat{\mathbf{h}}_{c,0} = \mathrm{E}\left[\frac{\mathbf{r}_{P,0}(i)}{\alpha(i-1)}\right] = \mathbf{H}_{\mathrm{IBI}}(D)\mathbf{c}_D, \tag{2}$$

$$\hat{\mathbf{h}}_{c,1} = \mathrm{E}\left[\frac{\mathbf{r}_{P,1}(i)}{\alpha(i)}\right] = \mathbf{H}_{\mathrm{ISI}}(D)\mathbf{c}_D. \tag{3}$$

Since $\mathbf{H}_{\mathrm{ISI}}(D)+\mathbf{H}_{\mathrm{IBI}}(D)=\mathbf{H}_{\mathrm{CIRC}}(D)$ is circular and diagonalizable in the frequency domain combining equations (2) and (3) and using the commutativity of the convolution yields:

$$\begin{aligned}\hat{\mathbf{h}}_c &= \hat{\mathbf{h}}_{c,1}+\hat{\mathbf{h}}_{c,0} = \mathbf{H}_{\mathrm{CIRC}}(D)\mathbf{c}_D \\ &= \mathbf{C}_D\mathbf{h}_D = \mathbf{F}_D^H\tilde{\mathbf{C}}_D\mathbf{F}_D\mathbf{h}_D,\end{aligned}$$

where $\mathbf{C}_D$ is a $D\times D$ circulant matrix with first row $row_0(\mathbf{C}_D) = [c_0, c_{D-1}, c_{D-2}, \cdots, c_1]$ and $\tilde{\mathbf{C}}_D = \mathrm{diag}\{\mathbf{F}_D\mathbf{c}_D\}$. Thus, an estimate of the CIR $\hat{\mathbf{h}}_D$ can be retrieved that way:

$$\hat{\mathbf{h}}_D = \mathbf{C}_D^{-1}\hat{\mathbf{h}}_c = \mathbf{F}_D^H\tilde{\mathbf{C}}_D^{-1}\mathbf{F}_D\hat{\mathbf{h}}_c.$$

Note that $\mathbf{c}_D$ is designed such that $\tilde{\mathbf{C}}_D$ is full rank.

We have detailed in this section a very simple method for blind estimation of the CIR only relying on a first order statistics: an expectation of the received signal vector.

Though the results presented above are based on the assumption that the channel does not vary, this method can be used to mitigate the effects of Doppler. Indeed this approach can be combined with the initial channel estimates derived from the preambles usually present at the start of the frame for either refining the channel estimates or tracking the channel variations. For WLANs this enables to operate at a mobility exceeding the specification of the standard (3m/s). In that case it often provides better results if the channel estimate is derived in the mean-square error sense rather than the zero forcing approached detailed in this section.

IV. Symbol Recovery

Once the channel is known, in order to retrieve the data two steps are usually performed: i) equalization of the received vector $\mathbf{r}_P(i)$, ii) soft decoding when forward error encoding is applied at the emitter.

A. Equalization schemes suited for PRP-OFDM

Several equalization strategies can be proposed for the received vector $\mathbf{r}_P(i)$:

- one can first reduce (1) to the ZP-OFDM case by simple subtraction of the known postfix convolved by the pseudo-circulant channel matrix: $\mathbf{r}_P^{\mathrm{ZP}}(i) = \mathbf{r}_P(i) - \alpha(i)\hat{\mathbf{H}}_{\beta_\mathbf{i}}\mathbf{c}_P$, with $\hat{\mathbf{H}}_{\beta_\mathbf{i}}$ being derived from the current channel estimate. In that case all known methods related to the ZP-OFDM can be applied. Among others let quote the corresponding ZF and MMSE equalizers provided in [8], [9]: $\mathbf{G}_{\mathrm{ZF}} = \mathbf{F}_N\mathbf{H}_o^{\dagger}$ and $\mathbf{G}_{\mathrm{MMSE}} = \mathbf{F}_N\mathbf{H}_o^H(\sigma_n^2\mathbf{I}+\mathbf{H}_o\mathbf{H}_o^H)^{-1}$, where $\mathbf{H}_o$ is the $P\times N$ matrix containing the N first columns of $\mathbf{H}_{\mathrm{ISI}}(P)$ and the frequency domain symbols $\tilde{\mathbf{s}}(i)$ are assumed uncorrelated and of unit variance. Note that other alternatives exist [14] and with an overlap-add (OLA) approach: ZP-OFDM-OLA, same performance and complexity as CP-OFDM is feasible.
- it is also possible to directly equalize (1) relying on the diagonalization properties of pseudo circulant matrices applied to $\mathbf{H}_{\beta_\mathbf{i}}$. We have:

$$\mathbf{H}_{\beta_i} = \mathbf{V}_\mathbf{P}^{-1}(i)\,\mathrm{diag}\left\{H(\beta_i^{-\frac{1}{P}}),\cdots,H(\beta_i^{-\frac{1}{P}}e^{j2\pi\frac{P-1}{P}})\right\}\mathbf{V}_\mathbf{P}(i)$$

where

$$\mathbf{V}_\mathbf{P}(i) = \left(\frac{1}{P}\sum_{n=0}^{P-1}|\beta_i|^{\frac{2n}{P}}\right)^{-\frac{1}{2}}\mathbf{F}_P\,\mathrm{diag}\left\{1,\beta_i^{\frac{1}{P}},\ldots,\beta_i^{\frac{P-1}{P}}\right\}$$

Throughout the paper, β_i is assumed to be a pure phase in order to preserve the overall block variance but for simplification sake let choose β_i as a M-PSK symbol: $\beta_i = e^{j2\pi\frac{m_i}{M}},\ m_i\in\{0,1,...,M-1\}$. In that condition (4) reduces to:

$$\mathbf{H}_{\beta_\mathbf{i}} = \mathbf{V}_\mathbf{P}^H(i)\,\mathrm{diag}\left\{H(e^{-j2\pi\frac{m_i}{MP}}),\ldots,H(e^{j2\pi\frac{(P-1)M-m_i}{PM}})\right\}\mathbf{V}_\mathbf{P}(i)$$

Thus diagonal

$$\mathbf{D}_i = \mathrm{diag}\left\{H(e^{-j2\pi\frac{m_i}{MP}}),\ldots,H(e^{j2\pi\frac{(P-1)M-m_i}{PM}})\right\}$$

is obtained for all m_i by a FFT of size PM of vector $(h_0,\cdots,h_{L-1},0,\rightarrow,0)^T$. The corresponding equalization matrices verifying the ZF and MMSE criteria are

$$\begin{aligned}\mathbf{G}_{\mathrm{ZF}}^{\mathrm{PRP}} &= \mathbf{F}_N\left[\mathbf{I}_N\mathbf{0}_{N,D}\right]\mathbf{H}_{\beta_\mathbf{i}}^{-1} \\ &= \mathbf{F}_N\left[\mathbf{I}_N\mathbf{0}_{N,D}\right]\mathbf{V}_\mathbf{P}^H(i)\mathbf{D}_i^{-1}\mathbf{V}_\mathbf{P}(i), \\ \mathbf{G}_{\mathrm{MMSE}}^{\mathrm{PRP}} &= \mathbf{F}_N\left[\mathbf{I}_N\mathbf{0}_{N,D}\right]\mathbf{R}_{\mathbf{s}_P,\mathbf{s}_P}\mathbf{H}_{\beta_\mathbf{i}}^H\mathbf{Q}^{-1} \\ &= \mathbf{F}_N\left[\mathbf{I}_N\mathbf{0}_{N,D}\right]\mathbf{R}_{\mathbf{s}_P,\mathbf{s}_P}\mathbf{V}_\mathbf{P}^H(i)\mathbf{D}_i^H\hat{\mathbf{Q}}^{-1}\mathbf{V}_\mathbf{P}(i),\end{aligned}$$

where $\mathbf{Q} = \sigma_n^2\mathbf{I}+\mathbf{H}_{\beta_\mathbf{i}}\mathbf{R}_{\mathbf{s}_P,\mathbf{s}_P}\mathbf{H}_{\beta_\mathbf{i}}^H$, $\hat{\mathbf{Q}} = \sigma_n^2\mathbf{I}+\mathbf{D}_i\hat{\mathbf{R}}_{\mathbf{s}_P,\mathbf{s}_P}\mathbf{D}_i^H$ and $\mathbf{R}_{\mathbf{s}_P,\mathbf{s}_P} = \mathrm{E}\left(\mathbf{s}_P(i)\mathbf{s}_P^H(i)\right)$, $\hat{\mathbf{R}}_{\mathbf{s}_P,\mathbf{s}_P} = \mathbf{V}_\mathbf{P}(i)\mathbf{R}_{\mathbf{s}_P,\mathbf{s}_P}\mathbf{V}_\mathbf{P}^H(i)$. For allowing an easier implementation, the following assumption is usually made:

$$\mathbf{G}_{\mathrm{MMSE}}^{\mathrm{PRP}}(i) \approx \mathbf{F}_N\left[\mathbf{I}_N\mathbf{0}_{N,D}\right]\mathbf{V}_\mathbf{P}^H(i)\mathbf{D}_i^H\left(\sigma_n^2\mathbf{I}+\mathbf{D}_i\mathbf{D}_i^H\right)^{-1}\mathbf{V}_\mathbf{P}(i)$$

This amounts to approximate $\mathrm{E}\left(\mathbf{s}_P(i)\mathbf{s}_P^H(i)\right)$ by $\sigma_s^2\mathbf{I}_P, \sigma_s^2 = 1$ and yields to nearly identical results up to 10^{-3} BER targeted for usual wireless systems.

Further improvements of the BER performance can be achieved by using *unbiased MMSE* equalizers as proposed by [15], [16]. As a conclusion to this section, it shall be pointed out that PRP-OFDM leads to a very simple modulation scheme on the transmitter side. In the receiver, a variety of demodulation and equalization approaches are possible, each characterized by different complexity/performance trade-offs.

B. Viterbi metric derivation for PRP-OFDM

In this subsection we assume that a bit interleaved convolutionally coded modulation is used at the emitter and explain how to derive the Viterbi metrics. For example for IEEE802.11a a rate $R=\frac{1}{2}$, constraint length $K=7$ Convolutional Code (CC) (o171/o133) is applied before bit interleaving over a single OFDM block followed by QAM mapping. Note that the approach detailed below is quite general and can be extended to other coding schemes.

According to (1), after equalization by any of the $N \times P$ matrices $\mathbf{G}$ presented above, the vector to be decoded can generally be expressed by:

$$\hat{\tilde{\mathbf{s}}} = \mathbf{G}\mathbf{r}_P(i) = \mathbf{G}_d\tilde{\mathbf{s}}_N(i) + \hat{\mathbf{n}}_N \tag{4}$$

where $\mathbf{G}_d$ is a diagonal weighting matrix and $\hat{\mathbf{n}}_N$ the total noise plus interference contribution which is assumed here Gaussian and zero-mean.

For maximum-likelihood decoding, usually a log-likelihood approach is chosen based on a multivariate Gaussian law leading to the following expression [17]:

$$\hat{\mathbf{d}} = \underset{\hat{\mathbf{d}}}{\operatorname{argmax}} \left\{ -\sum_{i=0}^{S-1} \left(\mathbf{G}_d m_N\left(\hat{\tilde{\mathbf{d}}}(i)\right) - \hat{\tilde{\mathbf{s}}}(i)\right)^H \mathrm{R}^{-1}_{\hat{\mathbf{n}}_N,\hat{\mathbf{n}}_N} \cdot \left(\mathbf{G}_d m_N\left(\hat{\tilde{\mathbf{d}}}(i)\right) - \hat{\tilde{\mathbf{s}}}(i)\right) \right\}$$

where vector $\hat{\mathbf{d}}$ contains an estimation of the original uncoded information bits, $\hat{\tilde{\mathbf{d}}}(i)$ gathers the corresponding bits after encoding, puncturing, etc. within the ith OFDM symbol. S is the number of OFDM symbols in the sequence to be decoded, $m_N(\cdot)$ is an operator representing the mapping of encoded information bits onto the N constellations, one for each carrier of the OFDM symbol.

Thus all what is needed for performing the decoding is an estimation of the noise covariance matrix $\mathrm{R}_{\hat{\mathbf{n}}_N,\hat{\mathbf{n}}_N}$ which requires the following derivations:

$$\begin{aligned}\hat{\tilde{\mathbf{s}}} &= \mathbf{G}\mathbf{r}_P(i) = \mathbf{G}\left(\mathbf{H}_{\beta_i}\begin{pmatrix}\mathbf{F}_N^H\tilde{\mathbf{s}}_N(i)\\ \alpha(i)\mathbf{c}_D\end{pmatrix} + \mathbf{n}_P(i)\right)\\ &= \mathbf{G}_d\tilde{\mathbf{s}}_N(i) + \mathbf{G}_f\tilde{\mathbf{s}}_N(i) + \alpha(i)\mathbf{G}_p\mathbf{c}_D + \mathbf{G}\mathbf{n}_P(i),\end{aligned}$$

where $\mathbf{G}_d$ is a $N \times N$ diagonal matrix and $\mathbf{G}_f$ a $N \times N$ full matrix with the main diagonal being zero such that

$$\mathbf{G}_d + \mathbf{G}_f = \mathbf{G}\mathbf{H}_{\beta_i}\begin{bmatrix}\mathbf{F}_N^H\\ \mathbf{0}_{D,N}\end{bmatrix} = \mathbf{G}\mathbf{H}_{\beta_i}\begin{bmatrix}\mathbf{I}_N\\ \mathbf{0}_{D,N}\end{bmatrix}\mathbf{F}_N^H$$

$\mathbf{G}_p$ is a $N \times D$ matrix containing the last D columns of the matrix $\mathbf{G}\mathbf{H}_{\beta_i}$ and $\mathbf{G}_f\tilde{\mathbf{s}}_N(i)$ represents the inter-symbol interference. Thus, the total noise plus interference vector is $\hat{\mathbf{n}}_N = \mathbf{G}_f\tilde{\mathbf{s}}_N(i) + \mathbf{G}\mathbf{n}_P(i) + \alpha(i)\mathbf{G}_p\mathbf{c}_D$ and its covariance is

$$R_{\hat{\mathbf{n}}_N,\hat{\mathbf{n}}_N} = \sigma_s^2\mathbf{G}_f\mathbf{G}_f^H + \sigma_n^2\mathbf{G}\mathbf{G}^H + \mathbf{G}_p\mathbf{c}_D\mathbf{c}_D^H\mathbf{G}_p^H$$

The trouble is that the overall noise covariance is not diagonal which yields to a very high complexity decoding scheme if no approximations are applied. One way to achieve a reasonable decoding complexity is to approximate $R_{\hat{\mathbf{n}}_N,\hat{\mathbf{n}}_N}$ by a matrix only containing its main diagonal elements. Then, standard OFDM VITERBI decoding is applicable with the modified proposed metrics.

V. Considerations for a proper design of the postfix

This sections provides recommendations on the design the PRP-OFDM postfix and the choice of the pseudo random weighting sequence.

First it is desirable to provide a flat spectrum without rays. In order to analyze the spectral properties of the PRP-OFDM signal since the signal is obviously not stationary but cyclostationary with periodicity P (duration of the OFDM block) [18], the order 0 cyclospectrum of the transmitted time domain sequence $s(k), k \in \mathbb{N}$ has to be calculated:

$$\mathrm{S}^{(0)}_{s,s}(z) \quad = \quad \sum_{k\in\mathbb{Z}} z^{-k}\frac{1}{P}\sum_{l=0}^{P-1}\mathrm{R}_{s,s}(l,k),$$

with $\mathrm{R}_{s,s}(l,k) = \mathrm{E}\left[s_{l+k}s_l^\star\right]$. Hereby, $\mathrm{R}_{s,s}(l,k)$ is given for the symbol $s(k = 0\dots P-1)$ as

$$\mathrm{R}_{s,s}(l,k) = \begin{cases}\mathrm{E}\left[s_{l+k}s_l^\star\right] & \text{for } k+l \geq 0 \text{ and } k+l < P\\ s_{l+k}s_l^\star\,\mathrm{E}\alpha & \text{for } k+l \geq mP \text{ and}\\ & k+l < mP+D, m \in \mathbb{Z}/\{0\}\\ 0 & \text{otherwise.}\end{cases}$$

with $\mathrm{E}_\alpha = \mathrm{E}\left[\alpha\left(\lfloor\frac{l+n}{P}\rfloor\right)\alpha^\star\left(\lfloor\frac{l}{P}\rfloor\right)\right]$. Now it is clear that it is desirable to choose $\alpha(i), i \in \mathbb{Z}$ such that $\mathrm{E}_\alpha = 0$ in order to clear all influence of the deterministic postfix in the second order statistics of the transmitted signal. This is achievable by choosing $\alpha(i)$ as a pseudo-random value.

In order to specify the content of D samples composing the postfix we can consider the following criteria:

i) minimize the time domain peak-to-average-power ratio (PAPR);
ii) minimize out-of-band radiations, i.e. concentrate signal power on useful carriers and
iii) maximize spectral flatness over useful carriers since the channel is not known at the transmitter (do not privilege certain carriers).

The resulting postfix is obtained through a multi-dimensional optimization involving a complex cost function. For concision sake, the complete procedure is not detailed in this paper. Note that if the PAPR criterion is not an issue, the solution is given by the Kaiser-window [19].

VI. Simulation results and conclusion

In order to illustrate the performances of our approach, simulations have been performed in the IEEE802.11a [2] or HIPERLAN/2 [3] WLAN context: a $N = 64$ carrier 20MHz bandwidth broadband wireless system operating in the 5.2GHz band using a 16 sample prefix or postfix. A rate $R = \frac{1}{2}$, constraint length $K = 7$ Convolutional Code (CC) (o171/o133) is used before bit interleaving followed by QPSK mapping.

Monte carlo simulations are run and averaged over 2500 realizations of a normalized BRANA [20] frequency selective channel without Doppler in order to obtain BER curves.

Fig.3 presents results where the CP-OFDM modulator has been replaced by a PRP-OFDM modulator. The curves compare the classical ZF CP-OFDM transceiver (standard

IEEE802.11a) and PRP-OFDM with the ZF, ZF-OLA (low complexity decoding) and MMSE equalizers over the $P = N + D$ carriers. Each frame processed contains 2 known training symbols, followed by 72 OFDM data symbols.

For the PRP-OFDM, after initial acquisition, the channel estimate is then refined by a MMSE based semi-blind procedure using an averaging window of 72 and 20 OFDM symbols. In the case of MMSE equalization, semi-blind refinement brings respectively a 1.5 dB and 0 dB gain for a BER of 10^{-3} over the reference CP-OFDM curve still 0.75 dB (averaging over 72 OFDM symbols) and 2 dB (averaging over 20 OFDM symbols) from the optimum performance reached with a perfect CIR knowledge. This gap can further be reduced by increasing the averaging window. ZF equalization performs poorly due to the occasional amplification of noise on certain carriers that is then spread back over all the carriers when changing the resolution of the frequency grid from $P = 80$ carriers back to $N = 64$. The ZF-OLA approach, however, avoids the noise correlation and leads to a acceptable performances: An averaging window of 72 OFDM symbols leads to nearly IEEE802.11a like performances which are achieved at a considerably reduced complexity compared to the MMSE approach. It hence is a suitable trade-off for low-cost hardware implementation.

Note that the merit of PRP-OFDM is not mainly to gain in SNR, but rather the ability to maintain the BER performance of the system quasi-constant in the presence of Doppler by postfix-based channel estimation.

In this contribution a new OFDM modulation has been presented based on a pseudo random postfix: PRP-OFDM, using known samples instead of random data. This multicarrier scheme has the advantage to inherently provide a very simple blind channel estimation exploiting this deterministic values. The same overhead as CP-OFDM is kept. Moreover several equalization approaches have been proposed with the same robustness granted by the ZP-OFDM receivers. Suboptimal arithmetical complexity efficient Viterbi decoding metrics have also been detailed.

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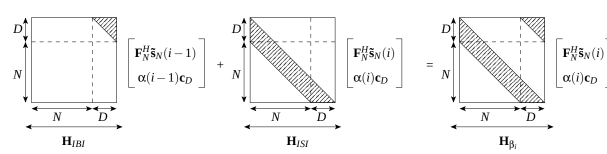

Fig. 2. Circularization for PRP-OFDM.

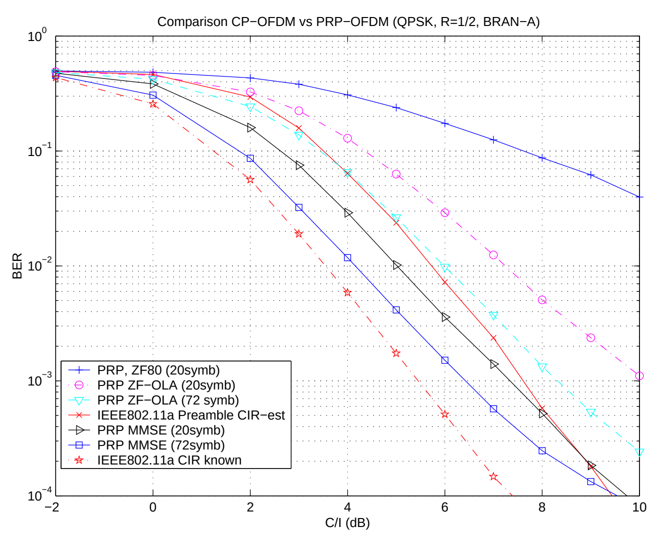

Fig. 3. BER for IEEE802.11a, BRAN channel model A, QPSK.