Broadcasting of a Common Source: Information-Theoretic Results and System Challenges

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Abstract — **We consider the problem of sending a common source to several users simultaneously at possibly different level of distortion. In this paper we review the classical information theoretic results on Broadcast Channels, then we examine the optimality of analog communication (single-letter coding) and we review the Hybrid Digital Analog (HDA) approach. Finally we motivate the need of all digital transmission and outline a joint source-channel coding approach based on layering.**

Classical Information Theoretic Approach. In 1972 Cover [1] formally introduced the concept of broadcast channel coding theory and discussed the basic problem of finding the capacity region $C(\Gamma) = \{(R_1, R_2, \ldots R_k) \in \mathcal{R}_+^k$ simultaneously achievable$\}$ where k is the number of users, R_i is the rate of user i and Γ is the average input cost constraint associated to average input cost function $\gamma(.)$. The case of broadcasting independent messages to each user over a degradated broadcast channel is the well studied and understood. Under this hypothesis Cover ([1] and references therein) has shown that by superimposing high-rate information on low-rate information, the achievable rates region dominates classical allocation policies such as time sharing or frequency multiplexing. Consider a two users degradated broadcast channel, superposition coding consists on embedding one high rate code in a lower rate code: a code for a channel noisier than the worst one is designed, considering a smaller number of codewords but also a higher tolerance to noise. This is exploited by packing in some extra message information that can be decoded by the best user only. The capacity region for the degradated broadcast channel $X \to Y_1 \to Y_2$ is the closure of the convex hull of the union of (R_1, R_2) satisfying $R_2 \leq I(U, Y_2)$ and $R_1 \leq I(X, Y_1|U)$ over all the $p(X, U)$ such that $E[\gamma(X)] \leq \Gamma$.

Broadcasting a common source. Suppose now that we want to broadcast a common source to several users simultaneously; the separation theorem is suboptimal since here the goal is to find a rate-distortion region simultaneously achievable for all the users, thus the tool is joint source-channel coding. An example of optimality of analog single letter mapping joint source-channel coding is when each channel is particularly matched to the source [3]; defining W_s and W_c the source and the channel bandwidth and $R(D)$ and $C(\Gamma)$ the source rate-distortion function and the channel capacity-cost function of each user, necessary and sufficient conditions for optimality are $W_s = W_c$, $R(D) = C(\Gamma)$ if and only if the cost function and the distortion measure are chosen among particular family of functions. Nevertheless the optimality achieving hypothesis may not be effective for some practical scenarios, such as the classical FM analog TV transmission where $W_c > W_s$. In this case coding of the source becomes necessary to exploit the additional channel bandwidth: nearly robust Hybrid Digital-Analog (HDA) joint source-channel coding have been proposed ([2] and references therein). Shamai et al. [4] show that systematic joint source channel coding (a type of bandwidth splitting HDA) is optimal for a wide class of sources and channels. Mittal et al. [2] computed the average squared-error distortion of several coding systems based on dimension (bandwidth) splitting and/or power splitting of the source; they are shown to outperform time-sharing systems, purely digital systems and in some cases systematic joint source-channel codes.

All Digital Systems. In today's systems the source is more and more digitally generated (HDTV). If we transmit the source uncoded we would require an enormous bandwidth; this motivate the need of lossy source coding; after compression the source becomes i.i.d meaning that channel coding is necessary to protect the information against noise. As a preliminary result we propose to study a joint source-channel coding approach based on layering. The source is coded in layers with a decreasing distortion function $D(R)$. Each user can decode a number of layers that depends on the quality of the channel. Following Shamai's [5] approach we want to find the optimal power allocation at the transmitter as a function of the fading statistic and of an outage probability (defined as the probability of not being able to decode the first layer), such that the average distortion is minimized, in both the cases of infinite and finite number of coding layers, subject to an average power constraint. This setting can model for example instantaneous broadcasting of fix images: future works will investigate the more complicated problem of video streaming where temporal dynamics have to be taken into account, and we will try to derive from theoretical results guidelines for implementable coding schemes.

References

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