On the capacity of MIMO Rice channels

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Abstract

The asymptotic –in the number of antennas– theoretic capacity of a MIMO (Multiple Input Multiple Output) system is derived when considering Rice distribution entries. Assuming perfect knowledge of the channel at the receiver, analytical expressions of the capacity are derived in the case of perfect and partial (based on the mean and limiting eigenvalue distribution of the mean) knowledge at the transmitter. Remarkably, the capacity depends only on a few meaningful parameters, namely, the limiting eigenvalue distribution of the mean matrix, the signal to noise ratio (SNR), the Ricean factor, and the system load. These results show in particular that, for a given SNR and in contrast to the SISO (Single Input Single Output) case, the MIMO Rice channel does not always outperform the MIMO Rayleigh channel in terms of capacity. Moreover, the results are also useful to quantify the effect of feedback on general MIMO systems.

I. Introduction

The problem of analyzing channel models is crucial for the efficient design of wireless systems [1]. Unlike the additive white Gaussian channel, the wireless channel suffers for constructive/destructive interference signaling [2]. This yields a randomized channel with arbitrary statistics. Recently [3], [4], the need to increase spectral efficiency has motivated the use of multiple antennas at both the transmitter and the receiver side. The pioneering works in [5] and [3] on multiple antennas at both the transmitter and the receiver site promise huge increases in the throughput of wireless communication systems. In the case of MIMO Rayleigh channels with n_t transmit antennas and n_r receive antennas, with perfect channel knowledge at the receiver and no channel knowledge at the transmitter, the ergodic capacity increase is min(n_r, n_t) bits per second per hertz for every 3dB increase at high SNR[1] [5]. However, for a general Ricean MIMO fading channel, linear growth with respect to the number of antennas of the ergodic capacity is still an open question. Indeed, although a Rice distribution is well known to enhance the performance with respect to the Rayleigh one in the SISO case, these results cannot be straightforwardly extended to the MIMO case and depend mainly on the characteristics of the line of sight component of the channel and on the channel state knowledge at the transmitter and the receiver.

Numerous contributions are already available on MIMO Ricean channels. The capacity of MIMO Ricean channels for various state of knowledge at the transmitter and the receiver and for various propagation models has already been considered in [6], [7], [8]: These analyses, based on simulations, have produced diverse conclusions depending on the assumptions made. This fact strengthens the need of a unified framework able to capture the essential parameters that characterize the capacity of a MIMO Rice channel. On the theoretical side, the available analysis has been mainly focused on rank 1 Rice fading. In [9], tight and upper bounds for the capacity of rank 1 Rice fading are derived when the transmitter has knowledge of the statistical

This research is supported in part by the Kplus programme of the Austrian government and by the European programme NEWCOM (Network of Excellence for Wireless Communications)

[1] While in the single antenna additive Gaussian channel, 1 bit per second per hertz can be achieved with every 3dB increase at high SNR.

properties of the fading process but does not know the instantaneous channel state. In [10], [11], the capacity of a Ricean MIMO channel with uncorrelated entries is studied: In the case of a diagonal line of sight matrix $\boldsymbol{A}$, the capacity was shown to be achieved by a Gaussian input vector of a diagonal covariance matrix. Moreover, the optimal covariance matrix of the Gaussian input vector is shown to have the same eigenvectors as $\boldsymbol{A}^H\boldsymbol{A}$. In [12], mutual information with uniform power allocation across the antenna elements is analyzed in the high SNR regime. It is shown that asymptotically in the SNR, this strategy of power allocation is optimal from a capacity perspective. In the case of a rank-1 Ricean MIMO channel, Hansel et al. show that the mutual information is asymptotically (in the number of antennas) non-central Gaussian [13]. Even though recent papers [14] have shown that the Rice distribution may incur a loss with respect to the i.i.d Rayleigh case, under which conditions this result is true is still an open problem as recently put into question in [15]. Hence, for general Rice models, results are still unknown and may seriously put into doubt the MIMO hype.

In this contribution, using asymptotic arguments, we derive the capacity of general Ricean MIMO channels assuming perfect knowledge of the channel at the receiver and various types of state of channel knowledge at the transmitter:

- Perfect channel knowledge at the transmitter.
- Knowledge of the channel mean matrix.
- Knowledge of the limiting singular value distribution of the channel mean at the transmitter

The results are based on random matrix theory [16] and are valid in the asymptotic regime, as the number of transmitting and receiving antennas increase but the ratio remains constant. The random matrices show self-averaging properties in asymptotic conditions, as both the matrix dimensions tend to infinity with a fixed ratio. This feature has been widely exploited in several contexts (e.g. code division multiple access systems, orthogonal frequency division modulation systems, and MIMO systems) and resulted in describing the system properties in terms of few macroscopic system parameters surprisingly valid for not so large systems. Our analysis predicts the behaviour of a general Ricean MIMO channel using only a few meaningful parameters, namely the asymptotic singular value distribution of the mean matrix $\boldsymbol{A}$, the Ricean factor K, the SNR, and the number of receive antennas per transmit antenna $\beta = \frac{n_r}{n_t}$. The analysis is all the more interesting as the results are striking in terms of closeness to simulations with reasonable size matrix (as far as capacity is concerned).

II. Channel Model and Notations

Throughout the paper the superscript H denotes the transpose conjugate of the matrix argument, $\boldsymbol{I}_n$ is the identity matrix of size $n \times n$ and $\mathbb{C}$ and $\mathbb{R}$ are the fields of complex and real numbers, respectively. $\mathrm{tr}(\cdot)$, $\|\cdot\|_F$, and $|\cdot|$ are the trace, the Frobenius norm, and the spectral norm of the argument, respectively (e.g. $\|\mathbf{A}\|_2 = \sqrt{\mathrm{tr}(\mathbf{A}\mathbf{A}^H)}$, $|\mathbf{A}| = \max_{\mathbf{x}\mathbf{x}^H \leq 1} \mathbf{x}^H\mathbf{A}\mathbf{A}^H\mathbf{x}$ with $\mathbf{x}$ arbitrary column vector). $\mathbb{E}(\cdot)$ is the expectation operator. δ_{ij} is the Kronecker symbol and $\delta(\lambda)$ is the Dirac's delta function.

We consider a point-to-point communication systems with n_t co-located transmitting antennas and n_r co-located receiving antennas. The channel is linear with flat fading and additive noise. Its discrete-time equivalent model is given by

$$\boldsymbol{y} = \sqrt{\frac{\rho}{n_t}}\boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}. \tag{1}$$

$\boldsymbol{x} \in \mathbb{C}^{n_t}$ is the column vector of transmitted signals with covariance matrix $\mathbb{E}(\boldsymbol{x}\boldsymbol{x}^H) = \boldsymbol{Q}$. $\boldsymbol{y} \in \mathbb{C}^{n_r}$ is the column vector of received signals. $\boldsymbol{n} \in \mathbb{C}^{n_r}$ is the column vector of the additive white Gaussian noise with circularly symmetric, zero mean and unitary variance entries. $\boldsymbol{H} \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix whose element h_{ij} represents the complex channel gain between the j-th transmit antenna and the i-th receive antenna. Without loss of generality, we assume that the average energy of the channel is normalized according to $\frac{\mathbb{E}(\mathrm{tr}(\boldsymbol{H}\boldsymbol{H}^H))}{n_r n_t} = 1$ so that ρ represents the SNR per receiving antenna. The complex entries of $\boldsymbol{H}$ are independently Gaussian distributed with identical variance and mean $\mathbb{E}(h_{ij}) = \mu_{ij}$. Denoting by K the Rice factor of the channel, we rewrite the channel matrix $\boldsymbol{H}$ as

$$\boldsymbol{H} = \sqrt{\frac{K}{K+1}}\boldsymbol{A} + \sqrt{\frac{1}{K+1}}\boldsymbol{B} \tag{2}$$

so as to separate the random component of the channel and the deterministic part:

- $\boldsymbol{A}$ represents the line of sight component of the channel such as $\|\boldsymbol{A}\|_F^2 = n_t n_r$ with entries $a_{ij} = \sqrt{\frac{K+1}{K}}\mu_{ij}$.
- $\boldsymbol{B}$ is the random component of the channel with Gaussian, independent and identically distributed entries. The complex element b_{ij} is circularly symmetric[2], with zero mean and unit variance.

Note that the model is general enough to take into account line of sight (LOS) and non line of sight (NLOS) cases. Indeed, as $K \to \infty$, (2) models a deterministic fading channel, whereas for $K = 0$ it describes a Rayleigh fading channel. In all the following, we will also assume that, as $n_r, n_t \to \infty$ with constant ratio $\beta = \frac{n_r}{n_t}$, the sequence of the empirical eigenvalue distribution of the matrix $\frac{\boldsymbol{A}\boldsymbol{A}^H}{n_t}$ converges in distribution to a deterministic limit function $F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}$. Note that within this assumption, one can take into account the case where the rank of $\mathbf{A}$ increases to $+\infty$ at the same rate than the number of antennas n_t.

Throughout this contribution, for any square matrix $\mathbf{C}$ we denote with $\widehat{\mathbf{C}}$ one of the possible diagonal similar matrices (through a singular value decomposition for example). We call "diagonal" an $m \times n$ matrix whose (i, j) component is zero whenever $i \neq j$, for $1 \leq i \leq m$ and $1 \leq j \leq n$. Given a rectangular matrix $\mathbf{C}$, $\widehat{\mathbf{C}}$ denotes a "diagonal" matrix obtained multiplying $\mathbf{C}$ on the left and on the right by unitary matrices. Finally, $J(\boldsymbol{H}, \boldsymbol{Q})$ denotes the mutual information per receiving antenna, between the input Gaussian vector $\boldsymbol{x}$ with covariance matrix $\boldsymbol{Q}$ and the output of the channel $\boldsymbol{H}$. In this case, it holds [17]

$$J(\boldsymbol{H}, \boldsymbol{Q}) = \frac{1}{n_r} \log_2 \det \left(\boldsymbol{I}_{n_r} + \frac{\rho}{n_t} \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H \right). \tag{3}$$

III. Main Results

In this section we analyze the capacity of the channel $\boldsymbol{H}$ per receiving antenna as the system size grows large, i.e. as $n_t, n_r \to \infty$ with constant ratio $\frac{n_r}{n_t} = \beta$. We refer to it as the asymptotic capacity per receiving antenna. The asymptotic capacity per receive antenna is determined assuming perfect knowledge of the channel at the receiver and for the following channel state information (CSI) at the transmitter:

- The transmitter has perfect knowledge of the channel. The correspondant asymptotic capacity per receiving antenna is denoted with C_1.
- The transmitter has perfect knowledge of the line of sight matrix $\boldsymbol{A}$. C_2 is the correspondent asymptotic capacity per receive antenna.
- The transmitter has perfect knowledge of the limiting eigenvalue of $\frac{\boldsymbol{A}\boldsymbol{A}^H}{n_t}$. In this case the natural choice for the transmitted vector is to distribute power equally over all transmitting antennas [18]. We denote by I_3 the correspondent mutual information per receive antenna.

[2]This condition can be easily removed. It is used here only to simplify the proof that the matrix $\frac{\boldsymbol{H}}{\sqrt{n_t}}$ satisfies the Lindberg condition.

A. *Perfect channel knowledge*

With perfect knowledge of the channel at the transmitter, the total transmit power can be allocated in the most efficient way over the different transmitters according to the water filling power allocation to achieve the highest possible bit rate [17].

Theorem 1: Given a channel $\boldsymbol{H}$, the channel capacity per receiving antenna converges almost surely, as $n_r = \beta n_t \to \infty$, to a deterministic value C_1 given by

$$C_1 = \frac{1}{\ln 2} \int_{\frac{1}{\mu^*}}^{+\infty} \ln \lambda \mu^* \mathrm{d}\, F_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(\lambda) \tag{4}$$

where μ^* satisfies the equation

$$\beta \int_{\frac{1}{\mu^*}}^{+\infty} \left(\mu^* - \frac{1}{\lambda}\right) \mathrm{d}\, F_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(\lambda) = \rho \tag{5}$$

and $F_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(\lambda)$ is the limit distribution function of the eigenvalues of $\frac{\boldsymbol{H}\boldsymbol{H}^H}{n_t}$, whose Stieltjes transform $m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z)$ is the unique solution of the fixed point equation

$$m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z) = \int \frac{\mathrm{d}\, F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda)}{\frac{K\lambda}{\beta m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z)+K+1} - z\left(\frac{\beta m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z)}{K+1} + 1\right) + \frac{1-\beta}{K+1}} \tag{6}$$

such that $\mathrm{Im}(m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z)) > 0$ for $\mathrm{Im}(z) > 0$.
Proof: See [19].

We recall that, if the distribution function $F(\lambda)$ has a continuous derivative, it is related to its Stieltjes transform $m(z) = \int \frac{dF(\lambda)}{\lambda - z}$ by

$$\frac{\mathrm{d}F}{\mathrm{d}\lambda} = \frac{1}{\pi} \lim_{y \to 0^+} \mathrm{Im}(m(\lambda + iy)). \tag{7}$$

Remarkably, C_1 is completely determined knowing only $F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda)$, β, ρ, and K and not the particular fluctuations of the fading. However, note that the ability to achieve C_1 requires that $\boldsymbol{H}$ is completely known at the transmitter (and in particular the eigenvectors on which to send the powers). Therefore, C_1 is achievable only if the channel fades slowly enough that the transmitter can have complete knowledge of the channel.

Remark

- In the case $K \to \infty$, equation (6) simplifies to

$$m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z) = \int \frac{\mathrm{d}\, F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda)}{\lambda - z} \tag{8}$$

  which is nothing else then the Stieljes transform of the distribution of the line of sight component.
- In the case $K \to 0$, equation 6 simplifies to

$$m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z) = \frac{1}{-z(\beta m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z) + 1) + (1 - \beta)} \tag{9}$$

which yields

$$\mathrm{d}\,F_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(\lambda) = \begin{cases} [1-\frac{1}{\beta}]^{+}\delta(\lambda) + \frac{1}{\pi\beta\lambda}\sqrt{\lambda - \frac{1}{4}(\lambda - 1 - \frac{1}{\beta})^2} & \text{if}(\sqrt{\frac{1}{\beta}}-1)^2 \leq \lambda \leq (\sqrt{\frac{1}{\beta}}+1)^2 \\ 0 & \text{otherwise} \end{cases}$$

where $[z]^{+} = \max(0, z)$. Note that although the computation of the capacity is independent of the channel realization, one needs however to know each realization to achieve it.

B. Perfect knowledge of the line of sight matrix $\boldsymbol{A}$

The channel capacity of a Ricean channel with channel mean matrix known at the transmitter is achieved by Gaussian transmitted vectors with covariance matrix having the same eigenvectors as the matrix $\frac{\boldsymbol{A}^H\boldsymbol{A}}{n_t}$ [10]. Therefore we focus our attention on transmitted vectors having these properties and derive from them the mutual information per receive antenna.

Theorem 2: Let $\mathbf{A}$ be the line of sight matrix of the channel $\boldsymbol{A}$ with the singular value decomposition $\mathbf{A} = \mathbf{V}\widehat{\mathbf{A}}\mathbf{U}^{H}$ and let $\widehat{\mathbf{Q}} \in \mathbb{R}^{n_r \times n_t}$ be a diagonal matrix. As $n_r, n_t \to \infty$ with $\frac{n_r}{n_t} \to \beta$, the mutual information of the channel $\mathbf{H}$ per receiving antenna with Gaussian inputs having covariance matrix $\mathbf{U}\widehat{\mathbf{Q}}\mathbf{U}^{H}$ converges almost surely to a deterministic value given by:

$$J(\mathbf{H}, \mathbf{U}\widehat{\mathbf{Q}}\mathbf{U}^{H}) = \frac{1}{\ln 2}\int_0^{\rho} \frac{1}{x}\left(1 - \frac{1}{x}m_2\left(-\frac{1}{x}\right)\right)\mathrm{d}\,x. \tag{10}$$

$m_2(z)$ is the unique function solution to the following system of equations:

$$\begin{cases} m_1(z) = \int \frac{\beta(m_1(z) - z(K+1))q(K+1)\,\mathrm{d}\,F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda)}{(K+1+\beta m_2(z)q)(m_1(z)-z(K+1)) + (K+1)K\lambda q} + \frac{(1-\beta)q_0(K+1)}{(K+1+\beta m_2(z)q_0)} \\ m_2(z) = \int \frac{(K+1+q\beta m_2(z))(K+1)\,\mathrm{d}\,F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda)}{(K+1+\beta m_2(z)q)(m_1(z)-z(K+1)) + \lambda q(K+1)K} \end{cases} \tag{11}$$

$q = q(\lambda, F(\lambda), K, \beta)$ denotes the diagonal entries of $\widehat{\mathbf{Q}}$ and $q_0 = q(0, F(\lambda), K, \beta)$.

Proof: See [19].

It follows as corollary of the previous theorem and from Lemma 2 in [10]:

Corollary 1: Let $\boldsymbol{H}, \boldsymbol{A}$ and $\widehat{\boldsymbol{Q}}$ be as in Theorem 2. The asymptotic channel capacity per receive antenna, as $n_r, n_t \to \infty$ with $\frac{n_r}{n_t} \to \beta$, converges almost surely to a deterministic value

$$C_2(\mathbf{H}) = \max_{\widehat{\boldsymbol{Q}}(\lambda, F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda), K, \beta)} J(\boldsymbol{H}, \mathbf{U}\widehat{\boldsymbol{Q}}\mathbf{U}^{H}) \tag{12}$$

with $J(\boldsymbol{Q}, \mathbf{U}\widehat{\boldsymbol{Q}}\mathbf{U}^{H})$ as in (10). $\widehat{\boldsymbol{Q}}(\lambda, F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda), K, \beta)$ satisfies the power constraint: $\frac{1}{n_t}\mathrm{tr}(\widehat{\boldsymbol{Q}}(\lambda, F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda), K, \beta)) \leq 1$.

If $\widehat{\boldsymbol{Q}}^{*}(\lambda, F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda), K, \beta)$ is the matrix achieving the maximum in (12), the achieving-capacity covariance matrix is $\boldsymbol{Q}^{*} = \mathbf{U}\widehat{\boldsymbol{Q}}^{*}(\lambda, F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}(\lambda), K, \beta)\mathbf{U}^{H}$.

Remark: Note that the solution does not correspond to any waterfilling solution on the mean matrix.

Interestingly, the capacity in (12) depends only on a few meaningful parameters, i.e. the limiting eigenvalue distribution $F_{\frac{A}{\sqrt{n_t}}}$, the Rice coefficient K, the ratio β, and the SNR ρ. Although the computation of the asymptotic capacity per receiving antenna requires only the limit eigenvalue distribution of the matrix $\frac{\boldsymbol{A}\boldsymbol{A}^H}{n_t}$, the channel capacity is achievable only if the line of sight matrix is completely known at the transmitter, since the achieving covariance matrix depends on the eigenvectors of $\frac{\boldsymbol{A}\boldsymbol{A}^H}{n_t}$. In practical communication systems this requires a feedback mechanism from the receiver. This feedback is feasible in practice as only statistical measures (mean of the channel) need to be sent back and not the varying channel realizations. Therefore, $C_2(\boldsymbol{H})$ is the capacity of practical interest, while $C_1(\boldsymbol{H})$ is an upper bound for $C_2(\boldsymbol{H})$ and is practically achievable only if the channel is quasi-static and the fading fluctuations can be tracked at the receiver and can be fed back to the transmitter.

C. Channel knowledge of the limiting eigenvalue distribution of $\frac{\boldsymbol{A}\boldsymbol{A}^H}{n_t}$

Since the transmitter has no channel knowledge of the eigenvector structure, the transmitted power is equally distributed among all the antennas and uniformly radiated on all directions. The corresponding asymptotic mutual information per receiving antenna is given in the following theorem.

Theorem 3: As $n_t, n_r \to \infty$ with $\frac{n_r}{n_t} \to \beta$, the asymptotic mutual information per receiving antenna with Gaussian input entries and covariance matrix $\boldsymbol{Q} = \boldsymbol{I}_{n_t}$ converges almost surely to a deterministic value

$$I_3 = \frac{1}{\ln 2}\int_0^{\rho} \frac{1}{x}\left(1 - \frac{1}{x} m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}\left(-\frac{1}{x}\right)\right) \mathrm{d}\, x \tag{13}$$

with $m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z)$ unique solution of the fixed point equation (6) and such that $\mathrm{Im}(m_{\frac{\boldsymbol{H}}{\sqrt{n_t}}}(z)) > 0$ for $\mathrm{Im}(z) > 0$.
Proof: See [19].

IV. NUMERICAL RESULTS

In this section we validate the theoretical results in Section III by showing that the average channel capacity of finite systems, with system size relatively small, matches the asymptotic capacity of the channel in the three cases of interest namely perfect knowledge of the channel at the transmitter, knowledge of the deterministic component of the channel at the transmitter, and no knowledge of the limiting eigenvalue distribution of the mean. Additionally, we use the results in Section III to study the behavior of MIMO channels and give answers to the initial questions that motivated this work:

- To compare the Rice MIMO channels to the Rayleigh MIMO channels and determine the impact of the Rice factor K on the channel capacity;
- To analyze how the rank of the line of sight matrix affects the system performance;
- To study the special case of MIMO channel with rank-1 line-of-sight matrix. This case received special attentions recently in the literature [13], [20];
- To determine the impact of β on the channel;

In the following we consider a Rice MIMO channel whose line-of-sight matrix has two distinct singular values $\sqrt{\lambda_1}$ and $\sqrt{\lambda_2}$. The limit empirical eigenvalue distribution of the matrix $\frac{\boldsymbol{A}\boldsymbol{A}^H}{n_t}$ is

$$F_{\frac{A}{\sqrt{n_t}}}(\lambda) = p_0\delta(\lambda) + p_1\delta(\lambda - \lambda_1) + p_2\delta(\lambda - \lambda_2). \tag{14}$$

$$p_0 = \begin{cases} 0 & \beta \le 1 \\ 1 - \frac{1}{\beta} & \beta > 1 \end{cases} \tag{15}$$

and p_1 and p_2 are completely defined, once fixed the ratio $\pi_{12} = \frac{p_1}{p_2}$, by the constraint $p_0 + p_1 + p_2 = 1$. The eigenvalues λ_1 and λ_2 are completely defined once their ratio $\lambda_{12} = \frac{\lambda_1}{\lambda_2}$ is fixed by the constraint $\|\boldsymbol{A}\|^2 = n_t n_r$, or equivalently

$$p_1 \lambda_1 + p_2 \lambda_2 = 1. \tag{16}$$

The function $q(\lambda)$ defining the eigenvalues of the capacity-achieving covariance matrix of the input vector is of the kind

$$q(\lambda) = q_0 \delta(\lambda) + q_1 \delta(\lambda - \lambda_1) + q_2 \delta(\lambda - \lambda_2) \tag{17}$$

with $q_0, q_1,$ and q_2 chosen so that they maximize (10) and satisfy the power constraint

$$\widetilde{p}_0 q_0 + \widetilde{p}_1 q_1 + \widetilde{p}_2 q_2 = 1. \tag{18}$$

$\widetilde{p}_0, \widetilde{p}_1,$ and $\widetilde{p}_2$ are the probabilities of the eigenvalues $0, \lambda_1,$ and λ_2 in the matrix $\frac{\boldsymbol{A}^H \boldsymbol{A}}{n_t}$. They satisfy the obvious relations $\widetilde{p}_1 = \beta p_1$, $\widetilde{p}_2 = \beta p_2$, and $\widetilde{p}_0 = 1 - \beta + \beta p_0$.

In Figure 1, 2, and 3 we considered a Rice MIMO channel with equal number of transmitting and receiving antennas, $\beta = 1$, $p_1 = \frac{1}{4}$, $p_2 = \frac{3}{4}$, $\lambda_1 = 3$, and $\lambda_2 = \frac{1}{3}$. We plotted the ergodic capacities C_1 and C_2 and the mutual information I_3 (markers in the figures) versus the Rice factor K for a 8×8 complex MIMO system at an SNR of 0, 5, and 10 dB, respectively. In the same figures, the lines show the corresponding asymptotic capacities as the system dimensions $n_t, n_r \to \infty$ with constant ratio. The ergodic capacities obtained by averaging the capacities of random generated 8×8 MIMO channels match the calculated curves of the asymptotic capacities. Note that the match between ergodic and asymptotic capacities is very good also for systems with a low number of antennas (2×2 MIMO systems) as shown later in figure 7.

A. *Effect of Channel State knowledge*

From the numerical analysis and Figure 1, 2, and 3 it is apparent that:

- C_1, C_2, and I_3 are not monotonic functions in K and the channel behavior depends heavily on the limit eigenvalue distribution $F_{\frac{\boldsymbol{A}}{\sqrt{n_t}}}$, the SNR, and the knowledge of the channel at the transmitter.
- In contrast to the case of a system with single transmitting and single receiving antenna, in which the Rice channel always outperforms the Rayleigh channel, in the MIMO case this is not true in general. Let us consider for instance Figure 2: If one has complete knowledge of the channel at the transmitter the Rayleigh MIMO channel outperforms the Rice channel. However, when the channel knowledge is limited to the line-of-sight matrix the capacity of the Rice channel is higher. Finally, if there is no feedback channel in the system and we assume only knowledge of the limiting eigenvalue distribution of $\boldsymbol{A}\boldsymbol{A}^H$ at the transmitter, the Rice channel has a lower mutual information than the Rayleigh channel.
- As already noticed in [12], the capacity gain due to feedback tends to disappear as the SNR becomes large and $\mathrm{C}_1 \to \mathrm{I}_3$ and $\mathrm{C}_2 \to \mathrm{I}_3$. This is better enlightened in Figure 4 and 5 where $\frac{C_1 - I_3}{I_3}$ and $\frac{C_2 - I_3}{I_3}$ versus K are plotted as a parametric function of ρ.

B. *Effect of line of sight matrix*

In order to analyze the effect of the line-of-sight matrix reduced rank we consider a 8×8 system with rank of the line of sight matrix varying from 1 to 8 and with identical non-zero singular values at $\rho = 10$ dB. This corresponds to consider an asymptotic system with $\beta = 1$

and $F_{\frac{\mathbf{A}}{\sqrt{n_t}}} = \frac{n}{8}\delta(\lambda) + \frac{8-n}{8}\delta(\lambda - \frac{8}{8-n})$ and $n = 1, \ldots, 8$. Figure 6 shows C_2 versus K as a parametric function of n. C_1 and I_3 have analogous behavior. Therefore, they are omitted. As K increases, the rank reduction of the line-of-sight matrix reduces the capacity due to the fact that the line-of-sight matrix becomes dominant with increasing K. Note that in all these cases, the transition phase with respect to K is around $[1, 10]$.

The Rice MIMO channel with rank-1 line-of-sight matrix has been widely studied under the assumption that the system size goes to infinity while keeping the line of sight matrix with constant rank equal to one. In this asymptotic regime the Rice MIMO channel behaves as a Rayleigh MIMO channel. However, this analysis is not able to catch the characteristics of a finite MIMO channel with rank-1 line of sight. For instance, in a 2×2 (4×4) MIMO channel only 50% (75%) of the line-of-sight matrix singular values are zero and not nearly 100% as previous contributions assume. The theoretical framework developed in Section III allows to analyze exactly the effect of rank-1 line-of-sight matrix in finite systems by modelling a system with $n \times n$ antennas as a system with $F_{\frac{\mathbf{A}}{\sqrt{n_t}}} = \frac{n-1}{n}\delta(\lambda) + \frac{1}{n}\delta(\lambda - n)$ and $\beta = 1$. Figure 7 shows the ergodic and asymptotic capacity C_1 per antenna versus K as a parametric function of n. The theoretical asymptotic curves describe perfectly the ergodic behavior of the rank-1 MIMO channel also for very small systems (e.g. $n = 2$).

C. *Effect of* β

The effect of β has been analyzed considering MIMO systems with equal total transmitted powers. According to the normalization made in Section II $P_t = n_t\rho$. Then, the SNR per receive antenna is given by $\frac{P_t}{n_t} = \frac{\rho}{\beta}$. The MIMO channel is described by (14) with $\pi_{12} = \frac{1}{3}$ and $\lambda_{12} = 9$, and $\beta = \frac{n}{8}$, $n = 1, \ldots, 16$. $\mathrm{SNR} = (10 - 10\log_{10}\beta)$ dB. In Figure 8 we plot C_1 versus K as a parametric function of β. Figure 8 shows that, while keeping the total transmitted power constant, the capacity per receiving antennas is a decreasing function of β. However, the total system capacity has to be an increasing function of β. In order to show the total system capacity $\widetilde{C}_{1t}$, which goes to infinity as the system size grows large, let us define $C_{1t} = \frac{\widetilde{C}_{1t}}{n_t}$. $C_{1t} = \beta C_1$ as the total capacity per transmitting antenna. Figure 9 shows C_{1t} versus the Ricean factor K as a parametric function of β. We can verify that the total system capacity increases as β increases.

V. Conclusions

In this contribution, the influence of line of sight components on the overall performance of MIMO systems has been considered. Although in the SISO case, it is well acknowledged that the capacity of Rice fading outperforms Rayleigh fading, in the MIMO case, this result does not hold[3]. Our analysis shows that the capacity depends only on the limiting behavior of the eigenvalues of the mean matrix, the ratio $\beta = \frac{n_r}{n_t}$, the SNR ρ and the Ricean Factor K through various fixed point equations depending on the type of state of knowledge available. For the case where only the line of sight matrix is known at the transmitter, the achieving capacity input covariance matrix was derived. Moreover, it was shown that for high SNR, the effect of feedback on the capacity diminishes as already highlighted previously in the case of Rayleigh fading. However, for low SNR, the effect of feedback is quite important and depends mainly on the Ricean factor K (typically for $K > 10$).

VI. Acknowledgment

The authors would like to thank Philippe Loubaton for very useful discussions.

[3]Note that the conclusion here differ with respect to [15] as we constrain ourselves in all the study to a power limited channel.

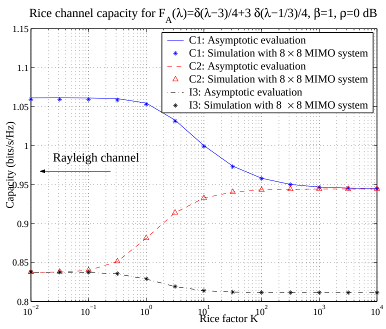

Fig. 1. Channel capacity comparison. C_1, C_2, and I_3 versus the Rice factor K for the channel with $F_{\frac{A}{\sqrt{n_t}}} = \frac{1}{4}\delta(\lambda - 3) + \frac{3}{4}\delta(\lambda - \frac{1}{3})$, $\beta = 1$, and $\rho = 0$ dB.

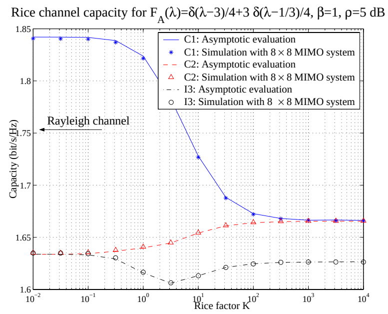

Fig. 2. Channel capacity comparison. C_1, C_2, and I_3 versus the Rice factor K for the channel with $F_{\frac{A}{\sqrt{n_t}}} = \frac{1}{4}\delta(\lambda - 3) + \frac{3}{4}\delta(\lambda - \frac{1}{3})$, $\beta = 1$ and $\rho = 5$ dB.

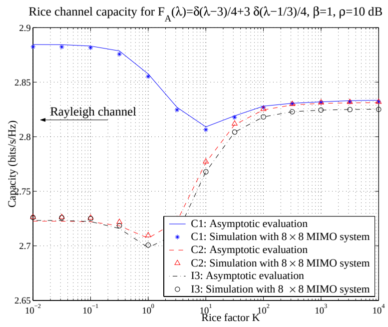

Fig. 3. Channel capacity comparison. C_1, C_2, and I_3 versus the Rice factor K for the channel with $F_{\frac{A}{\sqrt{n_t}}} = \frac{1}{4}\delta(\lambda - 3) + \frac{3}{4}\delta(\lambda - \frac{1}{3})$, $\beta = 1$ and $\rho = 10$ dB.

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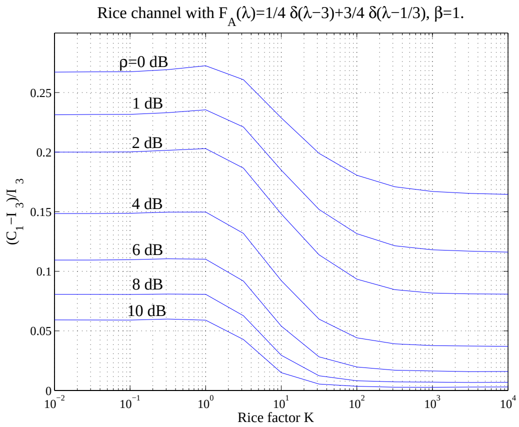

Fig. 4. $(\mathrm{C}_1 - \mathrm{I}_3)/\mathrm{I}_3$ versus the Rice factor K as parametric function of ρ for the channel with $F_{\frac{\mathbf{A}}{\sqrt{n_t}}} = \frac{1}{4}\delta(\lambda - 3) + \frac{3}{4}\delta(\lambda - \frac{1}{3})$, and $\beta = 1$.

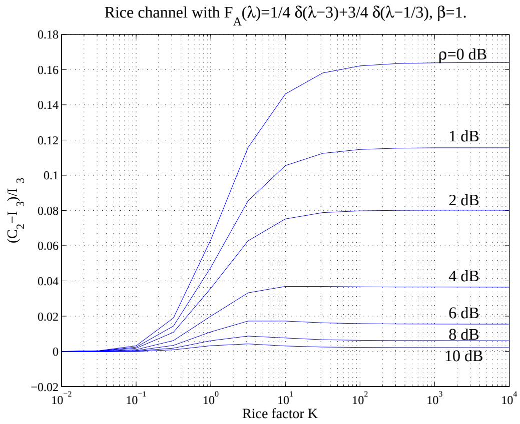

Fig. 5. $(\mathrm{C}_2 - \mathrm{I}_3)/\mathrm{I}_3$ versus the Rice factor K as parametric function of ρ for the channel with $F_{\frac{\mathbf{A}}{\sqrt{n_t}}} = \frac{1}{4}\delta(\lambda - 3) + \frac{3}{4}\delta(\lambda - \frac{1}{3})$, and $\beta = 1$.

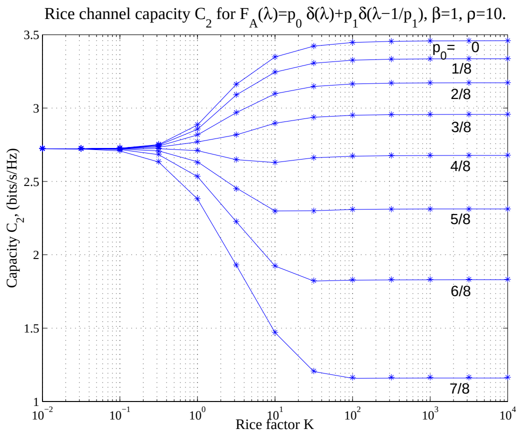

Fig. 6. C_2 versus the Rice factor K as parametric function of p_0 for the channel with $F_{\frac{\mathbf{A}}{\sqrt{n_t}}} = \frac{n}{8}\delta(\lambda) + \frac{8-n}{8}\delta(\lambda - \frac{8}{8-n})$, $n = 1, \ldots, 8$ $\beta = 1$, and $\rho = 10$ dB. The markers plot the ergodic capacity of finite 8×8 MIMO channels, the solid lines plot the asymptotic capacity.

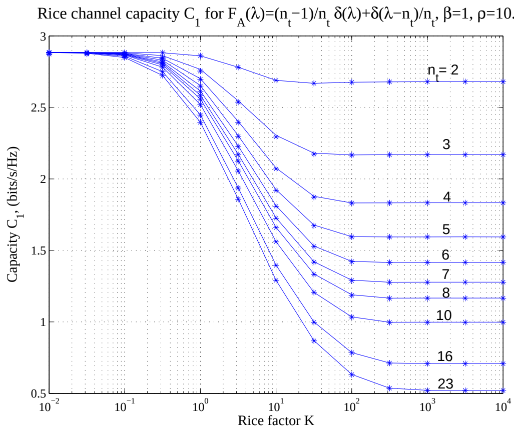

Fig. 7. C_1 versus the Rice factor K as parametric function of n_t for the channel with $F_{\frac{\mathbf{A}}{\sqrt{n_t}}} = \frac{n_t-1}{n_t}\delta(\lambda) + \frac{1}{n_t}\delta(\lambda - n_t)$, $\beta = 1$, and $\rho = 10$ dB. The markers plot the ergodic capacity for finite $n_t \times n_t$ MIMO channels, the solid lines plot the asymptotic capacity.

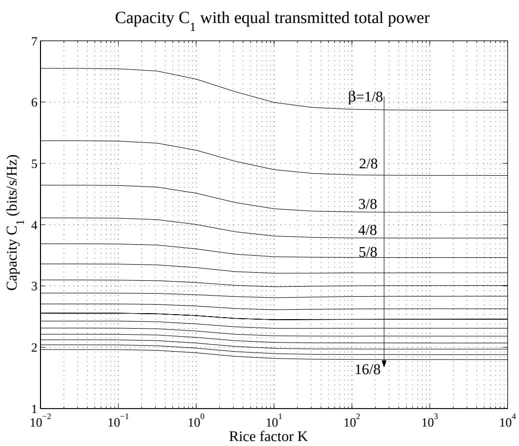

Fig. 8. C_1 versus the Rice factor K as parametric function of β with equal transmitted powers.

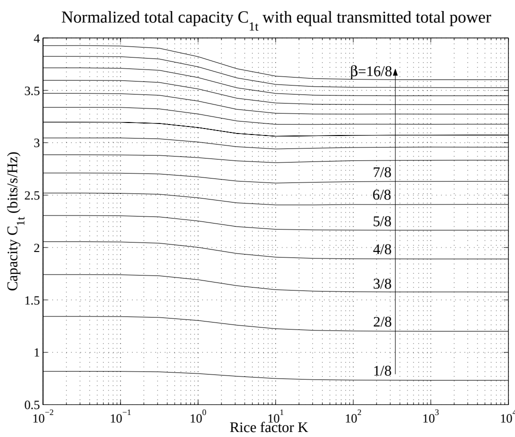

Fig. 9. Normalized total capacity C_{t1} versus the Rice factor K as parametric function of β with equal transmitted powers.