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Efficient Feedback Signaling using Multi-Channel Selection Diversity for Multi-User MIMO Systems

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***Abstract*—Recently, a number of techniques have been introduced to improve sum-rate performance of a wireless multiple-input multiple-output (MIMO) broadcast channel (BC). However, previous works have ignored the rate overhead associated with feedback of MIMO BC channel state information (CSI), which increases in proportion to the number of users in multi-user MIMO communications. Considering the limited amount of feedback signaling in a practical system, the effective feedback transmission as a form of partial CSI is required. In this paper, a novel scheme is proposed, of which the sum-rate performance approaches extremely close to the sum capacity of MIMO BC as the number of users increases, while the feedback overhead is designed to be reduced down to the number of active channel vectors. In our proposed scheme, multi-channel selection diversity (MCSD) is exploited to send partial CSI back to the transmitter.**

***Index Terms*—MIMO BC, Quantization, Feedback, Multi-Channel Selection diversity (MCSD)**

INTRODUCTION

Multiple-input multiple-output (MIMO) systems have been one of key techniques to achieve high rate and high reliability over wireless downlink channels. The investigation of the capacity region is of major concern in a MIMO broadcast channel (BC), where the base station (BS) has multiple transmit antennas and each mobile user has possibly multiple receive antennas [1]. In [2], it was shown that an achievable rate region for the multiple-input single-output (MISO) BC is obtained by applying the Costa precoding at the transmitter. Iterative water-filling with a sum-power constraint (SP-IWF) provides the optimum transmission policies for MIMO BC [3], [4], whereas the computational complexity is still an ongoing research area [5]. Effective approach with low complexity was proposed to exploit multi-user diversity, which is performed based on the greedy-type user selection instead of the optimal power allocation policy [6], [7]. However, prior work on computational complexity of sum-rate near optimal transmit schemes ignored the rate overhead associated with feedback of MIMO BC channel state information (CSI), to our best knowledge. In the limited channel feedback, i.e., partial CSI, most attention has been focused on single-user MIMO systems [8]. As a different approach for multi-user diversity based feedback reduction random beamforming was observed in [9], which may require infinite number of users to achieve sum capacity compared to the case using deterministic beams. In this paper, a novel scheme is proposed, of which the sum-rate performance approaches extremely close to the sum capacity of MIMO BC as the number of users increases, while the feedback overhead is designed to be reduced down to the number of active channel vectors. In our proposed scheme, multi-channel selection diversity (MCSD) is exploited to send partial CSI back to the transmitter. Simulation results indicate that the sum-rate performance of our novel scheme approaches extremely close to the sum capacity of MIMO BC with a few users.

Notations: The superscripts $()^T$ and $()^H$ stand for transpose, conjugate transpose, respectively. The cardinality of the set S is notated as |S|.

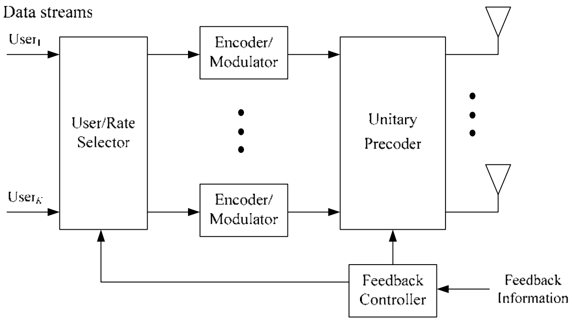

Figure 1. Schematic of the transmitter

System Model

Consider a K user wireless downlink communications system with multiple transmit antennas at the base station, as shown in Fig. 1, and multiple receive antennas for each user. We assume that the base station has t transmit antennas, the user k has r_k receive antennas, and the number of all receive antennas in the system is $r = \sum_{k=1}^{K} r_k$. Also, we model the channel as a frequency-flat block fading channel. Interference from neighboring cells is modeled as additive Gaussian noise, as we concentrate on the single cell model. The received signal of user k is expressed as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k$$

where the t x 1 input signal vector **x** is transmitted by the base station and is constrained to have power no greater than a sum-power constraint P, i.e., tr(E[$\mathbf{x}\mathbf{x}^H$]) $\leq$ P, and the t x 1 vector $\mathbf{z}_k$ represents the random additive noise for user k where $\mathbf{z}_k \sim CN(\mathbf{0},\mathbf{I})$. The channel $\mathbf{H}_k$ is a r_k x t matrix, whose entries are assumed to be independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero-mean and unit variance. Also, $\mathbf{H}_k$ is independent of $\mathbf{H}_j$ for all $j \neq k$.

In general, it is difficult for the base station to have the perfect knowledge of downlink channel state information (CSI) because the feedback link has delayed lossy feedback characteristics. Hence, the problem at hand is to find the transmit and receive structure that minimizes the feedback rate subject to the performance constraint such that the data throughput is kept as close as possible to the sum capacity.

Precoding MIMO-BC Scheme

Block QR Decomposition

We propose a multi-user MIMO scheme that is based on unitary beamforming and user selection diversity. It is assumed that t is the number of transmit antennas, r is the number of receive antennas, and K is the number of users. Beamforming using unitary transformation matrix **W** that is a function of the channel unitary matrices fed back from users is employed at the transmitter. The channel unitary matrix for feedback denotes the right-most matrix $\mathbf{V}_k$ obtained by SVD of the kth user channel $\mathbf{H}_k = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^H$. Each data stream for transmission is allocated to each beam vector of the unitary transform matrix, and the transmitter adjusts antenna rates independently. In the proposed system the channel is rotated using the right unitary matrix obtained by SVD of the each user channel, so as to reduce feedback overhead at the transmitter. MIMO channel is decomposed into multiple parallel MISO channels $\mathbf{F}_k$, which is referred to as the effective channel

$$\mathbf{F}_k = \mathbf{U}_k^H \mathbf{H}_k = \mathbf{D}_k \mathbf{V}_k^H$$

The row of the effective channel matrix $\mathbf{F}_k$ is also noted as the effective channel vector. In the transmitter, controlled beamforming is implemented by applying QR decomposition to the combination of the effective channels $\mathbf{F} = [\mathbf{F}_1^T, \ldots, \mathbf{F}_K^T]^T$. The effective BC **F** can then be treated as the multi-user MISO channel matrix. As in the algorithm of [6] for MISO, the QR decomposition is obtained using the Gram-Schmidt orthogonalization procedure to the rows of **F**. That is, geometrical projection is performed based on SVD decomposition, and then the finite dimensional subspace is determined by QR process. Using QR decomposition, the effective BC is represented as **F** = **RW**, where **R** is a *r* x *t* lower triangular matrix and **W** is a t x t matrix with orthonormal rows. The unitary matrix $\mathbf{W}^{H}$ is used for beamforming, and hence is applied to the transmitted signal

$$\begin{aligned} \mathbf{y} &= \mathbf{Fx} + \mathbf{z} \\ &= \mathbf{RWW}^{H}\mathbf{s} + \mathbf{z} \\ &= \mathbf{Rs} + \mathbf{z} \end{aligned}$$

where $\mathbf{y} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_K^T]^T$ and $\mathbf{z} = [\mathbf{z}_1^T, \ldots, \mathbf{z}_K^T]^T$. The sum-rate performance based on block QR decomposition is maximized by adopting MCSD which is described in the next subsection.

Multi-**Channel** Selective Diversity

Multi-user diversity is the promising solution to improve capacity gain while Costa precoding is the capacity-achieving strategy in MIMO BCs. In our proposed scheme, multi-channel based selective diversity (i.e., MCSD) is exploited in combination with Costa precoding for known interference cancellation, which means that the channel vectors of active users are selected and ordered to achieve diversity gain with the increase of the number of users and antennas therein, and interference cancellation using Costa precoding is processed at the transmitter to approach maximum sum-rate.

Let $S \subset \{1, \ldots, r\}$ be a subset of the effective channel vector indices that the BS selects for transmission using MCSD, and $\mathbf{F}(S) = [\mathbf{f}_1^T(S), \ldots, \mathbf{f}_{|S|}^T(S)]^T$ be the corresponding submatrix of **F**. The *t* x *t* unitary beamforming matrix $\mathbf{W}^{H}(S)$ is obtained by QR decomposition of the submatrix such that $\mathbf{F}(S) = \mathbf{R}(S)\mathbf{W}(S)$, where $\mathbf{W}(S) = [\mathbf{w}_1^T(S), \ldots, \mathbf{w}_{|S|}^T(S)]^T$ and $\mathbf{w}_i(S)$ is a 1 x *t* vector. Then, the achievable sum-rate of this system by Costa precoding is given by

$$\begin{aligned} R &\cong \max_{S} \sum_{i \in S} \log\left(1 + \frac{P}{|S|} |\mathbf{f}_i(S)\mathbf{w}_i^H(S)|^2\right) \\ &\le \max_{\sum_k tr(\mathbf{Q}_k) \le P, \mathbf{Q}_k \ge 0} \log\left|\mathbf{I} + \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{Q}_k \mathbf{H}_k\right| \end{aligned}$$

,

where each of the matrices $\mathbf{Q}_k$ is an r_k x r_k positive semi-definite covariance matrix. The selection process is partly performed in mobile users such that they select and feed back *l* active channels corresponding to the *l* largest eigenmodes, which reduces the feedback amount by a factor of *l*. The upper bound is the sum capacity of the MIMO BC as described above and the bound is achievable when the power *P* goes to infinity and the number of receive antennas is one for all receivers.

Candidate Schemes for Comparison

The sum-rate maximization can be solved efficiently by using SP-IWF, which achieves the sum capacity of a MIMO BC. On the other hand, time-division multiple-access (TDMA), where the BS transmits to only a single user at a time by using all transmit antennas, is a suboptimal solution when the BS has multiple transmit antennas, called TDMA-MIMO, while it achieves the sum capacity with only one transmit antenna. It is then shown that the maximum sum-rate of TDMA-MIMO is the largest single-user capacity of the *K* users, which is given by

$$C_{\text{TDMA-MIMO}} \triangleq \max_{i=1,\ldots,K} C(\mathbf{H}_i, P)$$

where $C(\mathbf{H}_i, P)$ denotes the single-user capacity of the *i*-th user subject to power constraint *P*.

Performance Analysis

In this section, the performance analysis is presented. We remind that the entries of {$\mathbf{H}_k$} are assumed to be i.i.d. zero-mean complex-Gaussian random variables. The proofs of the following lemmas and theorems are presented in [10].

Theorem 1 (Optimizing transmit covariance matrix) The objective of the transmit covariance matrix design is to find a covariance matrix set that maximizes the system throughput, subject to the sum power constraint and the unknown-interference free constraint. The transmit covariance matrix satisfying this objective is obtained by QR decomposition of $\mathbf{F}$.

Lemma 1 We assume that user k is not allowed to know CSI of all other users. That is, any information related to this CSI is not delivered from the transmitter as well as not exchanged between users. In this case, the optimal processing for user k is SVD-based (single-user) water-filling, in which the receive beamforming is performed with the left unitary matrix of the user k's channel.

Lemma 2 We consider a user that performs receive beamforming by the left unitary matrix of the corresponding channel. The average throughput of a MIMO BC with the user is no worse than the performance obtained based on non-cooperative reception across antennas, e.g., MMSE-DP.

Theorem 2 Receive beamforming with the left singular matrix offers the average throughput that is no worse than any fixed unitary matrix beam scheme.

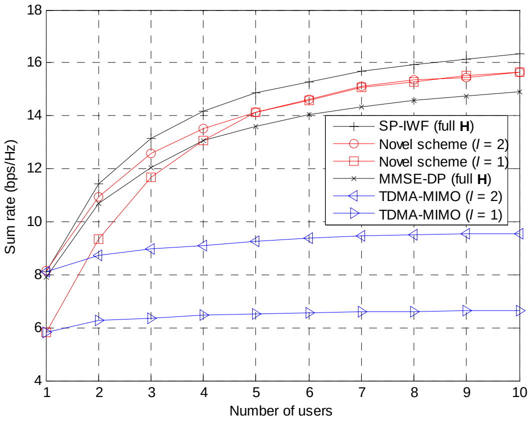

Figure 2. Ergodic sum-rate comparison when t = 4 and r = 2

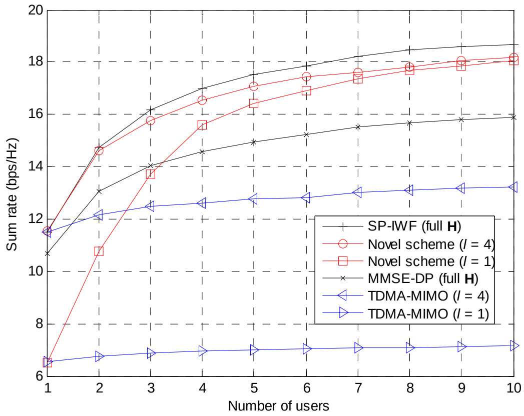

Figure 3. Ergodic sum-rate comparison when t = 4 and r = 4

Numerical Results

In this section, numerical results are presented. In Figs. 2 and 3, we compare the ergodic sum-rate performance of different MIMO downlink strategies. The signal-to-noise ratio (SNR) is assumed to be 10dB. Given the number of users, TDMA-MIMO achieves the maximum sum-rate corresponding to the largest single-user capacity, which shows relatively a small gain in proportion to the number of users. When the number of the active channel vectors is equal to the number of the effective channel vectors and one user is assumed, the performance of the proposed novel scheme is the same as that of TDMA-MIMO since in both cases receivers feed back the effective channel matrix $\mathbf{F}_k = \mathbf{D}_k\mathbf{V}_k^H$, instead of the full channel matrix $\mathbf{H}_k$. As the number of users becomes large enough, the performance of the novel scheme approaches close to the sum capacity, which can be driven by SP-IWF. Both figures show sum-rate improvement of 2 bps/Hz over MMSE dirty paper (MMSE-DP) scheme with full channel feedback and a gap of 0.4 bps/Hz from SP-IWF, in which MMSE-DP scheme exploits Costa precoding based on MMSE QR decomposition modified slightly from Caire's zero forcing dirty paper (ZF-DP) coding in [2].

In our proposed scheme, different feedback scenarios are examined. In Fig. 2, each user has two eigenmodes, i.e., two effective channel vectors, available since four transmit and two receive antennas are assumed. The sum-rate of the novel scheme with feedback of one active channel vector (one eigenvector multiplied by the corresponding eigenvalue that is the largest one) gets tightly close to the performance having feedback of two active channel vectors when the number of users is five. Contrastingly, TDMA-MIMO with one vector never gets close to TDMA-MIMO with two vector. Four transmit and four receive antennas are considered in Fig. 3, where two feedback signaling (i.e., one, four active channel vectors) are examined for the novel and TDMA-MIMO schemes. Both figures show that the novel scheme with reduced feedback, i.e., with the fewer active channel vectors, achieves slightly lower rate performance with small number of users compared to the scheme with full effective channel vector. However, the performance approaches extremely close to the upper bound as the number of users increases. Therefore, in the proposed scheme feedback of active channel vectors is shown to have the equivalent sum-rate performance with feedback of full effective channel vectors, resulting in the outstanding feedback robustness. That is, the feedback signaling per user can be significantly reduced with the increase of the number of users.

Conclusion

In this paper, we have proposed a multiuser MIMO transmission scheme that is efficient in terms of computational complexity and feedback overhead while obtaining near the maximum sum-rate of BC. Our novel scheme has employed *the block QR decomposition* at the transmitter, which reduces the computational complexity of designing transmit covariance matrices. Using MCSD in combination with known interference cancellation (Costa precoding), the proposed scheme with partial channel information at the transmitter has still achieved the near-optimal sum capacity, which was not observed in TDMA-MIMO. Numerical results have shown that the gain of sum-rate is 2 bps/Hz over the conventional MMSE-DP scheme with *full channel feedback* and the gap from SP-IWF is 0.4 bps/Hz.

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REFERENCES

[1] H. Sato, “An outer bound on the capacity region of the broadcast channel,” *IEEE Trans. Inform. Theory*, vol. 24, pp. 374–3778, May. 1978.

[2] G. Caire and S. Shamai, “On the achievable throughput of a multiantenna gaussian broadcast channel,” *IEEE Trans. Inform. Theory*, vol. 49, no. 7, pp. 1691–1706, July 2003.

[3] W. Yu, W. Rhee, S. Boyd, and J. Cioffi, “Iterative water-filling for gaussian vector multiple access channels,” *IEEE Trans. Inform. Theory*, vol. 50, no. 1, pp. 145–151, January 2004.

[4] N. Jindal, W. Rhee, S. Vishwanath, S. Jafar, and A. Goldsmith, “Sum power iterative water-filling for multi-antenna Gaussian broadcast channels,” Submitted to *IEEE Trans. on Information Theory*, July. 2004.

[5] W. Yu, “A dual decomposition approach to the sum power Gaussian vector multiple access channel sum capacity problem,” in *Conference on Information Sciences and Systems (CISS)*, Mar. 2003.

[6] Z. Tu and R. S. Blum, “Multiuser diversity for a dirty paper approach,” *IEEE Commun. Lett.*, vol. 7, pp. 370–372, Aug 2003.

[7] M. Airy, A. Forenza, R. W. Heath, Jr., and S. Shakkottai, “Practical Costa precoding for the multiple antenna broadcast channel,” Accepted to *Proc. IEEE GLOBECOM 2004.*

[8] D. J. Love, R. W. H. Jr., W. Santipach, and M. L. Honig, “What is the value of limited feedback for mimo channels?” *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54–59, Oct. 2004.

[9] M. Sharif and B. Hassibi, “On the capacity of mimo broadcast channel with partial CSI,” in *Proc. Asilomar conf.*, Pacific Grove, CA, Nov. 2003.

[10] J. S. Kim, H. Kim, and K. B. Lee, “Limited Feedback Signalling for MIMO Broadcast Channels,” submitted to *IEEE SPAWC 2005*.

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