A Low-Complexity Approach to Space-Time Coding for Multipath Fading Channels

Mari Kobayashi and Giuseppe Caire *

Institut EURECOM,

2229 Route des Cretes, B.P. 193,

06904 SOPHIA ANTIPOLIS CEDEX, France

E-mail: {mari.kobayashi, giuseppe.caire}@eurecom.fr

October 7, 2004

*This work was supported by France Telecom. The content of this paper was partially presented in WPMC'2003, Yokosuka, Japan, in 2003.

Abstract

We consider a single-carrier multi-input single-output (MISO) wireless system where the transmitter is equipped with multiple antennas and the receiver has a single antenna. For this setting, we propose a space-time coding scheme based on the concatenation of trellis-coded modulation (TCM) with time-reversal orthogonal space-time block coding (TR-STBC). The decoder is based on reduced-state joint equalization and decoding, where a minimum mean-square-error decision-feedback equalizer is combined with a Viterbi decoder operating on the TCM trellis without trellis state expansion. In this way, the decoder complexity is independent of the channel memory and of the constellation size. We show that, in the limit of large block length, the TCM-TR-STBC scheme with reduced-state joint equalization and decoding can achieve the full diversity offered by the MISO multipath channel. Remarkably, simulations show that the proposed scheme achieves full diversity for short (practical) block length and simple TCM codes. The proposed TCM-TR-STBC scheme offers similar/superior performance with respect to the best previously proposed schemes at significantly lower complexity and represents an attractive solution to implement transmit diversity in high-speed TDM-based downlink of 3rd generation systems, such as EDGE and UMTS.

Keywords: Space-Time Coding, Trellis-Coded Modulation, Joint Equalization and Decoding.

1 Motivations

In classical wireless cellular systems user terminals are miniaturized handsets and typically cannot host more than a single antenna. On the other hand, base-stations can be easily equipped with multiple antennas. Hence, we are in the presence of a multi-input single-output (MISO) channel. For pedestrian users in a urban environment, the propagation channel is typically slowly-fading and frequency-selective. For single-carrier transmission, as used in current 3rd generation standards [1, 2], frequency-selectivity generates inter-symbol interference (ISI). In systems that do not make use of spread-spectrum waveforms, such as GPRS and EDGE [2] or certain modes of wideband CDMA [1] using very small spreading factors, ISI must be handled by linear/decision-feedback equalization or maximum-likelihood sequence detection [3].

Due to the slowly-varying nature of the fading channel, a codeword spans a limited number of fading degrees of freedom. In the absence of reliable channel state information at the transmitter, the word-error probability (WER) is dominated by the so-called information outage event: namely, the event that the transmitted coding rate is above the mutual information of the channel realization spanned by the transmitted codeword [4]. In such conditions, the WER can be greatly improved by using space-time codes (STCs), i.e., coding schemes whose codewords are transmitted across the time dimension as well as the space-dimension introduced by the multiple transmit antennas [5].

In a frequency-selective MISO Rayleigh fading channel with M transmit antennas and P independent (separable) multipath components, it is immediate to show that the best WER behavior achievable by STC for high SNR is $O(\text{SNR}^{-d_{\max}})$, where $d_{\max} \triangleq MP$ is the maximum achievable *diversity order* of the channel, equal to the number of fading degrees of freedom. We hasten to say that in this work we focus on MISO channels and on STC design for achieving maximum diversity. Obviously, the STC scheme proposed in this paper can be trivially applied to the case of multiple receiver antennas (MIMO channel). However, for

$N_r > 1$ antennas at the receiver our scheme (as well as the competitor schemes mentioned in the following) would not be able to exploit the spatial multiplexing capability of the channel, i.e., the ability of creating up to $\min\{M, N_r\}$ parallel channels between transmitter and receiver, thus achieving much higher spectral efficiency. The optimal tradeoff between the achievable spatial *multiplexing gain* and *diversity gain* in frequency-flat MIMO channels was investigated in [6] and the analysis has been recently extended to the frequency-selective case in [7].

The design of space-time codes (STCs) for single-carrier transmission over frequency-selective MISO channels has been investigated in a number of recent contributions [8–11]. Maximum-likelihood (ML) decoding in MISO frequency-selective channels is generally too complex for practical channel memory and modulation constellation size. Hence, research has focused on suboptimal low complexity schemes. We may group these approaches into two classes. The first approach is based on mitigating ISI by some MISO equalization technique, and then designing a space-time code/decoder for the resulting flat fading channel. For example, the use of a linear minimum mean-square-error (MMSE) equalizer combined with Alamouti's space-time block code [12] has been investigated in [11]. However, this scheme does not achieve in general the maximum diversity order offered by the channel. The second approach is based on designing the STC by taking into account the ISI channel and then performing joint equalization and decoding. For example, trellis coding and bit-interleaved coded modulation (BICM) with turbo equalization have been proposed in [9, 13]. Time reversal orthogonal space-time block codes (TR-STBC) [8] (see also [9]) converts the MISO channel into a standard single-input single-output (SISO) channel with ISI, to which conventional equalization/sequence detection techniques, or turbo-equalization, can be applied. Turbo-equalization schemes need a soft-in soft-out MAP decoder for the ISI channel, whose complexity is exponential in the channel impulse response length and in the constellation size. For example, MAP symbol-by-symbol detection implemented by the BCJR algorithm [14] runs on a trellis with $|\mathcal{X}|^{L-1}$ states, where $|\mathcal{X}|$ denotes the size of the transmitted signal constellation $\mathcal{X} \subset \mathbb{C}$, and L denotes the channel impulse response length (expressed in symbol intervals).

In this work we consider the concatenation of TR-STBC with an outer trellis coded modulation (TCM) [15]. At the receiver, we apply reduced-state sequence detection based on joint MMSE decision-feedback equalization (DFE) and decoding. The decisions for the MMSE-DFE are found on the surviving paths of the Viterbi decoder acting on the trellis of the TCM code. Since the joint equalization and decoding scheme works on the trellis of the original TCM code, without trellis state expansion due to the ISI channel, the receiver complexity is independent of the channel length and of the constellation size. This makes our scheme applicable in practice even for large signal constellations and channel impulse response length as specified in 2nd and 3rd generation standards, while the schemes proposed in [9, 10, 13] are not.

We show that, in the limit of large block length, the TCM-TR-STBC scheme with reduced-state joint equalization and decoding can achieve the full diversity offered by the MISO multipath channel. Remarkably, simulations show that the proposed scheme achieves full diversity for short (practical) block length and simple TCM codes.

A significant advantage of the proposed TCM-TR-STBC scheme is that TCM easily implements adaptive modulation by adding uncoded bits (i.e., parallel transitions in the TCM trellis) and by expanding correspondingly the signal constellation [15]. Since the constellation size has no impact on the decoder complexity, variable-rate (adaptive) modulation can be easily implemented. This fact has particular relevance in the implementation of high-speed downlink schemes based on dynamic scheduling, where adaptive modulation is required [16]. Remarkably, simulations show that the TCM-TR-STBC scheme achieves WER performance at least as good as (if not better than) previously proposed schemes [9, 10, 13], that are more complex and less flexible in terms of variable-rate coding implementation.

The rest of the paper is organized as follows. Sections 2 and 3 describe our concatenated TCM-TR-STBC scheme and the low-complexity reduced-state joint equalization and decoding scheme. In Section 4 we derive two approximations to the WER of the proposed scheme. Numerical results are presented in Section 5 and Section 6 concludes the paper.

2 The concatenated TCM-TR-STBC scheme

System model. The channel from the i-th transmit antenna to the receive antenna is formed by a pulse-shaping transmit filter (e.g., a root-raised cosine pulse [3]), a multipath fading channel with P separable paths, an ideal low-pass filter with bandwidth $[-N_s/(2T_s), N_s/(2T_s)]$ where $N_s \geq 2$ is an integer and T_s is the symbol interval, and a sampler taking N_s samples per symbol. We assume that the fading channels are random but constant in time for a large number of symbol intervals (quasi-static assumption). We also assume that the overall channel impulse response spans at most L symbol intervals, corresponding to $N_g \triangleq LN_s$ receiver samples. Let $(s_i[0], \ldots, s_i[N-1], 0, \ldots, 0)$ denote the sequence of symbols transmitted over antenna i, where we add a tail of $L-1$ zeros in order to avoid inter-block interference. The discrete-time complex baseband equivalent MISO channel model can be written in vector form as

$$\mathbf{r} = \sum_{i=1}^{M} \mathcal{H}(\mathbf{g}_i)\mathbf{s}_i + \mathbf{w} \tag{1}$$

where $\mathbf{r} \in \mathbb{C}^{N_s(N-1)+N_g}$, $\mathbf{w} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{I})$ is the complex circularly-symmetric Additive White Gaussian Noise (AWGN), $\mathbf{s}_i = (s_i[0], \ldots, s_i[N-1])^{\mathsf{T}}$, and $\mathcal{H}(\mathbf{g}_i) \in \mathbb{C}^{(N_s(N-1)+N_g)\times N}$ is the convolution matrix obtained from the overall sampled channel impulse response $\mathbf{g}_i \in \mathbb{C}^{N_g}$ as follows: the n-th column of $\mathcal{H}(\mathbf{g}_i)$ is given by

$$(\underbrace{0, \ldots, 0}_{nNs}, g_i[0], \ldots, g_i[N_g-1], \underbrace{0, \ldots, 0}_{N_s(N-n-1)})^{\mathsf{T}} \tag{2}$$

for $n = 0, \ldots, N-1$.

Time-reversal STBC. TR-STBC [8] is a clever extension of orthogonal space-time block codes based on generalized orthogonal designs (GOD) [17–19] to the frequency-selective channel. As we shall briefly review in the following, the TR-STBC turns a frequency-selective MISO[1] into a standard SISO channel with ISI, by simple linear processing given by matched filtering and combining.

A $[T, M, k]$-GOD is defined by a mapping $\mathbf{S} : \mathbb{C}^k \to \mathbb{C}^{T\times M}$ such that, for all $\mathbf{x} \in \mathbb{C}^k$, the corresponding matrix $\mathbf{S}(\mathbf{x})$ satisfies $\mathbf{S}(\mathbf{x})^{\mathsf{H}}\mathbf{S}(\mathbf{x}) = |\mathbf{x}|^2\mathbf{I}$. Moreover, the elements of $\mathbf{S}(\mathbf{x})$ are linear combinations of elements of $\mathbf{x}$ and of $\mathbf{x}^*$.

Let $\mathbf{S}$ be a $[T, M, k]$-GOD with the following additional property: A1) the row index $\{1, \ldots, T\}$ set can be partitioned into two subsets, $\mathcal{T}_1$ and $\mathcal{T}_2$, such that all elements of the t-th rows with $t \in \mathcal{T}_1$ are given by $a_{t,i}x_{\pi(t,i)}$, for $i = 1, \ldots, M$, and all elements of the t-th rows with $t \in \mathcal{T}_2$ are given by $a_{t,i}x^*_{\pi(t,i)}$, for $i = 1, \ldots, M$, where $a_{t,i}$ are given complex coefficients and $\pi : \{1, \ldots, T\} \times \{1, \ldots, M\} \to \{1, \ldots, k\}$ is a given indexing function.

Given a $[T, M, k]$-GOD $\mathbf{S}$ satisfying property A1) and two integers $N \geq 1$ and $L \geq 1$, we define the associated TR-STBC $\mathbf{T}$ with parameters $[T, M, k, N, L]$ as the mapping $\mathbb{C}^{N\times k} \to \mathbb{C}^{T(N+L-1)\times M}$ that maps the k vectors $\{\mathbf{x}_j \in \mathbb{C}^N : j = 1, \ldots, k\}$ into the matrix $\mathbf{T}(\mathbf{x}_1, \ldots, \mathbf{x}_k)$ defined as follows. For all $t \in \mathcal{T}_1$, replace the i-th element of $\mathbf{S}$ by the vector $a_{t,i}\mathbf{x}_{\pi(t,i)}$ followed by $L-1$ zeros. For all $t \in \mathcal{T}_2$, replace the i-th element of $\mathbf{S}$ by the vector $a_{t,i}(\circ\mathbf{x}_{\pi(t,i)})$ followed by $L - 1$ zeros, where the complex conjugate time-reversal operator $\circ$ is defined by

$$\circ(v[0], \ldots, v[N-1])^{\mathsf{T}} = (v^*[N-1], \ldots, v^*[0])^{\mathsf{T}}$$

The time-reversal operator satisfies the following elementary properties: B1) Let $\mathcal{H}(\mathbf{g})$ be a convolution matrix as defined by (2) and $\mathbf{s} \in \mathbb{C}^N$. Then, $\circ(\mathcal{H}(\mathbf{g}) \circ \mathbf{s}) = \mathcal{H}(\circ\mathbf{g})\mathbf{s}$. B2) Let $\mathbf{g}$ and $\mathbf{h}$ be two impulse responses of length N_g, then $\mathcal{H}(\mathbf{g})^{\mathsf{H}}\mathcal{H}(\mathbf{h}) = \mathcal{H}(\circ\mathbf{h})^{\mathsf{H}}\mathcal{H}(\circ\mathbf{g})$.

In order to transmit the k blocks of N symbols each over the MISO channel defined by (1) by using a TR-STBC scheme with parameters $[T, M, k, N, L]$, the columns of $\mathbf{T}(\mathbf{x}_1, \ldots, \mathbf{x}_k)$ are transmitted in parallel, over the M antennas, in $T(N + L - 1)$ symbol intervals. Due to

[1] Extension to MIMO is straightforward, but as anticipated in Section 1, it is less relevant due to the significant spectral efficiency loss of orthogonal STBCs in MIMO channels.

the insertion of the tails of $L-1$ zeros, the received signal can be partitioned into T blocks of $N_s(N-1)+N_g$ samples each, without inter-block interference. If $t \in \mathcal{T}_1$, the t-th block takes on the form

$$\mathbf{r}_t = \sum_{i=1}^{M} a_{t,i}\mathcal{H}(\mathbf{g}_i)\mathbf{x}_{\pi(t,i)} + \mathbf{w}_t \tag{3}$$

If $t \in \mathcal{T}_2$, by using property B1) and the fact that when $\mathbf{w}_t \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{I})$ then $\circ\mathbf{w}_t$ and $\mathbf{w}_t$ are identically distributed, the t-th block takes on the form

$$\circ\mathbf{r}_t = \sum_{i=1}^{M} a^*_{t,i}\mathcal{H}(\circ\mathbf{g}_i)\mathbf{x}_{\pi(t,i)} + \mathbf{w}_t \tag{4}$$

We form the observation vector $\tilde{\mathbf{r}}$ by stacking blocks $\{\mathbf{r}_t : t \in \mathcal{T}_1\}$ and $\{\circ\mathbf{r}_t : t \in \mathcal{T}_2\}$. The resulting vector can be written as

$$\tilde{\mathbf{r}} = \mathcal{Q}(\mathbf{g}_1, \ldots, \mathbf{g}_M)\begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_k \end{bmatrix} + \mathbf{w} \tag{5}$$

where $\mathbf{w} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{I})$. The matrix $\mathcal{Q}(\mathbf{g}_1, \ldots, \mathbf{g}_M)$ has dimensions $T(N_s(N-1)+N_g) \times Nk$ and it is formed by Tk blocks of size $(N_s(N-1)+N_g) \times N$. The (t,j)-th block is given by $a_{t,i}\mathcal{H}(\mathbf{g}_i)$ for $t \in \mathcal{T}_1$ and $j = \pi(t,i)$, or by $a^*_{t,i}\mathcal{H}(\circ\mathbf{g}_i)$ for $t \in \mathcal{T}_2$ and $j = \pi(t,i)$. From the orthogonality property of the underlying GOD $\mathbf{S}$ and from property B2) it is straightforward to show that

$$\mathcal{Q}(\mathbf{g}_1, \ldots, \mathbf{g}_M)^{\mathsf{H}}\mathcal{Q}(\mathbf{g}_1, \ldots, \mathbf{g}_M) = \begin{bmatrix} \mathbf{\Gamma} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma} & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{\Gamma} \end{bmatrix} \tag{6}$$

where we define the combined total channel response

$$\mathbf{\Gamma} \triangleq \sum_{i=1}^{M} \mathcal{H}(\mathbf{g}_i)^{\mathsf{H}}\mathcal{H}(\mathbf{g}_i) \tag{7}$$

where $\mathbf{\Gamma}$ is a $N \times N$ Hermitian symmetric non-negative definite Toeplitz matrix. Therefore, by passing the received signal $\tilde{\mathbf{r}}$ through the bank of matched filters for the channel impulse responses $\mathbf{g}_i$ and combining the matched filter outputs (sampled at the symbol rate), the k blocks of transmitted symbols are completely decoupled. The equivalent channel for any of these blocks (we drop the block index from now on for simplicity) is given by

$$\mathbf{y} = \mathbf{\Gamma x} + \mathbf{z} \tag{8}$$

where $\mathbf{z} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{\Gamma})$.

The TR-STBC scheme has turned the MISO frequency-selective channel into a standard SISO channel with ISI, and the channel model (8) represents the so-called *Sampled Matched-Filter* output of the equivalent SISO channel, in block form. Notice that the noise $\mathbf{z}$ is correlated.

Concatenation with TCM. We wish to concatenate an outer code defined over a complex signal constellation $\mathcal{X} \subset \mathbb{C}$ with an inner TR-STBC scheme. For outer coding we choose standard TCM [15,20–22] for the following reasons: 1) it is very easy to implement variable-rate coding by adding uncoded bits, expanding the signal set correspondingly, and increasing the number of parallel transitions in the same basic encoder trellis; 2) they can be easily decoded by the Viterbi Algorithm (VA) which is particularly suited to the low-complexity joint equalization and decoding scheme proposed in the next section; 3) after the TR-STBC combining, we are in the presence of an ISI channel whose impulse response is given by the coherent combination of the M channel impulse responses of the underlying MISO channel. Due to the inherent diversity combining, the effect of fading is reduced and it makes sense to choose the outer coding scheme in a family optimized for classical AWGN-ISI channels [22]. Since TCM is standard, we will not discuss further details here for the sake of space limitation.

In the proposed TCM-TR-STBC scheme, the blocks of symbols $\{\mathbf{x}_1, \ldots, \mathbf{x}_k)$ of the TR-STBC transmit matrix $\mathbf{T}(\mathbf{x}_1, \ldots, \mathbf{x}_k)$ are obtained by interleaving the output sequence produced by a TCM encoder. As we shall see in the next section, a block interleaver with suitable depth D is necessary in order to enable the low-complexity joint equalization and decoding scheme to work efficiently. We consider a row-column interleaver formed by an array of size $N \times D$, where the symbols produced by the TCM encoder are written by rows, and the columns form the blocks $\mathbf{x}_j$ mapped into the TR-STBC transmit matrix.

Figure 1(a) shows the block diagram of the proposed concatenated scheme for $M = 3$, based on the rate-3/4 STBC with block length $T = 4$ defined by

$$\mathbf{S}(x_1, x_2, x_3) = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ -x_3^* & 0 & x_1^* \\ 0 & -x_3 & x_2 \end{bmatrix}$$

The sequence generated by the TCM encoder is arranged in the interleaving array by rows. The resulting D vectors of length N are mapped onto the $T(N + L - 1) \times M$ TR-STBC transmit matrix (this is shown transposed in Figure 1(a) where the shadowed areas correspond to zeros). The spectral efficiency of the resulting concatenated scheme is given by $\eta = \frac{N}{N+L-1} R_{\text{stbc}} R_{\text{tcm}}$, where R_{stbc} is the rate [symbol/channel use] of the underlying STBC, and R_{tcm} is the rate [bit/symbol] of the outer TCM code. The factor $\frac{N}{N+L-1}$ is the rate loss due to the insertion of the zero-padding, and can be made small by letting $N \gg L$.

3 Reduced-state joint equalization and decoding

ML decoding of the overall concatenated scheme is too complex, since it would require running a VA on an expanded trellis, where the number of states depends on the channel length and on the constellation size. To overcome this problem, we propose a reduced-state joint equalization and decoding approach based on the *per-survivor processing* (PSP) principle [23], similar to the scheme proposed in [24] for trellis STCs over the frequency-flat MIMO channel. The block diagram of the receiver is shown in Figure 1(b). An MMSE-DFE deals with the causal part of ISI by using the reliable decisions found on the survivors of the

VA operating on the trellis of the underlying TCM code. The non-causal part of the ISI is mitigated by the forward filter of the MMSE-DFE.

In order to compute the MMSE-DFE forward filter with linear complexity in the channel length L and in the TR-STBC block size N, we use the block formulation based on Cholesky factorization of [25]. For the sampled matched filter channel model in vector form, given in (8), we compute the Cholesky factorization

$$N_0\mathbf{I} + \mathbf{\Gamma} = \mathbf{B}^{\mathsf{H}}\mathbf{\Delta}\mathbf{B} \tag{9}$$

where $\mathbf{B}$ is upper triangular with unit diagonal elements and $\mathbf{\Delta} = \mathrm{diag}(\sigma[N-1], \dots, \sigma[0])$ is a diagonal matrix with positive real diagonal elements. The feedback filter matrix is equal to $\mathbf{B} - \mathbf{I}$, which is strictly causal. The Schur algorithm computes this factorization with linear complexity in L and N by considering the banded Toeplitz structure of $\mathbf{\Gamma}$ where each row of contains at most $2L - 1$ non-zero elements [25]. The MMSE-DFE forward filter is given by

$$\mathbf{F} = \mathbf{\Delta}^{-1}\mathbf{B}^{-H} \tag{10}$$

The output of this filter can be obtained efficiently by applying back-substitution to $\mathbf{y}$, yielding linear complexity in L and N.

Let $\{z[j]\}$ be the sequence of symbol-rate samples obtained after forward filtering and block deinterleaving. Due to the structure of the interleaver, the decisions in the decision-feedback section of the equalizer can be found on the survivors of the VA acting on the original TCM trellis (i.e., without state expansion). The resulting VA is fully defined by its branch metric. Consider the q-th parallel transition at the j-th trellis step, extending from state s and merging to state s'. The corresponding branch metric is given by

$$m_{s,s',q}[j] = \left| z[j] - \left(1 - \frac{1}{\sigma[n]}\right) x(s,s',q) - \sum_{\ell=1}^{L'} b_{n,\ell}\, \widehat{x}_{j-\ell D}(s) \right|^2 \tag{11}$$

where $L' = \min\{L, n\}$, $n = \lfloor j/D \rfloor$, $x(s, s', q)$ is the constellation symbol labeling the q-th parallel transition of the trellis branch $s \to s'$, $\widehat{x}_{n-\ell D}(s)$ are the tentative decisions found on the surviving path terminating in state s, and $(b_{n,1}, \ldots, b_{n,L})$ are the coefficients of the MMSE-DFE feedback filter where $b_{n,\ell}$ is the $(N-1-n, N-1-n+\ell)$-th element of the matrix $\mathbf{B}$.

Thanks to the interleaving depth D, the tentative decisions are found at least D trellis steps before the symbol of interest (step j in the trellis). If D is larger than the Viterbi decoding delay (typically 5 or 6 times the code constraint length), the corresponding decisions are reliably obtained from the Viterbi decoder output [24]. As a matter of fact, simulations show that the scheme is extremely robust and, even if D is much smaller than the typical Viterbi decoding delay, the WER performance of the proposed scheme is almost identical to that of a genie-aided scheme that makes use of ideal feedback decisions. The minimal D for which ideal-feedback performance is attained depends on the specific code and should be optimized by extensive simulation.

4 WER analysis

In this section, we provide two approximations to the WER of the proposed TCM-TR-STBC scheme. Both approximations are based on the assumption of a genie that helps the equalization and decoding scheme.

Matched Filter Bound (MFB). Assuming that a genie eliminates the whole ISI and that deinterleaving suffices to decorrelate the Gaussian noise, (8) is turned into the ISI-free AWGN channel

$$y^{(MFB)}[j] = \sqrt{\gamma_0}x[j] + w[j] \tag{12}$$

where $w[j] \sim \mathcal{N}_{\mathbb{C}}(0, N_0)$ is AWGN, $E[|x[j]|^2] = \mathcal{E}$ and $\gamma_0 = \sum_{i=1}^{M} |\mathbf{g}_i|^2$. The corresponding SNR is given $\gamma_0\mathcal{E}/N_0$. The coefficient γ_0 can be expressed by using the eigen decomposition of the covariance matrix of $\mathbf{g}_i$, given by $\mathbf{R}_g \triangleq \mathrm{E}[\mathbf{g}_i\mathbf{g}_i^{\mathsf{T}}]$, that we assume independent of i for simplicity. We let $\mathbf{R}_g = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathsf{H}}$ where $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_P\}$ contains the non-zero eigenvalues on the diagonal and $\mathbf{U} \in \mathbb{C}^{N_g \times P}$ has orthonormal columns. The number P of positive eigenvalues of $\mathbf{R}_g$ represents the number of fading effective degrees of freedom of the multipath channel, i.e., the number of *separable paths*.[2] In the rest of this paper we assume Rayleigh fading, uncorrelated scattering, and that the channel impulse responses of different antennas are statistically independent. We use the Karhunen-Loeve decomposition

$$\mathbf{g}_i = \mathbf{U}\mathbf{\Lambda}^{1/2}\mathbf{h}_i$$

where $\mathbf{h}_i = (h_i[1], \ldots, h_i[P])^{\mathsf{T}}$ are complex circularly symmetric Gaussian vectors with i.i.d. components $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. It follows that

$$\begin{aligned} \gamma_0 &= \sum_{p=1}^{P} \lambda_p \left(\sum_{i=1}^{M} |h_i[p]|^2 \right) \\ &= \sum_{p=1}^{P} \lambda_p \alpha[p] \end{aligned} \tag{13}$$

where the $\alpha[p]$'s are i.i.d. central Chi-squared random variables with $2M$ degrees of freedom.

The WER conditioned with respect to γ_0 under the MFB assumption is upper bounded by

$$P_w^{(MFB)}(e|\gamma_0) \leq K \sum_d A_d Q\left(\sqrt{\frac{\mathcal{E} d^2 \gamma_0}{2N_0}}\right) \tag{14}$$

where $K = DN$ denotes the frame length in trellis steps and A_d is the average number of *simple* error events at *normalized* squared Euclidean distance d^2.[3] This function can be evaluated numerically by using the Euclidean distance enumerator $\{A_d\}$ of the TCM code. In practice, the (possibly truncated) distance enumerator can be computed by several algorithms depending on geometrical uniformity of the TCM code under examination [20, 26,27]. In order to obtain the average WER over the realization of the channel γ_0, we cannot average the conditional union bound (14) term by term because the union bound averaged

[2] Notice that we have not made any constraining assumption about the channel delay-intensity profile [3]. Therefore, this definition applies to both diffuse and discrete multipath models.

[3] Having put in evidence the average symbol energy $\mathcal{E}$, we define the normalized Euclidean distance d between two code sequences $\mathbf{x}$ and $\mathbf{x}'$ by $d^2 = |\mathbf{x} - \mathbf{x}'|^2/\mathcal{E}$.

over the fading statistics may be very loose or even not converge if an infinite number of terms is taken into account in the union bound summation (see [28]). Then, we follow the approach of [28] and obtain

$$P_w^{(MFB)}(e) \leq \mathrm{E}_{\gamma_0}\left[\min\left\{1, K\sum_d A_d Q\left(\sqrt{\frac{\mathcal{E}d^2\gamma_0}{2N_0}}\right)\right\}\right] \tag{15}$$

where the expectation is with respect to the statistics of γ_0, that can be easily obtained by numerical integration. Since we have used a union upper bound in the MFB lower bound, (15) is neither a lower nor an upper bound. Rather, it provides a useful approximation for the actual WER $P_w(e)$.

Genie-Aided MMSE-DFE Gaussian approximation (MMSE-DFE-GA). Here we assume that a genie provides exact decisions on each survivor (i.e., the MMSE-DFE works under the ideal feedback assumption). The channel presented to the VA can be modeled as

$$y^{(GAB)}[j] = \sqrt{\beta}x[j] + w[j] \tag{16}$$

where $E[|w[j]|^2] = 1$, and $\mathcal{E}\beta$ is the signal to interference plus noise ratio (SINR) at the output of the MMSE-DFE under the ideal feedback assumption, given by [29]

$$\beta\mathcal{E} = \exp\left\{\int_{-1/2}^{1/2} \ln\left(1 + \frac{\mathcal{E}}{N_0}\Gamma(f)\right) df\right\} - 1 \tag{17}$$

where $\Gamma(f) \triangleq \sum_{i=1}^{M} G_i(f)$ and where $G_i(f)$ is the discrete-time Fourier transform of the symbol-rate sampled autocorrelation function of the i-th channel impulse response $\mathbf{g}_i$. The SINR expression (17) is obtained in the limit for large block length ($N \to \infty$), that makes the vector model (8) stationary. Since the term $w[j]$ in (16) contains both noise and anti-causal ISI, we make a Gaussian approximation and let $w[j] \sim \mathcal{N}_{\mathbb{C}}(0,1)$. The approximated error probability for this model can be derived exactly in the same manner as for the MFB, by replacing the SNR $\gamma_0\mathcal{E}/N_0$ in (15) by $\beta\mathcal{E}$. Unfortunately, the expectation with respect to β must be evaluated by Monte-Carlo average, since the pdf of β cannot be given in closed form. Remarkably, simulations show that this approximation is very tight and predicts very accurately the WER of the TCM-TR-STBC scheme under the actual joint equalization and decoding scheme (i.e., without ideal decision feedback).

Achievable diversity. The maximum achievable diversity in the MISO channel with M independent antennas and P separable paths is obviously given by $d_{\max} = MP$. Consider the single-input single-output channel with ISI obtained by including the TR-STBC encoding (at the transmitter) and combining (at the receiver) as part of the channel. Standard results of information theory show that the maximum information rate achievable by signals with frequency-flat power spectral density is given by [30]

$$I_G(\mathcal{E}/N_0) \triangleq \int_{-1/2}^{1/2} \log_2 \left(1 + \frac{\mathcal{E}}{N_0}\Gamma(f)\right) df \tag{18}$$

For the quasi-static fading model considered in this paper, it follows that the best possible WER for any code, in the limit of large block length, is given by the *information outage probability*

$$P_{\text{out}}(\mathcal{E}/N_0, \eta) = \Pr\left(I_G(\mathcal{E}/N_0) \leq \eta\right) \tag{19}$$

and, by following the argument of [6, 7], that the high-SNR slope of the outage probability curve, defined by the limit

$$\lim_{\mathcal{E}/N_0 \to \infty} \frac{-\log P_{\text{out}}(\mathcal{E}/N_0, \eta)}{\log \mathcal{E}/N_0} \tag{20}$$

is given by $d_{\max} = MP$.

It is also well-known that the information rate (18) can be achieved by Gaussian codes, block interleaving, and by joint MMSE-DFE equalization and decoding (see for example the tutorial presentation in [31, Sect. VII.B] and references therein). We conclude that, in the limit of large interleaving depth D and $N \gg L$, MMSE-DFE equalization and decoding with ideal Gaussian (capacity achieving) codes achieves maximum diversity. Our low-complexity decoding scheme can be seen as a practical version of the asymptotically optimal scheme and differs in two key aspects that make it practical: 1) it uses a very short interleaving depth D;

2) it uses very simple off-the-shelf TCM codes. Short D implies unreliable feedback decision. Simulations show that the PSP approach is able to mitigate this effect and that full diversity is easily achieved by our scheme under no ideal feedback assumption.

5 Numerical Results

In order to evaluate the performance of the proposed scheme, simulations have been performed in the following conditions. Two ISI channel models are considered: a symbol-spaced P-path channel with the equal strength paths and the pedestrian channel B [32] for the TD-SCDMA 3rd generation standard [33]. Classical Ungerboeck TCM codes are used with different signal constellations and spectral efficiencies. WER curves are plotted versus either E_b/N_0 or SNR in dB, where we define $\text{SNR} \triangleq \frac{M\mathcal{E}}{N_0}$ as the total transmit energy per channel use over the noise power spectral density or, equivalently, as the SNR at the receiver antenna, in agreement with standard STC literature. In the following, the simulated WER curves for the actual per-survivor processing decoder are denoted by "PSP" with an interleaving depth D , the simulated WER curves for a genie-aided decoder that makes use of ideal feedback decisions are denoted by "Genie", the MFB approximation is denoted by "MFB" and the MMSE-DFE Gaussian Approximation is denoted by "MMSE-DFE-GA".

Comparison with other schemes Figure 2 compares the TCM-TR-STBC scheme with previously proposed schemes for $\eta = 2$[bit/channel use], $M = 2$ and $P = 2$ equal strength ISI channel. The corresponding information outage probability is shown for comparison. The ST-BICM schemes of [9, 13], employing turbo equalization and decoding based on a BCJR algorithm for the ISI channel and for the trellis code, yield performance similar to ours. However, these schemes have much higher receiver complexity.[4] In the case of [9], the

[4]In order to obtain an implementation-free complexity estimate, we assume that the complexity of the BCJR and of the PSP algorithms are essentially given by their trellis complexity (number of branches per

memory-one ISI channel with 8PSK modulation has trellis complexity 64 and the 16-state convolutional code of rate-2/3 used in the BICM scheme has trellis complexity 64. Five iterations are required, yielding a total complexity of $5 \times 128 = 640$ branches per coded symbol. In the case of [34], the memory-two MISO ISI channel with 4PSK modulation has trellis complexity 256 and the 16-state TCM space-time code used has trellis complexity 64. Five iterations are required, yielding a total complexity of $5 \times 320 = 1600$ branches per coded symbol. Our scheme, with a 64-state rate-2/3 8PSK TCM code and no iterative processing, has trellis complexity of 256 branches per coded symbol.

Some aspects of the TCM-TR-STBC scheme. In Figure 3, we evaluate the impact of the number of separable paths on the WER with $M = 2$ for a spectral efficiency of 2[bit/channel use]. A 4-state 8PSK Ungerboeck TCM is used. As the number of paths increases, the slope of the curves becomes steeper and gets closer and closer to that of an unfaded ISI-free AWGN channel (TCM performance in standard AWGN). Since Ungerboeck TCM codes are optimized for the AWGN channel, this fact justifies the choice of these codes for the concatenated scheme. The performance of the actual PSP decoder lies in between the MFB and the MMSE-DFE-GA approximations. We have also simulated the performance of a genie-aided decoder that makes use of ideal feedback decisions. We notice that the performance of the PSP decoder coincide with that of the genie-aided decoder, showing that the effect of non-ideal decisions in the MMSE-DFE is negligible in the proposed PSP scheme already for interleaving depth $D = 4$.

In Figure 4, we investigated the effect of the number of transmit antennas for the 4-path equal strength ISI channel. The 4-state 8PSK Ungerboeck TCM is used, which yields a spectral efficiency of 2 [bit/channel use] for $M = 1, 2$. Since a full-rate GOD does not exist for $M = 4, 8$, the corresponding spectral efficiency is 1.5, 1[bit/channel use] respectively. By

coded symbol). Hence, we evaluate the receiver complexity as the overall trellis complexity times the number of equalizer/decoder iterations.

increasing the number of the transmit antennas, the actual WER performance gets closer to the MFB approximation and for 4 and 8 antennas the system achieves the MFB. This shows that the effect of ISI is reduced by increasing the system transmit diversity. In fact, the matrix $\mathbf{\Gamma}$ defined in (7) is given by the sum of M independent Toeplitz matrices $\mathcal{H}(\mathbf{g}_i)^{\mathsf{H}}\mathcal{H}(\mathbf{g}_i)$ where the diagonal terms are real and positive while the off-diagonal terms are complex and added non-coherently with different phases. Hence, as M increases $\mathbf{\Gamma}$ becomes more and more diagonally dominated.

Figure 5 shows the performance of our PSP scheme compared to the information outage probability for different modulation schemes (increasing spectral efficiency) over 4-path equal-strength ISI channel for $M = 2$. The 4-state Ungerboeck TCM codes are used over different constellations and the resulting spectral efficiencies are 1, 2, 3, 4 [bit/channel use] for QPSK, 8PSK, 16QAM, 32 cross respectively. For all spectral efficiencies the gap between the outage probability and the WER of the actual schemes is almost constant. This fact is due to the optimality of the underlying Alamouti code for the 2-antennas MISO channel in the sense of the diversity-multiplexing tradeoff of [6].

Figure 6 shows a $M = 4$ antenna system over the pedestrian channel B. The TCM-TR-STBC scheme is obtained by concatenating a 16-state Ungerboeck TCM code with the TR-STBC obtained from the rate-3/4 GOD with parameters $[T = 8, M = 4, k = 6]$ [18]. The spectral efficiencies for QPSK, 8PSK, 16QAM, 32cross are 0.75, 1.5, 2.25, 3[bit/channel use]. Even on a realistic channel model where the number of separable paths P is much smaller than the length of the channel impulse response, the proposed scheme shows the same slope of the information outage probability at high SNR, which shows that the maximum diversity $d_{\max} = MP$ is achieved. However, unlike the result in Figure 5, the gap to outage probability increases as the spectral efficiency becomes large. This fact is well-known and it is due to the non-optimality of GODs for $M > 2$ [6].

6 Conclusion

We proposed a concatenated TCM-TR-STBC scheme for single-carrier transmission over frequency selective MISO fading channels. Thanks to a reduced-state joint equalization and decoding approach, our scheme achieves much lower complexity with similar/superior performance than previously proposed schemes for the same spectral efficiency. Moreover, since the receiver complexity is independent of the modulation constellation size and Ungerboeck TCM schemes implement very easily different spectral efficiencies with the same encoder, by introducing parallel transitions and expanding the signal constellation, our scheme is suitable for implementing adaptive modulation with low complexity. This is a key component in high-speed downlink transmission with transmitter feedback information.

We wish to conclude with a simple numerical example inspired by a 3rd generation system setting, showing that very high data rates with high diversity can be easily achieved with the proposed scheme. Consider a MISO downlink scenario such as TD-SCDMA [33]. This system is based on slotted quasi-synchronous CDMA at 1.28 Mchip/s ($\sim$ 2MHz bandwidth). A slot, of duration 675 μs, is formed by two data-bearing blocks of 352 chips that are separated by 144 chips of midamble for channel estimation. At the end of the second block, 16 chips of guard interval are added for slot separation. With 128 chips plus 16 chips of guard interval (total 144 chips) we can estimate easily 4 channels of length 16 chips in the frequency domain, using an FFT of length 128 samples. We can use the rate-3/4 TR-STBC for $M = 4$ antennas with a 8-PSK TCM code. Using blocks of $N = 76$ [symbols], $L = 17$, and $R_{tcm} = 2$ [bit/channel use], the resulting spectral efficiency is: $\eta = \frac{3}{4}\frac{76\times 8}{864}R_{tcm} = 1.056$ [bit/chip]. This yields 1.35 Mb/s on a single carrier. On three carriers (equivalent to the 5 MHz of the European UMTS), we obtain 4.05 Mb/s, well beyond the "dream" target of 2Mb/s of high-speed links in 3rd generation systems. We conclude that the TCM-TR-STBC scheme represents a valid candidate for the high data rate downlink of TD-SCDMA.

References

[1] J.G.Holma and A.Toskala, "WCDMA for UMTS," *Wiley*, 2000.

[2] "Digital cellular communications system (Phase 2+)," *ETSI/3GPP, Sophia-Antipolis, France, Technical Specifications 3GPP TS 05.01-05*, 2001.

[3] John G.Proakis, "Digital Communications," *McGraw-Hill Book Company*, 1997.

[4] E. Biglieri, J. Proakis, , and S. Shamai, "Fading channels : information-theoretic and communications aspects," *IEEE Trans. on Inform. Theory*, vol. 44, pp. 2619–2692, October 1998.

[5] V.Tarokh, N.Seshadri, and A.R.Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. on Inform. Theory*, vol. 44, pp. 744 – 765, March 1998.

[6] L.Zheng and D.N.C.Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. on Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[7] A.Medles, "Coding and Advanced Signal Processing for MIMO Systems," *Ph.D thesis, Ecole Nationale Supérieure des Télécommunications*, April 2004.

[8] E.Lindskog and A.Paulraj, "A transmit diversity scheme for channels with intersymbol interference," *Communications, 2000. ICC 2000. 2000 IEEE International Conference on*, vol. 1, pp. 307 –311, 2000.

[9] S.Zhou and G.B.Giannakis, "Single-Carrier Space-Time Block Coded Transmissions over Frequency-Selective Fading Channels," *IEEE Trans. on Inform. Theory*, vol. 49, pp. 164–179, January 2003.

[10] H.El Gamal, A.R.Hammons, Y.Liu, M.P.Fitz, and O.Y.Takeshita, "On the Design of Space-Time and Space-Frequency for MIMO Frequency Selective Fading Channels," *Submitted to IEEE Trans. Info. Theory*, April 2001.

[11] W.Choi and J.M.Cioffi, "Multiple input/multiple output(MIMO) equalization for space-time block coding," *in Proc. IEEE Pacific Rim Conf. Commun., Computers and Sig. Proc.*, pp. 341–344, 1999.

[12] S.M.Alamouti, "A Simple Transmit Diversity Techniques for Wireless Communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[13] A.O.Berthet, R.Visoz, and J.J.Boutros, "Space-time BICM versus space-time trellis code for MIMO block fading multipath AWGN channel," *Proc. in ITW2003*, March 2003.

[14] L.R.Bahl, J.Cocke, F.Jelinek, and J.Raviv, "Optimal decoding," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[15] G. Ungerboeck, "Trellis-coded modulation with redundant signal sets. I:Introduction," *IEEE Commun.Mag.*, vol. 25, no. 2, pp. 5–11, 1987.

[16] T.E.Kolding et al., "High Speed Downlink Packet Access : WCDMA Evolution," *IEEE Vehic. Tech. Society News*, pp. 4–10, February 2003.

[17] O.Tirkkonen, "Square-matrix embeddable space-time block codes for complex signal constellations," *IEEE Trans. on Inform. Theory*, vol. 48, no. 2, pp. 384–395, February 2002.

[18] Xue-Bin Liang, "Orthogonal designs with maximal rates," *submitted to IEEE Trans. on Inform. Theory*, October 2002.

[19] V.Tarokh, H.Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. on Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[20] E.Biglieri, D.Divsalar, P.J.McLane, and M.K.Simon, "Introduction to Trellis-Coded Modulation with Applications," *Macmillan*, 1997.

[21] G.Ungerboeck, "Channel coding with multilevel/phase signals," *IEEE Trans. on Inform. Theory*, vol. 28, pp. 55–67, 1982.

[22] G.D.Forney and G.Ungerboeck, "Modulation and coding for linear Gaussian channels," *IEEE Trans. on Inform. Theory*, vol. 44, pp. 2384–2415, 1998.

[23] R.Raheli, A.Polydoros, and C.-K.Tzou, "Per-Survivor Processing: A general approach to MLSE in uncertain environments," *IEEE Trans. on Commun.*, vol. 43, no. 2, pp. 354–364, February 1995.

[24] G.Caire and G.Colavolpe, "On space-time coding for quasi-static multiple-antenna channels," *IEEE Trans. on Inform. Theory*, vol. 49, no. 6, pp. 1400–1416, June 2003.

[25] G.K.Kaleh, "Channel Equalization for Block Transmission Systems," *IEEE J. Select. Areas Commun.*, vol. 13, no. 1, pp. 110–121, January 1995.

[26] C.Schlegel, "Trellis coding," *IEEE Press*, 1997.

[27] R.D.Wesel, "Reduced Complexity Trellis Code Transfer Function Computation," *Proc. of Commun. Theory Mini-Conference*, pp. 37–41, June 1999.

[28] E.Malkamaki and H.Leib, "Evaluating the performance of convolutional codes over block fading channels," *IEEE Trans. on Inform. Theory*, vol. 45, no. 5, pp. 1643–1646, July 1999.

[29] J.M.Cioffi, G.P.Dudevoir, M.V.Eyuboglu, and G.D.Forney, "MMSE Decision-Feedback Equalizers and Coding-Part I: Equalization Results," *IEEE Trans. on Commun.*, vol. 43, no. 10, pp. 2582–2594, October 1995.

[30] T. Cover and J. Thomas, *Elements of information theory*, Wiley, New York, 1991.

[31] R. Zamir, S. Shamai, and U. Erez, "Nested linear/lattice codes for structured multiterminal binning," *IEEE Trans. on Inform. Theory*, vol. 48, pp. 1250–1276, June 2002.

[32] ITU, "Rec.ITU-RM 1225, Guidelines for evaluation of radio transmission technologies for IMT-2000," 1997.

[33] "TD-SCDMA," *http://www.tdscdma-forum.org.*

[34] Y.Liu, M.P.Fitz, and O.Y.Takeshita, "Space-Time Codes Performance Criteria and Design for Frequency Selective Fading Channels," *ICC'2001 Proceeding*, vol. 9, pp. 2800–2804, 2001.

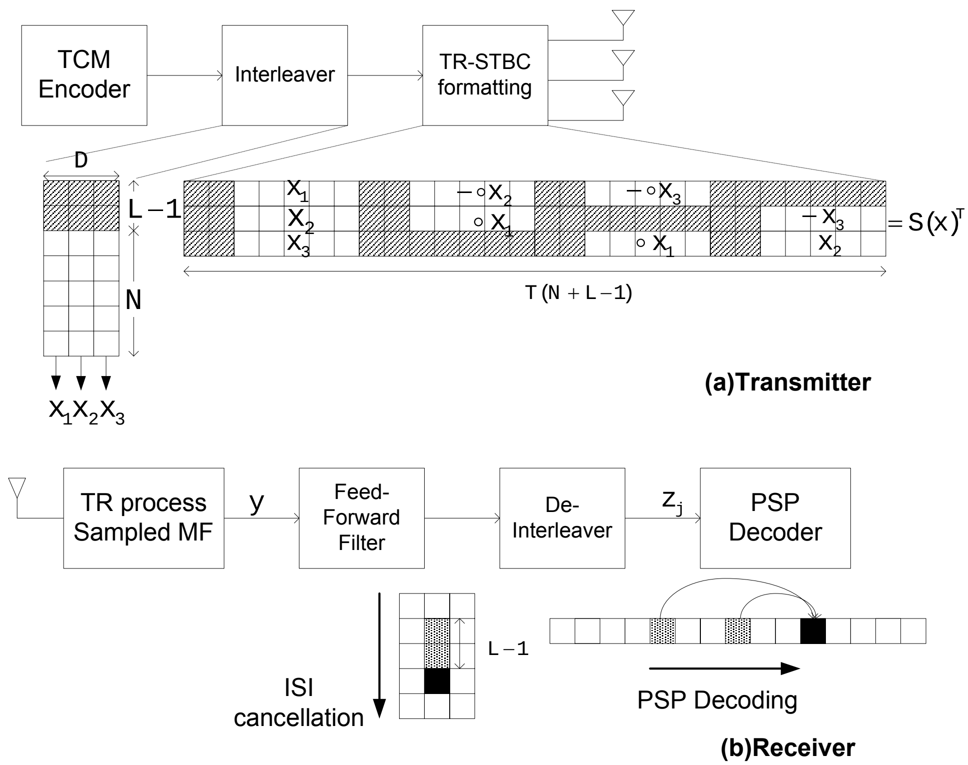

Figure 1: Block diagram of the TCM-TR-STBC scheme for $M = 3$.

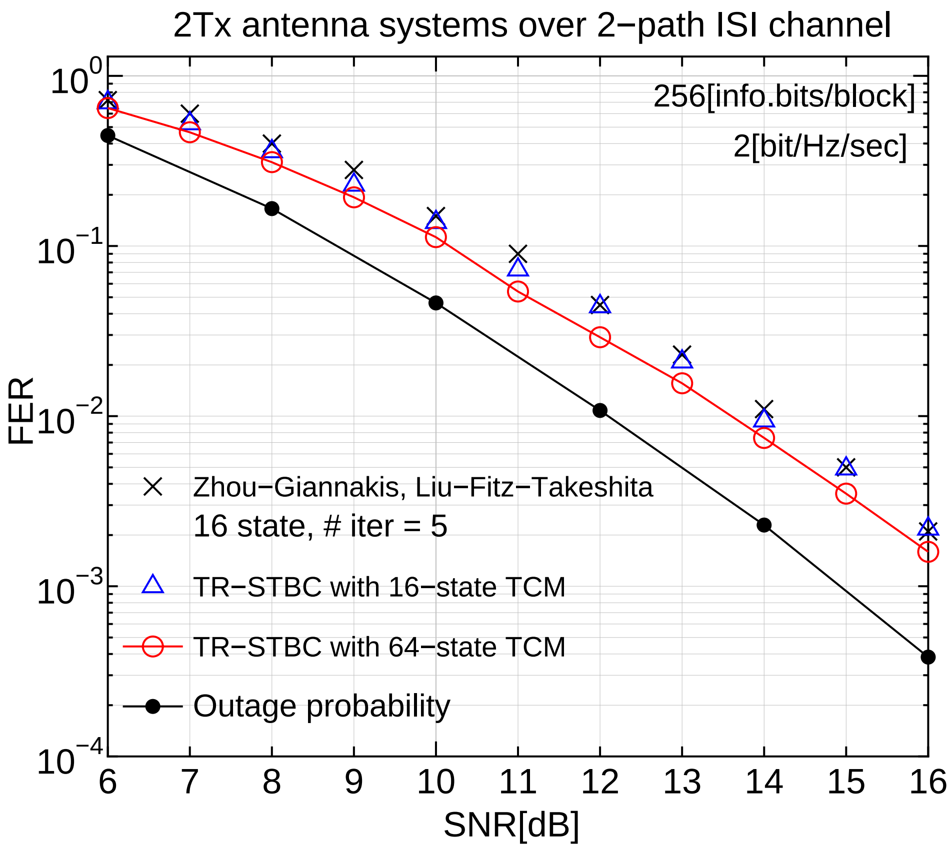

Figure 2: Comparison with previously proposed STC schemes.

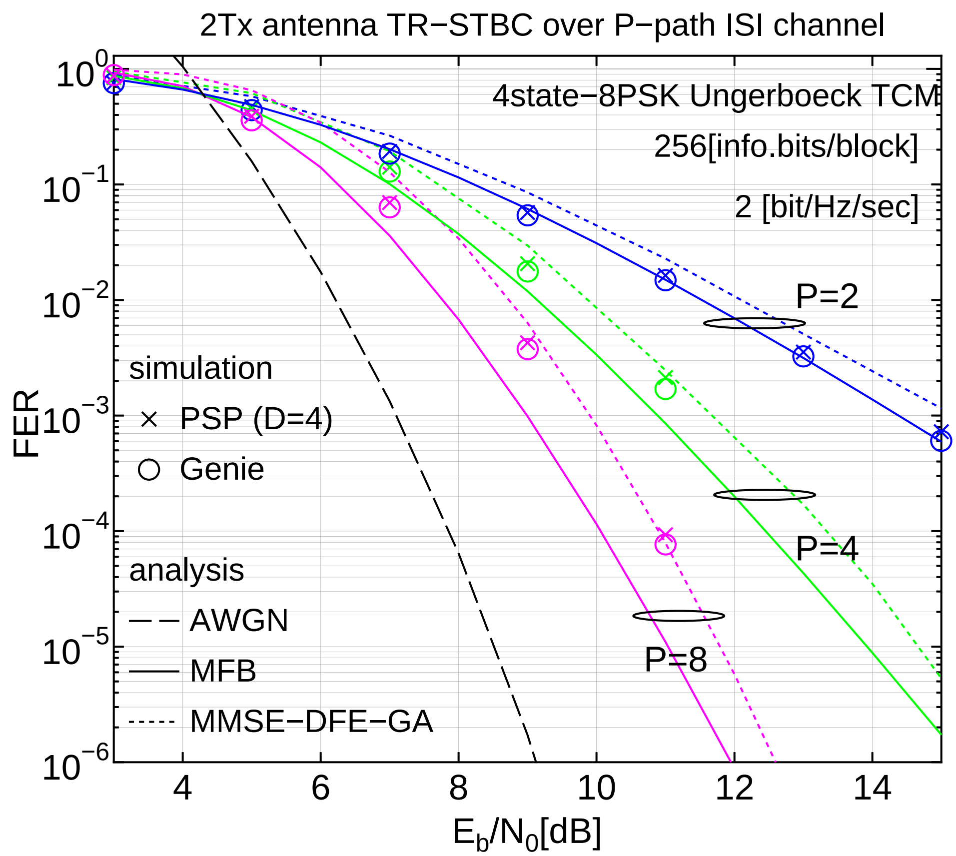

Figure 3: Performance of the TCM-TR-STBC scheme for $M = 2$ and increasing number of paths.

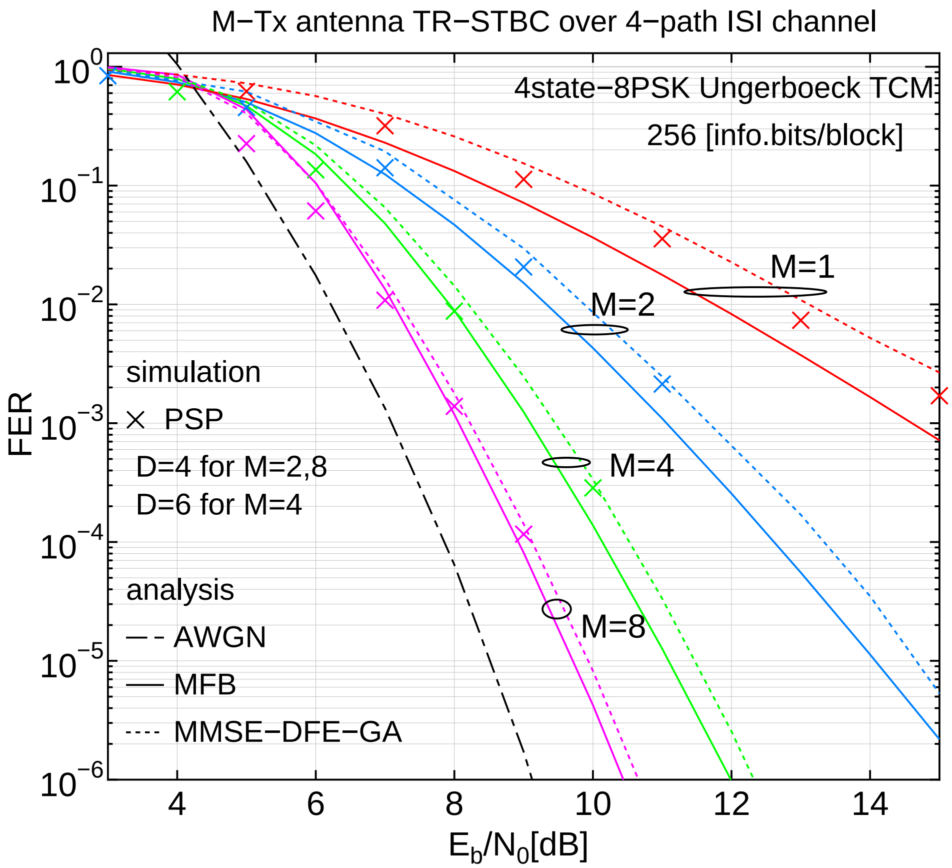

Figure 4: Performance of the TCM-TR-STBC scheme for $P = 4$ and increasing number of transmit antennas.

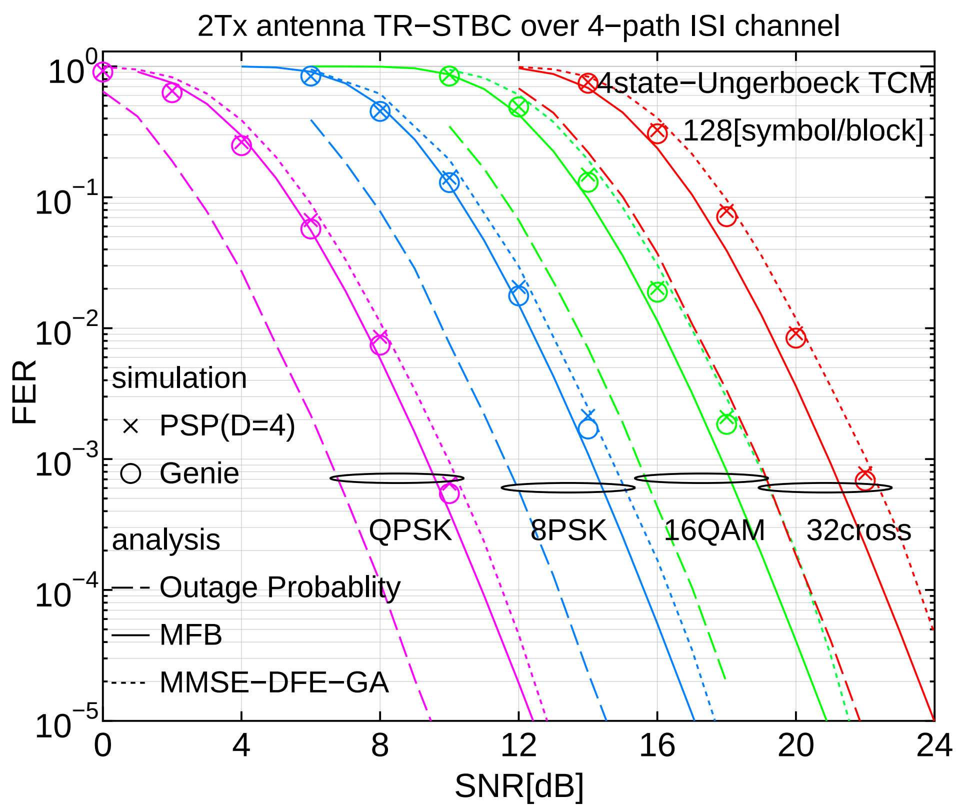

Figure 5: Comparison with outage probability for $M = 2$ and $P = 4$.

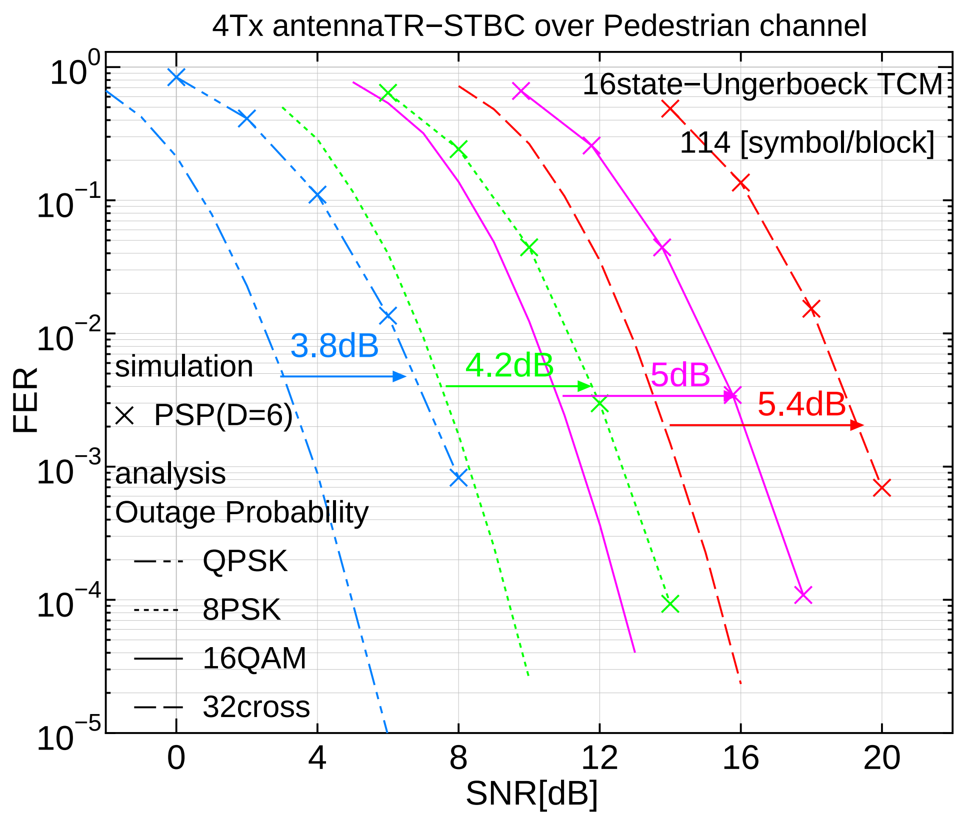

Figure 6: Performance over the pedestrian B channel, with $M = 4$ transmit antennas.