

Design Issues for MIMO Systems in Various Fading Situations

Maxime Guillaud,
Mérrouane Debbah
Institut Eurécom,
Mobile Communications Dept.,
Sophia Antipolis, France
guillaud@eurecom.fr,
debbah@eurecom.fr

Claude Oestges
Microwave Laboratory,
Université Catholique de
Louvain,
Louvain-la-Neuve, Belgium
oestges@emic.ucl.ac.be

Bernard H. Fleury
Department of Communication
Technology,
Institute of Electronic
Systems, Aalborg University,
Aalborg, Denmark
bfl@cpk.auc.dk

ABSTRACT

This contribution provides a fresh look at the asymptotic (in terms of number of antennas) design of multiple-antenna wireless systems, with the goal of giving useful insights on the deployment of future MIMO systems. For the i.i.d. Gaussian and double-directional models, we provide guidelines in terms of the repartition of the antennas between the transmitter and the receiver, and study the influence of array geometry on the ergodic mutual information per antenna. Furthermore, we compare the LOS and NLOS situations, and evaluate the relative importance of the LOS component and path loss in Ricean fading, as well as in the low-rank Ricean component case¹.

Keywords

Antenna array, random matrix theory, Rayleigh fading, Rice fading, double-directional model, LOS/NLOS

Categories and Subject Descriptors

H.1.1 [Systems and Information Theory]: General systems theory; G.3 [Probability and Statistics]: Multivariate statistics

1. INTRODUCTION

A common question that arises in Multiple-Input Multiple-Output (MIMO) systems concerns the allocation of the antennas between transmitter and receiver. Consider a system with n_t transmit and n_r receive antennas, and suppose that the total number $n = n_t + n_r$ of antennas is constrained, what is the optimal proportion $\gamma = \frac{n_r}{n}$ that optimizes the average mutual information in this type of system? Such

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considerations depend of course on the state of knowledge at hand and as a consequence on the type of model derived (see for example [1, 2, 3, 4]). The goal of this paper is to give a comprehensive overview of design issues related to MIMO systems. The analysis is conducted in the asymptotic regime (as n grows to infinity) for insight purposes. However, the averaging effect has been shown to kick in at a very low number of antennas (see [5] for details), and therefore that the results presented here remain useful for systems of moderate size. We consider the MIMO versions of various commonly used flat-fading models (Gaussian and Rayleigh with independent components, double-directional, and Ricean with low rank), and show the impact of antenna allocation, array geometry and Rice component, on the ergodic mutual information per antenna, when the transmitter has no knowledge about the channel state.

2. ANTENNA ALLOCATION IN LARGE SYSTEMS

Let us consider a frequency-flat MIMO channel, denoted by a $n_r \times n_t$ complex matrix \mathbf{H} , and an additive noise \mathbf{n} with n_r complex Gaussian independent identically distributed (i.i.d.) components of variance σ_n^2 . The input-output relationship between the transmitted signal \mathbf{x} and the received signal \mathbf{y} is $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$. When the channel variations are relatively slow w.r.t. the symbol rate, the mutual information of \mathbf{x} and \mathbf{y} for a fixed channel realization \mathbf{H} is a meaningful measure of the achievable rate over this channel. We recall that the mutual information in this case has been shown [6] to be $I^M = \log \det \left(\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{H} \mathbf{R}_x \mathbf{H}^H \right)$, where \mathbf{R}_x is the covariance of \mathbf{x} . Note that in general, the optimal transmit covariance is a function of \mathbf{H} . However, we will focus on the case where the transmitter has no information about the current channel realization. In this case, \mathbf{R}_x can still be chosen such as to maximize the expected I^M for a given distribution of \mathbf{H} .

2.1 Gaussian i.i.d. Channel Model

Let us consider the case of the i.i.d. Gaussian channel model. In this case, the optimal transmit covariance is $\mathbf{R}_x = \mathbf{I}_{n_t}$. Letting $\rho = \frac{\text{tr}(\mathbf{R}_x)}{\sigma_n^2}$ denote the signal to noise ratio (SNR), the asymptotic mean mutual information (in nats) normalized to the total number of antennas, is given by [7]:

$$\mathbb{E} \left[\frac{I^M}{n} \right] = -\frac{1}{1+\gamma} \alpha + \frac{1}{1+\frac{1}{\gamma}} \log(1+\rho-\rho\alpha) + \frac{1}{1+\gamma} \log(1+\rho\gamma-\rho\alpha) \quad (1)$$

with

$$\alpha = \frac{1}{2} \left[1 + \gamma + \frac{1}{\rho} - \sqrt{\left(1 + \gamma + \frac{1}{\rho}\right)^2 - 4\gamma} \right]. \quad (2)$$

$\mathbb{E} \left[\frac{I^M}{n} \right]$ is a function of γ and ρ . In general, the previous equation has no analytical expression for its maximum value. For each SNR ρ , one has to search for the optimum value of this function. Note that

1. In the case where $\rho \rightarrow \infty$, the optimum value is $\gamma = 1$. This should be related to the fact that the mutual information scales as $\min(n_t, n_r) \log(\rho)$.
2. For low values of the SNR, it can be easily shown that

$$\mathbb{E} \left[\frac{I^M}{n} \right] (\gamma, \rho) = \frac{\gamma\rho}{1+\gamma} + O(\rho). \quad (3)$$

In this case, the optimum value is $\gamma \rightarrow \infty$. The shift toward the receiving antennas can be understood intuitively: in this regime, the mutual information scales linearly with the SNR (and not with the degrees of freedom) and therefore one should focus on increasing the total received SNR (which can be done by increasing the number of receiving antennas).

For the general case, we have plotted in Figure 1 the mean mutual information versus γ for several values of the SNR ρ . In each case, a maximum value occurs (between 1 and $+\infty$) which depends on ρ as shown in Figure 2.

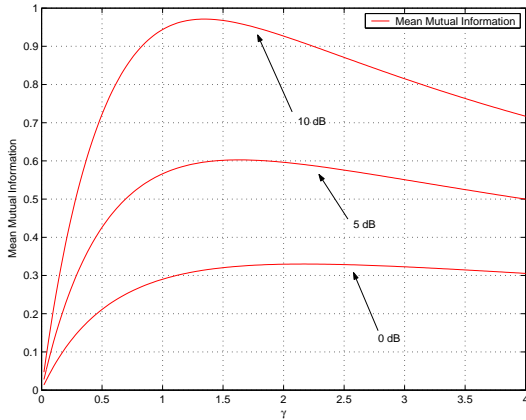


Figure 1: Mean Mutual Information vs. γ for the i.i.d. Gaussian model at various SNR.

2.2 Double directional model

Departing from the Gaussian i.i.d. model of section 2.1, let us now consider the double directional model of Figure 3. It is a double-bounce model with $s_r \leq n_r$ scatterers at the receiving side and $s_t \leq n_t$ scatterers at the transmitting

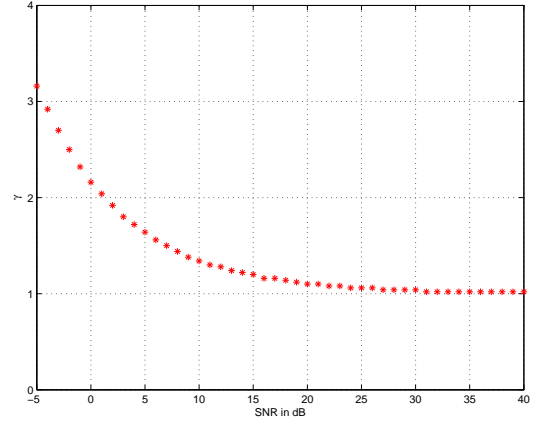


Figure 2: Optimum γ vs. SNR for the i.i.d. Gaussian model.

side. The scatterers are defined by their (angular) position and their attenuation (see [8] for more details). The MIMO matrix has the form

$$\mathbf{H} = \frac{1}{\sqrt{s_r s_t}} \Phi_{n_r \times s_r} \mathbf{P}^r \frac{1}{2} \Theta_{s_r \times s_t} \mathbf{P}^t \frac{1}{2} \Psi_{s_t \times n_t}, \quad (4)$$

where $\Theta_{s_r \times s_t}$ is an $s_r \times s_t$ i.i.d. Gaussian matrix with unit variance components. This general model has been shown to include the Kronecker model, Sayeed's virtual representation and the keyhole channel as particular cases. The steering matrices $\Phi_{n_r \times s_r}$ and $\Psi_{s_t \times n_t}$ represent respectively the directions of arrival (DoAs) from scatterers near the receiver to the receiving antennas and the directions of departure (DoDs) from the transmitting antennas to scatterers near the transmitter:

$$\Phi_{n_r \times s_r} = \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ e^{j2\pi \frac{d(n_r-1) \sin(\phi_1)}{\lambda}} & \dots & e^{j2\pi \frac{d(n_r-1) \sin(\phi_{s_r})}{\lambda}} \end{pmatrix} \quad (5)$$

and

$$\Psi_{s_t \times n_t} = \begin{pmatrix} 1 & \dots & e^{j2\pi \frac{d(n_t-1) \sin(\psi_1)}{\lambda}} \\ \vdots & \ddots & \vdots \\ 1 & \dots & e^{j2\pi \frac{d(n_t-1) \sin(\psi_{s_t})}{\lambda}} \end{pmatrix}. \quad (6)$$

For the sake of simplicity, let us focus on the case where the

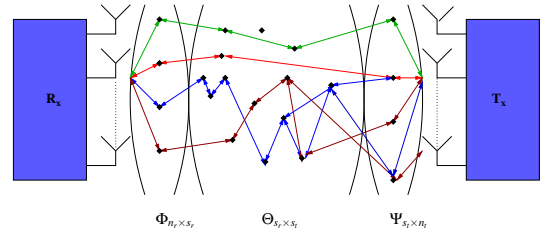


Figure 3: Double Directional model.

scatterers are maximally distant from each other, and have equal power. Therefore, the angle are distributed according to the Fourier directions ($\psi_k = \frac{2k\pi}{s_t}$, $k = 1 \dots s_t$, and $\phi_k =$

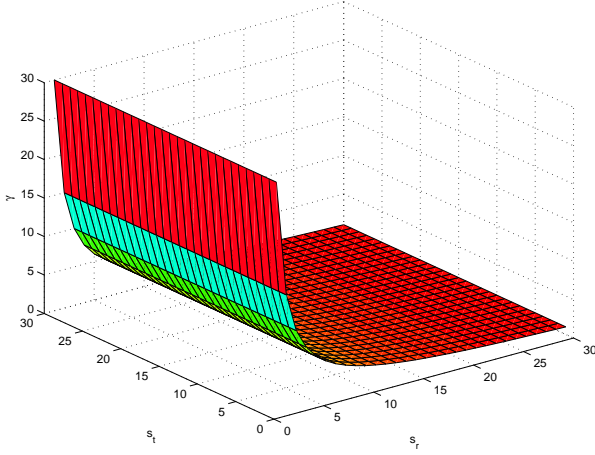


Figure 4: Optimum γ versus s_r and s_t for the double directional model, at 10dB SNR.

$\frac{2k\pi}{s_r}$, $k = 1 \dots s_r$), and all steering directions have equal gain ($\mathbf{P}^r = \mathbf{I}_{s_r}$ and $\mathbf{P}^t = \mathbf{I}_{s_t}$). In this case, denoting $\xi_t = \frac{s_t}{n_t}$ and $\xi_r = \frac{s_r}{s_r}$, the following result from [8] holds:

$$\begin{aligned} \mathbb{E} \left[\frac{I^M}{n} \right] &= \frac{1}{1 + \frac{1}{\gamma} \xi_r} \log(1 + \rho \xi_r - \rho \xi_r \alpha) \\ &+ \frac{\xi_t}{1 + \gamma} \log(1 + \rho \frac{\gamma}{\xi_t} - \rho \xi_r \alpha) - \frac{\xi_t}{1 + \gamma} \alpha, \end{aligned} \quad (7)$$

where

$$\alpha = \frac{1}{2} \left[1 + \frac{\gamma}{\xi_t \xi_r} + \frac{1}{\rho \xi_r} - \sqrt{\left(1 + \frac{\gamma}{\xi_t \xi_r} + \frac{1}{\rho \xi_r}\right)^2 - \frac{4\gamma}{\xi_t \xi_r}} \right] \quad (8)$$

In the general case, there is no explicit expression for the optimum value of γ . However, at very low SNR ($\rho \rightarrow 0$),

$$I^M = \frac{\gamma \rho}{1 + \gamma} + O(\rho) \quad (9)$$

which favors a shift towards the number of receiving antenna in this regime.

Figure 4 shows $\gamma_{opt} = \arg \max_{\gamma} \mathbb{E} \left[\frac{I^M}{n_t} \right]$ versus s_r and s_t for $\rho=10$ dB. The figure shows that γ_{opt} is in fact a function of s_r only. This is due to the fact that $s_t \leq n_t$ and therefore n_t has no effect on the multiplexing gain (s_t being the limiting factor). As s_r decreases, γ increases to increase the received SNR, since the multiplexing gain is any case limited by $\min(s_r, s_t)$. Note that the values of γ are between 1 and $+\infty$.

3. ANTENNA GEOMETRY IN LARGE SYSTEMS

Let us consider the mutual information statistics for the double directional model in the limit of large systems. In this section, we shall make as little assumptions as possible about the scattering matrices $\Phi_{n_r \times s_r}$ and $\Psi_{s_t \times n_t}$, with the ultimate goal of maximizing mutual information by optimizing array geometries. As a consequence, the considered antenna arrays are not necessarily uniform linear, and therefore we do not transmit and receive only on Fourier direc-

tions anymore. We will henceforth work under the following assumptions:

- When the size of matrix $\Phi_{n_r \times s_r} \mathbf{P}^r \frac{1}{2}$ grows large with $\xi_r = \frac{n_r}{s_r}$ remaining fixed, the empirical eigenvalue distribution S_{s_r, n_r} of $\frac{1}{s_r} \mathbf{P}^r \frac{1}{2} \Phi_{n_r \times s_r}^H \Phi_{n_r \times s_r} \mathbf{P}^r \frac{1}{2}$ converges in distribution to a fixed distribution:

$$S_{s_r, n_r}(\lambda) = \frac{1}{s_r} |\{j : \lambda_j \leq \lambda\}| \mapsto S_{\text{doa}}(\lambda); \quad (10)$$

- When the size of matrix $\mathbf{P}^t \frac{1}{2} \Psi_{s_t \times n_t}$ grows large with $\xi_t = \frac{s_t}{n_t}$ remaining fixed, the empirical eigenvalue distribution S_{s_t, n_t} of $\frac{1}{s_t} \mathbf{P}^t \frac{1}{2} \Psi_{s_t \times n_t}^H \Psi_{s_t \times n_t} \mathbf{P}^t \frac{1}{2}$ converges in distribution to a fixed distribution S_{dod} :

$$S_{s_t, n_t}(\lambda) = \frac{1}{s_t} |\{j : \lambda_j \leq \lambda\}| \mapsto S_{\text{dod}}(\lambda). \quad (11)$$

Let $f_{\text{dod}}(\lambda) = \frac{dS_{\text{dod}}(\lambda)}{d\lambda}$ and $f_{\text{doa}}(\lambda) = \frac{dS_{\text{doa}}(\lambda)}{d\lambda}$ denote the respective densities of the eigenvalues of $\frac{1}{s_t} \mathbf{P}^t \frac{1}{2} \Psi_{s_t \times n_t}^H \Psi_{s_t \times n_t} \mathbf{P}^t \frac{1}{2}$ and $\frac{1}{s_r} \mathbf{P}^r \frac{1}{2} \Phi_{n_r \times s_r}^H \Phi_{n_r \times s_r} \mathbf{P}^r \frac{1}{2}$. The asymptotic mutual information per receive antenna is given [9] by

$$\begin{aligned} \mathbb{E} \left[\frac{I^M}{n} \right] &= \frac{1}{1 + \gamma} \int_0^{+\infty} \log(1 + \rho \lambda \Gamma) f_{\text{doa}}(\lambda) d\lambda \\ &+ \frac{1}{1 + \frac{1}{\gamma}} \int_0^{+\infty} \log(1 + \rho \lambda \Upsilon) f_{\text{dod}}(\lambda) d\lambda - \frac{\rho}{1 + \gamma} \Gamma \Upsilon \end{aligned} \quad (12)$$

where Γ and Υ are the solutions of

$$\begin{cases} \Gamma = \gamma \int_0^{+\infty} \frac{\lambda}{1 + \rho \lambda \Upsilon} f_{\text{doa}}(\lambda) d\lambda, \\ \Upsilon = \int_0^{+\infty} \frac{\lambda}{1 + \rho \lambda \Gamma} f_{\text{dod}}(\lambda) d\lambda. \end{cases} \quad (13)$$

Interestingly, what really governs the transmission limits of different scenarios are only the properties of the eigenvalues of the steering matrix. This result is extremely useful, as it shows that only the limiting eigenvalue distribution of the steering directions and their respective powers matters: in other words, two antenna configuration can yield the same throughput as long as they give rise to the same eigenvalue distribution for the steering matrix.

Furthermore, since this result can constitute a good approximation even in the finite-dimension case, note that eqs. (12) and (13) can be rewritten for the finite case from the eigenvalues $\lambda_1^{\text{dod}}, \dots, \lambda_{s_t}^{\text{dod}}$ of $\frac{1}{s_t} \mathbf{P}^t \frac{1}{2} \Psi_{s_t \times n_t}^H \Psi_{s_t \times n_t} \mathbf{P}^t \frac{1}{2}$ and $\lambda_1^{\text{doa}}, \dots, \lambda_{s_r}^{\text{doa}}$ of $\frac{1}{s_r} \mathbf{P}^r \frac{1}{2} \Phi_{n_r \times s_r}^H \Phi_{n_r \times s_r} \mathbf{P}^r \frac{1}{2}$, as

$$\begin{aligned} \mathbb{E} \left[I^M \right] &= \frac{1}{\xi_t} \sum_{i=1}^{s_t} \log(1 + \rho \lambda_i^{\text{dod}} \Gamma) \\ &+ \xi_r \sum_{i=1}^{s_r} \log(1 + \rho \lambda_i^{\text{doa}} \Upsilon) - \rho n_t \Gamma \Upsilon \end{aligned} \quad (14)$$

with

$$\Gamma = \gamma \frac{1}{s_r} \sum_{i=1}^{s_r} \frac{\lambda_i^{\text{doa}}}{1 + \rho \lambda_i^{\text{doa}} \Upsilon}, \quad (15)$$

$$\Upsilon = \frac{1}{s_t} \sum_{i=1}^{s_t} \frac{\lambda_i^{\text{dod}}}{1 + \rho \lambda_i^{\text{dod}} \Gamma}. \quad (16)$$

Based on this result, a scenario that could be devised as a future evolution of wireless communication systems is the following: imagine a set of reconfigurable antennas that can move on a grid. At the beginning of the communication, the antennas are positioned arbitrarily. Once the transmission starts, the position of the antennas (for fixed scatterers) on the grid are then optimized using the previous formulas in order to increase mutual information. The optimization can either use instantaneous information about the scatterers' position (angles of arrival, distance, ...) or can be carried out statistically (*e.g.* adaptively adjusting inter-antenna spacing). This is once more a viable scenario from a software defined radio perspective and gives means for future research in the field of antenna design. The antenna design problem can therefore be related to an eigenvalue optimization problem.

4. MULTIPLE ANTENNAS VS. SINGLE ANTENNA: IS THERE A CONTRADICTION?

Although a Rice distribution is well known to enhance the performance with respect to the Rayleigh one in the Single-Input Single-Output (SISO) case, these results cannot be straightforwardly extended to the MIMO case. Indeed, consider the following example:

EXAMPLE 1. *Suppose that the channel matrix is deterministic with equal entries 1 (this is the limit case of a Rice distribution with variance 0). The mutual information per transmitting antenna with input Gaussian entries and covariance matrix $\mathbb{E}(\mathbf{x}\mathbf{x}^H) = \mathbf{I}_{n_t}$ is:*

$$\frac{I^M}{n_t} = \frac{1}{n_t} \log \det(\mathbf{I}_{n_r} + \rho \frac{\mathbf{H}\mathbf{H}^H}{n_t}) \quad (17)$$

$$= \frac{1}{n_t} \sum_{i=1}^{n_r} \log(1 + \rho \lambda_i), \quad (18)$$

where $\lambda_1, \dots, \lambda_{n_r}$ are the eigenvalues of $\frac{\mathbf{H}\mathbf{H}^H}{n_t}$. In this case, since it is a rank-one matrix, it has one single eigenvalue equal to n_r and the mutual information is given by:

$$\frac{I^M}{n_t} = \frac{1}{n_t} \log(1 + \rho n_r) \xrightarrow{n_r \rightarrow +\infty} 0, \quad (19)$$

where the limit is taken as $\gamma = \frac{n_r}{n_t}$ tends to a fixed value.

This example shows that the line of sight component has a dramatic effect on the mutual information since it is well known that in the zero mean i.i.d. Gaussian case, the mutual information per transmitting antenna is constant (see Section 2.1).

In fact, the analysis of Rice MIMO models is quite important as it determines the way antennas should be placed. In light of the previous result, one could conclude that in MIMO situations, it would be better to avoid Line-of-Sight situations, and that effectively "hiding" the antenna array behind scatterers (under a table for example) would increase channel capacity!

4.1 Some Considerations on MIMO Rice Channels

As a consequence, a more in-depth analysis should be conducted for determining the parameters governing the performance achievable in the presence of Rice fading with respect

to the i.i.d. Gaussian case. Before going into detail, let us introduce more precisely the MIMO Rice model. It essentially consists in a weighted superposition of Line-of-Sight (LOS) and Non-Line of Sight (NLOS) components. Therefore, the complex entries of \mathbf{H} are independently Gaussian distributed with identical variance and mean $\mathbb{E}(h_{ij}) = m_{ij}$. Letting K denote the Rice factor of the channel, we rewrite the channel matrix \mathbf{H} as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}} \mathbf{H}^{\text{LOS}} + \sqrt{\frac{1}{K+1}} \mathbf{H}^{\text{NLOS}} \quad (20)$$

in order to separate the random component of the channel from the deterministic part:

- \mathbf{H}^{LOS} represents the line of sight component of the channel such as $\|\mathbf{H}^{\text{LOS}}\|_F^2 = n_t n_r$ with entries $h_{ij}^{\text{LOS}} = \sqrt{\frac{K+1}{K}} \mu_{ij}$.
- \mathbf{H}^{NLOS} is the random component of the channel with Gaussian, independent and identically distributed entries. The complex element h_{ij}^{NLOS} is circularly symmetric, with zero mean and unit variance.

This model is general enough to take into account line of sight (LOS) and non line of sight (NLOS) cases, since as $K \rightarrow \infty$, eq. (20) models a deterministic channel, whereas for $K = 0$ it describes a Rayleigh i.i.d. fading channel.

We would like to predict the mutual information of a general Ricean MIMO channel using only a few meaningful parameters, namely the asymptotic eigenvalue distribution of the mean matrix \mathbf{H}^{LOS} , the Ricean factor K , the SNR, and the number of receive antennas per transmit antenna $\gamma = \frac{n_r}{n_t}$. This situation has been studied by several authors, see *e.g.* [10, 11].

Assumption: As $n_r, n_t \rightarrow \infty$ while the ratio $\gamma = \frac{n_r}{n_t}$ remains constant, the sequence of the empirical eigenvalue distribution of the matrix $\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}$ is assumed to converge in distribution to a deterministic limit function $F_{\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}}(\lambda)$.

In this case, let us recall some important results of Cotatellucci *et al.* [11]:

THEOREM 1. *As $n_r, n_t \rightarrow +\infty$ while $\gamma = \frac{n_r}{n_t}$ tends to a fixed value, the asymptotic mutual information per transmit antenna assuming Gaussian, spatially white (covariance \mathbf{I}_{n_t}) input, converges almost surely to a deterministic value:*

$$\lim_{n \rightarrow +\infty} \mathbb{E} \left[\frac{I^M}{n} \right] = \frac{1}{1 + \gamma} \int_0^\rho \frac{1}{x} \left(1 - \frac{1}{x} m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}} \left(-\frac{1}{x} \right) \right) dx, \quad (21)$$

where $m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z)$ is the unique solution of the fixed point equation

$$m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) = \int_0^{+\infty} \frac{F_{\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}}(\lambda)}{\frac{K\lambda}{\gamma m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) + K + 1} - z \left(\frac{\gamma m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z)}{K+1} + 1 \right) + \frac{1-\gamma}{K+1}} d\lambda \quad (22)$$

such that $\text{Im}(m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z)) > 0$ for $\text{Im}(z) > 0$.

$$m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) = \int_0^{+\infty} \frac{1}{\lambda - z} F_{\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}}(\lambda) d\lambda \quad (23)$$

is the Stieltjes transform of the limit distribution function $F_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}$ of the eigenvalues of $\frac{\mathbf{H}\mathbf{H}^H}{n_t}$, and is implicitly – through eq. (22) – a function of the limit distribution $F_{\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}}(\lambda)$ of the eigenvalues of the LOS component.

Remarkably, in this case, the asymptotic mutual information per antenna is completely determined knowing only γ , ρ , K , and the eigenvalues of $\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}$, but not the particular fluctuations of the fading.

4.1.1 LOS versus NLOS situations

- In the LOS case $K \rightarrow \infty$, the Ricean component dominates, and eq. (22) logically simplifies into

$$m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) = \int_0^{+\infty} \frac{1}{\lambda - z} F_{\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}}(\lambda) d\lambda, \quad (24)$$

which is nothing else than the Stieltjes transform of the distribution of the line of sight component.

- In the Rayleigh ($K \rightarrow 0$, NLOS) case, eq. (22) becomes

$$m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) = \frac{1}{-z \left(\gamma m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) + 1 \right) + (1 - \gamma)}, \quad (25)$$

which yields (using the notation $[z]^+ = \max(0, z)$)

$$F_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(\lambda) = [1 - \frac{1}{\gamma}]^+ \delta(\lambda) + \begin{cases} \frac{1}{\pi\gamma\lambda} \sqrt{\lambda - \frac{1}{4}(\lambda - 1 - \frac{1}{\gamma})^2} \\ \text{if } (\sqrt{\frac{1}{\gamma}} - 1)^2 \leq \lambda \leq (\sqrt{\frac{1}{\gamma}} + 1)^2, \\ 0 \text{ otherwise.} \end{cases} \quad (26)$$

In this case, one obtains the classical semi-circle law distribution corresponding to the i.i.d. zero mean Gaussian channel, with the Dirac delta term accounting for the null eigenvalues when \mathbf{H} is tall ($\gamma > 1$).

In those two extreme cases, we verified that the results provided by Theorem 1 are consistent with previous knowledge.

4.1.2 Finite-dimension systems

Although formally valid only in the asymptotic regime, results on capacity of MIMO channels have been shown [5] to hold even for relatively low number of antennas. Therefore, let us apply the previous results to finite dimension cases by replacing the integral in eq. (22) by a discrete sum. This yields

$$m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) = \frac{1}{n_r} \sum_{i=1}^{n_r} \frac{1}{\frac{K\lambda_i^{\text{LOS}}}{\gamma m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) + K + 1} - z \left(\frac{\gamma m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z)}{K + 1} + 1 \right) + \frac{1 - \gamma}{K + 1}} \quad (27)$$

where $\lambda_1^{\text{LOS}}, \dots, \lambda_{n_r}^{\text{LOS}}$ are the eigenvalues of $\frac{\mathbf{H}^{\text{LOS}}\mathbf{H}^{\text{LOS}H}}{n_t}$. Similarly, in finite dimension, (23) reduces to

$$m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z) = \frac{1}{n_r} \sum_{i=1}^{n_r} \frac{1}{\lambda_i - z}. \quad (28)$$

Therefore, the average mutual information is given by

$$\mathbb{E}[I^M] = n_t \int_0^\rho \frac{1}{x} \left(1 - \frac{1}{n_r x} \sum_{i=1}^{n_r} \frac{1}{\lambda_i + x^{-1}} \right) dx, \quad (29)$$

where the $\lambda_1, \dots, \lambda_{n_r}$ are the solution of

$$\sum_{i=1}^{n_r} \frac{1}{\lambda_i - z} = \sum_{i=1}^{n_r} \left[\frac{K\lambda_i^{\text{LOS}}}{\frac{\gamma}{n_r} \sum_{i=1}^{n_r} \frac{1}{\lambda_i - z} + K + 1} - z \left(\frac{\gamma}{n_r(K + 1)} \sum_{i=1}^{n_r} \frac{1}{\lambda_i - z} + 1 \right) + \frac{1 - \gamma}{K + 1} \right]^{-1} \quad (30)$$

such that $\text{Im}(m_{\frac{\mathbf{H}\mathbf{H}^H}{n_t}}(z)) > 0$ for $\text{Im}(z) > 0$.

4.1.3 The infamous diversity versus path loss trade-off

In the light of the previous results, should the user “hide under the table” to communicate? In fact, the result depends mostly on how the mean matrix (through its limiting singular value decomposition) is structured, as revealed by eq. (22). Even the situation presented in Example 1 is tricky: one cannot compare Rice and Rayleigh at the same SNR ρ .

Indeed, in the case of Line of Sight, the path loss incurred by the Rice distribution is less dramatic than in the case of non-line of sight and as a consequence favors the Rice model. Therefore, in order to make sensible decisions in terms of system design, we need to consider the trade-off between the number of degrees of freedom of the channel (through the rank of \mathbf{H}) and the path loss factor (which influences ρ). Let us approach this issue by considering the following two extreme cases:

Rank-one channel The $n_r \times n_t$ Rice matrix has rank one (with zero variance) and the path loss factor is a function of the distance r between the transmit and receive antenna arrays. Letting $\rho = \frac{\rho_{\text{max}}}{r^2}$ (path loss model in free space), we consider a rank-one channel model, *i.e.* $\mathbf{H} = \sqrt{\rho} \mathbf{h}_r \mathbf{h}_t^H$ with $\mathbf{h}_r = [1 \dots 1]^T$ and $\mathbf{h}_t = [1 \dots 1]$. This situation can be regarded as an extreme case of a pinhole channel [12]. In this case, the total mutual information at high SNR is given by $I^M = \log(\frac{n_r \rho_{\text{max}}}{r^2})$.

Rayleigh channel The Rayleigh case has a path loss factor $\rho = \frac{\rho_{\text{max}}}{r^l}$ ($l \geq 2$). In this case, the total mutual information at high SNR is given [6] by: $I^M = n_r \log(\frac{\rho_{\text{max}}}{r^l})$.

For a given SNR and number of antennas, the exponent of r is the dominant factor in the analysis of the situation if $l > 2$, and in this case the rank-one (Line-of-Sight) case is always the most favorable when r tends to $+\infty$, whereas the Rayleigh situations yields a higher mutual information at short distances.

At high SNR, the path loss factor influences the slope of the mutual information (with respect to the distance). The intersection of the two cases is given by: $n_r \log(\rho_{\text{max}}) - n_r l \log(r) = \log(\rho_{\text{max}}) + \log(n_r) - 2 \log(r)$, which yields for a high number of antennas ($n_r \rightarrow +\infty$): $r \sim \rho_{\text{max}}^{\frac{1}{l}}$.

In practice, this trade-off between diversity and the path loss is hard to analyze, and the simplifying assumptions that were used above are not expected to hold for all situations.

In particular, the assumptions on the rank of the Rice matrix and the SNR are not always fulfilled. However, these examples still remain meaningful as extreme cases.

5. CONCLUSIONS

We presented an overview of issues faced during the design of a multiple-antenna wireless system, when the goal is to maximize the efficiency of the system (in terms of achieved ergodic mutual information per antenna). Various fading situations have been considered (Gaussian i.i.d., Ricean, double-directional and limited-rank models), as well as the trade-off between LOS and NLOS situations, and guidelines in terms of the repartition of the antennas between the transmitter and the receiver, as well as in terms of array geometry, have been established. In particular, the concept of a reconfigurable array that adapts its geometry to the channel conditions is proposed.

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