Performance Evaluation of Supervised PARAFAC Receivers for CDMA Systems

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***Abstract*— This paper evaluates the performance of two PARAFAC-based receiver structures wireless communication systems based on code division multiple access (CDMA). Motivated by the multidimensional nature of the wireless signal, the PARAllel FACtor tensor decomposition (PARAFAC) is an interesting approach to mitigate multiuser interferences and to reconstruct the transmitted signal of each user at the base station receiver. Furthermore, the PARAFAC decomposition is a simple and elegant multidimensional decomposition that offers easily interpretable signal models for the considered systems. The two supervised receiver structures differ on the type of spreading code sequence employed at the transmitter, which can be either a conventional Hadamard-Walsh sequence or a truncated Hadamard-Walsh sequence. Performance evaluation of the two proposed receivers is made from the analysis of the Bit Error Rate (BER) and the convergence speed.**

***Index Terms*— Tensor decomposition, communications systems, array processors, identification, equalization.**

I. INTRODUCTION

During the last years, some algorithms have been proposed to use the PARAllel FACtor decomposition (PARAFAC) for signal processing in the context of wireless communication systems. Mainly developed and used in chemometrics and psychometrics, PARAFAC is a tensor decomposition technique that merged as an attractive tool for system modeling, blind beamforming, multiuser channel estimation/equalization and signal separation for wireless communications [1].

In the multiple access context, PARAFAC has been used to operate in a blind way with the objective to separate co-channel user's signals and to reliably estimate their channel impulse responses [2], [3]. The PARAFAC decomposition has also been used recently as an unified modeling tool for some wireless communication systems such as CDMA, OFDM and oversampled wireless systems, [4], [5]. It has also been applied to problems of blind multiuser equalization in [6]. Blind PARAFAC receivers exhibit an inherent scaling and permutation ambiguity problem, like most of blind techniques. To overcome this ambiguity problem, constraints linked to the specific characteristics of the considered wireless communication systems are imposed on the signal models.

The objective of this paper is to evaluate the performance of the PARAFAC receivers for CDMA wireless systems under practical system constraints. Two PARAFAC-based receivers are presented. The first one is based on the knowledge of the spreading code matrix and the second one is based on the use of a known training sequence.

The paper is organized as follows. In Section 2, the PARAFAC model is introduced. The data model is presented in Section 3. The system considerations are discussed in Section 4. Section 5 presents the proposed CDMA receivers. The simulation results and the conclusions are presented respectively in Section 6 and Section 7.

II. PARAFAC DECOMPOSITION

Given a three-way array $\underline{\mathbf{X}}$ of dimension $\mathbf{I} \times \mathbf{J} \times \mathbf{K}$, the standard three-way PARAFAC model decomposes each element $x_{i,j,k}$ into trilinear components as shown in (1):

$$x_{i,j,k} = \sum_{r=1}^{R} a_{i,r} b_{j,r} c_{k,r}. \tag{1}$$

To simplify the mathematical analysis of the PARAFAC decomposition, some matrix representations can be used. Let us define the loading matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ with respective elements $a_{i,r}$, $b_{j,r}$ and $c_{k,r}$. The matrix representation that we will consider in this paper is called the *unfolded* form. The tensor is decomposed in matrix-slice forms as [2]:

$$\begin{aligned} \mathbf{X}_{i\cdot\cdot} &= \mathbf{B}D_i[\mathbf{A}]\mathbf{C}^T \quad i = 1, \ldots, I, \\ \mathbf{X}_{\cdot j\cdot} &= \mathbf{C}D_j[\mathbf{B}]\mathbf{A}^T \quad j = 1, \ldots, J, \\ \mathbf{X}_{\cdot\cdot k} &= \mathbf{A}D_k[\mathbf{C}]\mathbf{B}^T \quad k = 1, \ldots, K, \end{aligned} \tag{2}$$

where $D_i[.]$ is the diagonal operator based on the ith row of a given matrix. These matrix slices can be concatenated to construct the three different unfolded matrices given by:

$$\begin{aligned} \mathbf{X}_1 &= (\mathbf{A} \diamond \mathbf{B})\mathbf{C}^T \\ \mathbf{X}_2 &= (\mathbf{B} \diamond \mathbf{C})\mathbf{A}^T \\ \mathbf{X}_3 &= (\mathbf{C} \diamond \mathbf{A})\mathbf{B}^T \end{aligned} \tag{3}$$

where $\diamond$ denotes the Khatri-Rao (column-wise Kronecker) product. The unfolded matrices ($\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_3$) have respective dimensions $(JI \times K)$, $(KJ \times I)$ and $(IK \times J)$, and they are defined as the concatenation of the respective matrix slices. For example, $\mathbf{X}_1 = [\mathbf{X}_{1\cdot\cdot}^T \cdots \mathbf{X}_{I\cdot\cdot}^T]^T$ is one of the unfolded matrices. Similarly, $\mathbf{X}_2 = [\mathbf{X}_{\cdot 1\cdot}^T \cdots \mathbf{X}_{\cdot J\cdot}^T]^T$ and $\mathbf{X}_3 = [\mathbf{X}_{\cdot\cdot 1}^T \cdots \mathbf{X}_{\cdot\cdot K}^T]^T$ are defined in the same way as $\mathbf{X}_1$.

A. Uniqueness Property

One of the major advantages of the PARAFAC decomposition is its uniqueness property. Different from the two-dimensional (2D) models, that may suffer from rotational ambiguity problems, the trilinear model is unique, up to scaling and permutation ambiguities [7].

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It is not an easy task to exactly define all the necessary and sufficient conditions to respect the uniqueness property. Several conditions have been proposed, but the unknown characteristic of the tensor's rank makes hard the definition of the necessary and sufficient conditions to guarantee the uniqueness of its decomposition. The least restrictive condition for uniqueness was proposed by Kruskal [7] by means of the k-rank (*Kruskal-rank*) of the loading matrices. Introduced by Harshman & Lundy [8], the k-rank of a matrix $\mathbf{A}$ represents the maximum number of columns (r) such that *every* set of r columns of $\mathbf{A}$ are linearly independent and this does not hold for a set of $r+1$ columns. According to Kruskal a multi-way array $\underline{\mathbf{X}}$, decomposed as a function of matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, is unique if

$$k_A + k_B + k_C \geq 2(R+1), \tag{4}$$

where R is the number of factors of the PARAFAC decomposition and k_A, k_B and k_C are respectively the k-rank of the loading matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$. Sidiropoulos et al [2] extended the Kruskal proof and showed that the Kruskal condition is valid for complex tensors. Necessary and sufficient conditions of uniqueness have been recently proposed in [9].

The meaning of the uniqueness property is that any matrices $\overline{\mathbf{A}}$, $\overline{\mathbf{B}}$ and $\overline{\mathbf{C}}$ satisfying the PARAFAC model, are linked to $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ by:

$$\begin{aligned} \overline{\mathbf{A}} &= \mathbf{A}\Pi\Delta_1 \\ \overline{\mathbf{B}} &= \mathbf{B}\Pi\Delta_2, \\ \overline{\mathbf{C}} &= \mathbf{C}\Pi\Delta_3 \end{aligned} \tag{5}$$

where Π is a permutation matrix and $\Delta_{i=1,2,3}$ are diagonal matrices, with $\Delta_1\Delta_2\Delta_3 = \mathbf{I}$.

B. Alternating Least Square Algorithm (ALS)

The ALS is an iterative technique that can be used to estimate the loading matrices of the PARAFAC model. Based on the principle of grouping the parameters into several sets, the ALS consists in an iterative estimation in a least square sense of each set of parameters until convergence. In a generic way, the three-way array $\mathbf{X}_{i=1,2,3}$ is expressed by a function $f(\mathbf{A}, \mathbf{B}, \mathbf{C})$, and the ALS algorithm estimates each component matrix by assuming the knowledge of the two others, as shown in (6):

$$\begin{aligned} &\min_{\mathbf{C}} \quad \|\mathbf{X}_1 - (\mathbf{Z}_1)\mathbf{C}^T\|^2 \\ &\min_{\mathbf{A}} \quad \|\mathbf{X}_2 - (\mathbf{Z}_2)\mathbf{A}^T\|^2, \\ &\min_{\mathbf{B}} \quad \|\mathbf{X}_3 - (\mathbf{Z}_3)\mathbf{B}^T\|^2 \end{aligned} \tag{6}$$

where $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ are respectively defined as $(\widehat{\mathbf{A}} \diamond \widehat{\mathbf{B}})$, $(\widehat{\mathbf{B}} \diamond \widehat{\mathbf{C}})$ and $(\widehat{\mathbf{C}} \diamond \widehat{\mathbf{A}})$, and $\widehat{\mathbf{A}}$, $\widehat{\mathbf{B}}$ and $\widehat{\mathbf{C}}$ represent the previously estimated matrices.

Besides the attractive simplicity of the ALS, the convergence to the global optimum can not be always guaranteed because the algorithm can stop in a local minimum of the criterium. This characteristic is intimately dependent on the data nature and the initialization.

III. Data Model

The CDMA system considered in this work is a simple wireless system where the multidimensional received signal is modeled as a multi-way array and decomposed as a PARAFAC model. Let us consider a CDMA system with Q synchronized users, M receiver antennas and L chips per symbol in a quasi-static flat fading channel. In absence of noise, the discrete-time base-band representation of the received signal at the m-th receiver antenna of the n-th symbol in the l-th chip sampling is given by

$$x_{m,n,l} = \sum_{q=1}^{Q} h_{m,q} s_{n,q} c_{l,q}, \tag{7}$$

where $h_{m,q}$, $s_{n,q}$ and $c_{l,q}$ denote respectively the channel attenuation factor between the q-th user and the m-th receiver antenna, the n-th transmitted symbol of the q-th user and the l-th chip code of the q-th user. Using the PARAFAC model notation of (1), we can define the three loading matrices $\mathbf{H}$, $\mathbf{S}$ and $\mathbf{C}$ of the PARAFAC model as the matrices respectively constituted by the elements $h_{m,q}$, $s_{n,q}$ and $c_{l,q}$. The matrix $\mathbf{H}$ of dimension $M \times Q$ is the multiple-input multiple-output channel matrix, constructed with the channel elements $h_{m,q}$. The matrix $\mathbf{S}$ of dimension $N \times Q$ is a symbol matrix, where its q-th column contains the data sequence of the q-th user. The matrix $\mathbf{C}$ is a code matrix with dimension $(L \times Q)$, where each column q is the code of the q-th user.

For the decomposition process, when applying the ALS algorithm, the two proposed receiver structures are based on the knowledge of some properties of the loading matrices. They are described as follow:

- The first receiver structure assumes that the spreading codes of all users are known. We propose two variants for this structure. In the first one (RX1), the channel and symbol matrices are estimated. After the convergence, hard decision is performed to estimate the transmitted sequence. In the second variant (RX2), at each ALS step, the entries of the symbol matrix are projected onto the finite modulation alphabet of the transmitted symbols. Note that since the code matrix is known, permutation ambiguity is eliminated from the estimation process. Scaling ambiguity is eliminated by normalizing the channel matrix.
- In the second structure, the users spreading codes are not known, but a training sequence is employed in order to estimate the channel and the code matrices. In this case, each entry of the code matrix is projected onto the finite-alphabet of the set of codewords during the ALS algorithm. Similarly to the first receiver, permutation and scaling ambiguities are eliminated by the training sequence knowledge that we have.

IV. System Assumptions

For simplicity, we make some assumptions:

- The system is frame-based, which means that the signal processing elaborated by the receiver is done after the reception of each data packet;

- The wireless channel is a quasi-static flat fading channel. The flat fading assumption can easily be achieved by the insertion of guard chips or by the use of techniques like Orthogonal Frequency Division Multiplexing (OFDM).
- The signals captured by different antennas of the receiver array are assumed to be uncorrelated. This assumption is valid for an antenna array that has an antenna separation exceeding half of the wavelength of the wireless signal;
- Each receiver antenna employs an automatic gain controller that normalizes the received signal. This normalization is important to avoid scaling ambiguities in the receiver structures RX1 and RX2.

A. Uniqueness

The uniqueness property of the PARAFAC model, as described in Section II-A, means that the decomposition process is unique, implying that we can estimate the exact loading matrices of the multi-way array. Let us consider the signal model of the CDMA system as described in (7). The Kruskal condition that respects the uniqueness property is given by

$$k_H + k_S + k_C \geq 2(Q + 1), \tag{8}$$

where k_H, k_S and k_C are respectively the k-rank of the matrices $\mathbf{H}$, $\mathbf{S}$ and $\mathbf{C}$, and Q is the number of users.

Analyzing the loading matrices and considering the system characteristics, we can conclude that:

- The channel between each user and each receiver antenna suffers from an independent fading, which implies that $\mathbf{H}$ is full k-rank [1].
- The data sequence generated by each user is uncorrelated from the data sequence of the other users. Assuming that the number of symbols per frame is much greater than the number of users ($N \gg Q$), the matrix $\mathbf{S}$ has full k-rank.
- The code matrix $\mathbf{C}$ is generated so that each user has different codes, based on the considered code set. Two different kinds of code are considered: Hadamard-Walsh (HW) and truncated Hadamard-Walsh (THW) codes. For HW codes, the matrix $\mathbf{C}$ is always full k-rank. For THW codes, the matrix $\mathbf{C}$ is full k-rank when $L \geq Q$. These characteristics are shown in Appendix.

Under these assumptions, we can rewrite (8) as

$$min(M, Q) + min(N, Q) + min(L, Q) \geq 2(Q + 1),$$

which shows uniqueness conditions related with system parameters.

V. PARAFAC CDMA RECEIVERS

To evaluate the performance of the PARAFAC receiver structures, two CDMA systems are considered. Differently from the blind approach of [2], the CDMA receivers considered in this paper always have the information of one of the loading matrices (either the code matrix or the symbol matrix). This eliminates the intrinsic permutation ambiguity. The scale ambiguity is eliminated exploiting the knowledge of the alphabet used on the symbol matrix and code matrix.

A. Code Knowledge-based Receiver

Based on the conventional CDMA system, this receiver exploits the knowledge of the code matrix. Under this assumption, each frame is analyzed independently from the others, and each frame processing generates two estimated matrices: the channel matrix and the symbol matrix. This receiver is analyzed in terms of code length, number of users and number of receiver antennas. Two proposed structures (RX1 and RX2) are considered for performance evaluation.

B. Training Sequence-based Receiver

This receiver uses a training sequence at the beginning of each packet transmission. In this case, the channel matrix is estimated by using the training sequence information. After that, unknown symbols are transmitted and the channel matrix and the code matrix are considered static.

This receiver is analyzed in terms of training sequence length, code length, number of users and number of receiver antennas. For simplicity, the receiver knows the set of codewords. For this reason, only the second proposed receiver structure (RX2) is considered. This structure projects the estimated code matrix on the finite set of codewords.

VI. SIMULATIONS

The performance of the proposed receiver schemes is evaluated in this section by means of computer simulations. We employ binary-phase-shift-keying (BPSK) modulated symbols and each run represents a transmitted frame of 1000 symbols. The ALS initialization is done by using the knowledge of one of the loading matrices and the two other matrices are randomly initialized. The results are evaluated for different numbers of receiver antennas, different code lengths, different numbers of users, under the assumption of a flat fading channel. For the supervised case, the training sequence is generated with a length equal to the number of users. Performance evaluation is done in terms of Bit-Error-Rate (BER) and convergence speed. All the simulations are based on the Monte Carlo technique with 100 experiments. The performance results shown in the figures are the average performance over all the users. A brief summary of the simulation parameters is given in Table I.

TABLE I

SIMULATION PARAMETERS.

Symbol modulation	BPSK
# users (Q)	10
# symbols per frame (N)	1000
# receiver antennas (M)	2 and 4
# chips per code (L)	$\geq$ # users
# training symbols (Symbtr)	= # users
# Monte Carlo experiments	100

[1]The matrix whose columns are drawn independently from an absolutely continuous distribution is full k-rank with probability one, because any combination of columns can be thought as another random matrix with columns drawn independently from an absolutely continuous distribution.

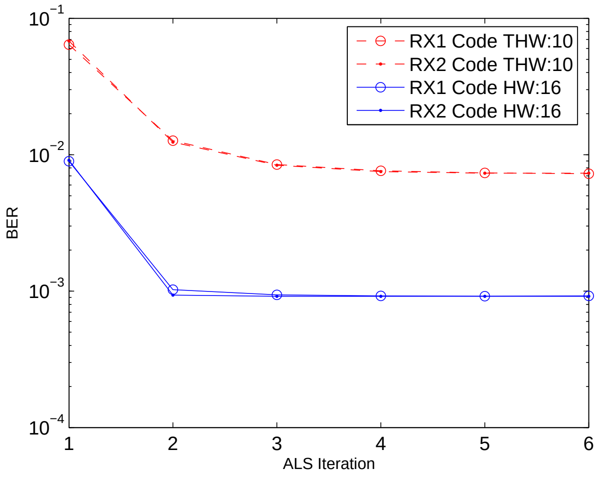

Fig. 1. ALS convergence for 4 antennas, 10 users and SNR of 5db.

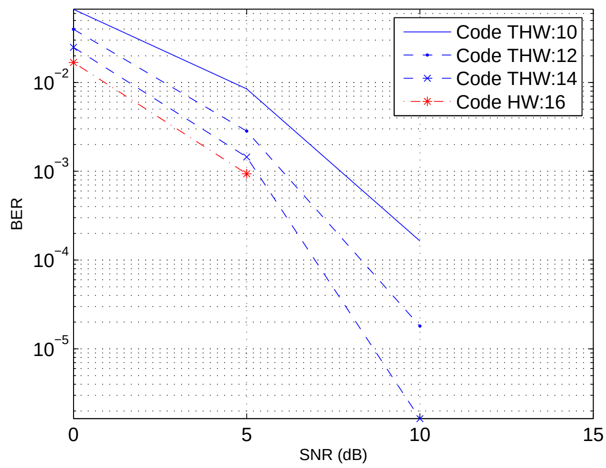

Fig. 3. BER versus SNR for structure RX1 with 10 users and different codes.

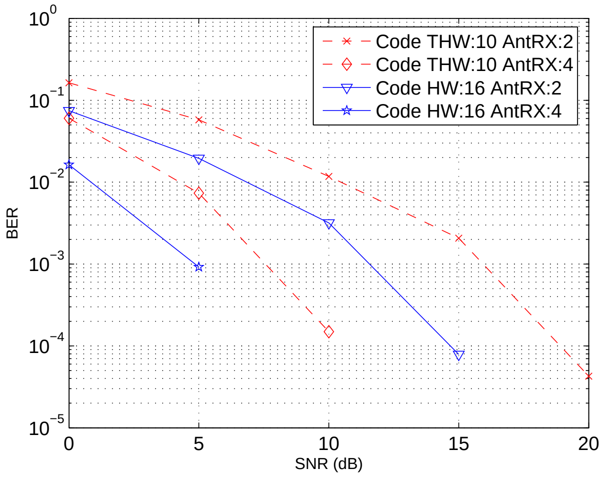

Fig. 2. BER versus SNR for the structure RX1 with 10 users.

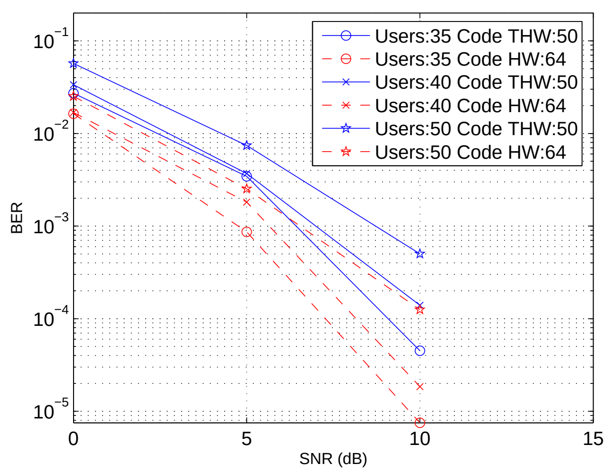

Fig. 4. BER versus SNR for different number of users, HW and THW codes.

A. CDMA Receiver with Code Knowledge

In Fig. 1, the ALS convergence is evaluated for a SNR of 5dB for the two structures (RX1 and RX2) with two different codes (code THW with 10 chips and code HW with 16 chips) in a conventional CDMA system. It can be seen that the ALS convergence is very fast, convergence being achieved after very few iterations, and the two structures do not show any performance difference.

For performance evaluation, we only consider the structure RX1 because it gives the same performance as the RX2 structure with less complexity. Figure 2 compares the performance for two numbers of receiver antennas ($M = 2$ and $M = 4$) and two different code structures, which are Hadamard-Walsh (HW) and Truncated Hadamard-Walsh (THW). It can be seen that a satisfactory BER performance is achieved. The receiver performance improves where the number of receiver antennas is increased. These results were expected because multiple antennas at the receiver provide spatial diversity, adding more redundancy to the received signal.

Regarding the influence of the code length, all the figures show a better performance when the code length increases. In Fig. 3, it can be seen that the system efficiency increases with the code length. In other words, when we increase the code length, we increase the symbol redundancy, which in turn increases the system performance. It is important to note that the HW code is orthogonal, and it represents the best code with length $L = 16$.

In order to evaluate the impact of the number of users, we present in Fig. 4 the performance of the system that employs 35, 40 and 50 users with different codes. As we expected, the increase of the number of users degrades the receiver performance, but the system continues to be able to separate/recover the transmitted signals.

B. Supervised CDMA Receiver

For the supervised CDMA receiver, the ALS convergence is achieved after the first iteration. As was observed in all the simulations, this convergence can be justified by the

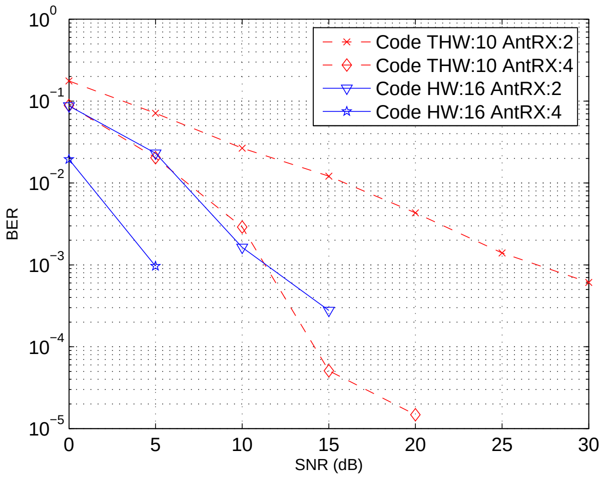

Fig. 5. BER versus SNR with a training sequence of 10 symbols, 10 users.

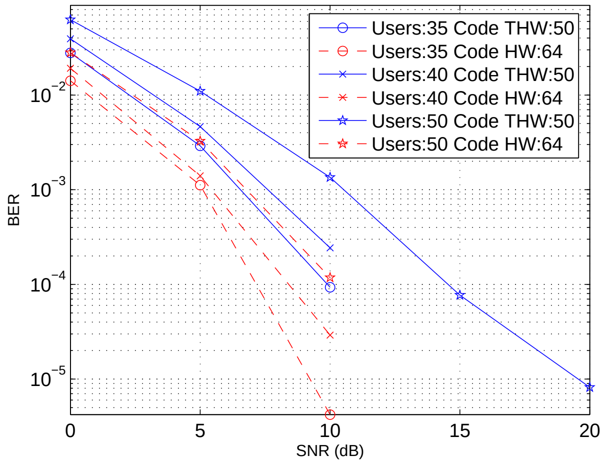

Fig. 7. BER versus SNR with a training sequence of 64 symbols.

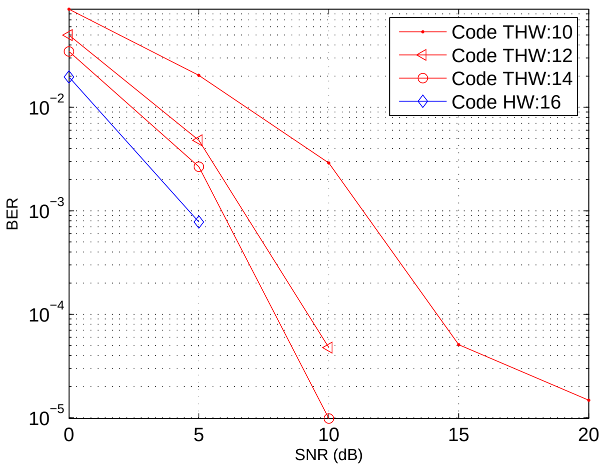

Fig. 6. BER versus SNR with a training sequence of 10 symbols.

knowledge of the training symbol matrix and the codewords ensemble at the receiver, i.e., after the first iteration, the symbol matrix is exactly known and the estimated code matrix columns are projected on the known set of possible codewords. In Fig. 5, we compare the performance as a function of the number of receiver antennas ($M = 2$ and $M = 4$) and as a function of the code type (THW with 10 chips and HW with 16 chips). Like for the PARAFAC receiver with code knowledge, Fig. 5 shows that when we increase the number of receiver antennas, the receiver performance is improved. In Fig. 6, it can be seen that an increased code length results in an improved receiver performance.

In order to evaluate the impact of the number of users, Fig. 7 shows the performance of a system with 35, 40 and 50 users, considering different code structures. Note that an increase on the number of users degrades the receiver performance, but for high SNR, the signal separation/recovery is remarkable.

VII. Conclusion and Perspectives

In this paper, we have presented two PARAFAC-based receivers for CDMA systems. The receivers exhibit satisfactory BER performance for two different system configurations (code knowledge or training sequence) and remarkable convergence speed, showing reasonable performance with low complexity. A perspective of this work includes the use of non-finite-alphabet codes which is full k-rank by definition. It is also expected to take the dispersion and time-varying channel into account for performance evaluation.

Appendix

The k-rank is extremely important for the PARAFAC uniqueness property. In this section, we show that the considered code matrices (Hadamard-Walsh and truncated Hadamard-Walsh) are full k-rank.

A. Hadamard-Walsh Code Matrices

The Hadamard-Walsh code matrices is a code family that has a particular orthogonal structure. The cross correlation between any two Hadamard-Walsh codes of the same code matrix is zero, implying that the columns are mutually orthogonal, i.e., for two different codewords ($\mathbf{c}_p$ and $\mathbf{c}_q$) with length N, generated by the same Hadamard-Walsh matrix, we have that

$$\sum_{i=0}^{N-1} \mathbf{c}_p(i) \cdot \mathbf{c}_q(i) = 0 \qquad \forall p \neq q. \tag{9}$$

Because of this property, the Hadamard-Walsh matrices are always full k-rank.

B. Truncated Hadamard-Walsh Code Matrices

The truncated Hadamard-Walsh code matrix is built based on the Hadamard-Walsh structure. Different from the Hadamard-Walsh matrix, the truncated one has not any constraint on the code length (the Hadamard-Walsh has always length $N = 2^n$). For this reason, the truncated Hadamard-Walsh does not satisfy the orthogonal property between different codewords. Besides this characteristics, it is possible to prove that a truncated Hadamard-Walsh code matrix is full k-rank when the code length is greater than one value that depends on the number of users, i.e., for Q users, we define a value n such that $(2^{n-1} < Q \leq 2^n)$ and the code length L is chosen to be greater than (2^{n-1}). To exemplify the problem, let us consider a 4×4 Hadamard-Walsh matrix:

$$\mathbf{C}^{(4)} = \left| \begin{array}{rrrr} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{array} \right|,$$

where each column represents an user codeword. We will denote the four code words respectively as $\mathbf{c}_1^{(4)}$, $\mathbf{c}_2^{(4)}$, $\mathbf{c}_3^{(4)}$ and $\mathbf{c}_4^{(4)}$. Now, let us consider a 5×5 truncated Hadamard-Walsh matrix:

$$\mathbf{C}^{(5)} = \left| \begin{array}{rrrrr} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 \end{array} \right|.$$

Rewriting the matrix in terms of the $\mathbf{C}^{(4)}$ codewords, we have:

$$\mathbf{C}^{(5)} = \left| \begin{array}{ccccc} \mathbf{c}_1^{(4)} & \mathbf{c}_2^{(4)} & \mathbf{c}_3^{(4)} & \mathbf{c}_4^{(4)} & \mathbf{c}_1^{(4)} \\ 1 & 1 & 1 & 1 & -1 \end{array} \right|.$$

Assuming that the codewords of the matrix $\mathbf{C}^{(5)}$ are linear dependent, we have that:

$$\alpha \left| \begin{array}{c} \mathbf{c}_1^{(4)} \\ 1 \end{array} \right| + \beta \left| \begin{array}{c} \mathbf{c}_2^{(4)} \\ 1 \end{array} \right| + \gamma \left| \begin{array}{c} \mathbf{c}_3^{(4)} \\ 1 \end{array} \right| + \xi \left| \begin{array}{c} \mathbf{c}_4^{(4)} \\ 1 \end{array} \right| + \zeta \left| \begin{array}{c} \mathbf{c}_1^{(4)} \\ -1 \end{array} \right| = 0$$

or equivalently:

$$(\alpha + \zeta)\mathbf{c}_1^{(4)} = -\beta\mathbf{c}_2^{(4)} - \gamma\mathbf{c}_3^{(4)} - \xi\mathbf{c}_4^{(4)} \quad (10)$$

$$\alpha + \beta + \gamma + \xi - \zeta = 0. \quad (11)$$

By definition, $\mathbf{c}_1^{(4)}$, $\mathbf{c}_2^{(4)}$, $\mathbf{c}_3^{(4)}$ and $\mathbf{c}_4^{(4)}$ are linear independent (orthogonal codewords). This implies that the only solution of (10) is the trivial solution, where β, γ, ξ and $(\alpha + \zeta)$ are equal to zero, i.e., the columns built with different orthogonal codewords are independent.

Assuming the proposed conditions for the code matrix, which has codes with length greater than $L > (2^{n-1} + 1)$, where n is an integer given by a relation with the number of users $(2^{n-1} < Q \leq 2^n)$. With these constraints, for a generic system with a given number of users, each of the orthogonal codewords with length $(L > (2^{n-1} + 1))$ is used to build at maximum two different columns of the truncated Hadamard-Walsh matrix. Let us have a truncated Hadamard-Walsh code matrix $\mathbf{C}^{(s)}$ with a given dimension $((2^{n-1} + 1) \times Q)$, which the number of users and the code length respect the proposed conditions. This matrix is constructed based on the Hadamard-Walsh matrix $\mathbf{C}^{(2^{n-1})}$ and given by:

$$\mathbf{C}^{(s)} = \left| \begin{array}{cccc} \mathbf{C}^{(2^{n-1})} & \mathbf{c}_1^{(2^{n-1})} & \cdots & \mathbf{c}_{Q-2^{n-1}}^{(2^{n-1})} \\ \mathbf{r}^T & -1 & \cdots & -1 \end{array} \right|,$$

where $\mathbf{c}_i^{(2^{n-1})}$ is the ith column of the matrix $\mathbf{C}^{(2^{n-1})}$ and $\mathbf{r}$ is a column vector of ones with dimension $(2^{n-1} \times 1)$.

As we can see in the above equation, due to the structure of the truncated Hadamard-Walsh matrix, the columns beginning with the same orthogonal codeword vector ($\mathbf{c}_i^{(2^{n-1})}$) are linear independent because of the insertion of the new row of elements. The independence of the codewords guarantees that the code matrix is full k-rank for every code matrix constituted by Q users, and L chips where n satisfies both assumptions $(2^{n-1} < Q \leq 2^n$ and $L \geq 2^{n-1})$.

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