Orthogonal Linear Beamforming in MIMO Broadcast Channels

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***Abstract*— The problem of joint linear beamforming and scheduling in a MIMO broadcast channel is considered. We show how Orthogonal Linear Beamforming (OLBF) can be efficiently combined with a low-complexity user selection algorithm to achieve a large portion of the multiuser capacity. The use of orthogonal transmission enables the transmitter to calculate exact signal-to-interference plus noise ratio (SINR) values during the user selection process. The knowledge of multiuser interference proves to be of particular importance for user scheduling as both the number of users in the cell and the average signal-to-noise ratio (SNR) decrease. The sum capacity of our scheme is characterized in the low-SNR regime, providing analytical results on the performance gain over Zero-Forcing Beamforming (ZFBF). Numerical results show gains over both suboptimal and optimal ZFBF techniques in different scenarios.**

I. Introduction

Multiple-input multiple-output (MIMO) communication systems have the potential to offer high spectral efficiency as well as link reliability. In point-to-point multiple antenna systems, it is well known that the capacity increases linearly with the minimum of the number of transmit/receive antennas, irrespective of the availability of channel state information (CSI) at the base station [1], [2]. In MIMO broadcast channels, it was shown [3] that the capacity can be boosted by exploiting the spatial multiplexing capability of transmit antennas and transmit to multiple users simultaneously, by means of Space Division Multiple Access (SDMA), rather than trying to maximize the capacity of a single-user link.

As the capacity-achieving dirty paper coding (DPC) approach [4] is difficult to implement, many more practical downlink transmission techniques have been proposed. Downlink linear beamforming, although suboptimal, has been shown to achieve a large portion of DPC capacity, exhibiting the best tradeoff between complexity and performance [5], [6], [7], [8]. However, finding the optimal beamforming vectors is a non-convex optimization problem, and the optimal solution for a downlink channel with K users is given by exhaustive search over all possible combinations. Evidently, the complexity of the above problem becomes prohibitively high for large K.

In this paper, we propose a system based on joint orthogonal linear beamforming and scheduling in MIMO broadcast channels, coined as Orthogonal Linear Beamforming (OLBF). In order to avoid user selection based on exhaustive search, we propose a low-complexity user selection technique valid under the constraint of using orthogonal beamforming vectors for transmission. Orthogonal transmission enables calculation of the exact signal-to-interference plus noise ratio (SINR) values during the user selection process in a compact and computationally efficient manner. This SINR expression was introduced as a scalar feedback metric for MIMO broadcast channels with limited feedback [9], [10], and can be interpreted as an upper bound on the SINR. Note that a similar metric is also reported in [11]. In the case of full channel knowledge and suboptimal user scheduling, the use of orthogonal beamformers allows to have a precise control on the multiuser interference at the transmitter. In order to improve the system performance, this knowledge proves to be of particular importance for decreasing number of users and average signal-to-noise ratio (SNR). In addition, the exact per-user contribution to the sum rate can be computed at each selection step.

Simulation results show performance improvements with respect to zero-forcing beamforming (ZFBF) and transmit matched filtering (TxMF) in realistic networks with low to moderate number of users. The proposed algorithm is also compared with the ZFBF scheme with Greedy User Selection (ZFBF-GUS) introduced in [8]. We show that in systems with low to moderate number of active users, the proposed scheme exhibits sum-rate gains over ZFBF-GUS in the low-SNR regime. However, as the average SNR and number of users increase, suboptimal ZFBF techniques like [8] can provide higher rates. One of our main results is to show that in the regime of low number of users, orthogonal SDMA offers better performance than optimal zero-forcing beamforming. Analytical results are provided and corroborated through numerical simulations.

The paper is organized as follows. Section II introduces the system model. Linear beamforming strategies are discussed in Section III. The proposed Orthogonal Linear Beamforming approach is described in Section IV. Section V provides a capacity scaling analysis of OLBF and ZFBF in the low-SNR regime. Simulation results are given in Section VI. Finally, conclusions are drawn in Section VII.

II. System Model

We consider a multiple antenna broadcast channel consisting of M antennas at the transmitter and K single-antenna receivers. The received signal y_k of the k-th user is mathematically described as

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \quad k = 1, \ldots, K \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{M \times 1}$ is the transmitted signal, $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$ is the channel vector, and n_k is additive white Gaussian noise at receiver k. We assume that n_k is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian with zero mean and unit variance. The transmitted signal is subject to an average transmit power constraint P, i.e., $\mathbb{E}\{\|\mathbf{x}\|^2\} = P$. We consider equal power allocation over each transmit beam, and an homogeneous network where all users have the same average signal-to-noise ratio (SNR). Due to the noise variance normalization to one, P takes on the meaning of average SNR.

Let $\mathbf{H} \in \mathbb{C}^{K \times M}$ refer to the concatenation of all channels, $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \ldots \mathbf{h}_K]^H$, where the k-th row is the channel of the k-th receiver ($\mathbf{h}_k^H$). Define $\mathcal{Q}$ as the set of all possible subsets of cardinality M of disjoint indices among the complete set of user indices $\mathcal{K} = \{1, \cdots, K\}$. Let $\mathcal{S} \in \mathcal{Q}$ be one such group of M users selected for transmission at a given time slot. Then $\mathbf{H}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathbf{s}(\mathcal{S})$, $\mathbf{y}(\mathcal{S})$ are the concatenated channel vectors, unit-norm beamforming vectors, uncorrelated data symbols and received signals respectively for the set of scheduled users $\mathcal{S}$. When concatenating the beamforming matrix $\mathbf{W}(\mathcal{S})$ prior to transmission, the signal model can be described as follows

$$\mathbf{y}(\mathcal{S}) = \mathbf{H}(\mathcal{S})\mathbf{W}(\mathcal{S})\mathcal{P}\mathbf{s}(\mathcal{S}) + \mathbf{n} \tag{2}$$

where $\mathcal{P}$ is an $M \times M$ diagonal matrix with entries equal to $\sqrt{P/M}$, as equal power allocation is used. Note that the use of OLBF and normalized beamforming vectors implies that the matrix $\mathbf{W}(\mathcal{S})$ is unitary, i.e., $\mathbf{W}(\mathcal{S})\mathbf{W}(\mathcal{S})^H = \mathbf{W}(\mathcal{S})^H\mathbf{W}(\mathcal{S}) = \mathbf{I}_M$. At the k-th mobile, the received signal is given by

$$y_k = \sqrt{\frac{P}{M}} \sum_{i \in \mathcal{S}} \mathbf{h}_k^H \mathbf{w}_i s_i + n_k, \quad k = 1, \ldots, K \tag{3}$$

We consider an i.i.d. block Rayleigh flat fading channel, whose parameters are considered invariant during each coded block, but are allowed to vary independently from block to block. We focus on the ergodic sum rate, which means that the capacity is averaged over the fading distribution, and thus the block size does not affect our results. We also assume that the number of mobiles is greater or equal to the number of transmit antennas, i.e., $K \geq M$, implying the use of a user selection algorithm.

Notation: We use bold upper and lower case letters for matrices and column vectors, respectively. $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ stand for transpose, Hermitian transpose, and pseudo-inverse, respectively. $\mathbb{E}(\cdot)$ denotes the expectation operator. The Euclidean norm of the vector $\mathbf{x}$ is denoted as $\|\mathbf{x}\|$, and the $\log_2(\cdot)$ refers to the base 2 logarithm.

III. Linear Beamforming Strategies

In this paper, we focus on joint downlink linear beamforming and scheduling with the objective of maximizing the system sum rate. The optimal solution can be conceptually given by exhaustive search over all possible user sets. In a system employing exhaustive search, the scheduler selects the set of users that maximize the sum rate as follows

$$\mathcal{S}^* = \arg\max_{\mathcal{S} \in \mathcal{Q}} \sum_{k \in \mathcal{S}} \log_2 \left[1 + SINR_k(\mathcal{S})\right] \tag{4}$$

where the values $SINR_k(\mathcal{S})$ have to be computed for each possible set $\mathcal{S}$ and user k, given a certain beamforming strategy. In MIMO broadcast channels, the most commonly used linear beamforming techniques are zero-forcing beamforming (ZFBF) and minimum mean squared error beamforming (MMSE-BF). In ZFBF, the transmit beamformer is computed as

$$\mathbf{W}(\mathcal{S}) = \frac{1}{\lambda}\mathbf{H}(\mathcal{S})^H(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H)^{-1} \tag{5}$$

where $\lambda = \frac{1}{P} tr\left[(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H)^{-1}\right]$, and $tr(\cdot)$ is the trace operator. In MMSE-BF, the beamformer is given by

$$\mathbf{W}(\mathcal{S}) = \mathbf{H}(\mathcal{S})^H(\alpha\mathbf{I} + \mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H)^{-1} \tag{6}$$

where α is chosen such that $tr\left(\mathbf{W}(\mathcal{S})\mathbf{W}(\mathcal{S})^H\right) = P$. For simplicity, we have not considered optimal power allocation in equations (5) and (6). When combining the above linear beamforming approaches with user scheduling in order to find the optimal user set as described in equation (4), the beamformers need to be computed for each user set $\mathcal{S}$. In order to avoid exhaustive user search, suboptimal scheduling approaches such as greedy user selection algorithms [7], [8], [12] can be implemented instead. The idea behind these approaches is to pre-select a reduced number of users according to different criteria, e.g. orthogonality properties as in [7], [8], hence reducing the search space. Once scheduling is performed, the base station computes the beamformers for transmission and is able to determine each user's achievable SINR. The inconvenience of these approaches is that in order to precisely know the SINR of a user, the beamformers have to be computed first, which is in general a computationally complex operation. Hence, suboptimal scheduling techniques rely on criteria other than the exact SINR values. In certain scenarios, such as systems with low number of users or in low SNR conditions, precise knowledge of SINR can help to increase the system performance, as we show in a later section.

TABLE I

ALGORITHM A

Step 0 Select first scheduled user and beamforming vector

$k_1 = \arg\max_{k\in\mathcal{K}} \|\mathbf{h}_k\|$

$\mathbf{w}_{k_1} = \overline{\mathbf{h}}_{k_1}$

Set $\mathcal{S}^* = \{k_1\}$

Step 1 Gram-Schmidt orthogonalization

Compute orthonormal basis $\mathbf{W}$ from $\mathbf{w}_{k_1}$

Step 2 Loop

For $i : 2\ldots M$ repeat

Step 2.1 Set $SINR^i_{max} = 0$

Step 2.2 Loop

For $k : 1\ldots K, k \notin \mathcal{S}^*$ repeat

Step 2.2.1 Compute $\rho_k = \left|\overline{\mathbf{h}}_k^H \mathbf{W}(i)\right|$

Step 2.2.2 Compute $SINR_k = \frac{\|\mathbf{h}_k\|^2\rho_k^2}{\|\mathbf{h}_k\|^2(1-\rho_k^2)+\frac{M}{P}}$

Step 2.2.3 If $SINR_k > SINR^i_{max}$

$SINR_k \rightarrow SINR^i_{max}$ and $k_i = k$

Step 2.3 $k_i \rightarrow \mathcal{S}^*$

We introduce OLBF as an alternative technique to perform linear beamforming that, as we show, can provide good performances. One of the advantages of using OLBF is that the base station can easily compute the SINR values for each user during the scheduling process.

IV. Orthogonal Linear Beamforming

In order to avoid the prohibitively high complexity of exhaustive search, we use a low-complexity user selection approach. The base station schedules M among K users for downlink transmission with the purpose of maximizing the sum rate and under the constraint of orthogonal beamforming.

The user selection criterion consists of scheduling the users with the largest SINR values. In order to express each user's SINR, we use the following simplified expression [11]

$$SINR_k = \frac{\|\mathbf{h}_k\|^2 \rho_k^2}{\|\mathbf{h}_k\|^2 (1-\rho_k^2) + M/P} \tag{7}$$

where ρ_k is the alignment between the k-th user instantaneous normalized channel vector $\overline{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ (channel direction) and the corresponding beamforming vector $\mathbf{w}_k$, defined as $\rho_k = \left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|$. Note that a similar metric is also reported in [9], [10]. Since the linear beamformers used for transmission are orthogonal, the beamforming vectors $\mathbf{w}_j$, $\forall\ j \neq k$ span the null space of $\mathbf{w}_k$. Hence, as shown in [13], [14], the multiuser interference can be simplified as $I = \frac{P}{M}\|\mathbf{h}_k\|^2(1-\rho_k^2)$.

In what follows, we propose two algorithms with different computational complexity. As we later show through simulations, *Algorithm B* shows better performance than *Algorithm A* at the expense of higher processing complexity.

A. Algorithm A

An outline of *Algorithm A* is provided in Table I. The proposed suboptimal transmission scheme has reduced complexity, in the sense that it has

- Simple beamforming strategy: given K users, only 1 possible transmission set is taken in consideration.
- Reduced user search space: greedy algorithm with linear complexity of order O(K) is used. Hence, combinatorial search over the entire index set $\mathcal{Q}$ is avoided.

Given M possible users to be scheduled out of K active users, the user scheduled on the first beam, denoted as k_1, is the one that exhibits the largest channel norm, i.e.,

$$k_1 = \arg\max_{k\in\mathcal{K}} \|\mathbf{h}_k\| \tag{8}$$

Once the best user is identified, its beamforming vector is given as

$$\mathbf{w}_{k_1} = \overline{\mathbf{h}}_{k_1} \tag{9}$$

so that transmit matched filtering is performed for the first user. Hence, this user observes an alignment of $\rho_{k_1} = 1$, and thus $SINR_{k_1} = \frac{P}{M}\|\mathbf{h}_{k_1}\|^2$, independently of the users scheduled on the remaining beams. Note also that the first user is selected as the best over K users, therefore exploiting all multiuser diversity gain. The remaining $M-1$ beamforming vectors are found by the following procedure. Since orthogonal transmission is to be performed, the already selected vector $\mathbf{w}_{k_1}$ corresponds to a basis vector of the orthonormal basis $\mathbf{W} \in \mathbb{C}^M$ to be used for transmission. Hence, the complete set of beamforming vectors can be found by applying *Gram-Schmidt* orthogonalization method [15]. Let $\mathbf{W}(i)$ be the i-th column of the beamforming matrix $\mathbf{W}$ and define $\mathbf{W}(1) = \mathbf{w}_{k_1}$. Once the beamforming vectors are determined, the user scheduled on the i-th beam, $i = 2\ldots M$, corresponds to the user that maximizes the SINR expression

$$k_i = \arg\max_{k\in\mathcal{K}-\{k_1,\ldots,k_{i-1}\}} SINR_k \quad s.t. \mathbf{w}_k = \mathbf{W}(i) \tag{10}$$

Note that selection of the k_i user does not affect the expression of $SINR_{k_j}$, for $j \neq i, j = 2\ldots M$, since the beamforming vectors in OLBF are already determined in the first step, and therefore the SINR expression of k-th user is only a function of $\mathbf{h}_k$ and ρ_k as equation (7) shows. In the selection process, exact SINR values are computed for the given beamforming vectors. This ensures that, even though the transmit directions are fixed (after selecting k_1), the exact knowledge of interference will allow to capture a large portion of the sum rate even when the number of users is reduced.

The expected sum rate of the scheduled user set $\mathcal{S}^* = \{k_1, k_2, \ldots, k_M\}$ is given by

$$R(\mathcal{S}^*) = \sum_{i=1}^{M} \log_2 [1 + SINR_{k_i}] \tag{11}$$

TABLE II
ALGORITHM B

Initialize Set $\mathcal{S}^* = \emptyset$ and $R(S^*) = 0$
For $k : 1 \ldots K$ repeat
Step 0 Set $k_1 = k$
$\mathbf{w}_{k_1} = \bar{\mathbf{h}}_k$
$\mathcal{S} = \{k_1\}$
Step 1 Gram-Schmidt orthogonalization
Compute orthonormal basis $\mathbf{W}$ from $\mathbf{w}_{k_1}$
Step 2 Loop
For $i : 2 \ldots M$ repeat
Step 2.1 Set $SINR^i_{max} = 0$
Step 2.2 Loop
For $k : 1 \ldots K, k \notin \mathcal{S}$ repeat
Step 2.2.1 Compute $\rho_k = \left|\bar{\mathbf{h}}_k^H \mathbf{W}(i)\right|$
Step 2.2.2 Compute $SINR_k = \frac{\|\mathbf{h}_k\|^2 \rho_k^2}{\|\mathbf{h}_k\|^2 (1-\rho_k^2) + \frac{M}{P}}$
Step 2.2.3 If $SINR_k > SINR^i_{max}$
$SINR_k \rightarrow SINR^i_{max}$ and $k_i = k$
Step 2.3 $k_i \rightarrow \mathcal{S}$
Step 3 $R(\mathcal{S}) = \sum_{j \in \mathcal{S}} \log\left[1 + SINR_j\right]$
Step 4 If $R(\mathcal{S}) > R(\mathcal{S}^*)$, $R(\mathcal{S}) \rightarrow R(\mathcal{S}^*)$ and $\mathcal{S} \rightarrow \mathcal{S}^*$

B. Algorithm B

An outline of *Algorithm B* is provided in Table II. This algorithm exhibits increased complexity compared to *Algorithm A*, which can be summarized as follows

- Simple beamforming strategy: given K users, only K possible transmission sets are taken in consideration
- Reduced user search space: user selection algorithm of complexity of order O(K^2) (hence, combinatorial search over $\mathcal{Q}$ is avoided).

Hence, *Algorithm B* also counts on a finite set of possible beamformers (thus limiting the complexity) but does not perform a greedy selection procedure. This algorithm is equivalent to *Algorithm A*, but instead of selecting the first user as in equation (8), all users are considered as possible candidates, i.e., $k_1 = k, k = 1, \ldots, K$. In other words, the procedure described in *Algorithm A* is performed K times. Each time, a new set of beamforming vectors is computed, given that $\mathbf{w}_{k_1} = \bar{\mathbf{h}}_{k_1}$ changes from user to user, and its corresponding rate is computed by using eq. (11). Let $R^B_m(\mathcal{S})$ denote the sum rate for a user set $\mathcal{S}$ at m-th iteration of the algorithm, where $m = 1, \ldots, K$. The set of beamforming vectors $\mathbf{W}(\mathcal{S}^*)$ and scheduled users $\mathcal{S}^*$ are those having the maximum sum rate $R^B_m(\mathcal{S})$ among the K iterations of the algorithm. Thus the resulting sum rate of *Algorithm B*, denoted as R_B, is given by

$$R_B\left(\mathcal{S}^*\right) = \max_{m=1,\ldots,K} R^B_m(\mathcal{S}) \quad (12)$$

At this point, we should note that in this paper we focus on the region of low to moderate number of users, which is of particular interest in real scenarios. A sum rate analysis of our proposed algorithms for $K \rightarrow \infty$, which is omitted due to space limitations, can show that both our schemes achieve asymptotically the optimum sum rate scaling of $M \log\log K$. This is also evident as random opportunistic beamforming [16], which has been shown to achieve the DPC capacity scaling [5], is a pessimistic lower bound on the performance of our scheme.

V. PERFORMANCE ANALYSIS

In this section, we study the sum rate performance of OLBF (algorithm A) at low-power regime, and its capacity growth is compared with that of zero-forcing beamforming with equal power allocation. For simplicity, we assume that $K = M$, thus the results are independent of the user selection strategy. The analytical tool used for the characterization of capacity at asymptotically low SNR was proposed by Verdú [17]. At low SNR, the capacity $C(SNR)$ (in nats/dimension) can be approximated by the second-order Taylor series expansion:

$$C(SNR) = \dot{C}(0)SNR + \frac{\ddot{C}(0)}{2}SNR^2 + o(SNR^2) \quad (13)$$

with $\dot{C}(0)$ and $\ddot{C}(0)$, the first and second derivative, respectively, of the function $C(SNR)$ at SNR = 0.

The sum capacity of zero-forcing beamforming with equal power allocation $C_{ZFBF}(SNR)$ is given by

$$C_{ZFBF}(SNR) = \mathbb{E}\left\{\sum_{i \in \mathcal{S}^*} \log\left(1 + \frac{SNR}{M}\left|\gamma_i\right|^2\right)\right\} \quad (14)$$

where $\gamma_i = \mathbf{h}_i^H \mathbf{w}_i$ is the effective channel of i^{th} user, and $\mathbf{w}_i$ the zero-forcing beamformer, corresponding to the i^{th} column of matrix $\mathbf{W}(\mathcal{S}) = \mathbf{H}(\mathcal{S})^{\dagger}$. Note that $\left|\gamma_i\right|^2$ is a chi-square random variable with two degrees of freedom for all i (denoted $\chi^2_{(2)}$).

The derivatives of the sum capacity in nats are equal to

$$\dot{C}_{ZFBF}(0) = \mathbb{E}\left\{\left|\gamma_i\right|^2\right\} = 1 \quad (15)$$

and

$$\ddot{C}_{ZFBF}(0) = -\frac{\mathbb{E}\left\{\left|\gamma_i\right|^4\right\}}{M} \quad (16)$$

Similarly, when orthogonal linear beamforming is used, the sum capacity $C_{OLBF}(SNR)$ is given by

$$C_{OLBF}(SNR) = \mathbb{E}\left\{\log\left(1 + \frac{SNR}{M}\left\|\mathbf{h}_{k_1}\right\|^2\right)\right\}$$
$$+\mathbb{E}\left\{\sum_{i \in \mathcal{S}-\{k_1\}} \log\left(1 + SINR_{k_i}\right)\right\} \quad (17)$$

with $\left\|\mathbf{h}_{k_i}\right\|^2 \sim \chi^2_{(2M)}$.

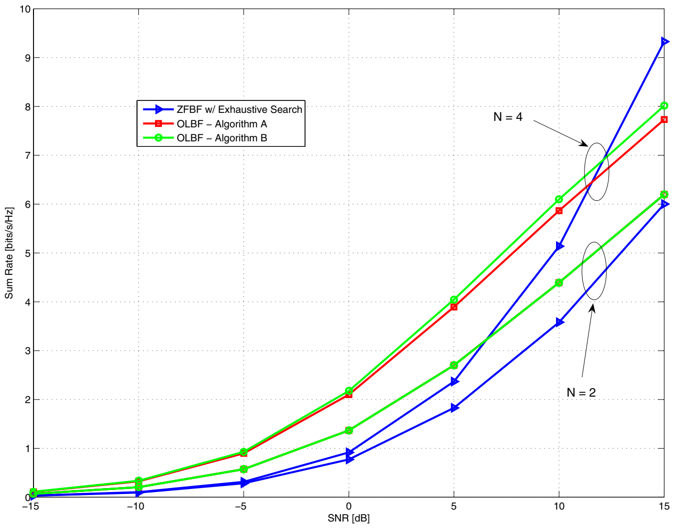

Fig. 1. Sum rate as a function of the SNR for $M = 2, 4$ transmit antennas and $K = M$ users.

The derivatives of the sum capacity in nats are equal to

$$\dot{C}_{OLBF}(0) = 2 - 1/M \tag{18}$$

and

$$\ddot{C}_{OLBF}(0) = -\frac{\mathbb{E}\left\{\|\mathbf{h}_k\|^4\right\}}{M} + \frac{M-1}{M^2}\mathbb{E}\left\{\|\mathbf{h}_k\|^4\left(1-\rho_k^2\right)^2\right\} \tag{19}$$

Note that for isotropically distributed channels, $\|\mathbf{h}_k\|$ and ρ_k are independent random variables. As a first-order approximation, as $SNR \to 0$, the capacity grows linearly with SNR, i.e. $C_{ZFBF} \approx SNR$ and $C_{OLBF} \approx (2 - 1/M)SNR$, meaning that the capacity scaling (for fixed M) at low SNR satisfies

$$\lim_{SNR \to 0} \frac{C_{ZFBF}(SNR)}{SNR} = 1 \tag{20}$$

and

$$\lim_{SNR \to 0} \frac{C_{OLBF}(SNR)}{SNR} = 2 - 1/M \tag{21}$$

By taking the capacity scaling ratio $\Lambda = \frac{C_{OLBF}(SNR)}{C_{ZFBF}(SNR)}$, we conclude that a system employing OLBF provides at low SNR a gain of $10\log_{10}(\frac{2M-1}{M})$ dB compared to a system based on zero-forcing beamforming, or equivalently, a factor of $\Lambda = (2M-1)/M$ in rate (nats/s/Hz) for the same power.

Figure 1 shows a sum rate comparison between OLBF and ZFBF versus the average SNR for $M = 2$ and $M = 4$ transmit antennas. We can observe the consistency of the simulation results with the above analysis, since the sum capacity gap between OLBF and ZFBF increases with the number of transmit antennas (by a factor of $2 - 1/M$) at low SNR.

VI. Simulation Results

In this section, we evaluate the performance of the proposed algorithms through simulations, for $M = 2$ transmit antennas, and we compare their sum rate with three alternative transmission techniques for the MIMO downlink: zero-forcing beamforming (ZFBF) with exhaustive user search, transmit matched filtering (TxMF) with exhaustive user search, and zero-forcing with greedy user selection (ZFBF-GUS) [8]. We consider in the simulated ZFBF approaches simple power normalization as described in equation (5) instead of optimal power allocation techniques. Transmit matched filtering consists of using as beamforming vectors the channel realizations of scheduled users, normalizing the transmitted power of the resulting beamforming matrix.

Figure 2 shows a performance comparison between ZFBF with exhaustive search, TxMF with exhaustive search and the proposed OLBF suboptimal techniques in the low SNR regime. In this scenario, the proposed algorithm outperforms matched filtering for the simulated range of active users. We can observe that the proposed scheme with orthogonal beamforming shows even better performance than zero-forcing where the user selection is performed via exhaustive search. However, as the number of active users in the cell increases, the gap between ZFBF and OLBF becomes smaller.

In Figure 3, instead of comparing with optimal scheduling techniques as done in Figure 2, we focus on another suboptimal technique, ZFBF-GUS [8]. It is a fair comparison, since the proposed OLBF approaches and ZFBF-GUS both rely on suboptimal scheduling algorithms. However, our proposed scheduling algorithm is computationally much less complex than [8], which involves computation of matrix inversions. We observe that knowledge of the interference exploited during the user selection process is particularly beneficial as the total number of users K and the average SNR decrease. In this region, ZFBF-GUS exhibits a performance degradation. Knowledge of the interference between users that do not have good spatial separability helps to improve the task of the scheduler. Hence, the proposed scheme with joint beamforming and scheduling can effectively select users for transmission in sparse networks where moderate number of users is present, providing good average rates through a low-complexity design. However, as the average SNR increases, ZFBF-GUS can provide higher rates except for the case when the number of transmit antennas equals the number of users, $M = K$.

VII. Conclusion

In this paper, we have shown how Orthogonal Linear Beamforming (OLBF) can be efficiently combined with a low-complexity user selection algorithm to achieve a large portion of the multiuser capacity. The use of orthogonal transmission enables the transmitter to calculate exact SINR values

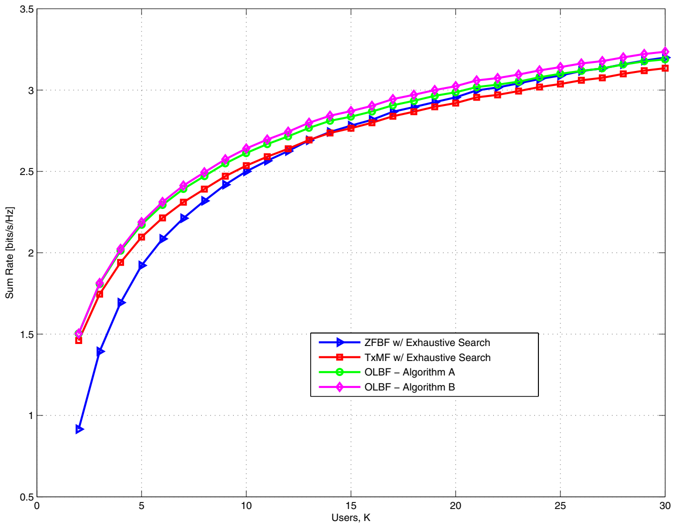

Fig. 2. Sum rate as a function of the number of users for $M = 2$ transmit antennas, and average $SNR = 0$ dB.

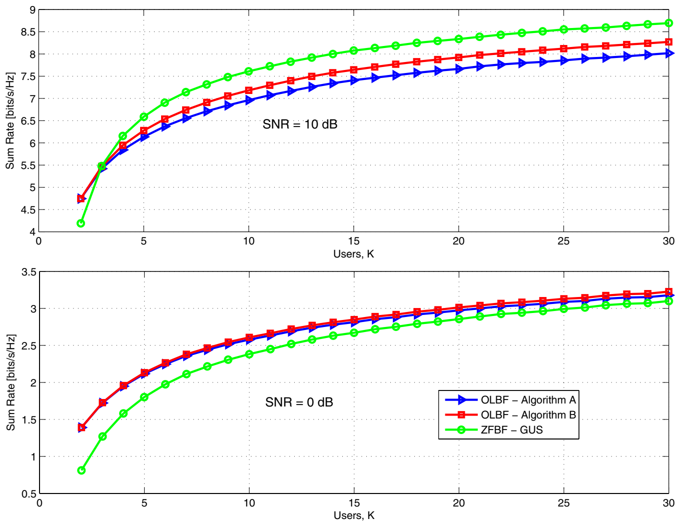

Fig. 3. Sum rate comparison of suboptimal beamforming approaches as a function of the number of users for $M = 2$ transmit antennas, for a) average $SNR = 0$ dB (below) and b) average $SNR = 10$ dB (above).

during the user selection process. The proposed suboptimal algorithms provide performance gains with respect to optimal ZFBF and TxMF, as both the number of users in the cell and the average SNR decrease. Sum rate comparison with a ZFBF technique based on greedy user selection further shows the benefits of OLBF in cells with low to moderate number of active users in low SNR environments, highlighting the importance of multiuser interference knowledge for user scheduling.

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