Blind equalization and source separation with MSK inputs

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ABSTRACT

Channel equalization and identification appear as key issues in improving wireless communications. It is known that the linearization of the GMSK modulation (used in the European standard GSM) leads to a continuous phase OQPSK which can be considered as a Minimum Shift Keying (MSK) modulation. Thus methods of equalization and identification when the channel is input by MSK modulated signals is worth to look at. Most algorithms consider MSK signals as two independent white binary PAM staggered signals; this is not the case in our approach. Here, MSK signals are seen, after sampling at baud rate, as colored complex discrete signals. Even if this view of MSK modulation is quite simple, it has never been utilized with the purpose of blind equalization. The particular statistics of such signals are studied, yielding an original closed-form analytical solution to blind equalization, both in the mono-variate case (SISO or SIMO) and in the source separation problem (MIMO). Simulations show a good behavior of the algorithms in terms of Bit Error Rate (BER) as a function of SNR, both in the case of blind equalization and source separation.

Keywords: MSK modulation, Source Separation, Blind Channel Equalization, GSM standard, Multiuser

1. INTRODUCTION

The GSM European standard is based on Gaussian Minimum Shift Keying (GMSK) modulation[10,9] which efficiently reduces the interference between two contiguous frequency bands. GMSK is obtained by a Gaussian filtering of the frequency waveform of a MSK modulation. Furthermore, it has been shown by Laurent[1] that GMSK could be linearly approximated, as far as phase is concerned, by MSK modulation. Therefore, the study of MSK sources turns out to be of prior importance in improving GSM communications.

MSK sources are often treated via their trellis representation which leads to Viterbi-like algorithms (necessarily sub-optimal in the multiuser case) based on known learning sequences; this also requires the channel to be identified in a first stage. Another approach is to split them into statistically independent BPSK sources. The advantage of such a representation is that it allows the use of standard algorithms. But the inconvenience is that the three features of the discrete-time complex envelope of the MSK modulation are not simultaneously taken into account, namely: its constant modulus, its 4-state discrete distribution, and its memory.

In this paper, we handle MSK sources in a new manner. They are considered as colored signals with a particular structure wich allows analytical treatment for blind equalization and source separation. In addition, it is worth noting that, because GSM uses a constant envelope modulation, the received sequence does not need to be synchronized prior to blind equalization or identification.

The paper is organized as follows. In section 2 the MSK and the GMSK modulations are described, in section 3 the statistical properties of MSK signals are presented. Section 4 is dedicated to the single input (source) case, closed form solutions to blind equalization are described and illustrated by computer experiments. The source separation problem (MIMO case) is addressed eventually in section 5. Computer results are reported in the case of MSK sources.

Notation in the following, a roman letter like x denotes any scalar, a bold face letter like $\mathbf{x}$ is a vector, $(^T)$ denotes matrix transposition, $(^*)$ complex conjugation and $(^\dagger)$ Hermitian transposition.

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2. MSK DEFINITIONS

2.1. Definitions of the modulation

It is well known[8] that Minimum Shift Keying (**MSK**) is a Continuous Phase Frequency Shift Keying (**CP-FSK**), which associates each binary element with frequencies equal to $f_c \pm 1/4T_s$ where f_c is the carrier frequency and T_s is the symbol duration :

$$f(t) = f_c + \frac{1}{4T_s}\sum_k \varepsilon(k)g(t - kT_s)$$

The frequency offset used here, $\pm 1/4T_s$, leads to the minimum frequency separation that can ensure orthogonality, hence the name MSK. Another characteristic of MSK is its narrow spectrum, granted by its continuous phase and its constant modulus envelope.

MSK can also be defined as a particular 4-QAM modulation. Indeed, such a modulation writes :

$$x(t) = \sqrt{2}\sum_k g_c(t - kT)s_c(u_k)cos(2\pi f_c t) - g_s(t - kT)s_s(u_k)sin(2\pi f_c t)$$

where g_c and g_s are waveforms and s_c and s_s are functions that associate an amplitude to each binary elements to be sent. If now the following waveforms are used :

$$\begin{aligned} g_c(t) &= \sqrt{\frac{2}{T}}\,cos\left(\frac{\pi t}{T}\right), \quad t \in [T/2,\ 3T/2] \\ g_s(t) &= \sqrt{\frac{2}{T}}\,sin\left(\frac{\pi t}{T}\right), \quad t \in [0,\ T] \end{aligned}$$

with $T = 2T_s$, then a MSK signal is obtained.

Obviously, each definition corresponds to a different trellis but we will see in the next paragraph that both baseband sampled signals can merge into the same model.

2.2. General definition of the baseband signal

It is quite obvious, from the CP-FSK definition of MSK signals, that their phase increase or decrease of $\pi/2$ every symbol duration so that the sampled baseband MSK signal $\{x(n)\}$ verifies the following recursion :

$$x(n+1) = x(n)\ exp\left(i\frac{\pi}{2}\varepsilon(n)\right)$$

where $\varepsilon(n)$ equals to ± 1 and $x(0)$ represents the initial phase of the signal; so $x(n)$ can also be written :

$$x_n = x_0\ exp\left(i\frac{\pi}{2}\sum_{i=0}^{n-1}\varepsilon_i\right) \tag{1}$$

Thus, even though MSK is a two-dimensionnal modulation, it is completly described by the single binary sequence $\{\varepsilon(n)\}$. However, the QAM-MSK definition shows that one can handle MSK signals as two binary sequences at period $2T_s$, one being delayed by T_s. This is the current manner of treating MSK sources.

2.3. Definition of GMSK

Gaussian Minimum Shift Keying (**GMSK**) is derived from MSK and further reduces the levels of the spectrum sidelobes by passing the MSK modulating data waveform through a premodulation Gaussian pulse-shaping filter. Moreover, the Gaussian filtering improves the power efficiency of the modulated signal by introducing ISI in the transmitted signal. It has been shown in Ishizuka[11] that the degradation caused remained not severe unless the $3dB$-bandwidth-bit-duration product (BT_s) of the filter is less than 0.3 if the Gaussian filter is defined by :

$$h_G(t) = \frac{1}{\sigma\sqrt{2\pi}}exp\left(-\frac{t^2}{2\sigma^2}\right)$$

with $\sigma = \sqrt{ln(2)}/2\pi B$. Figure 1 presents the constellation of MSK and GMSK signals and shows that they are quite similar. Therefore, the performances obtained for MSK signals will be sligthly better than in the case of GMSK as the eye of the former will be open earlier.

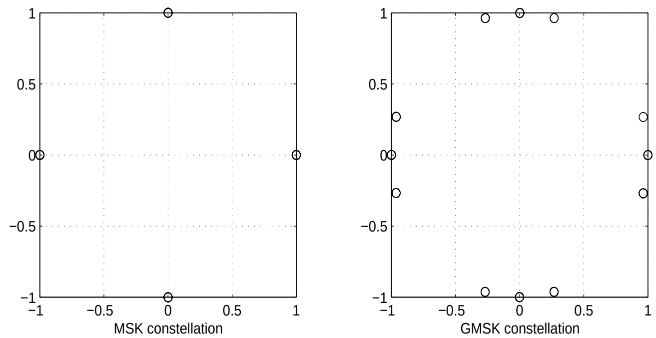

Figure 1. MSK and GMSK constellations

3. MSK SIGNAL STATISTICS

We now give the statistics of a baseband sampled MSK signal. The computation of its statistics was done with the definition (1) and assuming that the process $\{\varepsilon(n)\}$ and $x(0)$ are two independent random processes which are respectively i.i.d $\{+1, -1\}$ and uniformly distributed in $[-\pi, \pi[$. With these assumptions, it is quite easy to show that the first and third statistics are equal to zero.

3.1. Second order statistics

Since the mean of a MSK signal is equal to zero, the second order moments and cumulants are equal. Hence, we have to compute :

$$E\left[x(n)x(n-\ell)^*\right] = E\left[|x(0)|^2 \prod_{k=1}^{n} \prod_{u=1}^{n-l} exp\left(i\frac{\pi}{2}\varepsilon(k)\right) \; exp\left(-i\frac{\pi}{2}\varepsilon(u)\right)\right]$$

The second-order circular and non-circular moments are then :

$$\begin{aligned} E\left[x(n)x(n-\ell)^*\right] &= \begin{cases} 1 & \text{if } \ell = 0 \\ 0 & \text{otherwise} \end{cases} \\ E\left[x(n)x(n-\ell)\right] &= 0 \end{aligned}$$

But the computation of these expectations conditionnaly to the initial phase state $x(0)$ is of greater interest. Indeed, even though the circular conditional expectation doesn't change, the non-circular expection turns out to be very attractive :

$$E\left[x(n)x(n-\ell)|x(0)\right] = \begin{cases} (-1)^n x(0)^2 & \text{if } \ell = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The conditional non-circular expectation becomes cyclostationary with period $2T_s$. In fact, if the expectation is removed, equation (2) remains true. In other words, one obtains the alternate series $(-1)^n x(0)^2$. This will be the basis of the coming algorithms.

3.2. Fourth order statistics

3.2.1. Moments

The fourth order moments are given by :

$$E\left[\prod_{j=1}^{4} x(n_j)^{(*)_j}\right] = E\left[\prod_{j=1}^{4} x(0)^{(*)_j} \prod_{u=1}^{U_j} exp\left(\pm_j i\frac{\pi}{2}\varepsilon(u)\right)\right]$$

where $(*)_j$ stands for an optional conjugation and $\pm_j$ is equal to one if $x(n_j)$ is not conjugated and to -1 otherwise.

Since $E\left[x_0^4\right] = E\left[x_0^* x_0^3\right] = 0$ and $E\left[|x_0|^4\right] = 1$, it is quite obvious that only the moments where exactly two variables are conjugated may be non zero. Then the problem becomes symmetric and with no loss of generality one can calculate the moments $E\left[x(n_1)x(n_2)x^*(n_3)x^*(n_4)\right]$. Then three different cases appear :

Case 1 *:* $n_1 = n_2 = N_1$ and $n_3 = n_4 = N_2$ and $N_1 \leq N_2$

$$E\left[x(N_1)^2 x(N_2)^{*2}\right] = \prod_{N_1+1}^{N_2} exp(-i\pi) = (-1)^{N_2-N_1}$$

Case 2 *:* $n_1 = n_3 = N_1$ and $n_2 = n_4 = N_2$ and $N_1 \leq N_2$

$$E\left[|x(N_1)|^2 |x(N_2)|^2\right] = 1$$

Case 3 *:* $n_1 = n_2 = n_3 = n_4 = N$

$$E\left[|x(N)|^4\right] = 1$$

The other moments are deduced by conjugation.

3.2.2. Cumulants

We now deduce the fourth-order cumulants from the above moments. Since $E\left[x(n)\right] = 0$ and with the above results one can prove that the cumulants which may not be necessary equal to zero are :

$$\begin{aligned} C\left[x(n_1)x(n_2)x(n_3)^*x(n_4)^*\right] &= E\left[x(n_1)x(n_2)x(n_3)^*x(n_4)^*\right] \\ &\quad - E\left[x(n_3)^*x(n_1)\right]E\left[x(n_4)^*x(n_2)\right] - E\left[x(n_4)^*x(n_1)\right]E\left[x(n_3)^*x(n_2)\right] \end{aligned}$$

We still observe the following three different cases :

Case 1 *:* $n_1 = n_2 = N_1$ and $n_3 = n_4 = N_2$ and $N_1 \leq N_2$

$$C\left[x(N_1)^2 x(N_2)^{*2}\right] = (-1)^{N_2-N_1}$$

Case 2 *:* $n_1 = n_3 = N_1$ and $n_2 = n_4 = N_2$ and $N_1 \leq N_2$

$$C\left[|x(N_1)|^2 |x(N_2)|^2\right] = 0$$

Case 3 *:* $n_1 = n_2 = n_3 = n_4 = N$

$$C\left[|x_N|^4\right] = -1$$

Thus, the first case proves that MSK signals are colored at fourth order.

4. SINGLE INPUT CASE

4.1. SISO equalization

Assume that the output $\{y(n)\}$ of a FIR filter $\{h(m)\}$ excited by a MSK source signal $\{x(n)\}$ is observed, in presence of an additive Gaussian noise $\{w(n)\}$:

$$y(n) = \sum_{m=0}^{M-1} h(m)^* x(n-m) + w(n)$$

The equalization problem consists of finding a FIR filter $\{f(l)\}$ that transforms $\{y(n)\}$ into an almost MSK signal, $\{z(n)\}$.

Equation (1) shows that the square of a MSK signal alternates between $+1$ and -1 if its initial phase is equal to 0 or π. Yet, the initial phase can be pulled in the channel part, without restricting the generality. In other words, the equalization problem is to find $\{f(l)\}$ such that the square of $z = f * y$ alternates between $+1$ and -1. This constraint on $z(n)$ can be written in the following compact form[2] :

$$(z(n))^2 = (-1)^n = \mathbf{f}^\dagger \mathbf{y}(n;L)\,\mathbf{y}(n;L)^T \mathbf{f}^* \tag{3}$$

where $\mathbf{y}(n;L) \stackrel{\text{def}}{=} [y(n), y(n-1), \ldots y(n-L+1)]^T$ and $\mathbf{f} \stackrel{\text{def}}{=} [f(0), f(1), \ldots f(L-1)]^T$. Now equation (3) can be written even more compactly for every n as :

$$\mathbf{f}^{\otimes\dagger}\, \mathbf{y}(n;L)^{\otimes} = (-1)^n$$

where $\mathbf{f}^{\otimes}$ is a $L(L-1)/2 \times 1$ vector containing the squares f_i^2 and the cross terms $f_i f_j$ weigthed by $\sqrt{2}$, without repetition, and in a given order. The terms are arranged in the same manner in a $L(L-1)/2 \times 1$ vector $\mathbf{y}^{\otimes}_{n:L}$.

When n ranges from 1 to N, the N equations obtained can be merged into a single system :

$$\mathbf{f}^{\otimes\dagger}\left[\mathbf{y}(1;L)^{\otimes T}, \cdots, \mathbf{y}(N;L)^{\otimes T}\right] \stackrel{\text{def}}{=} \mathbf{f}^{\otimes\dagger} Y(1:N;L) = \mathbf{d}$$

where $Y(1:N;L)$ is a $L(L+1)/2 \times N$ data matrix, and $\mathbf{d}$ is the $N \times 1$ vector whose entries are $\left[(-1), (-1)^2, \cdots, (-1)^N\right]$. The solution of the above system, in the Least Square (LS) sense is given by :

$$\mathbf{f}^{\otimes}_{LS} = \left[Y(1:N;L)\,Y(1:N;L)^\dagger\right]^{-1} Y(1:N;L)\,\mathbf{d}^\dagger \tag{4}$$

Now consider the operation **unvecs** that builds, from a vector $\mathbf{f}^{\otimes}$ of size $L(L+1)/2$, the symmetric complex matrix unvecs($\mathbf{f}^{\otimes}$) with diagonal entries $f(i)^2$, and as (i,j)-entries the cross products $f(i)f(j)$. Then, in the noiseless case, unvecs($\mathbf{f}^{\otimes}$) is the a rank-one matrix $\mathbf{f}\,\mathbf{f}^T$. In presence of noise, the rank of unvecs($\mathbf{f}^{\otimes}_{LS}$) is not 1 anymore, but its dominant eigenvector gives a good estimate of the corresponding vector $\mathbf{f}$.

This gives the core of the SISO blind equalization algorithm described in Comon's patent.[2] Its main advantage is that the solution is not iterative, and can thus be computed theoretically with data samples as short as $N = L$. In practice, as will be shown in figure 3, excellent performance is already obtained for data length $N = 157$, that is, a single GSM burst!

4.2. SIMO equalization

Suppose now that the outputs $\{y_k(n)\}$ of K sensors, excited by a single MSK source $\{x(n)\}$, are observed in presence of additive Gaussian noises $\{w_k(n)\}$:

$$\mathbf{y}(n) = \sum_{m=0}^{M-1} \mathbf{h}(m)^\dagger \mathbf{x}(n-m) + \mathbf{w}(n)$$

where $\mathbf{y}(n) = [y_1(n), \cdots, y_K(n)]^T$, $\mathbf{w} = [w_1(n), \cdots, w_K(n)]^T$, $\mathbf{h}(m) = [h_1(m), \cdots, h_K(m)]^T$ and M denotes the largest channel length. Therefore, we now have to find K filters $\{f_k(\ell)\}$ such that

$$z(n) = \sum_{k,\ell} f_k(\ell)^*\, y_k(n-\ell)$$

verify the MSK constraint $z(n)^2 = (-1)^n$.

The corresponding filters can be computed in the same manner as in the SISO case. Denote :

$$\begin{aligned} \mathbf{f} &= \left[\mathbf{f}_1^T, \cdots, \mathbf{f}_K^T\right]^T \\ \mathbf{y}(n;L) &= \left[y_1(n;L)^T, \cdots, y_K^T(n;L)\right]^T \end{aligned}$$

with $\mathbf{f}_k = [f_k(0), \cdots, f_k(L-1)]^T$, the solution is then given by the dominant eigenvector of unvecs($\mathbf{f}^{\otimes}_{LS}$).

4.3. Computer results in the SISO case

Computer experiments have been run using the channel impulse response shown in figure 2, which looks like the GSM urban channel. Actually, it has been generated by the following AR2 filter :

$$H(z-1) = \frac{A}{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}$$

with $A = \exp(i\pi/7)$, $z_0 = 0.7\,\exp(i\pi/3)$ and $z_1 = z_0^* \exp(i\pi/9)$. This choice has been made in order to make it feasible to compute the exact channel inverse, for the purpose of comparisons. But our algorithm does not require the channel to be AR.

The analytical algorithm has been tested over 10000 runs and for a source duration equal to 157 symbols (the duration of a GSM burst). The performance shown is the Bit Error Rate (BER) for various SNR's. The solid line in figure 3 corresponds to our blind equalizer, whereas the dashed line corresponds to the mere channel inversion. This shows that the analytical equalization performs as well as (and even slightly better than) the zero forcing equalization. This behavior is excellent in comparison with the number of symbols used (most blind equalizers are iterative, and require dozens of thousands of samples to converge). A comparison with the MMSE equalization should yield a similar performance.

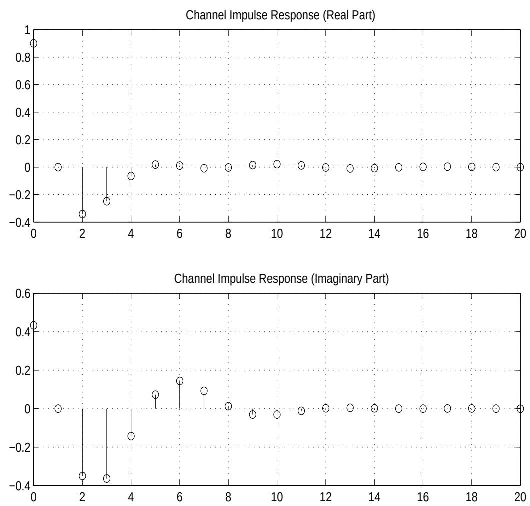

Figure 2. Channel Impulse response

5. MIMO CASE

5.1. Source Separation

5.1.1. Algorithm

Assume that P source signals impinge on an array of $K \geq P$ sensors. Then, in a narrow band context and if the medium is linear, the signals observed on the array can be modeled as a *static* mixture :

$$\mathbf{y} = A\mathbf{x} + \mathbf{w}$$

where $\mathbf{w}$ stands for additive noise and A is a $K \times P$ matrix. This problem has been extensively studied by Cardoso[35] or Comon,[4] among others, but the case of MSK sources, as formulated in this paper, is quite new.

It is well known that P discrete sources with constellation $\mathcal{C}$ can be separated if the vector $\mathbf{x}$ describes all the Possible P-uplets in $\mathcal{C}^{\mathcal{P}}$. But due to the inherent structure of MSK sources described in section 2, these signals do not verify this necessary condition. Thus standard algorithms, such as CMA,[7] cannot recover them and we must resort to other means to perform the separation.

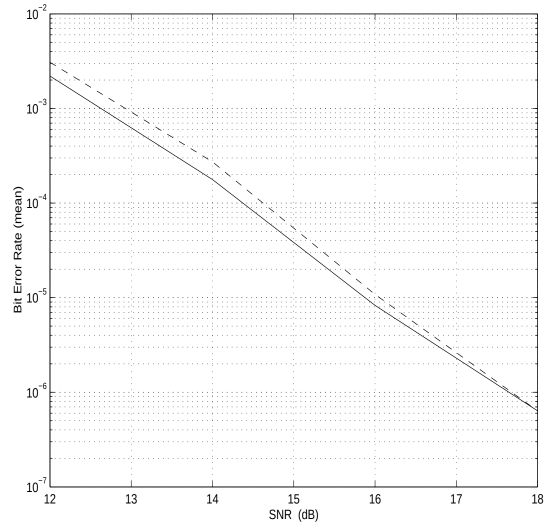

Figure 3. Performance of Blind SISO equalization

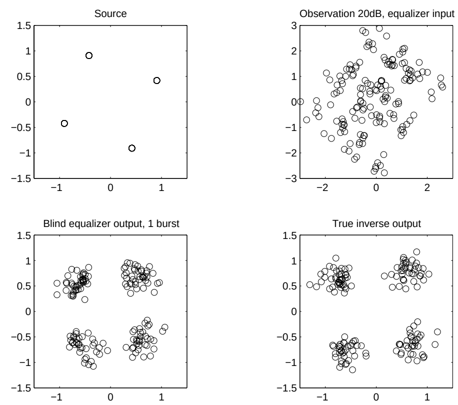

Figure 4. Typical example for SNR=20dB

As we have more sensors than sources and if we suppose the mixture is non singular, the source signals can be computed by looking for the vector $\mathbf{b}_p$ such that $\{z_p(n) = \mathbf{b}_p^\dagger \mathbf{y}(n)\}$ is almost MSK, that is $\mathbf{b}_p^\dagger \mathbf{y}(n)\mathbf{y}(n)^T \mathbf{b}_p^* = (-1)^n$.

The case $\mathbf{b}_p^T \mathbf{y}(n)\mathbf{y}(n)^T \mathbf{b}_p = 1$, for the separation of BPSK sources, has already been studied.[6] Thus the following must be seen as an extension of this previous work to MSK sources.

Consider the economical SVD of $\mathbf{y}$: $\mathbf{y} = U\Sigma V^\dagger$. The constraint on $\{z(n)\}$, to be almost MSK, rewrites $\mathbf{f}_p^T \mathbf{v}_n^* \mathbf{v}_n^\dagger \mathbf{f}_p = (-1)^n$ with $V = [\mathbf{v}_1, \cdots, \mathbf{v}_N]^T$. When n ranges from 1 to N, the N equations obtained can be merged into a single system :

$$V^{\otimes} \mathbf{f}^{\otimes *} = \begin{bmatrix} -1 \\ \vdots \\ (-1)^N \end{bmatrix}$$

with $V^{\otimes} = \left[v_1^{\otimes *}, \cdots, v_N^{\otimes *} \right]^T$.

A similar system was analytically solved by Van der Veen[6] for a right-hand side equal to $[1 \cdots 1]^T$ and gave the estimates of the P impinging BPSK sources. The estimates of the P MSK sources can be computed by transforming our right-hand side vector $\left[-1 \cdots (-1)^N\right]$ into $[1 \cdots 1]$ by a matrix premultiplication, and then run the abovementioned algorithm. For this purpose, define the transformation :

$$Q\, V^{\otimes}\mathbf{f}^{\otimes *} = Q \begin{bmatrix} -1 \\ \vdots \\ (-1)^N \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

where

$$Q = \frac{1}{2} \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \ddots & \vdots \\ 0 & 0 & -1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & (-1)^N \\ 0 & \cdots & 0 & 0 & 2.(-1)^N \end{pmatrix}$$

5.1.2. Computer results

Simulations have been carried out using a uniformly spaced linear array of 4 sensors impinged by 3 MSK sources with angles $\theta_1 = -30°$, $\theta_2 = 10°$and $\theta_3 = 20°$. The element spacing was $\lambda/2$, where λ is the wavelength of the propagating waveforms. We tested our algorithm with datalength $N = 157$ (the duration of a GSM burst), over 3000 trials and for various SNRs. The results in terms of Bit Error Rate for each source signal, the first in solid line, the second in dashdotted line and the third in dashed line, are presented in figure 5.

The performances are quite good if we consider the datalength and the poor SINR. Moreover, one remarks that the closest sources have the worst BER, which exhibits a good behavior as far as we search for the most isolated source.

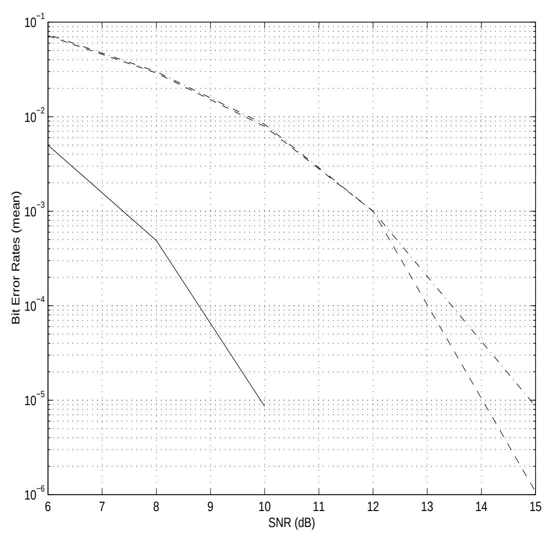

Figure 5. Performance of source separation

5.2. MIMO Equalization

Convolutive mixtures can be treated quite similarly to the case of static mixtures. Indeed, assume that one observes the outputs $\{y_k(n)\}$ of K sensors excited by P statistically independent sources $\{x_p(n)\}$. In presence of Gaussian additive noises we have :

$$y_k(n) = \sum_{p=1}^{P} \sum_{m=0}^{M-1} h_{kp}(m)^* x_p(n-m) + w_k(n)$$

where M denotes the largest channel length.

Now, the goal is to find KP filters f_{pk} such that the

$$z_p(n) = \sum_{k=1}^{K} \sum_{\ell=0}^{L-1} f_{pk}(\ell)^* y_k(n-\ell) \tag{5}$$

are statistically independent MSK signals (L denotes the largest inverse filter length). Equation (5) can be rewritten in a compact form :

$$z_p(n) = \mathbf{f}_p^\dagger \mathbf{y}$$

with $\mathbf{f}_p = \left[\mathbf{f}_{p1}^T, \cdots, \mathbf{f}_{pK}^T\right]^T$ and $\mathbf{y} = \left[y_1(n;L)^T, \cdots, y_K(n;L)^T\right]^T$. Then the MSK constraint $z_p(n)^2 = (-1)^n$ rewrites $\mathbf{f}^\dagger \mathbf{y}\mathbf{y}^T \mathbf{f}^* = (-1)^n$ that has already been studied in section 5.1. Therefore, the algorithm used for static mixtures can also perform MIMO equalization. This has been experimentally verified but a statistical performance analysis has not been carried out yet.

6. CONCLUSION

In this paper, we have presented a new way to handle MSK signals. This new formulation of baseband MSK sources leads to analytical algorithms able to perform blind equalization (SISO or MIMO) as well as blind source separation. All these algorithms are based on the fact that the square of a MSK signal alternates between $+1$ and -1. The performance of blind SISO equalization are excellent and source separation also performs quite well. Moreover, these algorithms also work with GMSK sources that are actually used in the GSM European standard; the results concerning blind channel identification are currently completed and will be presented in an upcoming paper.

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