Orthogonal Space-Time Block Codes for Analog Channel Feedback

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Abstract— [1] **In this paper, we propose to use complex orthogonal space-time block coding (COSTBC) in analog transmission with application to channel feedback. We prove that an equivalent complex orthogonal channel can be generated by COSTBC and then the matched filter bounds on the signal-to-noise ratio via multiple-input multiple-output channels are achieved by maximal ratio combining (MRC). Simulation shows that COSTBC-MRC analog schemes outperforms spatial-multiplexing oriented analog schemes and uncoded random vector quantization schemes with respect to mean-squared errors (MSE).**

I. Introduction

Continuous-amplitude discrete-time transmission, which is called *analog transmission*, is an interesting alternative for the feedback of channel knowledge. In the case of quasi-static multi-input multi-output (MIMO) channels, space-time coding considerations with issues of diversity and spatial multiplexing arise also for analog transmission. Using analog transmission and linear coding for channel feedback to obtain the channel state information at the transmitter (CSIT) has been studied in [1]–[4] and so on. In [1], Tejera and Utschick present that analog linear coding can do better on the distortion decay rate comparing to the other two possible digital approaches via a single-input single-output (SISO) channel subject to the constraint of limited delay. In [2], Marzetta and Hochwald propose to use linear analog modulation for feeding back channel information in frequency-division duplex (FDD) systems. The studied scenario is multi-user single-input multi-output (MU-SIMO), where no transmit diversity possibility exists due to incoordination among users. In [3], we suppose to use spatial-multiplexing space-time block coding (SMSTBC) to do analog channel feedback via a peer-to-peer MIMO channel and compare two channel feedback schemes with zero-forcing (ZF) receivers. Such a full-multiplexing coder without exploitation of transmit diversity is fast but is not optimum with respect to mean-squared error (MSE).

For linear coding in analog transmission, the matched filter bound (MFB), which refers to maximum spatial diversity combining, is different to the one in digital transmission in the light of affected objects. It is well-known that in digital transmission, MFB is an exponentially upper bound on the probability of error [5] and can be achieved by orthogonal space-time coding with maximum likelihood (ML) detection or other means. For linear receivers in analog transmission, MFB is a factor coefficient upper bound on SNR which closely relates to MSE.

If the equivalent channel constructed by an actual channel and a code is complex orthogonal, as which Alamouti space-time code for two transmit antennas [6] can make, the MFB on SNR can be easily achieved by employing maximal ratio combining (MRC) at the receiver. This motivates us to search a family of STC for any transmit antenna number, which can make equivalent channels complex orthogonal by linear processing. Fortunately, we find that such equivalent channels can be generated by the rate $1/2$ complex orthogonal space-time block coding (COSTBC) and in [7], Tarokh *et al.* have given a generalization of COSTBC designs, which can be implemented to MIMO systems for arbitrary transmit antenna number. Consequently, the optimum spatial diversity performance of linear analog transmission is achieved, although it sacrifices spatial multiplexing.

Note that the quantization error of numerical computation and binary storage at transceivers are neglected in this paper.

Remark: in the following text, $\boldsymbol{X}^\dagger$ denotes the conjugate transpose transformation, $\boldsymbol{X}^T$ denotes the transpose transformation, $\boldsymbol{X}^*$ denotes the conjugate transformation, $||\boldsymbol{X}||_F^2$ denotes the square of the Frobenius norm, and $\Re(\boldsymbol{X})$ denotes taking real part of $\boldsymbol{X}$.

II. Channel Model, MFB and COSTBC

In this section, we present the linear channel model and the MFB on SNR for MIMO systems. We prove that an equivalent complex orthogonal channel is generated by COSTBC and the MFB on SNR is achieved by unbiased COSTBC-MRC and COSTBC-LMMSE. Moreover, we provide solutions for special cases of applying COSTBC to channel feedback.

A. Channel model

Assume a frequency-flat block-Rayleigh-fading additive-white-noise MIMO channel with N_t inputs and N_r outputs.

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The channel model is represented by

$$\mathbf{y} = \boldsymbol{H}\mathbf{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{H}$ is the channel matrix of size $N_r \times N_t$, $\boldsymbol{x}$ is the length N_t vector of transmitted symbols subject to the transmit power constraint P_t per symbol-period, $\boldsymbol{n}$ is the length N_r vector of additive white noise whose elements are independently identical distributed $\mathcal{CN}(0, \sigma_n^2)$ r.v.'s, and $\mathbf{y}$ is the length N_r vector of received symbols.

B. MFB

Under the assumption that a L_b-symbol block code is used to transmit L_s continuous-amplitude source symbols, the total energy for the received block at the receiver should be $L_b P_t ||\boldsymbol{H}||_F^2$ under the assumption of maximum spatial diversity combining. In order to combine maximum diversity, each source symbol should be transmitted from all transmit antennas, which means there are at least $N_t - 1$ replicas per source symbol. Thus, the matched filter bound on SNR per source symbol

$$\mathrm{SNR}_{\mathrm{MFB}} = \frac{L_b P_t ||\boldsymbol{H}||_F^2}{N_t L_s \sigma_n^2}. \tag{2}$$

C. About COSTBC

Constructing equivalent complex orthogonal channels:

Theorem 1: Complex orthogonal space-time coding generates an equivalent complex orthogonal channel matrix for MISO systems.

Proof: Suppose for transmitting a length k real row vector $\boldsymbol{s}$, we have a $N_t \times k$ ($N_t \leq k$) rate 1 real orthogonal space-time code (OSTBC) $\mathcal{O}_{N_t,k}$, which can be generated from the general OSTBC design in [7],

$$\mathcal{O}_{N_t,k}\mathcal{O}_{N_t,k}^T = ||\boldsymbol{s}||^2 \boldsymbol{I}_{N_t}. \tag{3}$$

Then, according to [7], if elements of $\boldsymbol{s}$ are complex numbers, a rate $1/2$ complex orthogonal space-time code (COSTBC) $\mathcal{G}_{N_t,2k}$ can be constructed by $\mathcal{O}_{N_t,k}$ and its conjugate $\mathcal{O}_{N_t,k}^*$,

$$\mathcal{G}_{N_t,2k} = \begin{pmatrix} \mathcal{O}_{N_t,k} & \mathcal{O}_{N_t,k}^* \end{pmatrix}. \tag{4}$$

By using $\mathcal{G}_{N_t,2k}$, the channel model via a MISO channel can be represented by

$$\begin{pmatrix} \boldsymbol{y}^{(1)} & \boldsymbol{y}^{(2)} \end{pmatrix} = \boldsymbol{h} \begin{pmatrix} \mathcal{O}_{N_t,k} & \mathcal{O}_{N_t,k}^* \end{pmatrix} + \begin{pmatrix} \boldsymbol{n}^{(1)} & \boldsymbol{n}^{(2)} \end{pmatrix}, \tag{5}$$

where

$$\boldsymbol{y}^{(1)} = \boldsymbol{h}\mathcal{O}_{N_t,k} + \boldsymbol{n}^{(1)}, \tag{6}$$

$$\boldsymbol{y}^{(2)} = \boldsymbol{h}\mathcal{O}_{N_t,k}^* + \boldsymbol{n}^{(2)}. \tag{7}$$

It can also be written as

$$\begin{pmatrix} \boldsymbol{y}^{(1)^T} \\ \boldsymbol{y}^{(2)^\dagger} \end{pmatrix} = \boldsymbol{H}'\boldsymbol{s}^T + \begin{pmatrix} \boldsymbol{n}^{(1)^T} \\ \boldsymbol{n}^{(2)^\dagger} \end{pmatrix} \tag{8}$$

where $\boldsymbol{H}'$ is the equivalent $2k \times k$ channel matrix that can be represented by

$$\boldsymbol{H}' = \begin{pmatrix} \boldsymbol{H}_a \\ \boldsymbol{H}_a^* \end{pmatrix}. \tag{9}$$

Then,

$$\boldsymbol{H}'^\dagger \boldsymbol{H}' = 2\boldsymbol{H}_a^\dagger \boldsymbol{H}_a. \tag{10}$$

From (10), we can straightforwardly deduce if $\boldsymbol{H}_a$ is complex orthogonal, then $\boldsymbol{H}'$ is complex orthogonal as well. Then complex orthogonality of $\boldsymbol{H}_a$ is to be proved first.

From (5), (8) and (9), we have

$$\boldsymbol{s}\boldsymbol{H}_a^T = \boldsymbol{h}\mathcal{O}_{N_t,k}. \tag{11}$$

If $\boldsymbol{s}$ is real, by multiplying each side's conjugate part on both sides of the equation above, in the light of (3), we get the following equation for any $\boldsymbol{h}$ and real $\boldsymbol{s}$,

$$\begin{aligned} \boldsymbol{s}\boldsymbol{H}_a^T\boldsymbol{H}_a^*\boldsymbol{s}^T &= ||\boldsymbol{h}||^2||\boldsymbol{s}||^2 \\ &= \boldsymbol{s}||\boldsymbol{h}||^2\boldsymbol{s}^T. \end{aligned} \tag{12}$$

Thus,

$$\boldsymbol{H}_a^T\boldsymbol{H}_a^* = ||\boldsymbol{h}||^2\mathbf{I}_{N_t}, \tag{13}$$

i.e. $\boldsymbol{H}_a$ is complex orthogonal. Hence, from (10)

$$\boldsymbol{H}'^\dagger\boldsymbol{H}' = 2||\boldsymbol{h}||^2\boldsymbol{I}_{N_t}, \tag{14}$$

i.e. the equivalent channel matrix $\boldsymbol{H}'$ is complex orthogonal. ∎

MRC receiver: A MIMO channel is composed of N_r sub MISO channels. By COSTBC at the transmitter, under the assumption that the receiver knows perfect CSI, matched filters based on equivalent complex orthogonal sub-channel matrices are employed at each receive antenna. By MRC, their outputs are summed up. Supposing for the i-th receive antenna, the received signal vector is $\mathbf{y}_i$ and the equivalent channel matrix is $\mathbf{H}_i'$, after unbiased MRC, we get

$$\widehat{\boldsymbol{s}}^T = \frac{1}{2||\boldsymbol{H}||_F^2}\sum_{i=1}^{N}\boldsymbol{H}_i'^\dagger \begin{pmatrix} \boldsymbol{y}_i^{(1)^T} \\ \boldsymbol{y}_i^{(2)^\dagger} \end{pmatrix}. \tag{15}$$

By (8) and (14), we see that

$$\widehat{\boldsymbol{s}}^T = \boldsymbol{s}^T + \frac{1}{2||\boldsymbol{H}||_F^2}\sum_{i=1}^{N}\boldsymbol{H}_i^\dagger \begin{pmatrix} \boldsymbol{n}_i^{(1)^T} \\ \boldsymbol{n}_i^{(2)^\dagger} \end{pmatrix}. \tag{16}$$

LMMSE receiver: Using a LMMSE receiver based on equivalent MISO subchannels, under the assumption of i.i.d. additive white Gaussian noises and i.i.d. white Gaussian source symbols, we get

$$\widehat{\boldsymbol{s}}^T = \frac{1}{\frac{N_t}{\rho} + 2||\boldsymbol{H}||_F^2}\sum_{i=1}^{N}\boldsymbol{H}_i'^\dagger \begin{pmatrix} \boldsymbol{y}_i^{(1)^T} \\ \boldsymbol{y}_i^{(2)^\dagger} \end{pmatrix} \tag{17}$$

where ρ is the transmit SNR, P_t/σ_n^2.

By analyzing (16) and (17), we see that the receive SNRs of both receivers achieve the matched filter bound on SNR,

$$\mathrm{SNR}_{\mathrm{MFB}} = \frac{2P_t||\mathbf{H}||_F^2}{N_t\sigma_n^2}. \tag{18}$$

when $N_r > \rho(N_t N_r)$: In terms of [7, Theorem 4.1.2], for constructing a rate 1 real OSTBC for k symbols from the generalized orthogonal design, transmit antenna number must be not greater than $\rho(k)$ which is the number of matrices

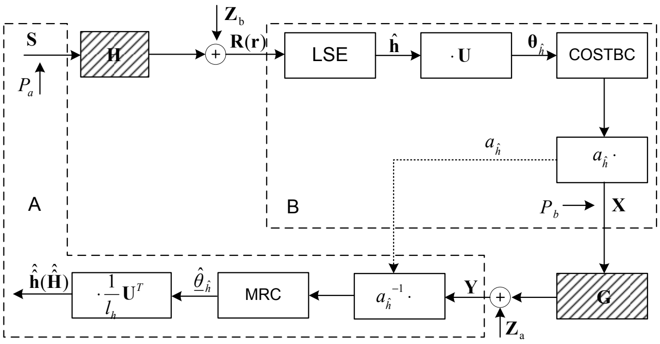

Fig. 1. The diagram of the COSTBC-MRC analog feedback scheme

in a Hurwitz-Radon family of k. In the application to analog channel feedback, supposing there are $N_t N_r$ channel coefficients to be fed back and $N_t > \rho(N_t N_r)$, we can construct a COSTBC by repeating the parameters T times to make $N_t \le \rho(TN_t N_r)$ and then regard the vector after repetition as a $TN_t N_r$-length source vector. Mean estimation (ME) is used to recover transmitted channel coefficients after equalization. Although the feedback interval is prolonged, the total transmit energy is manifolded T times and the SNR of such a scheme thus achieves the MFB on SNR as well.

III. Application to the Analog Channel Feedback

In this section, we detail an analog feedback scheme applied COSTBC-MRC and analyze its performance with respect to MSE.

A. Scheme Description

Fig.1 presents the block diagram of the COSTBC-MRC analog feedback scheme.

We assume the transmitter A has N_a antennas and the receiver B has N_b antennas, all noises are additive and the channel is quasi-static. The training interval is Q_h symbol-periods. The space-orthogonal $N_a \times Q_h$ training matrix $\mathbf{S}$ is transmitted from A to B under the transmit power constraint P_a. Namely,

$$\boldsymbol{S}\boldsymbol{S}^{\dagger} = \frac{Q_h P_a}{N_a}\boldsymbol{I}_{N_a}. \tag{19}$$

The $N_b \times N_a$ transmit channel matrix $\boldsymbol{H}$ can be written in the form of a $N_a N_b$-length row vector

$$\boldsymbol{h} = \begin{pmatrix} \boldsymbol{H}_{(\cdot,1)} & \cdots & \boldsymbol{H}_{(\cdot,N_b)} \end{pmatrix} \tag{20}$$

where $\boldsymbol{H}_{(\cdot,i)}$ is the channel coefficient row vector of the MISO subchannel for the i-th receive antenna at B. Hence, we can write the training-signal-transmission model as

$$\boldsymbol{r} = \boldsymbol{h}\begin{pmatrix} \boldsymbol{S} & & O \\ & \ddots & \\ O & & \boldsymbol{S} \end{pmatrix} + \boldsymbol{z}_b \tag{21}$$

where $\boldsymbol{z}_b$ is the additive noise vector. $\begin{pmatrix} \boldsymbol{S} & & O \\ & \ddots & \\ O & & \boldsymbol{S} \end{pmatrix}$ is a block-diagonal matrix composed of N_b $\boldsymbol{S}$, which is denoted by $\boldsymbol{S}'$ in the following.

The least-square estimator (LSE) based on training is employed at B to estimate $\boldsymbol{H}$ ($\boldsymbol{h}$),

$$\widehat{\boldsymbol{H}} = \frac{N_a}{Q_h P_a}\boldsymbol{R}\boldsymbol{S}^{\dagger}, \tag{22}$$

i.e.,

$$\widehat{\boldsymbol{h}} = \frac{N_a}{Q_h P_a}\boldsymbol{r}\boldsymbol{S}'^{\dagger}. \tag{23}$$

At A, the channel information of feedback channel $\boldsymbol{G}$ is obtained by LSE as well. The channel coefficients of $\boldsymbol{H}$ are fed back from B to A. We suppose that channel coefficients are linear coded by COSTBC, scaled to the block transmit power constraint at B and then fed back to A, *i.e.*, the direction of the source vector is transmitted in a linear way, and the scaling factor $a_{\hat{h}}$ is fed back in a non-linear reliable way to de-scale linear fed-back channel coefficients for reobtaining the magnitude of the source vector.

In the diagram in Fig.1, the matrix $\boldsymbol{U}$ at B is composed of l identity matrices,

$$\boldsymbol{U} = \begin{pmatrix} \boldsymbol{I}_{N_a N_b} & \cdots & \boldsymbol{I}_{N_a N_b} \end{pmatrix}, \tag{24}$$

which in fact does repetition on $\boldsymbol{h}$ corresponding to system requirement. After the repetition, the inter-media row vector $\boldsymbol{\theta}_{\hat{h}}$ is generated,

$$\boldsymbol{\theta}_{\hat{h}} = \hat{\boldsymbol{h}}\boldsymbol{U}. \tag{25}$$

We notice that

$$\boldsymbol{U}\boldsymbol{U}^T = l\boldsymbol{I}_{N_a N_b}. \tag{26}$$

Hence, at A, the final result $\widehat{\widehat{\boldsymbol{h}}}$ can be obtained by

$$\widehat{\widehat{\boldsymbol{h}}} = \frac{1}{l}\hat{\boldsymbol{\theta}}_{\hat{h}}\boldsymbol{U}^T. \tag{27}$$

Remark: For transmitting an analog (discrete-time continuous-amplitude) complex symbol, such as $x = x_c + jx_s$, the corresponding signal waveform $s(t)$ at a transmit antenna can be expressed as

$$s(t) = \Re[(x_c + jx_s)g(t)e^{j2\pi f_c t}] \tag{28}$$

where x_c and x_s are the information-bearing signal amplitudes, $g(t)$ is the signal pulse and f_c is the carrier frequency.

B. Estimation Error

We assume the transmitter perfectly knows the scaling factor by nonlinear feedback and the imperfectness of estimated $\boldsymbol{H}$ at A is due to white additive noises in phases of transmitting the training matrix for estimating $\boldsymbol{H}$, linearly feeding back channel coefficient and transmitting the training matrix for estimating $\boldsymbol{G}$. The noises are respectively denoted as $\boldsymbol{Z}_a$, $\boldsymbol{Z}_b$, $\boldsymbol{Z}_g$ in the form of matrix, and as $\boldsymbol{z}_a$, $\boldsymbol{z}_b$, $\boldsymbol{z}_g$ in the form of row vector. Suppose the feedback interval T is $2lN_a N_b$ symbol-periods.

After LSE at B,

$$\widehat{\boldsymbol{h}} = \boldsymbol{h} + \widetilde{\boldsymbol{h}} \tag{29}$$

where

$$\widetilde{\boldsymbol{h}} = \frac{N_a}{Q_h P_a} \boldsymbol{z}_b \boldsymbol{S}'^{\dagger}. \tag{30}$$

Since the energy of source information after COSTBC is $2lN_b||\widehat{\boldsymbol{h}}||^2$ and the block transmit power constraint is $2lN_aN_bP_t$, the scaling factor

$$a_{\hat{h}} = \sqrt{\frac{N_a P_b}{||\widehat{\boldsymbol{h}}||^2}}. \tag{31}$$

For the i-th receive antenna at A, the equivalent feedback channel model is

$$\begin{pmatrix} \boldsymbol{y}_i^{(1)^T} \\ \boldsymbol{y}_i^{(2)^\dagger} \end{pmatrix} = a_{\hat{h}} \boldsymbol{G}'_i \boldsymbol{\theta}_{\hat{h}}^T + \begin{pmatrix} \boldsymbol{z}_{a,i}^{(1)^T} \\ \boldsymbol{z}_{a,i}^{(2)^\dagger} \end{pmatrix} \tag{32}$$

where the $2lN_aN_b \times lN_aN_b$ matrix $\boldsymbol{G}'_i$ is the equivalent complex orthogonal channel matrix of $\boldsymbol{G}_{(\cdot,i)}$.

Considering the feedback-channel state information known at A is also imperfect, after de-scaling and MRC, from (15), we get

$$\widehat{\boldsymbol{\theta}}_{\hat{h}}^T = \frac{1}{2a_{\hat{h}}||\widehat{\boldsymbol{G}}||_F^2} \sum_{i=1}^{N_b} \widehat{\boldsymbol{G}}_i'^{\dagger} \begin{pmatrix} \boldsymbol{y}_i^{(1)^T} \\ \boldsymbol{y}_i^{(2)^\dagger} \end{pmatrix}. \tag{33}$$

Then, from (27) and (33), the equivalent matrix of estimate error

$$\begin{aligned} \boldsymbol{e}^T &= \widehat{\widehat{\boldsymbol{h}}}^T - \boldsymbol{h}^T \\ &= \frac{N_a}{Q_h P_a} \boldsymbol{S}'^* \boldsymbol{z}_b^T - \frac{1}{2l||\widehat{\boldsymbol{G}}||_F^2} \boldsymbol{U} \Big(\sum_{i=1}^{N_a} \widehat{\boldsymbol{G}}_i'^{\dagger} \widetilde{\boldsymbol{G}}'_i \boldsymbol{U}^T \widehat{\boldsymbol{h}}^T \\ &\quad - \frac{1}{a_{\hat{h}}} \sum_{i=1}^{N_a} \widehat{\boldsymbol{G}}_i'^{\dagger} \begin{pmatrix} \boldsymbol{z}_{a,i}^{(1)^T} \\ \boldsymbol{z}_{a,i}^{(2)^\dagger} \end{pmatrix} \Big) \end{aligned} \tag{34}$$

where $\widehat{\boldsymbol{G}}'_i$ is the equivalent matrix of channel estimate $\boldsymbol{g}_i$ and $\widetilde{\boldsymbol{G}}'_i$ is the equivalent matrix of $\widetilde{\boldsymbol{g}}_i$. Since $\widehat{\boldsymbol{G}}'_i = \boldsymbol{G}'_i + \widetilde{\boldsymbol{G}}_i$, $\widehat{\boldsymbol{G}}'_i$ and $\widetilde{\boldsymbol{G}}'_i$ are also complex orthogonal matrices of size $2lN_aN_b \times lN_aN_b$.

For simplicity, we assume there is no error in estimating $\boldsymbol{G}$, *i.e.* $\widetilde{\boldsymbol{G}} = 0$, and noises $\boldsymbol{z}_a$ and $\boldsymbol{z}_b$ are white additive with variance $\sigma_{z_a}^2$ and $\sigma_{z_b}^2$ respectively. Under this assumption, the average mean-squared error per channel coefficient

$$\begin{aligned} \overline{\epsilon^2} &= \frac{\mathbb{E}||\boldsymbol{e}||^2}{N_a N_b} \\ &= \frac{N_a \sigma_{z_b}^2}{Q_h P_a} + \frac{N_a N_b \sigma_{z_a}^2}{T||\boldsymbol{G}||_F^2 a_{\hat{h}}^2} \\ &= \frac{N_a}{Q_h \rho_t} + \frac{N_b}{T||\boldsymbol{G}||_F^2 \rho_f} \left(||\boldsymbol{H}||_F^2 + \frac{N_a^2 N_b}{Q_h \rho_t} \right) \end{aligned} \tag{35}$$

where $\rho_t = P_a/\sigma_{z_b}^2$, $\rho_f = P_b/\sigma_{z_a}^2$, and $T = 2lN_aN_b$. It indicates that by increasing SNRs, the training interval or the feedback interval can decrease MSE on the imperfect CSIT, which corresponds to common sense.

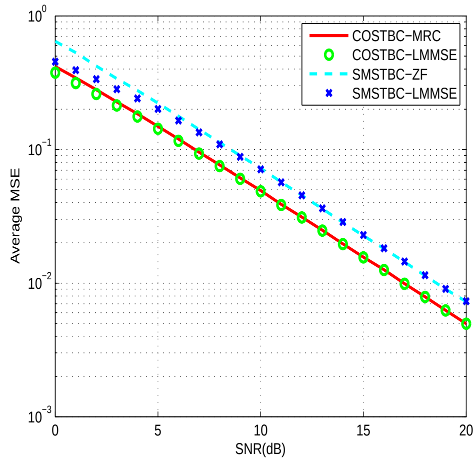

Fig. 2. Graph of COSTBC vs. SMSTBC in analog feedback. MIMO 4×2, training lengths $Q_h = Q_g = 10$, the feedback interval $T = 32$ symbol periods, $\rho_t = \rho_f = \rho_g$, Mont Carlo runs = 10,000 times.

IV. Compare to Other Feedback Methods

In this section, we compare our COSTBC-MRC analog feedback scheme to several other feedback schemes by simulation results. It is illustrated that COSTBC-MRC has the lowest MSE amongst them.

A. Compare to Spatial-Multiplexing STBC Analog Feedback Methods

Another space-time coding technique for analog transmission is the spatial-multiplexing STBC. That is, for N_t transmit antennas, N_t independent data symbols are one-to-one transmitted per symbol period and no specific codebook exists. In the case of a long transmit interval, for achieving smaller MSE, symbols are to be repeated and then at the receiver, mean estimation is to be done after equalization.

Under the assumption that the channel state information is known at the receiver and the source symbols are i.i.d. zero-mean r.v.'s, zero-forcing (ZF) or linear mean-square error (LMMSE) equalization methods can be employed at the receiver to recover transmitted symbols. Note that ZF equalization is subject to the constraint that the transmit antenna number N_t must not be greater than the receiver antenna number N_r; LMMSE equalization is subject to the constraint that the receiver is supposed to know the spatial covariance matrices of noises.

In Fig.2, we plot curves of average MSE per channel coefficient of COSTBC and SMSTBC analog feedback schemes. We consider three noises, – the noise in estimating the feedback channel $\boldsymbol{G}$, the noise in estimating the transmit channel $\boldsymbol{H}$ at the receiver, the noise in feeding back the channel estimate $\widehat{\boldsymbol{H}}$. We assume all elements of noise and channel matrices are symmetric $\mathcal{CN}(0,1)$ r.v.'s. For simplicity of plotting, three SNRs are supposed to be equal. The curves are generated by 10,000 Mont Carlo runs.

It shows that COSTBC analog feedback schemes have about 1.6dB advantage over SMSTBC schemes in MSE at 20dB SNR. There is merely the trivial difference between MSE curves of COSTBC-MRC and COSTBC-LMMSE when SNR is pretty low; when SNR is decent, it is hardly to see. Considering doing LMMSE requires more information at the receiver, we thus suggest the COSTBC-MRC analog feedback scheme for linear analog transmission.

B. Compare to Random Vector Quantization Transmission

In recent literature, random vector quantization (RVQ), as a digital way, is widely used to feed back the channel information (see [8], [9], etc.). An interesting analogy between RVQ and linear analog transmission is both of them can only be used to transmit an analog vector's direction (here, we assume source symbols in linear analog systems are scaled to the transmit power constraint). That is, to rebuild the source vector, its magnitude is supposed to be transmitted in another way.

For comparing RVQ to COSTBC analog transmission with respect to MSE, assuming the magnitude is genie-aided transmitted to the receiver for rebuilding the analog vector, our criterion to select an index i from the unit vector quantization codebook $\mathcal{W}$ is to select a codeword $\boldsymbol{w}_i$ which satisfies $\boldsymbol{w}_i = \arg\max_{\boldsymbol{w}\in\mathcal{W}} \Re\left(\frac{\boldsymbol{h}}{||\boldsymbol{h}||}\boldsymbol{w}^\dagger\right)$ and $\boldsymbol{h}$ is the row complex analog vector to be transmitted. This criterion is derived from

$$||\boldsymbol{h}-\boldsymbol{w}||^2 = ||\boldsymbol{h}||^2\left(2-2\Re\left(\frac{\boldsymbol{h}}{||\boldsymbol{h}||}\boldsymbol{w}_i^\dagger\right)\right). \quad (36)$$

In Fig.3, we compare the COSTBC-MRC analog transmission technique to the RVQ technique with respect to MSE by simulation. Assume a length 2 complex vector is to be transmitted during an interval of 4 symbol periods by b bits per symbol via a 2×2 MIMO channel. A unit random vector codebook $\mathcal{W}$ of size 2^{2b} is generated. The index is selected according to the aforementioned criterion, divided into 2 symbols and mapped to the constellation. Generated constellation points are transmitted after COSTBC. The maximum likelihood detection is employed at the receiver. For simplicity and fairness, no channel coding and labeling is considered here for RVQ methods, which we call *uncoded*. Thus, for uncoded RVQ methods, not only quantization error exists but decision error exists as well.

From Fig.3, we can see that an adaptive RVQ scheme has advantage over fixed RVQ schemes due to characteristics of MSE curves of RVQ methods. For an adaptive RVQ, with increasing SNR, the codebook becomes larger and larger and the constellation becomes denser and denser. But even for an uncoded adaptive RVQ sheme, our simulation shows that it is at least 4.7dB inferior to the COSTBC-MRC analog scheme in terms of MSE in the scenario we set, although the digital way is of significantly greater complexity.

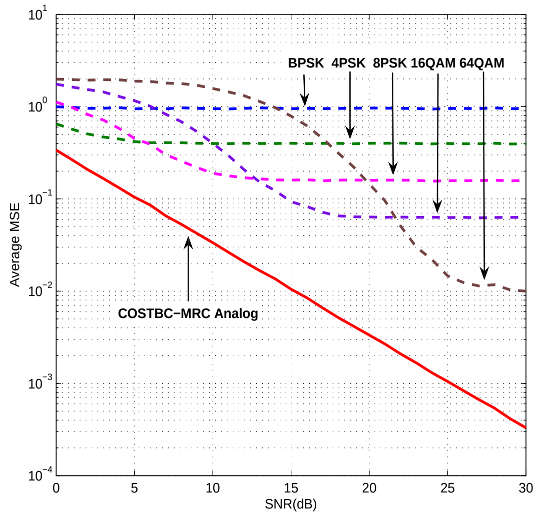

Fig. 3. COSTBC vs. RRVQ in transmitting continuous-amplitude symbols. MIMO 2×2, the complex analog vector's length $= 2$, the transmit interval $T = 4$ symbol periods, Mont Carlo runs = 20,000 times.

V. Conclusion

We have proposed to apply rate 1/2 complex orthogonal space time block coding (COSTBC) to analog transmission. Such a space-time coding method can generate an equivalent complex orthogonal channel for a MISO channel and thus achieves the matched filter bound on SNR for analog transmission over a MIMO channel. We present a detailed channel information analog feedback scheme applied COSTBC-MRC and analyze its MSE to indicate how factors work in such a scheme. The COSTBC-MRC method are compared to several other possible analog and RVQ (digital) methods and it shows that COSTBC-MRC has the lowest MSE amongst them.

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