Uplink Distributed Binary Power Allocation for Cognitive Radio Networks

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***Abstract*— Motivated by the desire for efficient spectral utilization, we present a novel algorithm for power allocation for sum rate maximization in cognitive radio context while preserving a guaranteed QoS for the primary network. To this effect, we propose a distributed cognitive radio coordination that maximizes the Cognitive Radio Network (CRN) sum rate while minimizing the interference to the primary users (PU). Our goal is to realize spectrum sharing by optimally allocating secondary users (SU) transmit powers in order to maximize the total SU throughput under interference and noise impairments. Both theoretical and simulation results under realistic wireless network settings are shown to exhibit interesting features in term of CRN deployment while maintaining QoS for the primary system.**

I. INTRODUCTION

Spectrum utilization can be improved by making secondary users (cognitive users) access spectrum holes unoccupied by any primary user (licensed user) at the right location and the right time [1]. In current cognitive radio protocol proposals, the SU device listens to the wireless channel and determines, either in time or frequency, which part of the spectrum is unused. It then adapts its signal to fill this void in the spectrum domain. Thus, a SU device transmits over a certain time or frequency band only when no other user does, like in [2]. In the same context, it was shown in [3] how we can improve the overall system spectral efficiency compared to classical approaches by considering orthogonal cognitive communications. The contribution of some recent studies [4] and [5] has however also extended cognitive protocols to allow the SU to transmit *simultaneously* with the PU in the same frequency band. This is exactly the setup in this work, where the cognitive radio behavior is generalized to allow secondary users to transmit simultaneously with PU *as long as the level of interference to primary users remains within an acceptable range*. A particularly noteworthy target in the context of cognitive radio, when we seek to optimize the system capacity, is to guarantee a QoS to primary system. In this contribution, we will propose a way to efficiently protect primary systems from cognitive device interference based on outage probability We adopt this setting and consider a CRN in which primary and secondary users attempt to communicate, subject to mutual interference. Our goal is to realize PU-SU spectrum sharing by optimally allocating SU transmit powers in order to maximize the total SU throughput under interference and noise impairments, and short term (minimum and peak) power constraints, while preserving the QoS of the primary system. In particular, it is of interest to determine, in a *distributed* manner, the optimal noise/interference threshold above which SUs can decide to transmit without affecting the primary users' QoS. In fact, in a realistic network, centralized system coordination is hard to implement, especially in fast fading environments and in particular if there is no fixed infrastructure for SUs, i.e., no back-haul network over which overhead can be transmitted between users.

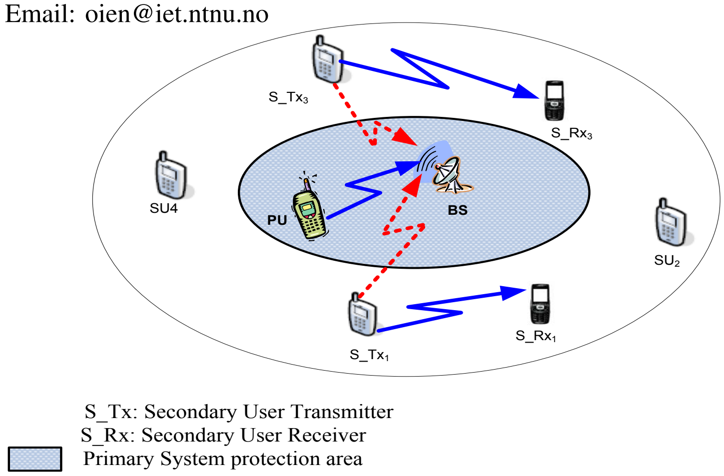

Fig. 1. The Cognitive Radio Network with one primary user (PU) and $M = 4$ secondary transmitters attempting to communicate with their respective receivers in an ad-hoc manner during an uplink transmission of the primary user, subject to mutual interference.

The rest of the paper is organized as follows. The next section describes the cognitive radio network. In Section III, the proposed distributed power control algorithm is investigated in both the high and low SINR regimes, respectively, including primary users' QoS issues. Simulation results are provided in Section IV and Section V concludes the paper.

II. THE COGNITIVE RADIO NETWORK

A. The System Model

Consider an uplink of a CRN with one PU and M SUs randomly distributed over the system. Throughout the paper, we will use the following notation: the index of SU j lies between 1 and M, $h_{pu,n}$ denotes the channel gain from the

PU indexed by pu to the desired user n. The PU is assumed to operate with a power level equal to p_{pu} while the data destined from SU j is transmitted with power p_j. In the coverage area of the primary system, there is an *interference boundary* within which no SUs can communicate in an ad-hoc manner. Thus, as can be seen in Figure 1, for the impairment experienced by the primary system to be as small as possible, a SU must be able to detect very reliably whether it is far enough away from a primary base station (BS), i.e., in the area of possible cognitive radio operation. The expression of the PU instantaneous capacity at the BS is

$$C_{pu} = \log_2 \left(1 + \frac{p_{pu} \mid h_{pu,pu} \mid^2}{\sum_{j=1}^{M} p_j \mid h_{j,pu} \mid^2 + \sigma^2} \right) \tag{1}$$

where σ^2 is the ambient noise variance. On the other hand, by making SUs access the primary system spectrum, the j^{th} SU experiences interference from the PU and all neighboring co-channel SU links that transmit on the same band. Accordingly, the j^{th} SU instantaneous capacity is given by:

$$C_j = \log_2 \left(1 + \text{SINR}_j\right); \quad \text{for} \quad j = 1, ..., M \tag{2}$$

where

$$\text{SINR}_j = \frac{p_j \mid h_{j,j} \mid^2}{\sum_{\substack{k=1 \\ k \neq j}}^{M} p_j \mid h_{k,j} \mid^2 + p_{pu} \mid h_{pu,j} \mid^2 + \sigma^2}$$

SUs need to recognize their communication environment and adapt the parameters of their communication scheme in order to maximize the per-user cognitive capacity, expressed as

$$C_{SU} = \frac{1}{\tilde{M}} \sum_{j=1}^{\tilde{M}} C_j \,, \tag{3}$$

while minimizing the interference to the primary users, in a *distributed* fashion. The sum here is made over the $\tilde{M}$ SUs transmitting. Moreover, we assume that the coherence time is sufficiently large so that the channel stays constant over each scheduling period length. We also assume that SUs know the channel state information (CSI) of their own links, but have no information on the channel conditions of other SUs. No interference cancelation capability is considered. Power control is used for SUs both in an effort to preserve power and to limit interference and fading effects.

B. The Cognitive Radio protocol

Under this scheme, we allow SUs to transmit simultaneously with the PU as long as the interference from the SUs to the PU that transmits on the same band remains within an acceptable range. Specifically, we impose that SUs may transmit simultaneously with the PU as long as the PU in question does not have his QoS affected in terms of outage probability. We consider that PUs operate at a desired rate (depending on their respective QoS demands). From a practical point of view, the outage probability as well as the requested rate can be broadcasted, before the start of the communication, by the primary system base station and is used as a preamble for the PU to get informed which data rate is requested. This preamble can also be overheard by SUs who can then learn about these outage values.

III. Binary power control algorithm

Secondary users offer the opportunity to improve the system capacity over the system by detecting the PU activity and adapting their transmissions accordingly. Our goal within this work is to determine, under the assumption that the PU is oblivious to the presence of the cognitive users, what would be the cognitive system capacity (which can also be viewed as the total increase in system capacity due to the SUs' activity) and, at the same time, the maximum number of cognitive communication links allowed in such a system. The optimization problem can therefore be expressed as follows:

$$\text{Find } \{p_1*, ..., p_M*\} = \arg \max_{p_1, ..., p_M} C_{SU} \tag{4}$$

subject to:

$$\begin{cases} 0 \leq p_j \leq P_{max}, \quad \text{for} \quad j = 1, ..., M \\ \\ P_{out} = Prob \left\{ C_{pu} \leq R_{pu} \mid R_{pu}, q \right\} \leq q, \end{cases} \tag{5}$$

The key idea within the proposed iterative algorithm is, as in [6], is to subsequently limit p_j to $\{0, P_{max}\}$, i.e., to switch "off" transmission in SUs' links which do not contribute enough capacity to outweigh the interference degradation caused by them to the rest of the network. Though other SUs stay silent, they may be active during the next iteration for the PU in question. We propose an adaptation of the distributed algorithm which allows a subset of controlled size $\tilde{M}$ of the total number of SUs M to transmit simultaneously on the same sub-band. It turns out necessary to limit the number of SUs interfering with the primary user so as to guarantee a QoS for the primary system. Let Ψ be the set of indices of all presently active SUs. A SU should be deactivated if this action results in an increase in the cognitive capacity of SUs or if its transmission violates the PU outage constraint.

A. At high SINR regime

The CRN described in the previous subsection can be modeled by interference channels, due to the fact that SUs employ the same spectral resource in each link, giving rise to an interference-limited system. At high SINR regime, in all "on" SU, and assuming an interference-limited system, we obtain after simple manipulations[1]

$$\text{SINR}_m < \frac{\prod_{\substack{j \in \Psi \\ j \neq m}} \left(p_{pu} \mid h_{pu,j} \mid^2 + \sum_{\substack{k \in \Psi \\ k \neq j}} p_k \mid h_{k,j} \mid^2 \right)}{\prod_{\substack{j \in \Psi \\ j \neq m}} \left(p_{pu} \mid h_{pu,j} \mid^2 + \sum_{\substack{k \in \Psi \\ k \neq j \neq m}} p_k \mid h_{k,j} \mid^2 \right)}$$

[1]Due to the lack of space, we will not present all analytical derivations in this paper. The reader is referred to the journal version for additional information.

$$\Downarrow$$

$$\text{SINR}_m < \frac{\prod_{\substack{j\in\Psi \\ j\neq m}} \sum_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j}} p_k \mid h_{k,j} \mid^2}{\prod_{\substack{j\in\Psi \\ j\neq m}} \sum_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j\neq m}} p_k \mid h_{k,j} \mid^2} \quad (6)$$

Suppose that devices operate in a dense network, i.e. a large number of SUs is a distributed over a restricted geometrical area. It was shown in [7] that, based on the observation that interference to any user in a large dense network is only weakly dependent on the user's position, we can approximate the interference term in (6) by an average interference gain, (denoted by G_{su}^2) which is independent of the user location, multiplied by the total transmit power of active interferers:

$$\sum_{j=1}^{M} p_j \mid h_{n,j} \mid^2 \simeq G_{su}^2 \sum_{j=1}^{M} p_j \, , \quad \text{for all} \quad n \quad (7)$$

where G_{su}^2 is a constant depending only on the average amplitude of the SU channel gain. Accordingly, condition (6) becomes

$$\frac{p_m \mid h_{m,m} \mid^2}{\sum_{\substack{k\in\Psi\cup\{pu\} \\ k\neq m}} p_k \mid h_{k,m} \mid^2} < \frac{\prod_{\substack{j\in\Psi \\ j\neq m}} G_{su}^2 \sum_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j}} p_k}{\prod_{\substack{j\in\Psi \\ j\neq m}} G_{su}^2 \sum_{\substack{k\in\Psi\cup\{pu\} \\ k\neq j\neq m}} p_k} \quad (8)$$

As all "on" SU transmit with P_{max} and denoting by $\tilde{M} = \mathit{card}\{\Psi\}$, the m^{th} SU will be active only if

$$\frac{\mid h_{m,m} \mid^2}{\sum_{\substack{k\in\Psi\cup\{pu\} \\ k\neq m}} \mid h_{k,m} \mid^2} > \left(\frac{\tilde{M}}{\tilde{M}-1}\right)^{\tilde{M}-1} \quad (9)$$

As the number of SUs increases, we get (as in [6])

$$\lim_{\tilde{M}\to\infty} \left(\frac{\tilde{M}}{\tilde{M}-1}\right)^{\tilde{M}-1} = e = 2.718281...$$

Thus, for a large network size, a SU will be active if the user signal-to-interference ratio of the scheduled user is more than e, namely

$$\text{SIR}_m > e \quad (10)$$

B. At low SINR regime

The restriction to binary power levels yields in general only a negligible capacity loss. As stated before, it was shown in [6] that at low-SINR regime, i.e., where the approximation $\ln(1+x) \simeq x$ holds with good accuracy, binary power control is in fact always optimal. In the low SINR regime, after simple manipulations, the m^{th} SU will now be active if

$$\text{SINR}_m < \frac{\sum_{\substack{j\in\Psi \\ j\neq m}} p_j \mid h_{j,j} \mid^2}{P_{max} G_{su}^2 (\tilde{M}-1) + \sigma^2} \quad (11)$$

$$\simeq \frac{P_{max} G_{su}^2 (\tilde{M}-1)}{P_{max} G_{su}^2 (\tilde{M}-1) + \sigma^2}$$

where we use the same dense average network assumptions as in (7). Suppose, as in the high SINR regime, that we are in an interference-limited context. This would suggest that $\sigma^2 \ll P_{max} G_{su}^2 (\tilde{M}-1)$ in the RHS of (11). Thus, a SU will be active if the user SIR ratio of the scheduled user is more than 1:

$$\text{SIR}_m > 1 \quad (12)$$

We thus confirm, as intuition would expect, that SUs under better SINR conditions would transmit only above a higher threshold than in the low-SINR regime.

C. Primary system QoS issues

Because SUs enjoy economies of scale and do not pay for expensive spectrum licenses, unlicensed spectrum can offer a less expensive and more readily deployable form of wireless service than licensed spectrum. Nevertheless, this should be obtained at a tradeoff for QoS and protection from secondary system interference. In the current section, we study how to guarantee a QoS to the PU by means of an outage constraint. The notion of *information outage probability* defined as the probability that the capacity of the user is below the transmitted code rate, namely:

$$P_{out} \triangleq Prob\{C_{pu} \leq R_{pu}\} \leq q, \quad (13)$$

We introduce the PU pathloss gain profile estimate G_{pu}^2 based on the following decomposition:

$$h_{pu,pu} \triangleq G_{pu}^2 * h'_{pu,pu}$$

where $h'_{pu,pu}$ represents the normalized channel impulse response tap. This gives us the following PU outage probability expression:

$$P_{out} = Prob\left\{ \log_2\left(1 + \frac{p_{pu} G_{pu}^2 \mid h'_{pu,pu} \mid^2}{\sum_{j=1}^{\tilde{M}} p_j \mid h_{j,pu} \mid^2 + \sigma^2}\right) \leq R_{pu} \right\} \leq q$$

$$\simeq Prob\left\{ \mid h'_{pu,pu} \mid^2 \leq \left(2^{R_{pu}} - 1\right) \left(\frac{\tilde{M} P_{max} G_{su}^2 + \sigma^2}{p_{pu} G_{pu}^2}\right) \right\} \leq q$$

From now on we assume that the channel gains are i.i.d Rayleigh distributed. However, such an approach can be immediately translated into results for any other probability distribution function of the channel by replacing by the appropriate probability distribution function.

$$P_{out} \simeq \int_0^{\left(2^{R_{pu}} - 1\right) \left(\frac{\tilde{M} P_{max} G_{su}^2 + \sigma^2}{p_{pu} G_{pu}^2}\right)} \exp(-t) dt \leq q$$

Finally, we get the following outage constraint:

$$P_{out} \simeq 1 - \exp\left[- \left(2^{R_{pu}} - 1\right) \left(\frac{\tilde{M} P_{max} G_{su}^2 + \sigma^2}{p_{pu} G_{pu}^2} \right) \right] \leq q$$

and the maximum number of "on" SU that transmit with P_{max} is given by

$$0 \leq \tilde{M} \leq \frac{-\log(1-q)}{(2^{R_{pu}} - 1)} \cdot \frac{p_{pu} G_{pu}^2}{P_{max} G_{su}^2} - \frac{\sigma^2}{P_{max} G_{su}^2} \qquad (14)$$

The LHS in (14) prevents from obtaining a negative number of users when the SNR decreases significantly. The formula in (14) points out that that the number of SUs allowed to transmit increases as their SNR (SNR $= \frac{G_{su}^2 P_{max}}{\sigma^2}$) increases.

IV. Numerical Results

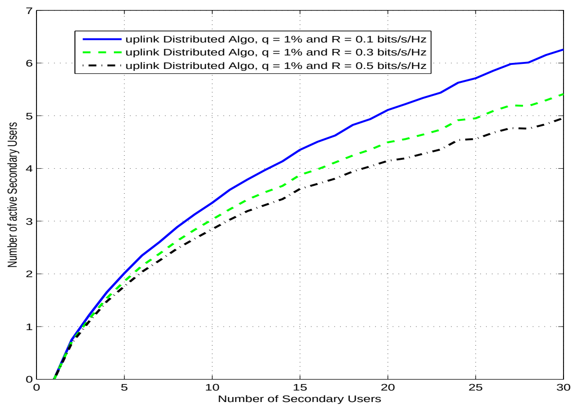

Fig. 2. Number of active secondary users vs. number of SUs for different rates and outage probability.

To go further with the analysis, we resort to realistic network simulations. We consider a cognitive radio network as described in Fig. 1 with one PU and M secondary transmitters attempting to communicate with their respective receivers during an uplink transmission of the primary user, subject to mutual interference. Specifically, a hexagonal cellular system functioning at 1.8 GHz with a primary cell of radius $R = 1000$ meters and a primary protection area of radius $R_p = 600$ meters is considered. Secondary transmitters may communicate with their respective receivers of distances $d < R_p$ from the BS. Channel gains are based on the COST-231 path loss model [8] including log-normal shadowing with standard deviation of 10 dB, plus fast-fading assumed to be i.i.d. circularly symmetric with distribution $\mathcal{CN}(0, 1)$. The peak power constraint is given by $P_{max} = 1$ Watt. As intuition would expect, Figure 2 shows that the lower the transmission rate is, the higher number of active SUs we get for a given value of outage probability. Moreover, it is clear that increasing the number of SUs yields improvements in the number of active users. This can be explained by the fact that multi user diversity yields more opportunity to SUs to satisfy equation (14). As an example, at a rate $R = 0.1$ bits/s/Hz and an outage probability $q = 1\%$, we get 3 and 5 active SUs for 10 and 20 potential SUs, respectively. Figure 3 shows however that the SUs' cognitive capacity increases as the number of SUs increases due to multi-user diversity till a certain value where interference impairment are more important. The current curve confirms that in CRN, when one attempts to maximize the number of "on" SUs, the cognitive capacity degrades asymptotically. Hence, there is a fundamental trade-off between per-user cognitive capacity maximization and number of active SUs maximization.

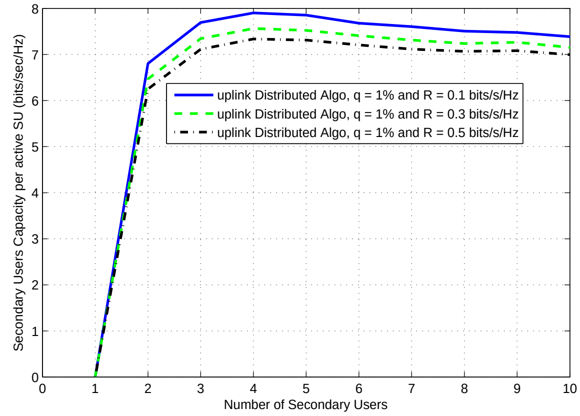

Fig. 3. Secondary user capacity per user vs. number of SUs.

V. Conclusion

In this paper, we have explored the idea of combining multi-user diversity gains with spectral sharing techniques to maximize the secondary user rate while maintaining a QoS to a primary user. We first derived a distributed algorithm under a cognitive capacity maximization criterion using binary power allocation. Then, we showed that the proposed approach exhibits a significant number of cognitive users able to transmit while minimizing interference to guarantee QoS for the primary user. Simulation results were carried out based on a realistic network setting.

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