Multiuser MIMO Downlink: Multiplexing Gain Without Free Channel Information

Umer Salim
Mobile Communications Department
Eurecom Institute
06560 Sophia Antipolis, France
umer.salim@eurecom.fr

Dirk Slock
Mobile Communications Department
Eurecom Institute
06560 Sophia Antipolis, France
dirk.slock@eurecom.fr

***Abstract*— A multiuser MIMO system is considered with no initial assumption of channel state information (CSI) at any of the receivers or the BS transmitter. For a system working under frequency-division duplexing (FDD) mode, simple practically realizable transmission strategy is proposed which provides necessary channel state information to both sides with minimal resource utilization for downlink (DL) data transmission. For the given transmission strategy, high SNR degrees of freedom (DOF) for the DL channel are specified. If the users have data to transmit in the uplink (UL) direction, with no extra burden of training, full multiplexing gain of the multiple-access channel (MAC) can be achieved. Thus the given strategy makes the system fully scalable for the data transmission in both directions.**

I. Introduction

In multiple-antenna downlink channels, capacity or achievable data rates can be excessively increased just by adding multiple antennas at the transmitting end. Thus if a base station (BS) has M transmit antennas and the number of users in the system is K with $K \geq M$, this downlink channel can support data rates M times larger than a single antenna BS, although all users may have single antenna each in both cases [1], [2], [3]. So under favorable conditions, the sum capacity of the downlink channel is comparable to the capacity of a point-to-point MIMO channel having the same number of transmit and receive antennas. Apart from this sum capacity aspect, there are two advantages of this broadcast channel. It requires mobile users to have a single antenna each so user terminals are quite inexpensive and simple. The second advantage is that point-to-point MIMO links are plagued by line-of-sight channel conditions where channel matrices are of reduced rank and they lose their multiplexing abilities. In a multiuser channel, naturally users are far apart so the assumption of independent channel for each user holds very well and the channel matrix is of full-rank with probability one and is much well-conditioned as compared to the channel matrix of a point-to-point MIMO link [4].

But these promising advantages of multiuser MIMO don't come for free. To realize these high throughputs, BS has to transmit to multiple users over the same bandwidth. Orthogonal transmission schemes such as time-division multiple access (TDMA), frequency-division multiple access (FDMA) and code-division multiple access (CDMA) are highly suboptimal as effectively BS will be transmitting to a single user over a particular resource. The other price to pay to achieve these high data rates is that BS must know the forward channel to all users [1]. This point is in sharp contrast to point-to-point MIMO. In point-to-point MIMO, channel state information at the transmitter (CSIT) only affects the power offset of the capacity. The slope of the capacity versus SNR curve, normally termed as the multiplexing gain or the degrees of freedom (DOF), remains unaffected by CSIT [5], [3].

We don't impose any initial assumption of channel knowledge on either side. But we don't prevent any side (transmitter and receiver) to learn/feedback the channel and subsequently use this information for precoding/decoding of data. Most of the initial results on the information theoretic capacity analysis of the broadcast channel came with the assumption of perfect channel state information at the transmitter (CSIT), and each user knows its own channel (CSIR). Inherently all channels are non-coherent and the users (receivers) need to estimate the channels implicitly (data driven) or explicitly by some kind of training (pilots transmission) to get CSIR. In frequency-division duplex (FDD) mode of operation, downlink (forward) channels are normally different from the uplink (reverse) channels. So the users need to feedback their estimated forward channel information on the reverse link. On the other hand, the acquisition of CSIT gets facilitated when the system operates under time-division duplex (TDD) mode. In this case, reciprocity implies that the forward channel matrix is the transpose of the reverse channel matrix [6]. As most of the current wireless systems use FDD mode, so we focus on FDD system in this contribution.

Notation: $\mathbb{E}$ denotes statistical expectation. Lowercase letters represent scalars, boldface lowercase letters represent vectors, and boldface uppercase letters denote matrices. $\mathbf{A}^\dagger$ denotes the Hermitian of matrix $\mathbf{A}$.

II. System Model

The system we consider consists of one BS having M transmitting antennas and K single-antenna user terminals. In the downlink, the signal received by k-th user can be expressed as

$$y_k = \mathbf{h_k}^\dagger \mathbf{x} + n_k, \qquad k = 1, 2, \ldots, K \tag{1}$$

where $\mathbf{h_1}, \mathbf{h_2}, \ldots, \mathbf{h_K}$ are the channel vectors of users 1 through user K with $\mathbf{h_k} \in \mathbb{C}^{M\times 1}$ ($\mathbb{C}^{M\times 1}$ denotes the M-

dimensional complex space), $\mathbf{x} \in \mathbb{C}^{M\times 1}$ denotes the M-dimensional signal transmitted by the BS and $n_1, n_2, \ldots, n_K$ are independent complex Gaussian additive noise terms with zero mean and unit variances. We denote the concatenation of the channels by $\mathbf{H}^\dagger = [\mathbf{h_1}\mathbf{h_2}\cdots\mathbf{h_K}]$, so $\mathbf{H}$ is the $K \times M$ forward channel matrix with k-th row equal to the channel of the k-th user ($\mathbf{h_k^\dagger}$). Similarly $\mathbf{G}$ is the $M \times K$ reverse channel matrix. Due to frequency division duplexing, we take them completely independent of each another. The BS has an average power constraint of P for the DL input and each user has the same power constraint of P_u.

The channel is assumed to be block fading having coherence length of T symbol intervals where fading remains the same, with independent fading from one block to the next [7]. The entries of the forward channel matrix $\mathbf{H}$ and the reverse channel matrix $\mathbf{G}$ are independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance. Initially all receivers and the BS transmitter are oblivious of the channel realization in each block.

In normal downlink scenarios, the number of users (K) will be more than the number of BS transmit antennas (M). It is well-known that with perfect CSIT and CSIR, downlink channel with M transmit antennas and K single antenna users with $K \geq M$ achieves the multiplexing gain (DOF) of M [8] i.e., the dominant term of the sum-capacity of this downlink channel is $M\log(P)$. Extra number of users does not contribute to increasing the multiplexing gain of this system although definite power gain can be achieved by scheduling over the users. In this contribution, our main point of concern is the multiplexing gain or the DOF of non-coherent downlink channel so we focus our attention on the case with $K = M$.

III. Transmission Strategy and Analysis

For a downlink channel having a transmitter equipped with M transmit antennas and M single antenna receivers with perfect CSIT and CSIR, the first order term of the sum capacity is $M\log(P)$ [8]. If we compare this to the capacity of the same downlink channel with only CSIR available where the dominant term of the sum capacity is only $\log(P)$, the difference in the sum capacity forcefully dictates that the transmitter must be fed back the DL channel information.

For our block fading channel with coherence length of T symbol intervals, we divide this interval in four phases, 1) Initial UL and DL training, 2) Uplink Feedback, 3) Final DL training and 4) coherent data transmission. The first phase is

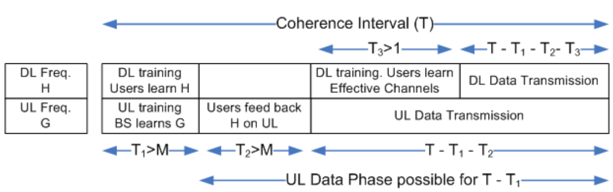

Fig. 1. Coherence interval Divided in Training and Data Phases

the initial UL and DL training phase where users transmit pilots on the UL frequency so that BS estimates the UL channels and BS transmits pilots on the DL frequency so that each user estimates its corresponding DL channel. But to multiplex data streams of multiple users, BS requires the information of DL channel which is fed back in the second feedback interval. Based upon this channel information, BS may choose some transmission strategy which could be a simple linear beamforming strategy like zero forcing (ZF), some non-linear strategies like vector perturbation or the optimal dirty paper coding (DPC). Thus the third phase is the downlink training phase where the BS transmits pilots so that users estimate their corresponding effective channels. When this third phase ends, both sides of the downlink channel have necessary channel state information albeit imperfect. Thus starting from a downlink channel with no CSIT and no CSIR, reaching up to the forth data phase, we have a downlink channel with imperfect CSIT and CSIR and hence in this data phase, BS may choose good transmission strategies and users can decode data coherently. The data rates obtained and their scaling with SNR show that these training phases are beneficial.

Below we give a detailed analysis of the transmission phases mentioned above.

A. Initial UL and DL Training Phase

In this training phase, users transmit pilot signals which are known at the BS. As there are M users, hence the length of this training interval is $T_1 \geq M$. For this uplink training, the use of orthogonal training sequences by all users is very attractive because in that case all users can transmit simultaneously to the BS with their full power without interfering with each other. Thus pilot signal matrix (combined from all users) is $\sqrt{T_1}\mathbf{A}$ where $\mathbf{A}$ is a $M \times T_1$ unitary matrix hence $\mathbf{A}\mathbf{A}^\dagger = \mathbf{I_M}$ where $\mathbf{I_M}$ denotes a $M \times M$ identity matrix. If $\mathbf{Y_u}$ denotes the $M \times T_1$ matrix of the received signal by M antennas of the BS in this training interval of length T_1, the system equation for this uplink training phase becomes

$$\mathbf{Y_u} = \sqrt{P_u T_1}\mathbf{G}\mathbf{A} + \mathbf{Z_u} \tag{2}$$

where $\mathbf{Z_u}$ is a $M \times T_1$ matrix having i.i.d. zero mean unit variance complex Gaussian noise entries. As pilot signal matrix $\mathbf{A}$ is known at the BS, it can formulate an MMSE estimate of the uplink channel matrix $\mathbf{G}$ which is given by

$$\hat{\mathbf{G}} = \frac{\sqrt{P_u T_1}}{P_u T_1 + 1}\mathbf{Y_u}\mathbf{A}^\dagger \tag{3}$$

And the estimation error corresponding to each UL channel entry is given by

$$\sigma^2_{1UL} = \mathbb{E}[|\mathbf{G}_{ij} - \hat{\mathbf{G}}_{ij}|^2] = \frac{1}{P_u T_1 + 1} \tag{4}$$

Similarly on the DL frequency, BS transmits pilot signals which are known to all users. Thus they are capable of estimating the DL channel realization. Each user has to estimate M channel coefficients which link M antennas of the BS to its single antenna. The channel vector for user k can be expressed as $\mathbf{h_k} = \check{\mathbf{h}}_\mathbf{k} + \dot{\mathbf{h}}_\mathbf{k}$ where $\check{\mathbf{h}}_\mathbf{k}$ is the estimated channel at k-th user and $\dot{\mathbf{h}}_\mathbf{k}$ is the estimation error vector both with i.i.d. Gaussian entries. The estimation error variance for any DL channel entry denoted by σ^2_{1DL} is given by

$$\sigma^2_{1DL} = \mathbb{E}[|\mathbf{H}_{ij} - \check{\mathbf{H}}_{ij}|^2] = \frac{1}{T_1 P/M + 1} \tag{5}$$

The estimation error variance for each UL (DL) channel entry goes inversely proportional to the training length T_1 and the power constraint of the user terminals P_u (the BS power constraint P).

B. Uplink Feedback Phase

In the initial training phase, BS has estimated the UL channel matrix $\mathbf{G}$ hence on the UL frequency band, users can transmit simultaneously and BS can decode the data of this MAC channel based upon its estimate $\hat{\mathbf{G}}$. We adopt analog feedback strategy at the users' side hence k-th user just feeds back M coefficients of $\check{\mathbf{h}}_\mathbf{k}$ assuming them to be perfect. As each user comes with a single antenna so the transmission of M complex coefficients will require this interval length to be at least M hence $T_2 \geq M$.

$$\mathbf{y}_{FB} = \sqrt{P_u}\mathbf{G}\mathbf{f} + \mathbf{z}_{FB} \tag{6}$$

Here $\mathbf{y}_{FB}$ represents the M-dimensional vector received by BS for one symbol interval during this feedback phase, $\mathbf{f}$ denotes the data sent by M users which actually consists of the DL channel coefficients estimated by the users in the first training interval. BS decodes $\mathbf{f}$ based upon its knowledge of $\hat{\mathbf{G}}$ considering that estimate as perfect. For the k-th user channel $\mathbf{h_k}$, the BS estimate is denoted by $\hat{\mathbf{h}}_\mathbf{k}$ and $\tilde{\mathbf{h}}_\mathbf{k}$ denotes the corresponding estimation error. The variance of each entry of $\tilde{\mathbf{h}}_\mathbf{k}$ is σ^2_{FB}. This estimation error gets augmented (as compared to σ^2_{1DL}) with the fact that k-th user transmitted $\check{\mathbf{h}}_\mathbf{k}$ instead of $\mathbf{h_k}$ and the BS had access to $\hat{\mathbf{G}}$ in place of $\mathbf{G}$.

C. BS Transmission Strategy: ZF Precoding

It is known that the dirty paper coding (DPC) is the capacity achieving transmission scheme for MIMO broadcast channel and achieves the full capacity region [9] but this scheme is complex and its implementation is quite tedious. So a lot of research has been carried out to analyze the performance of simpler linear precoding schemes. Zero forcing precoding, one of the simplest linear precoding strategy, has been shown to behave quite optimally at asymptotically high values of SNR and achieves the full DOF of a coherent downlink channel [8]. It means that the first order term of the sum capacity of the downlink channel remains same whether one employs DPC or ZF precoding at the BS. In this contribution we are mainly interested in analyzing the DOF obtainable with some simple transmission scheme hence BS uses ZF precoding based upon the knowledge of the DL channel matrix $\hat{\mathbf{H}}$ obtained through explicit feedback from users.

In ZF precoding, unit norm beamforming vector for user k (denoted as $\bar{\mathbf{v}}_\mathbf{k}$), is selected such that it is orthogonal to the channel vectors of all other users. Hence with perfect CSIT, each user will receive only the beam directed to it and no multi-user interference will be experienced. For the case in hand, where the BS has imperfect estimate of the channel matrix, there will be some residual interference. If we represent ZF beamforming matrix by $\bar{\mathbf{V}} = [\bar{\mathbf{v}}_\mathbf{1}\bar{\mathbf{v}}_\mathbf{2} \cdots \bar{\mathbf{v}}_\mathbf{M}]$, the transmitted signal $\mathbf{x}$ becomes $\mathbf{x} = \bar{\mathbf{V}}\mathbf{u}$ and the signal received by user k (1) can be expressed as

$$\begin{aligned} y_k &= \mathbf{h}_\mathbf{k}^\dagger \bar{\mathbf{V}}\mathbf{u} + n_k \\ &= \mathbf{h}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{k} u_k + \sum_{j \neq k} \mathbf{h}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{j} u_j + n_k \end{aligned} \tag{7}$$

Due to imperfect MMSE estimation at the BS and the choice of ZF beamforming unit vectors, we have

$$\mathbf{h}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{j} = \hat{\mathbf{h}}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{j} + \tilde{\mathbf{h}}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{j} = \tilde{\mathbf{h}}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{j} \tag{8}$$

hence the received signal at k-th user becomes

$$\begin{aligned} y_k &= \mathbf{h}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{k} u_k + \sum_{j \neq k} \tilde{\mathbf{h}}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{j} u_j + n_k \\ &= h_{k,k} u_k + \sum_{j \neq k} h_{k,j} u_j + n_k \end{aligned} \tag{9}$$

$h_{k,k}$ is the effective scalar channel for user k and $h_{k,j}$ are the coefficients which arise due to imperfect ZF beamforming as BS had no access to perfect channel realizations. Although users estimated their corresponding channel vectors in the first training phase to feed them back later, but they know nothing about their effective scalar channels.

D. Final DL Training Phase

We assume a very simple downlink training strategy. If the BS had the perfect knowledge of the forward channels to all users, due to ZF beamforming vectors each user would only receive the signal from the beam directed to it and no interference from any other beam would be observed. Here BS estimates the users' channels and therefore channel estimates and the corresponding ZF beamforming vectors are imperfect so each user receives some unwanted signal contribution from the beam directed to any other user. But this interference is of the order of the channel noise so for this DL training phase, BS activates all beams simultaneously for T_3 symbols times. In each symbol interval, every user receives through its effective scalar channel, the Gaussian noise of the channel and the interference due to imperfect channel estimates and ZF beamforming vectors.

$$y_k = h_{k,k} u_k + \sum_{j \neq k} h_{k,j} u_j + n_k \tag{10}$$

Based upon this received signal and the known pilots, k-th user can form the MMSE estimate of the effective scalar channel $h_{k,k}$ which is given by

$$\begin{aligned} \hat{h}_{k,k} &= \frac{\mathbb{E}[h_{k,k} y_k^\dagger]}{\mathbb{E}[y_k y_k^\dagger]} y_k \\ &= \frac{\sqrt{\frac{PT_2}{M}}}{\frac{PT_2}{M} + \frac{PT_2}{M}(M-1)\sigma^2_{FB} + 1} y_k \end{aligned} \tag{11}$$

As $\bar{\mathbf{v}}_\mathbf{k}$ is a unit vector independent of $\mathbf{h_k}$, so effective scalar channel $h_{k,k} = \mathbf{h}_\mathbf{k}^\dagger \bar{\mathbf{v}}_\mathbf{k}$ is zero mean complex Gaussian with unit variance. As a result, MMSE estimate $\hat{h}_{k,k}$ and the estimation error $\tilde{h}_{k,k}$ both are complex Gaussian

$$h_{k,k} = \hat{h}_{k,k} + \tilde{h}_{k,k} \tag{12}$$

$$\hat{h}_{k,k} \sim \mathcal{CN}\left(0, \frac{\frac{PT_3}{M}}{\frac{PT_3}{M} + \frac{PT_3}{M}(M-1)\sigma^2_{FB}+1}\right)$$

$$\tilde{h}_{k,k} \sim \mathcal{CN}\left(0, \frac{\frac{PT_3}{M}(M-1)\sigma^2_{FB}+1}{\frac{PT_3}{M} + \frac{PT_3}{M}(M-1)\sigma^2_{FB}+1}\right)$$

The estimation error variance in estimating this scalar effective channel is inversely proportional to the downlink power constraint. When this phase of downlink training ends, both the BS and all of the users have estimates for the channel and coherent transmission with imperfect CSIT and CSIR is possible.

The length of this final DL training phase T_3 is independent of the number of transmit antennas M at the BS and the number of users.

E. Coherent Data Phase

We adopt the strategy of independent data transmission to all users from the BS with power equally divided among them. So k-th user destined signal, u_k is Gaussian i.i.d. i.e, $u_k \sim \mathcal{CN}(0, P/M)$. The intuition is that in case of perfect CSIT and CSIR, Gaussian signals are the optimal ones.

After the training and the feedback phases described earlier, both the BS and all users have imperfect channel estimates. So with ZF beamforming employed, the signal y_k received by user k (9) may be expressed as

$$y_k = \hat{h}_{k,k}u_k + \tilde{h}_{k,k}u_k + \sum_{j\neq k} h_{k,j}u_j + n_k \tag{13}$$

The above equation differs a lot from (9) as there user k was unaware of its scalar channel $h_{k,k}$ but (13) effectively represents a point-to-point coherent channel with channel $\hat{h}_{k,k}$ known at user k, although there is Gaussian noise, some interference coming from the ZF beamforming vectors of other users and the noise due to imperfect estimation of the effective channel at user's side.

F. Lower Bound of the Achievable Rate

We are interested in calculating the achievable sum rate of this DL channel or its lower bound which could at least point to the number of DOF achievable. If we denote the rate obtained by k-th user as R_k, then it is the mutual information between u_k and y_k with channel $\hat{h}_{k,k}$ known

$$R_k = I(u_k; y_k) \tag{14}$$

In this case, the problem is that we cannot simply use the expression for the mutual information of known scalar channel because of the presence of interference terms whose distributions are unknown. If we combine the noise, the interference and the estimation error contribution in y_k (eq. (13)) in an effective additive noise w_k, then

$$w_k = \tilde{h}_{k,k}u_k + \sum_{j\neq k} h_{k,j}u_j + n_k \tag{15}$$

Due to the use of MMSE estimation in the downlink training, we remark that the signal is uncorrelated with the noise and all interfering terms.

$$\mathbb{E}[u_k(\tilde{h}_{k,k}u_k + \sum_{j\neq k} h_{k,j}u_j + n_k)^\dagger] = 0 \tag{16}$$

The above expectation is zero because of the property of uncorrelated MMSE estimation error, the use of independent signals for different users and that the noise is independent of everything else. Now once we have shown that all additive noise terms are uncorrelated with the desired signal, we can invoke Theorem 1 from [10] which states that the worst case uncorrelated noise has the zero mean Gaussian distribution. So we can replace the effective scalar additive noise w_k of unknown distribution with a noise of the same second moment but having Gaussian distribution, it will give a lower bound to the rate R_k of k-th user but we can instantly write the expression for the mutual information as

$$\begin{aligned} R_k &\geq \mathbb{E}_{\hat{h}_{k,k}} \log\left(1 + \frac{|\hat{h}_{k,k}|^2 \mathbb{E}|u_k|^2}{\mathbb{E}[w_k w_k^\dagger | \hat{h}_{k,k}]}\right) \\ &= \mathbb{E}_{\hat{h}_{k,k}} \log\left(1 + \frac{P}{M}\frac{|\hat{h}_{k,k}|^2}{\mathbb{E}[w_k w_k^\dagger | \hat{h}_{k,k}]}\right) \end{aligned} \tag{17}$$

IV. High SNR DOF of the Sum Rate

The rate for k-th user given in eq. (17) can further be lower bounded as

$$\begin{aligned} R_k &\geq \mathbb{E}_{\hat{h}_{k,k}} \log\left(\frac{P}{M}\frac{|\hat{h}_{k,k}|^2}{\mathbb{E}[w_k w_k^\dagger | \hat{h}_{k,k}]}\right) \\ &= \mathbb{E}_{\hat{h}_{k,k}} \log\left(\frac{P}{M}|\hat{h}_{k,k}|^2\right) - \mathbb{E}_{\hat{h}_{k,k}} \log\left(\mathbb{E}[w_k w_k^\dagger | \hat{h}_{k,k}]\right) \\ &\geq \mathbb{E}_{\hat{h}_{k,k}} \log\left(\frac{P}{M}|\hat{h}_{k,k}|^2\right) - \log\left(\mathbb{E}[w_k w_k^\dagger]\right) \end{aligned} \tag{18}$$

where the last inequality follows from the Jensen's inequality. With this, we only need to compute the 2nd moment of w_k, denoted as $\sigma_w^2 = \mathbb{E}[w_k w_k^\dagger]$.

As all of the users are symmetrically distributed, so the sum rate of this downlink channel is given by

$$R_{\text{sum}} = \frac{T - T_1 - T_2 - T_3}{T} M R_k \tag{19}$$

where we have incorporated the DOF loss in the sum rate due to one feedback and two training phases.

If we increase the first training phase duration T_1, it improves the quality of the $\hat{\mathbf{G}}$ at the BS and of $\hat{\mathbf{H}}$ at users but it gives only a gain in SNR offset which is logarithmic in nature but the coefficient $(T - T_1 - T_2 - T_3)$ reduces the DOF of the sum rate linearly with increase in T_1 so the optimal length of the first training phase should be the minimum possible at high SNR, hence $T_1 = M$.

For the second feedback phase in the UL direction, minimal length is M as each single antenna user needs at least M intervals to feedback M channel coefficients. About the third training phase in the downlink direction of length T_3, reasoning is not very different. With the increase in this training interval, users are better able to estimate their effective scalar channels which gives SNR gain, logarithmic in nature but increase in T_3 directly hits DOF due to the coefficient $(T-T_1-T_2-T_3)$ in front of the logarithm. So to exploit the maximum number of DOF at high SNR, the optimal (minimal) value of T_3 comes out to be 1. Hence adopting these values, the sum rate becomes

$$R_{\text{sum}} \geq \frac{T-2M-1}{T} M \left[\mathbb{E}_{\hat{g}_{k,k}} \log \left(\frac{P}{M} |\hat{g}_{k,k}|^2 \right) - \log(\sigma_w^2) \right] \tag{20}$$

For very large values of P (the BS power constraint) and if power constraints of users are of the same order as that of P, it can be shown that DOF are maximized with minimal training and feedback phases. So for limiting value of P, the lower bound to the multiplexing gain of the sum rate becomes

$$\lim_{P \to \infty} \frac{R_{\text{sum}}}{\log(P)} \geq \frac{T-(2M+1)}{T} M \tag{21}$$

If we compare this multiplexing gain to the multiplexing gain of the same downlink channel with full CSIR and no CSIT where DOF is only 1, we see that even for very practical values of the block coherence interval T in mobile environments, this lower bound $M[1-(2M+1)/T]$ is comparatively much larger and to make the BS learn the channel pays off very well.

An upper bound to the sum rate of non-coherent downlink channel can be obtained by letting all the users cooperate so that we get an $M \times M$ MIMO system. For this the multiplexing gain has been given to be $M(1-M/T)$ in [11]. But the problem is inherently different. In broadcast channel, the transmitter must know the DL channel. Due to FDD, the users first need to estimate DL channel to later feed it back. So it does not seem probable that DL multiplexing gains higher than given by our scheme can be achieved for this system.

V. UL Data Transmission

As the system in hand has two different frequency bands, out of which one is dedicated for UL transmission. Strictly speaking as soon as the first phase ends and BS obtains the estimate of the UL channel matrix $\mathbf{G}$, this information is sufficient to use the UL channel as MIMO MAC and full DOF can be obtained during the rest of the coherence interval $(T-T_1)$ [12]. But the users have to provide DL channel information $\mathbf{H}$ to the BS to use DL frequency resource properly which requires 2nd feedback interval. So after the second feedback phase, UL frequency band can be used as MIMO MAC which will give us multiplexing gain of $M(1-2M/T)$.

Remark 1: To obtain full multiplexing gain at high DL SNR, the UL power (users' power constraints) should be of the same order as of the DL SNR. As users receive some interference due to imperfect channel estimates at the BS so if the DL SNR goes on increasing for a fixed UL power (which means a fixed estimation error at the BS), the signal power and the interference power both will increase with DL SNR and the system will become interference limited causing the collapse of the DOF.

Remark 2: The channels of concern in this paper are fast fading channels which arise for fast moving mobile users e.g. for user speeds of 100Km/h, carrier frequency of 2GHz and coherence BW of 100KHz, coherence time will be about 100 symbol intervals [13]. So even for BSs having 8 or 16 antennas, training/feedback interval minimization becomes really necessary.

VI. Concluding Remarks

We studied the capacity of a multiuser MIMO downlink channel without any initial assumption of channel knowledge. We gave a complete transmission strategy, through which the BS and all users acquire necessary channel information. Sum rate analysis shows the achievability of significant multiplexing gain with this scheme in both UL and DL directions.

Acknowledgments

Eurecom's research is partially supported by its industrial members: BMW Group Research & Technology, Bouygues Telecom, Cisco, Hitachi, ORANGE, SFR, Sharp, STMicroelectronics, Swisscom, Thales. The research work leading to this paper has also been partially supported by the European Commission under the ICT research network of excellence NewCom++ of the 7th Framework programme and by the French ANR project APOGEE.

References

[1] G. Caire and S. Shamai (Shitz), "On the achievable throughput of a multiantenna gaussian broadcast channel," *IEEE Trans. on Information Theory*, vol. 49, pp. 1691–1706, July 2003.

[2] N. Jindal and A. Goldsmith, "Dirty paper coding versus tdma for mimo broadcast channels," *IEEE Trans. on Information Theory*, vol. 51, pp. 1783–1794, May 2005.

[3] N. Jindal, "Mimo broadcast channels with finite rate feedback," *IEEE Trans. on Information Theory*, vol. 52, pp. 5045–5060, November 2006.

[4] D. Gesbert, M. Kountouris, J. R. W. Heath, C. B. Chae, and T. Salzer, "From single user to multiuser communications: Shifting the mimo paradigm," *IEEE Sig. Proc. Magazine*, 2007.

[5] I. E. Telatar, "Capacity of multi-antenna gaussian channels," *European Transactions on Telecommunications*, pp. 585–595, November 1999.

[6] T. L. Marzetta, "How much training is required for multiuser mimo?," in *Proc. Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, USA*, November 2006, pp. 359–363.

[7] T. Marzetta and B. Hochwald, "Capacity of a mobile multiple-antenna communications link in rayleigh flat fading," *IEEE Trans. on Information Theory*, vol. 45, pp. 139–157, January 1999.

[8] N. Jindal, "A high snr analysis of mimo broadcast channels," in *Proc. IEEE Int. Symp. Information Theory, Adelaide, Australia*, 2005, pp. 2310–2314.

[9] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. on Information Theory*, vol. 52, pp. 3936–3964, September 2006.

[10] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple-antenna wireless links?," *IEEE Transactions on Information Theory*, vol. 49, pp. 2058–2080, April 2003.

[11] L. Zheng and D. N. C. Tse, "Communication on the grassmann manifold: A geometric approach to the noncoherent multiple-antenna channel," *IEEE Trans. on Information Theory*, vol. 48, pp. 359–383, February 2002.

[12] S. Murugesan, E. Uysal-Biyikoglu, and P. Schniter, "Optimization of training and scheduling in the non-coherent mimo multiple-access channel," *EEE Journal on Selected Areas in Communication*, vol. 25, pp. 1446–1456, September 2007.

[13] D. Tse and P. Viswanath, *Fundamentals of Wireless Communications*, Cambridge, U.K. Cambridge Univ. Press, 2005.