Hybrid Pilot/Quantization based Feedback in Multi-Antenna TDD Systems

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Abstract— The communication between a multiple-antenna transmitter and multiple receivers (users) with either a single or multiple-antenna each can be significantly enhanced by providing the channel state information at the transmitter (CSIT) of the users, as this allows for scheduling, beamforming and multiuser multiplexing gains. The traditional view on how to enable CSIT has been as follows: In time-division duplexed (TDD) systems, uplink (UL) and downlink (DL) channel reciprocity allows for the use of a training sequence in any given uplink slot, which is exploited to obtain an uplink channel estimate. This estimate is in turn recycled in the next downlink slot. In frequency-division duplexed (FDD) systems, which lack the UL and DL reciprocity, the CSIT is provided via the use of a dedicated feedback link of limited capacity between the receivers and the transmitter. In this paper, we focus on TDD systems and put this classical approach in question. In particular, we show that the traditional TDD setup above fails to fully exploit the channel reciprocity in its true sense. In fact, we show that the system can benefit from a combined CSIT acquisition strategy mixing the use of limited feedback and that of a training sequence. We demonstrate the potential of our approach in terms of improved CSIT quality under a global training and feedback resource constraint.

I. INTRODUCTION

Multiple-antenna transmitters and receivers are instrumental to optimizing the performance of bandwidth and power limited wireless communication systems. In the downlink (DL), in particular, the communication between a multiple-antenna enabled base station (BS) and one or more users with either a single or multiple antenna each can be significantly enhanced through the use of scheduling, beamforming and power allocation algorithms, be it in single user or multi-user mode (spatial division multiplexing). To allow for beamforming and/or multi-user multiplexing capability, the BS transmitter must however be informed with the channel state information of each of the served users [1], [2], except when the number of users reaches an asymptotic (large) regime in which case random opportunistic beamforming scheme can be exploited [3], [4]. This has motivated the proposal of many techniques for providing the channel state information at the transmitter (CSIT) in an efficient manner. Proposals for how to provide CSIT roughly fall in two categories depending upon the chosen duplexing scheme for the considered wireless network. In the case of time-division duplex (TDD) systems, it was proposed to exploit the reciprocity of the uplink (UL) and DL channels, so as to avoid the use of any feedback channel [5], [6]. The way reciprocity is exploited, in the current TDD systems, is through the use of a training sequence sent by the user on the UL, based on which the BS first builds an estimate of the UL channel which in turn serves as an estimate for the downlink channel in the next DL slot [5]. In frequency-division duplex (FDD) scenarios, the lack of channel reciprocity motivates instead the use of a dedicated feedback link in which the user conveys the information about the estimated downlink channel, back to the BS. Recently, several interesting strategies have been devised for how to best use a limited feedback channel and still provide the BS with exploitable CSIT [7], [8], [9].

Although in the past, the balance has weighed in the favor of FDD systems when choosing a duplexing scheme (in part because of heavy legacy issues in voice oriented 2G networks and also because of interference management between UL and DL), current discussions in the standardization groups indicate an increasing level of interest for TDD for upcoming wireless data-access networks (e.g.WiMax, etc.), caused partly by its advantages in maintaining system flexibility with respect to UL and DL traffic loads, and mostly because TDD systems are seen as more efficient in providing the CSIT required by several MIMO DL schemes, thanks to the channel reciprocity.

In this paper, we focus on the problem of CSIT acquisition in a TDD system. We take a step back and shed some critical light on the traditional approach above consisting in exploiting channel reciprocity via the use of training sequences exclusively. In fact we show that this approach fails to fully exploit the channel reciprocity. The key idea is as follows: when sending a training sequence in the UL of a traditional TDD system, the user allows the BS to estimate the channel by a classical channel estimator (it can be a least-square (LS) estimator or minimum mean square error (MMSE) based, just to name a few). However, note that the user itself has the knowledge of the channel coefficients (from the previous DL transmission slot) but, regretfully, does not exploit that knowledge in order to facilitate the CSIT acquisition by the BS. Instead, it uses this knowledge only for its own detection purposes.

Interestingly, by contrast, in FDD systems, the user exploits its channel knowledge by quantizing the channel and sending the result over a dedicated feedback link (actually UL bandwidth can be used for this feedback along with UL data transmission). In this case, UL training is used by the BS solely for UL data detection as this UL training cannot give any direct information to the BS about the DL channel coefficients. Actually in FDD systems, UL and DL portions of the bandwidth are normally selected quite apart and hence

channel realizations over respective bandwidths can be safely assumed to be independent of each other, so there is no channel reciprocity in FDD systems.

We propose that, while dealing with TDD systems, there is a clear opportunity to combine both forms of CSIT acquisition strategies. As a result, we investigate a scheme mixing classical channel estimation using training with the use of dedicated limited feedback carrying information about the quantized channel. This gives us a framework for fully utilizing the channel reciprocity in a TDD setup and it improves the classical trade-off between CSIT estimation accuracy and training/feedback resource usage. We characterize the optimal CSIT acquisition structure and then propose a sub-optimal outage rate based approach which helps us to optimize the fixed resource partitioning between training and quantized feedback phases. The results obtained confirm our intuition and demonstrate the potential of this hybrid (mix of training and quantized feedback) approach for upcoming TDD systems.

The paper is structured as follows: System model is given in section II, followed by classical CSIT acquisition in TDD and FDD systems in section III. Optimal training and feedback combining strategy is explained in section IV with outage based optimization framework in section V. Simulation results appear in section VI followed by conclusions in section VII.
Notation: $\mathbb{E}$ denotes statistical expectation. Lowercase letters represent scalars, boldface lowercase letters represent vectors, and boldface uppercase letters denote matrices. $\mathbf{A^T}$, $\mathbf{A}^\dagger$, $\mathbf{A}^{-1}$, $\mathbf{A}^+$ denote the transpose, the Hermitian, the inverse and the pseudo-inverse of matrix $\mathbf{A}$, respectively. For a vector $\mathbf{a}$, $||\mathbf{a}||$ and $\bar{\mathbf{a}}$ represent, respectively, its norm and unit-norm direction vector so that $\mathbf{a} = ||\mathbf{a}||\bar{\mathbf{a}}$.

II. SYSTEM MODEL AND CSIR ACQUISITION

We consider the two way communication in a cell between a single BS, equipped with M antennas, and a single antenna mobile user. The channel $\mathbf{h} \in \mathcal{C}^M$ is assumed to be flat-fading with independent complex Gaussian zero-mean unit-variance entries, where $\mathcal{C}^M$ represents the M-dimensional complex space. We assume block fading channel so each channel realization stays constant for T symbol intervals [10] which can be accordingly partitioned between UL and DL data transmissions. Extensions to OFDM(A) and multi-user systems are possible and straightforward.

The goal of this paper is to provide a reliable estimate of the DL channel to the BS, which in turn can be used for beamforming/precoding purposes. However we focus on the acquisition issue of the channel knowledge and not about its use in MIMO transmission schemes.

In the downlink, the received signal at the user for L symbol intervals is given by

$$\mathbf{y_{dl}} = \mathbf{X_{dl}h} + \mathbf{n_{dl}}, \tag{1}$$

where $\mathbf{X_{dl}} \in \mathcal{C}^{L \times M}$ is the signal transmitted by the BS during L symbol intervals (satisfying BS power constraint), $\mathbf{n_{dl}} \in \mathcal{C}^L$ is the complex Gaussian noise with independent zero-mean

unit-variance entries and $\mathbf{y_{dl}} \in \mathcal{C}^L$ is the observation sequence during this L-length interval.

If we want to use the above DL system equation for channel estimation, for identifiability of M-dimensional channel at the user's side, the length of the transmitted data (which would be the training sequence in this case) should be larger than M, the number of BS transmit antennas. Based upon the knowledge of the transmitted data $\mathbf{X_{dl}}$ (the training sequence) and the observed sequence $\mathbf{y_{dl}}$, the user can estimate the DL channel $\mathbf{h}$ using various techniques. The LS estimate would be

$$\hat{\mathbf{h}}_{\mathbf{LS}} = \mathbf{X_{dl}^+ y_{dl}}. \tag{2}$$

The user can make a better channel estimate using MMSE criteria, and the estimate is given by

$$\hat{\mathbf{h}}_{\mathbf{MMSE}} = \mathbf{X_{dl}^\dagger} \left(\mathbf{X_{dl} X_{dl}^\dagger} + \mathbf{I_L} \right)^{-1} \mathbf{y_{dl}}. \tag{3}$$

In the above equation, $\mathbf{I_L}$ is the identity matrix of L dimensions.

III. CLASSICAL CSIT ACQUISITION IN FDD AND TDD

We now briefly review the classical approaches for acquiring CSIT at the BS in FDD and TDD systems.

A. FDD Systems

In FDD systems, the mobile station first obtains the DL channel estimate $\hat{\mathbf{h}}$ as described in the previous section. Then it needs to quantize the channel using some pre-selected codebook. Quantization is a well-studied subject and a lot of well developed techniques are available in literature and have been implemented in practical wireless systems. If Q denotes the quantization function, then for the DL channel estimate $\hat{\mathbf{h}}$, its quantized version (which is actually the index of the closest channel code in the codebook) is given by $Q(\hat{\mathbf{h}})$. Afterwards user maps this index (sequence of bits) into a sequence of constellation symbols, using the mapping function denoted by S. Hence the feedback of the DL channel would be

$$\mathbf{x_q} = S(Q(\hat{\mathbf{h}})), \tag{4}$$

where $\mathbf{x_q} \in \mathcal{C}^{1 \times T_q}$ is the T_q dimensional row vector of normalized constellation symbols representing the information of the quantized channel. As we explained earlier, the bandwidths allocated for UL and DL transmissions in FDD systems are quite far apart and channel realizations are assumed to be independent. Hence to decode the feedback properly, BS first needs to estimate the UL channel ($\mathbf{h_u} \in \mathcal{C}^M$). If $\mathbf{x_u} \in \mathcal{C}^{1 \times T_a}$ denotes the normalized training sequence of length T_a in the UL direction, the signal received at the BS for T_a symbol intervals is given by

$$\mathbf{Y_a} = \sqrt{P} \, \mathbf{h_u x_a} + \mathbf{N_a}, \tag{5}$$

where $\mathbf{N_a} \in \mathcal{C}^{M \times T_a}$ represents the spatio-temporally white Gaussian noise and $\mathbf{Y_a} \in \mathcal{C}^{M \times T_a}$ is the received signal at M antennas of the BS during this T_a-length training interval. P represents the user's peak power constraint. After observing $\mathbf{Y_a}$, the BS can make an estimate $\hat{\mathbf{h}}_\mathbf{u}$ of the UL channel $\mathbf{h_u}$,

knowing $\mathbf{x_a}$. Same estimation techniques like LS or MMSE as described in the previous section can be applied.

Now if user transmits the quantized channel feedback $\mathbf{x_q}$ on the UL, BS can decode this information, having an estimate $\hat{\mathbf{h}}_\mathbf{u}$ of the UL channel $\mathbf{h_u}$.

$$\mathbf{Y_q} = \sqrt{P}\,\mathbf{h_u x_q} + \mathbf{N_q}, \qquad (6)$$

where $\mathbf{Y_q}$ and $\mathbf{N_q}$ are $M \times T_q$ matrices of the received signal and the noise respectively at M antennas of the BS during this explicit T_q length feedback interval. So based upon the estimate of UL channel $\mathbf{h_u}$ and the received feedback $\mathbf{Y_q}$, BS tries to recover the DL channel feedback (quantized version, $\mathbf{x_q}$) using the optimum (although relatively complex) maximum likelihood sequence estimation technique.

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} ||\mathbf{Y_q} - \sqrt{P}\,\hat{\mathbf{h}}_\mathbf{u} S(Q(\mathbf{h}))||^2 \qquad (7)$$

Actually the search space will be restricted to the codebook, hence the BS, at best, can estimate the quantized version of the channel.

B. TDD Systems

If the communication system is operating under TDD mode, DL and UL channels are reciprocal, hence $\mathbf{h_u} = \mathbf{h}$. So if a user transmits pilot sequence on the UL (like eq. (5)), the simple (UL) channel estimation at the BS furnishes CSIT due to UL and DL channel reciprocity. In the past, this has been the classical way of getting CSIT in TDD systems [5], [6]. We show that this CSIT acquisition through pilots alone, although very simple, is not the optimal resource utilization for TDD systems.

IV. OPTIMAL TRAINING AND FEEDBACK COMBINING IN TDD SYSTEMS

Our system of interest operates under TDD mode and to improve the CSIT quality is the issue that we focus on. The classical training based only CSIT acquisition ignores the fact that user knows the DL channel and CSIT acquisition based only on the quantized channel feedback does not use channel reciprocity whereas in TDD systems both can be exploited at the same time.

We propose a novel hybrid two stage CSIT acquisition strategy which exploits the channel reciprocity and user's channel knowledge at the same time. Working under a constraint of fixed resource available for CSIT acquisition (T_{fb} symbol intervals and user's power constraint of P), our strategy consists of dividing this interval in two phases. The first stage of this hybrid approach is the transmission of training sequence from the user to the BS for T_a symbol intervals and the received signal will be

$$\mathbf{Y_a} = \sqrt{P}\,\mathbf{h x_a} + \mathbf{N_a}. \qquad (8)$$

(See eq. (5) for the dimensions of all parameters.)
The optimal training based estimate based upon the observed signal $\mathbf{Y_a}$, knowing $\mathbf{x_a}$, will be

$$\hat{\mathbf{h}}_\mathbf{a} = \arg\min_{\mathbf{h}} ||\mathbf{Y_a} - \sqrt{P}\,\mathbf{h x_a}||^2 \qquad (9)$$

The second stage consists of transmission of quantized channel (digital feedback), already known at the user, for T_q symbol intervals and the received signal will be

$$\mathbf{Y_q} = \sqrt{P}\,\mathbf{h x_q} + \mathbf{N_q}, \qquad (10)$$

(See eq. (6) for the dimensions of all parameters.)
where $\mathbf{x_q} = S(Q(\mathbf{h}))$. This equation reveals the intriguing aspect that BS needs to acquire $\mathbf{h}$. Now $\mathbf{h}$ behaves as channel and also appears in $\mathbf{x_q}$. BS can try to decode only the quantized channel information based upon the knowledge of $\hat{\mathbf{h}}_\mathbf{a}$, eq. (9), obtained through training as

$$\hat{\mathbf{h}}_\mathbf{q} = \arg\min_{\mathbf{x_q}} ||\mathbf{Y_q} - \sqrt{P}\,\hat{\mathbf{h}}_\mathbf{a} \mathbf{x_q}||^2. \qquad (11)$$

The optimal CSIT will be obtained by the joint estimation and detection (of $\mathbf{h}$ and $\mathbf{x_q}$) based upon the observation of $\mathbf{Y_a}$ and $\mathbf{Y_q}$, knowing $\mathbf{x_a}$, involving an optimal split between the training and the quantized feedback phases (constrained as $T_a + T_q = T_{fb}$).

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}, \mathbf{x_q}, T_a} || \,[\mathbf{Y_a}\ \mathbf{Y_q}] - \sqrt{P}\,\mathbf{h}[\mathbf{x_a}\ \mathbf{x_q}]\, ||^2 \qquad (12)$$

The optimal solution requires a double minimization and does not seem to bear a closed form expression for $\hat{\mathbf{h}}$. Therefore accepting some sub-optimalities, we give a framework in the next section, based upon the idea of outage rate, which allows us to obtain a simplified solution for the hybrid approach.

V. OUTAGE BASED TRAINING AND FEEDBACK PARTITIONING

A. Definitions and Initial Setup

In this section, we give a strategy, under some assumptions, which allows optimizing the partitioning of the feedback resource between the training and the quantized feedback.

For the digital feedback part, we employ vector quantization rather than scalar quantization of individual channel coefficients. We restrict the user to quantize the unit-norm channel direction vector and feed it back. In single-user scenarios, channel direction information (CDI) is sufficient to beamform toward user which focuses all energy in the user's direction. Even in multi-user broadcast scenarios, CDI plays the key role as BS can decide to beamform to selected users (maximum ratio combining) or it can try to beamform so as to render the zero interference at each of the selected users (so called zero-forcing strategy) and all of this just requires the CDI of the selected users. For the quantization of channel directions, it has been shown that Grassmannian codebooks are the optimal choice but the formation of such codebooks is quite complicated from medium to large sized codebooks [11]. On the other hand, random vector quantization (RVQ) codebooks are easy to obtain, they have been shown to achieve very good performances and very nice closed form expressions for the quantization error are also known [8], [12]. So our analysis assumes the use of RVQ codebooks for CDI quantization.

The solution for the optimal CSIT estimate, $\hat{\mathbf{h}}$ in eq. (12), requires joint estimation and detection along with an optimization over the resource split between training and feedback.

Even if we focus separately on training based estimate $\hat{\mathbf{h}}_\mathbf{a}$ (given in eq. (9)) and digital feedback based estimate $\hat{\mathbf{h}}_\mathbf{q}$ (given in eq. (11)), two questions arise: i) how the CSIT acquisition interval T_{fb} should be split between training and feedback?, and ii) how the two estimates should be combined to get the final estimate?

Embracing some sub-optimalities, we propose the use of quantized feedback based estimate $\hat{\mathbf{h}}_\mathbf{q}$ as the final CSIT estimate $\hat{\mathbf{h}}$. Apparently it gives the impression that training based estimate $\hat{\mathbf{h}}_\mathbf{a}$ goes wasted but in reality quantized feedback $\mathbf{x_q}$, which provides $\hat{\mathbf{h}}_\mathbf{q}$, is decoded based upon this training based estimate $\hat{\mathbf{h}}_\mathbf{a}$. This answers the 2nd question above. For the 1st question, we have to wait till the next subsection.

The choice of selecting the digital feedback as the final CSIT estimate makes it clear that the quantization error will always sneak in the final estimate. Apart from this small error (depends upon how many bits are used for quantization), the incorrect detection of digital feedback will wreck a havoc for the final CSIT estimate. As the channel stays constant for each feedback interval, we are dealing with slow fading channels for which deep channel fades (causing outage) are the typical error events [13].

This two stage outage based strategy consists of first providing a training based estimate to the BS in the training interval of length T_a. In the second interval of length T_q, user sends the quantized version (using RVQ) of its unit-norm channel direction vector which we assume to be perfectly known at the user. We suppose that channel statistics are such that the user can send b bits per symbol interval with an outage probability of ϵ. So b is the ϵ-outage rate of the UL channel (see [13] for details). That means the user can send a total of $B = bT_q$ feedback bits at ϵ outage. Now the user should map B bits over T_q constellation symbols to transmit on the UL channel. Although the constellations used in practice have 2^b points where b must be a positive integer, for the time being we allow positive real values for b.

We define the CSIT error as the $\sin$ of the angle (θ) between the true channel direction vector $\bar{\mathbf{h}}$ and the BS estimated direction vector $\hat{\bar{\mathbf{h}}}$, denoted as $\sigma^2(\mathbf{h}, \hat{\mathbf{h}})$.

$$\sigma^2(\mathbf{h}, \hat{\mathbf{h}}) = \sin^2(\theta) = 1 - \cos^2(\theta) = 1 - |\bar{\mathbf{h}}^\dagger \hat{\bar{\mathbf{h}}}| \qquad (13)$$

When the channel is not in outage and the BS is able to decode the feedback correctly, there is only quantization error. On the other hand, when the channel is in outage (happens with probability ϵ), BS cannot decode the feedback information. Hence average CSIT error σ^2 can be written as

$$\begin{aligned} \sigma^2 &= (1-\epsilon)\sigma_q^2 + \epsilon\sigma_{\bar{\mathbf{h}}\neq\hat{\mathbf{h}}}^2(\mathbf{h}, \hat{\mathbf{h}}) \\ &\leq (1-\epsilon)\sigma_q^2 + \epsilon \\ &\leq \sigma_q^2 + \epsilon, \end{aligned} \qquad (14)$$

where σ_q^2 represents the mean-square quantization error and $\sigma_{\bar{\mathbf{h}}\neq\hat{\mathbf{h}}}^2(\mathbf{h}, \hat{\mathbf{h}})$ represents the mean-square CSIT error when the channel is in outage (which means feedback error occurs). The

first inequality is obtained as $\sigma_{\bar{\mathbf{h}}\neq\hat{\mathbf{h}}}^2(\mathbf{h}, \hat{\mathbf{h}})$ is upper-bounded by 1 and the second inequality is obtained by adding $\epsilon\sigma_q^2$.

We select this upper bound of mean-square CSIT error as the performance metric and minimize it following the constraints on the total feedback interval and user's power constraint, obtaining the optimal values for T_a, T_q and b (the number of bits/symbol - this parameter governs the constellation size and hence quantization error). The results thus obtained show the superiority of this hybrid approach over the classical training based only CSIT.

For RVQ, the exact expression for the mean-square quantization error σ_q^2 has been given in [12], [8] as

$$\sigma_q^2 = 2^B \beta\left(2^B, \frac{M}{M-1}\right), \qquad (15)$$

where B is the total number of bits used for feedback (i.e. codebook has 2^B codes) and β represents the beta function which is defined in terms of Gamma function as $\beta(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$. But we'll use a very simple and tight upper bound given in reference [8]

$$\sigma_q^2 \leq 2^{\frac{-B}{M-1}}. \qquad (16)$$

Putting the value of σ_q^2 using $B = bT_q$ in eq. (14), we get the upper bound of average CSIT error as

$$\sigma^2 \leq 2^{\frac{-bT_q}{M-1}} + \epsilon. \qquad (17)$$

This equation shows us the basic trade-off involved. If more outage is allowed in the system, the outage rate b will increase allowing the user to select a larger codebook (with 2^B code words where $B = bT_q$) but in this case final CSIT estimation error will be plagued by outages as for a lot of realizations BS won't be able to decode the feedback correctly. On the other hand, if almost no outage is allowed, this condition will ask the user to transmit at a pessimistic rate b corresponding to very bad channel conditions. Hence there will be fewer code words in the codebook and, in this case, CSIT estimate at the BS will be of poor quality due to large quantization error.

B. Training and Feedback Resource Split Optimization

Pilot sequence transmission from the user to the BS for an interval of length T_a, given in eq. (8), can be equivalently written in a simplified form as

$$\mathbf{y_a} = \sqrt{PT_a}\,\mathbf{h} + \mathbf{n_a}, \qquad (18)$$

where P is the user's power constraint and $\mathbf{y_a}, \mathbf{h}, \mathbf{n_a}$ are the received signal, the channel vector and the noise respectively, all column vectors of dimension M. BS can make MMSE estimate $\hat{\mathbf{h}}_\mathbf{a}$ of the channel $\mathbf{h}$ as

$$\hat{\mathbf{h}}_\mathbf{a} = \frac{\sqrt{PT_a}}{PT_a + 1}\mathbf{y_a}. \qquad (19)$$

As channel entries are i.i.d. standard Gaussian, MMSE estimation error $\tilde{\mathbf{h}}_\mathbf{a} = \mathbf{h} - \hat{\mathbf{h}}_\mathbf{a}$ has also i.i.d. Gaussian distributed

entries as $\tilde{\mathbf{h}}_{\mathbf{a}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_a^2 \mathbf{I_M}\right)$, where $\mathbf{I_M}$ represents the M-dimensional identity matrix and mean-square estimation error per channel entry σ_a^2 is given by

$$\sigma_a^2 = \frac{1}{PT_a + 1}. \qquad (20)$$

Similarly the estimate $\hat{\mathbf{h}}_{\mathbf{a}}$ has i.i.d. Gaussian entries and is distributed as $\hat{\mathbf{h}}_{\mathbf{a}} \sim \mathcal{CN}\left(\mathbf{0}, \frac{PT_a}{PT_a+1}\mathbf{I_M}\right)$.

Now we focus our attention on the digital feedback interval of the CSIT acquisition, given in eq. (10). The signal received during one symbol interval of this phase is given by

$$\mathbf{y_q} = \sqrt{P}\,\mathbf{h}x_q + \mathbf{n_q}, \qquad (21)$$

where x_q represents the scalar constellation symbol transmitted by the user and $\mathbf{y_q}, \mathbf{h}, \mathbf{n_q}$ are M-dimensional column vectors representing respectively the observed signal, channel and noise for this particular symbol interval. To decode this information, BS uses the estimate $\hat{\mathbf{h}}_{\mathbf{a}}$ that it developed during the training phase. So the above equation can be written as

$$\mathbf{y_q} = \sqrt{P}\,\hat{\mathbf{h}}_{\mathbf{a}}x_q + \sqrt{P}\,\tilde{\mathbf{h}}_{\mathbf{a}}x_q + \mathbf{n_q}. \qquad (22)$$

Average signal-to-noise-ratio (SNR) at the BS with channel $\hat{\mathbf{h}}_{\mathbf{a}}$ known can be computed to be

$$\mathrm{SNR}(\hat{\mathbf{h}}_{\mathbf{a}}) = \frac{P||\hat{\mathbf{h}}_{\mathbf{a}}||^2}{P\sigma_a^2 + 1}. \qquad (23)$$

Putting the value of σ_a^2, SNR will become

$$\mathrm{SNR}(\hat{\mathbf{h}}_{\mathbf{a}}) = \frac{P||\hat{\mathbf{h}}_{\mathbf{a}}||^2}{\frac{P}{PT_a+1} + 1}. \qquad (24)$$

We can do the magnitude scaling of the estimated channel vector $\hat{\mathbf{h}}_{\mathbf{a}}$ so as to convert $||\hat{\mathbf{h}}_{\mathbf{a}}||^2$ in a standard chi-square random variable having $2M$ degrees of freedom (DOF), denoted as χ_{2M}^2. So the SNR becomes

$$\mathrm{SNR}(\chi_{2M}^2) = \frac{P^2 T_a}{2(P + PT_a + 1)}\chi_{2M}^2. \qquad (25)$$

This equation has been obtained by dividing and multiplying the right hand side of the preceding equation by $\frac{PT_a}{2(PT_a+1)}$ as $\frac{2(PT_a+1)}{PT_a}||\hat{\mathbf{h}}_{\mathbf{a}}||^2$ actually represents the sum of squares of $2M$ real independent zero-mean unit-variance Gaussian variables which is a standard chi-square random variable with $2M$ DOF.

ϵ, the outage probability of this channel corresponding to outage rate b, can be written as

$$\begin{aligned}
\epsilon &= \mathbb{P}\left[\log\left(1 + \mathrm{SNR}(\chi_{2M}^2)\right) \le b\right] \\
&= \mathbb{P}\left[\log\left(1 + \frac{P^2 T_a}{2(P + PT_a + 1)}\chi_{2M}^2\right) \le b\right], \quad (26)
\end{aligned}$$

where $\mathbb{P}$ denotes the probability of an event. This relation can be inverted to obtain the outage rate b corresponding to the outage probability ϵ, as given below

$$b = \log\left(1 + \frac{P^2 T_a}{2(P + PT_a + 1)}F^{-1}(\epsilon)\right), \qquad (27)$$

where $F^{-1}(.)$ is the inverse of the standard cumulative distribution function (CDF) of χ_{2M}^2 distributed variable.

Finding the above expression, which relates the outage rate of digital feedback with the training length T_a, we can put formally our optimization problem (the minimization of the mean-square CSIT error given in eq. (17)) as follows:

$$\min_{T_a, \epsilon}\left[2^{\frac{-b(T_{fb}-T_a)}{M-1}} + \epsilon\right] \qquad (28)$$

where we have used $T_{fb} = T_a + T_q$ and b is given by eq. (27). The constraints for this minimization are:

$$T_a \ge 1 \quad , \quad T_a \le T_{fb} \qquad (29)$$
$$\epsilon \ge 0 \quad , \quad \epsilon \le 1 \qquad (30)$$

Apart from these, the user has to take care of its UL power constraint. The analytical solution to the above optimization problem does not bear closed form expression but its numerical optimization is quite trivial.

VI. SIMULATION RESULTS

Our simulation environment consists of a BS with $M = 4$ antennas and a single user with a single antenna. The channel model is same as described in Section II. The feedback interval T_{fb} is fixed to 20 symbol intervals for all simulations. The optimization of the objective function described in the previous section gives us the values for the optimal training length T_a and the optimal outage rate b for various values of user's power constraint, which is equal to the UL SNR as noise at every BS antenna has been normalized to have unit variance. We have plotted the optimal values of training length T_a, corresponding values of quantized feedback interval T_q and outage rate b in Fig. (1).
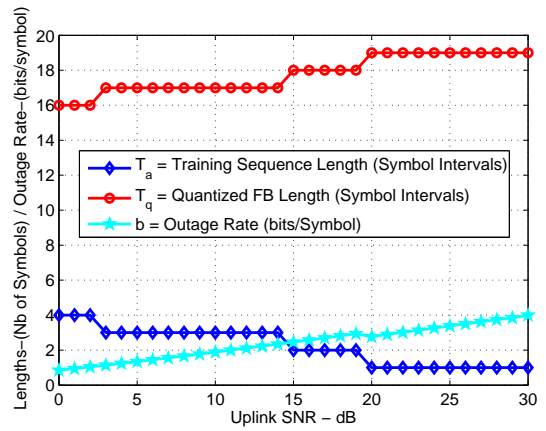

Fig. 1. Optimal Lengths and Outage Rate: $T_{fb} = 20$ and $M = 4$. With increase in SNR, both the length of quantized feedback interval T_q and the outage rate b increase gradually.

Knowing the values of b and T_q, computed based upon the optimal values of T_a and ϵ, allows us to compute the upper bound of the final CSIT error eq. (17). These values have been plotted in dB scale in Fig. 2. For comparison purpose, we have also plotted the CSIT error when classical training

spans the full interval reserved for CSIT acquisition. This plot clearly shows the interest of our hybrid two-staged CSIT acquisition strategy as, from medium to large SNR values, CSIT error incurred by this scheme is much less than the error obtained by training based only CSIT acquisition. At low SNR
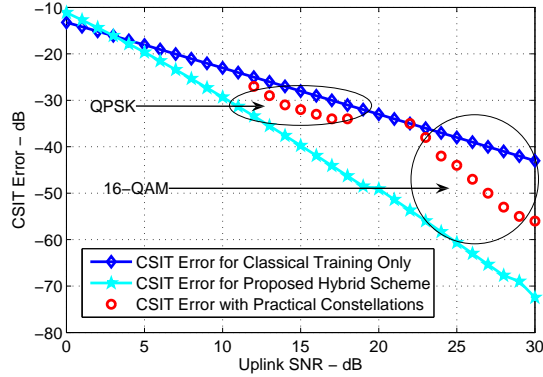

Fig. 2. Mean-Square CSIT Errors: $T_{fb} = 20$ and $M = 4$. Novel hybrid scheme performs much better than classical training based CSIT acquisition. Gains are significant even with naive use of practical constellations without any coding.

values, this two stage scheme performs worse than the training based traditional scheme. This happens because we have restricted our final estimate to come from the digital feedback. Here total feedback resource (SNR and T_{fb}) does not allow transmission of sufficient number of bits through the channel so quantization error is quite large. This gets aggravated due to poor training based estimate based upon which these bits are decoded, further degrading the performance. This degradation can be easily avoided by selecting an SNR threshold below which traditional training based scheme is employed.

A. Transmission with Practical Constellations

Although the plots show the potential of our hybrid approach over the classical training only scheme, with gains which are increasing in SNR, yet these results have been obtained using the real positive values for b. Constellations, used in practice, have number of points which are an integer power of 2. This practical limitation dictates that the ϵ-outage rate b should be an integer. The CSIT error, obtained by transmission of QPSK and 16-QAM symbols, has been plotted for certain SNR values. We remark that these constellations incur performance degradation as compared to the theoretical analysis. The reason is analysis might show that $b = 2.7$, in this case if we select smaller constellation i.e., QPSK ($b = 2$), we actually let the quantization error increase, if we select e.g. 16-QAM ($b = 4$), it will cause the outage probability to increase. In any case, increase in either the outage probability or the quantization error will cause to degrade the quality of the CSIT estimate at the BS.

This performance degradation can be avoided by selecting a larger constellation (higher b) but doing coding with the help of extra bits i.e., codebook size is not increased. In this way, all

extra bits are used as parity bits and help combat the channel noise and outages. This point is the focus of ongoing research and results will be presented elsewhere.

VII. CONCLUSIONS

We presented a novel approach of acquiring CSIT at the BS for the DL transmission in a reciprocal TDD MIMO communication system. Traditional CSIT acquisition in TDD systems relies on the use of training sequences in an UL slot. This approach fails to fully exploit the channel reciprocity by ignoring the shared knowledge of an identical channel between the BS and the user. Instead, we propose a strategy combining the use of a training sequence together with a limited feedback channel. For the transmission of the quantized feedback, ergodic rates are far beyond the reality due to stringent delay constraints. We characterize the optimal CSIT acquisition setup and give an outage-rate based approach which allows the optimum partitioning of resource between the training and the quantized feedback. Superior performance is demonstrated due to better exploitation of the reciprocity principle and CSIT quality improves significantly.

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