Cooperative Spectrum Sensing Technique Based on Sub Space Analysis for Cognitive Radio Networks

Bassem Zayen and Aawatif Hayar
Mobile Communications Group, Institut Eurecom
2229 Route des Cretes, P.O. Box 193, 06904 Sophia Antipolis, France
Email: {zayen, hayar}@eurecom.fr

***Abstract*—Recently, a new blind spectrum sensing technique based on signal space dimension estimation was developed for sensing the spectrum holes in the primary user's bands. The mean idea of this technique is that the number of significant eigenvalues of the covariance matrix of the received signal is directly related to the presence/absence of data in the signal. In this paper[1], we study the collaborative sensing as a means to improve the performance of the proposed spectrum sensing technique and show their effect on cooperative cognitive radio network. Specifically, we will present the performance evaluation and advantages of this method and propose an optimization method that compute only the first dominates eigenvalues in order to reduce the complexity of the proposed cooperative spectrum sensing algorithm. Simulations results and performances evaluation presented in this paper are based on experimental measurements captured by Eurecom RF Agile Platform.**

Keywords—Cognitive radio, cooperative spectrum sensing, sub space analysis, eigenvalues.

I. Introduction

Cognitive radio arises to be a tempting solution to spectral crowding problem by introducing the opportunistic usage of frequency bands that are not heavily occupied by licensed users [1]. One of the most important components of cognitive radio concept is the ability to measure, sense, learn, and be aware of the parameters related to the radio channel characteristics, availability of spectrum and power, user requirements and applications, and other operating restrictions [2]. In cognitive radio terminology, *primary users* can be defined as the users who have higher priority on the usage of a specific part of the spectrum. On the other hand, *secondary users*, which have lower priority, exploit this spectrum in such a way that they do not cause interference to primary users. Therefore, secondary users need to have cognitive radio capabilities, such as sensing the spectrum reliably to check whether it is being used by primary users, and to change radio parameters to exploit the unused part of the spectrum [3].

There are several spectrum sensing techniques that were proposed for cognitive radio. These techniques are mainly categorized in two family: Blind sensing techniques and signal specific sensing techniques. While the first don't need any prior knowledge about the transmitted signal, the second need some infirmations about the features of the signal such as carrier frequency, symbol period, modulation type, etc. This classification leads to decide whether one of these choices best fit the CR. The elaboration of sensing techniques that use some prior information about the transmitted signal is interesting in terms of performances. The most known signal specific sensing technique is the cyclostationary detector [4]. This detector offer high performances but needs a long time for computation since it have a high complexity. Others methods that exploit a recorded form of the covariance matrix are also derived in the literature [5]. In fact, by this way primary users are detected even in very low SNR. However this kind of detection is not interesting because only few transmitted signals are considered, that leads to consider a specific band to scan. On the other hand, completely blind sensing techniques that not consider any prior knowledge about the transmitted signal are more convenient to cognitive radio. Few methods that belong to this category were proposed, but all of them suffer from the noise uncertainty and fading channels. One of the most popular is the energy detector [6]. Despite its easy implementation and few complexities, the energy detector does not perform in low SNR and cannot differentiate between noise and signals. Moreover, this kind of detector will be inconvenient when the level of noise is completely unknown. Another blind technique was proposed in [7]. This technique exploits the dimension or the entropy of the received signal. Specifically, this method investigates the relationship between the behavior of the slope of signal dimension curve and the transition from an occupied band to an adjacent free band.

The estimation of traffic in a specific geographic area can be done locally, by one cognitive radio only, or information from different cognitive radios can be combined. In fact, cooperation is discussed as a solution to problems that arise in spectrum sensing due to noise uncertainty, fading, and shadowing. Cooperative sensing decreases the probability of miss-detections and the probability of false alarms considerably. In addition, cooperation can solve the hidden primary user problem and can decrease sensing time [8].

In this paper, we adopt the same framework to detect vacant sub-bands given in [7]. Specifically, we study the collaborative sensing as a means to improve the performance of the proposed spectrum sensing technique and show their effect on cooperative cognitive radio network. We will show that there is significant improvement in the performance for spectrum sensing in detecting a primary user by performing cooperative spectrum sensing, especially when the number of

[1]The work reported herein was partially supported by the European project SENDORA and the national project GRACE.

the cooperating cognitive users is increased in the network. In addition, we will present the the performance evaluation of this method by studying the complexity required for sensing. We will show that the major complexity of this method comes from the computation of the covariance matrix and the eigenvalues decomposition. Consequently, we propose a simplify method that compute only the first dominates eigenvalues in order to detect the presence of primary user.

This paper is organized as follows. After the presentation of spectrum sensing techniques based on signal space dimension estimation in Section II, the detection performances of this method are discussed in Section III. In Section IV, we present an optimization method to estimate eigenvalues of the covariance matrix and a comparison of the modified detector with the dimension analysis based detector is given. Finally, Section V concludes the paper.

II. Cooperative Spectrum Sensing Technique Based on Dimension Analysis

In this section, we give the main idea of the blind cooperative spectrum sensing technique based on dimension analysis. We consider a wireless cognitive radio network with a collection of users randomly distributed over the geographical area considered. By virtue of a scheduling protocol, K primary users and M pairs of secondary users are simultaneously selected from these users to communicate at a given time instant, while others remain silent. Spectrum sensing has been identified as key enabling secondary users to communicate and not interfere with primary user, by detecting in reliable way primary users signals. In fact, the individual secondary users make independent decisions about the presence of the primary signal in the frequency band that they are monitoring. Then, they communicate their decisions to a fusion center that makes the final decision about the occupancy of the band by fusing the decisions made by all cooperating radios. Cooperative sensing by using the spectrum sensing results from several cognitive users can be used to obtain more reliable spectrum sensing information. The channel model that will be used throughout this paper is given by (1):

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{s}$ of size $K \times 1$ being the transmit signal sent from primary users (i.e. k-th element is the transmitted signal by the k-th primary user), and $\mathbf{x}$ of size $M \times 1$ whose m-th element is the received signal by the m-th secondary user. $\mathbf{A}$ of size $M \times K$ being the transmission gain from a primary user k to a desired secondary user m, and $\mathbf{n}$ is the additive Gaussian noise. The goal of spectrum sensing is to decide between the following two hypotheses:

$$\mathbf{x} = \begin{cases} \mathbf{n} & \mathrm{H}_0 \\ \mathbf{A}\mathbf{s} + \mathbf{n} & \mathrm{H}_1 \end{cases} \tag{2}$$

We decide a spectrum band to be unoccupied if there is only noise in it, as defined in H_0; on the other hand, once there exists primary user signal besides noise in a specific band, as defined in H_1, we say the band is occupied. Let P_F be the probability of false alarm given by:

$$P_F = P(\mathrm{H}_1 \mid \mathrm{H}_0) = P(\mathbf{x} \text{ is present} \mid \mathrm{H}_0) \tag{3}$$

that is the probability of the spectrum detector having detected a signal under hypothesis H_0, and P_D the probability of detection expressed as:

$$\begin{aligned} P_D &= 1 - P_M = 1 - P(\mathrm{H}_0 \mid \mathrm{H}_1) \\ &= 1 - P(\mathbf{x} \text{ is absent} \mid \mathrm{H}_1) \end{aligned} \tag{4}$$

the probability of the detector having detected a signal under hypothesis H_1, where P_M indicates the probability of missed detection.

The mean idea of the blind spectrum sensing technique based on dimension analysis is that the number of significant eigenvalues of the covariance matrix of the received signal $\mathbf{x}$ is directly related to the presence/absence of data in the signal. The studied approach of this technique is based on the distribution of eigenvalues of the covariance matrix $\mathbf{R}$ given by:

$$\mathbf{R} = \Psi + \sigma^2 \mathbf{I} \tag{5}$$

where

$$\Psi = \mathbf{A}\mathbf{S}\mathbf{A}^H \tag{6}$$

with $\mathbf{S}$ denoting the covariance matrix of the transmitted signals, i.e., $\mathbf{S} = E\{\mathbf{s}\mathbf{s}^H\}$, and σ^2 denotes an unknown scalar. From our covariance matrix model given by equation (9), we define the following family of covariance matrix:

$$\mathbf{R}^{(m)} = \Psi^{(m)} + \sigma^2 \mathbf{I} \tag{7}$$

where $\Psi^{(m)}$ denotes a semi-positive matrix of rank m. Note that m ranges over the set of all possible number of DoF, i.e. $m = 1, ..., M$. Using linear algebra, we can express $\mathbf{R}^{(m)}$ as:

$$\mathbf{R}^{(m)} = \sum_{i=1}^{m} (\lambda_i - \sigma^2) \mathbf{V}_i \mathbf{V}_i^H \sigma^2 \tag{8}$$

where $\lambda_1, ..., \lambda_m$ and $\mathbf{V}_1, ..., \mathbf{V}_m$ are the eigenvalues and eigenvectors, respectively, of $\mathbf{R}^{(m)}$. Our goal here is to detect vacant sub-band over the spectrum band exploiting the significant eigenvalues. The significant eigenvalues are determined from the estimated covariance matrix $\hat{\mathbf{R}}$ defined by:

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}(t_i) \mathbf{x}(t_i)^H \tag{9}$$

where N is the length of the received signal by each secondary user and $\hat{\lambda}_1, \hat{\lambda}_2, ..., \hat{\lambda}_m$ are the eigenvalues of $\hat{\mathbf{R}}$. Therefore, if we have a noise (i.e. H_0 hypothesis), the covariance matrix of emitted signal $\mathbf{S}$ is equal to zero and we have $\hat{\lambda}_1 = \hat{\lambda}_2 = \cdots = \hat{\lambda}_m = \sigma^2$. On the other band, if we have data, the number of significant eigenvalues called p is less than the rank of the covariance matrix m and lower than 1. To determine the number of significant eigenvalues in [7], we use the Akaike information criterion (AIC) presented in [9]. In fact, the number of DoF, possibly the number of significant eigenvalues, is determined as the value of $m \in \{1, ..., M\}$ which minimizes AIC value.

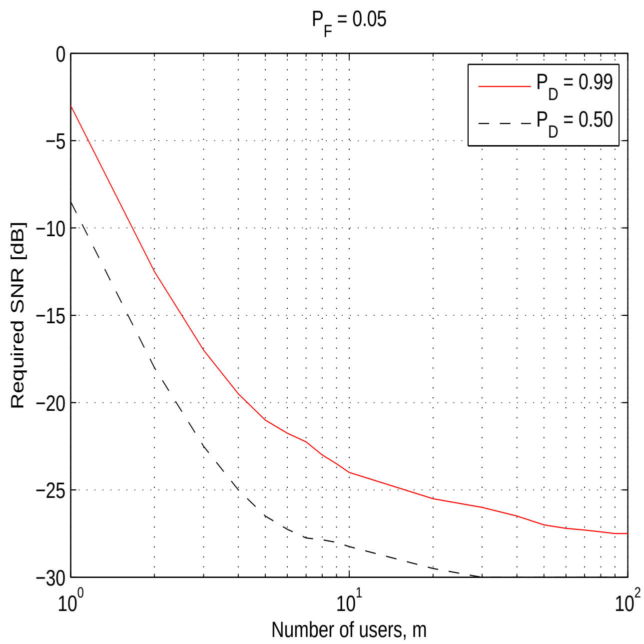

Fig. 1. The required SNR vs. the number of collaborating users m with $P_F = 0.05$ using blind spectrum detectors based on signal space dimension estimation for $P_D = 0.99$ and $P_D = 0.50$.

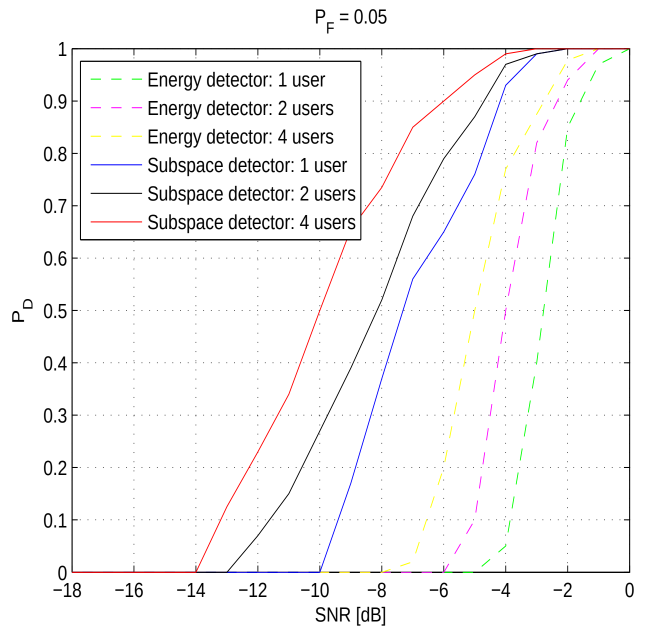

Fig. 2. Probability of detection vs. SNR for the blind cooperative spectrum detector based on signal space dimension estimation in comparison with energy detector for variable number of secondary users and $P_F = 0.05$.

III. Detection Performances of the Cooperative Sensing Method

In order to evaluate the performances of the spectrum sensing method, measurement by the RF Agile Platform at Eurecom are considered [10]. RF Agile Platform covers an RF band from 200MHz to 7.5GHz, with a maximum bandwidth of 20MHz. Concerning the transmitted power, the target is comparable to existing GSM terminals (+21dBm). On the receiver side, the noise figure is from 8 to 12dB, depending on the frequency band.

Fig. 1 provides plots of SNR versus the number of collaborating users m for different probability of detection. We assume here that the cooperating cognitive users use identical blind spectrum detector based on signal space dimension estimation. For each curve, the decision threshold is chosen such that $P_F = 0.05$. The results show that there is significant improvement in the performance for spectrum sensing in terms of SNR in detecting a primary user by performing cooperative spectrum sensing, especially when the number of the cooperating cognitive users is increased in the network. This is the main advantage gained by performing cooperative spectrum sensing by using the spectral sensing information obtained at the individual users. In fact, results indicate a significant improvement in terms of the SNR required for detection. In particular, to achieve $P_D = 0.99$, local spectrum sensing requires SNR $= -3$dB while collaborative sensing with $m = 10$ only needs SNR of -24dB for the individual users. In addition, we remark that the number of collaborating users increases with the value of probability of detection especially at low SNR region. As an example, having SNR $= -25$dB, more than 99% of the occupied sub-bands can be correctly detected with 17 users. On the other hand, for the same SNR, 50% of occupied sub-bands is detected with $m = 4$ collaborating users.

In Fig. 2 we present the detection performances of the cooperative spectrum sensing method for multiple users. This figure shows the probability of detection versus SNR ranging between -18dB and 0dB. The proposed detector is performed in comparison with a simple cooperative energy detector. In this case, the statistical test is the same as above and the $P_F = 0.05$. Each secondary user receives the same signal with different noise and SNR is the same for each users. Fig. 2 shows that the proposed detector outperforms the energy detector. For example, the probability of detection with 4 users is seen to rapidly approach 90% when SNR$= -6$ dB while the probability of detection with 2 users is seen to approach 78% for the same SNR. Performance gain of roughly 2dB is obtained from the cooperative sensing.

We study now the complexity required for this detector to derive their sensing algorithm. The complexity of this technique is computed according to the different steps of the algorithm, namely computation of the covariance matrix and its correspondent's eigenvalues. For the first part, noticing that the covariance matrix is block Toeplitz matrix and hermitian, then Nm^2 multiplications are sufficient. For the computation of eigenvalues, $O(m^3)$ multiplications are needed. AIC values are computed according to [7] with Nm^2 multiplications. The total complexity is therefore as follows:

$$C_{m,N} = 2Nm^2 + O(m^3) \tag{10}$$

The major complexity of this method comes from two parts: computation of the covariance matrix and the eigenvalues decomposition (i.e. $Nm^2+O(m^3)$). Usually, the dimension of covariance matrix is small. However, when cooperative sensing is established in order to detect the presence of primary users in very low SNR, the number of secondary user m must be high. That corresponds to a large covariance matrix. As consequence, the complexity $C_{m,N}$ is well increased. To avoid this problem, we propose a new eigenvalues detector. This sensing technique needs to compute only the first dominant vector from the covariance matrix using a low complex algorithm. This technique will present in the next section.

IV. Optimization Method

To avoid the complexity problem discussed in the last section, we propose here an optimization method that compute only the first dominates eigenvalues in order to reduce the complexity of the discussed cooperative spectrum sensing algorithm. The modified detector needs to compute only the first dominant vector of the covariance matrix using a low complex algorithm referred as Power method and given in [11]. If we consider the covariance matrix given by (9) with eigenvalues $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_m > 0$ corresponding to the orthonormal system of eigenvectors $\mathbf{V}_1, ..., \mathbf{V}_m$, we can estimate the first dominant eigenvalues λ_1 without going to the expense of computing the complete eigensystem of the covariance matrix $\mathbf{R}$.

Algorithm 1 Estimating the Largest Eigenvalues of the Covariance Matrix $\mathbf{R}$: Power Method

$k \leftarrow 0$
Initialize $\mathbf{U}$ ($||\mathbf{U}||_2 = 1$)
for $k = 0, 1, 2, ...$ **do**
  $k \leftarrow k + 1$
  $\mathbf{U} \leftarrow \mathbf{R}\mathbf{V}_1$
  $\lambda_1 \leftarrow \mathbf{V}_1^T\mathbf{U}$
  $\mathbf{V}_1 \leftarrow \mathbf{U}/||\mathbf{U}||_2$
end for

The power method for estimating the dominant eigenvalue of the covariance matrix is described in Algorithm 1. We first initialize the vector $\mathbf{U}$ of Euclidean norm unity. If $\mathbf{V}_1^T\mathbf{U}$ is pathologically small compared to some of the numbers $\mathbf{V}_i^T\mathbf{U}$ $(i > 1)$, then it will take many iteration k to have a good approximation of λ_1. In the other hand, if λ_2 is very near to λ_1, the final rate of convergence will be slow.

Algorithm 2 describe the modified blind cooperative spectrum sensing algorithm. In the first step, we initialize the position of the data signal to zero. Then, we scan the spectrum band of the received signal with the mean of sliding window [12], [7]. The number of the sliding windows is denoted by $nw = \frac{N}{T}$ where N is the size of the spectrum band and T is the size of windows. For the first analysis window, we compute the largest eigenvalue and the dominant eigenvector of the corresponding covariance matrix $\mathbf{R}_n$ using

Algorithm 2 Modified Signal Space Dimension Estimation Algorithm

$l \leftarrow 0$ ▷ Position of data signal
for $n = 1 : nw$ **do**
  Estimating λ_1 and $\mathbf{V}_1$ of $\mathbf{R}_n$
  for $k = 2 : m$ **do**
    $\mathbf{R}_n \leftarrow \mathbf{R}_n - \lambda_{k-1}\mathbf{V}_{k-1}\mathbf{V}_{k-1}^H$
    Estimating λ_k and $\mathbf{V}_k$ of $\mathbf{R}_n$
  end for
  $d \leftarrow \lambda_1 - \lambda_m$
  if $d > d_{max}$ **then**
    $l \leftarrow n$ ▷ Detection of data signal
  else break
  end if
end for

Algorithm 1. We compute after that eigenvalues and eigenvectors of $\mathbf{R}_n - \lambda_{k-1}\mathbf{V}_{k-1}\mathbf{V}_{k-1}^H$ for $k = \{1, 2, ..., m\}$. Once we get all significant eigenvalues, we compare $\lambda_1 - \lambda_m$ and the threshold d_{max}: If it is negative, the analysis window contain a noise, else we have an occupied window at $l = n$. Then, the analysis window will be shifted by T samples and we make the same approach for all analysis windows (i.e., $nw = \frac{N}{T}$) till the end of the band.

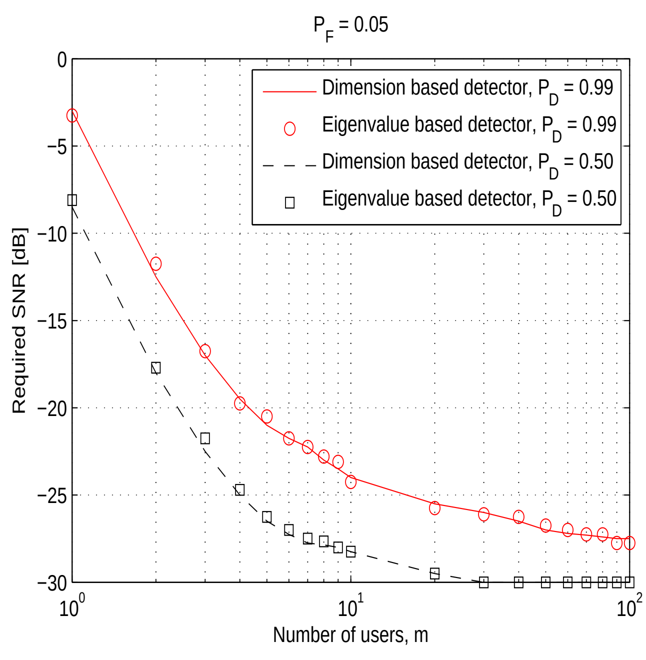

Fig. 3. The required SNR vs. the number of collaborating users m with $P_F = 0.05$ for the two blind spectrum detectors for $P_D = 0.99$ and $P_D = 0.50$.

We notice that, for this detector the presence of signal is depicted according to the criterion $\lambda_1 - \lambda_m$. In fact, we scan the sub-band of interest and compare its corresponding criterion to the threshold d_{max} in order to decide when the sub-band is free or not. The thresholds can be set based on some latest random matrix theories. The threshold is chosen based on the P_F and does not depend on noise power or some information about the transmitted signal. That's why eigenvalues based detector belong to the blind sensing technique in cognitive radio. For comparison, the required SNR versus the number of collaborating users m for the two blind spectrum detectors for $P_D = 0.99$ and $P_D = 0.50$ are plotted in Fig. 3. It is observed that the performances of the spectrum sensing method based on eigenvalues and the one based on dimension analysis are almost the same. Now, how about the complexity of the proposed algorithm?

The proposed sensing technique compute only the first dominant vector from the covariance matrix using the power method given by Algorithm 1. The contribution of the first eigenvalue and its correspondent eigenvector is then subtracted and the second dominant eigenvalue is derived. This process is repeated m times in order to evaluate the behavior of the first m dominant eigenvalues. Since the complexity of the power method is $O(m)$, and the total complexity of Algorithm 2 is:

$$C_{m,N} = Nm^2 + O(m) \tag{11}$$

From (10) and (11) we conclude that the proposed cooperative spectrum sensing method is less complex compared to the one based on dimension analysis. Generally, we found out that the proposed method outperforms the second one with 2 time complexity. The main advantage of the proposed detector is its simplicity; It computes the first dominant eigenvalues of the covariance matrix in an iterative manner. The complexity is then well decreased for large covariance matrix. In conclusion, this algorithm offers better complexity and does not affect significantly the sensing performances.

V. Conclusion

Spectrum is a very valuable resource in wireless communication systems. Cognitive radio, which is one of the efforts in utilization of the available spectrum more efficiently through opportunistic spectrum usage, become an exciting and promising concept. One of the important elements of the cognitive radio is sensing the available spectrum opportunities. In this paper, several blind cooperative spectrum sensing method based on dimension analysis is explained in detail. Specifically, collaborative sensing is considered as a solution to some problems in the presented sensing method. We showed that the complexity of this method comes from the computation of the covariance matrix and the eigenvalue decomposition. Furthermore, the spectrum sensing concept is re-evaluated by proposing a simplify signal space dimension estimation based detector that compute only the first dominates eigenvalues in order to detect the presence of primary user. The sensing detector of spectrum space create new opportunities and challenges for this type of cooperative spectrum sensing while it solves some of the traditional problems requiring the complexity to known.

References

[1] J. Mitola. Cognitive radio for flexible mobile multimedia communications. *IEEE International Workshop on Mobile Multimedia Communications*, 1999.

[2] J. Mitola. Cognitive radio: An integrated agent architecture for software defined radio. *Doctor of Technology, Royal Inst. Technol. (KTH)*, Stockholm, Sweden, 2000.

[3] A. Sahai and D. Cabric. Spectrum sensing: fundamentals limits and practical challenges. *IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Network(DySPAN)*, November 2005.

[4] A.V. Dantawate and G.B. Giannakis. Statistical tests for presence of cyclostationarity. *IEEE Transactions on Signal Processing*, 42(9):2355–2369, Sept. 1994.

[5] M.A. Lagunas-Hernandez, M.A. Rojas, and P. Stoica. New spectral estimation based on filterbank for spectrum sensing. *IEEE International Conference on Acoustics, Speech and Signal Processing*, April 2008.

[6] H. Urkowitz. Energy detection of unknown deterministic signals. *Proceeding of the IEEE*, 55(4):523–531, Apr. 1967.

[7] B. Zayen, A. Hayar and K. Kansanen. Blind spectrum sensing for cognitive radio based on signal space dimension estimation. *IEEE International Conference on Communications*, June 14-18 Dresden, Germany, 2009.

[8] S. M. Mishra, A. Sahai and R. W. Brodersen. Cooperative sensing among cognitive radios. *IEEE Int. Conf. Communications*, 4:16581663, Jun. 2006.

[9] H. Akaike. Information theory and an extension of the maximum likelihood principle. *Second International Symposium on Information Theory, Budapest*, pages 267–281, Budapest, Hungary, 1973.

[10] L. Maurer, T. Dellsperger, T. Burger, D. Nussbaum, R. Knopp and H. Callewaert. Medium term evolution for reconfigurable rf transceivers. *2006 Software "Defined Radio Technical Conference and Product Exposition"*, November 13-17 Orlando, 2006.

[11] D.P. O'Learly, G.W. Stewart, and J. Vandergraft. Estimating the largest eigenvalue of a positive definite matrix. *Mathematics of Computation*, 33:12891292, October 1979.

[12] B. Zayen, A. Hayar and D. Nussbaum. Blind spectrum sensing for cognitive radio based on model selection. *CrownCom*, May 15-17 Singapore, 2008.