Performance of Reed Solomon Error-Correcting Codes On Fading Channels

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Abstract - In this paper, our aim is to evaluate the performance of Reed Solomon (RS) codes when used to protect Asynchronous Transfer Mode (ATM) cells (424 bits) over fading channels. The well-known Gilbert-Elliott model (GE) represents the memory-fading channel. Its parameters like transition probabilities are related to physical quantities associated with fading statistics (Doppler normalized frequency, fading margin,..). Analytical expressions for code-word error probability (P_{cw}), residual error rate (R_{er}) and cell loss ratio (C_{LR}) are derived. The GE channel model is used to compare the performance of a large set of codes including those lengths that are 7,15,31,63 and 127. Contrarily to situations where binary symmetric channel is applied, the performance obtained is improved using short codes and imperfect interleaving. The Doppler normalized frequency and the variation of signal-to-noise ratio affect noticeably performance parameters. Also, theoretical and simulation results are compared to verify and to validate the accuracy of the GE model. In fact, simulation results obtained for a certain class of parameters seem to be in excellent agreement with analytical formulas.

I. Introduction

Since ATM has been internationally accepted as transport technique for broadband integrated services digital networks (B-ISDN), the interconnection between wireless systems and wired ATM networks becomes a critical issue.

Wireless ATM (WATM) is viewed as mandatory access technology to broadband networks in order to provide users with performed integrated services. In fact, users become more and more greedy; they intend to access to new multimedia services which require flexible bandwidth allocation, service type selection for a wide range of applications, high speed transmission and Quality of Service (QoS) on demand. WATM seems to possess all the capabilities required to successfully providing a networking platform [1][2][3]. In fact, the most important benefit of ATM is its flexibility.

Nevertheless, the integration of wireless networks into B-ISDN/ATM networks introduces a number of issues which arise from the inherent mismatch between wired and wireless links in terms of transmission speed and bit error rates (BER). In fact, wireless links are notorious for their unreliability and poor BER in the range of 10^{-4} and 10^{-5}, which varies considerably in time and space contrarily to ATM networks where the BER is around 10^{-9} [4].

The performance of wireless links is characterized by the inherent problems: 1) limited bandwidth, 2) high error rates and 3) high variation of 1 and 2.

Indeed, error control for high speed wireless ATM networks is an important research topic even wireless communication channels are highly affected by unpredictable factors like co-channel interference, adjacent channel interference, propagation path loss and multipath fading. Channel coding is used in communication systems to reduce the effect of noise introduced by the channel. To cope with this limitation, basic techniques should be used. In the case of WATM, FEC (Forward Error Correction) mechanism, convolutional coding, interleaving, multi-carrier modulation, diversity reception and ARQ (Automatic Repeat reQuest) retransmission are considered to target BER of service requirements. In fact, the challenge is to design efficient error control techniques that satisfy different QoS with the best use of available resources. In this context, we focus on FEC mechanism to achieve error control tasks based on using RS codes.

Actually, studies of the performance of error-correcting codes are most often based on situations where the channel is assumed to be memoryless.

The received signal envelope in wireless systems is Rayleigh distributed. It is described as a correlated Rayleigh signal. Due to the correlation, deep fades usually cause bursts of bit errors in a packet [5]. More realistic models for such kind of channels are those which capture memory as fading models, Markov models, etc. Among these models, GE model provides a useful discrete model where its parameters can be calculated from statistics of fades [6]. The simplicity of this model allows making performance analysis of error correcting codes through both simulations and exact or approximate calculations.

Our paper is organized as follows. Section 2 provides a description of GE model. Section 3 outlines the performance analysis where analytical expressions for code-word error probability (P_{cw}), residual error rate (R_{er}) and cell loss ratio (C_{LR}) are derived for RS error correcting codes. In section 4, the GE channel model is used to compare the performance of a large set of RS codes. Furthermore, simulation results are compared with analytical ones to evaluate the accuracy of the theoretical model. Finally, Section 5 provides concluding remarks and perspectives.

II. Wireless channel model or System model

Obviously, the error control technique that should be used is dependent on wireless channel characteristics. Furthermore, studies of the performance of error-correcting codes are most often based on situations where the channel is assumed to be memoryless. This allows performing easily theoretical analysis. In situations where memory is considered, the analytical results, which are very complicated, are few and performance tasks are then often achieved via simulations.

In this study, we assume that the channel error behaviour can be described by the well-known Gilbert-Elliott model. It is one of the simplest models proposed to capture the effects of memory channels. It has been defined by Gilbert [7] and Elliott [8].

In the GE model, the channel is viewed as a Binary Symmetric Channel (BSC) with memory determined by a two state Markov chain. It is a first-order discrete-time and stationary Markov chain. This model describes the channel error statistics and is shown schematically in Figure 2.1.

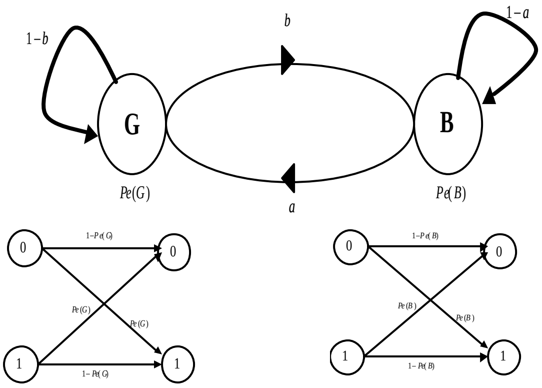

Figure 2.1: Gilbert-Elliott Model

The GE model contains two states, one state (state ‘*G*’ in Figure 2.1) is considered the ‘good’ state with low Additive White Gaussian Noise (AWGN) distributed error probability *Pe(G)* and the other state (state ‘*B*’ in Figure 2.1) is considered the ‘bad’ state where the error rate *Pe(B)* is high. The transition probabilities are denoted a and b . b is the transition probability from state ‘*G*’ to state ‘*B*’ and a is the transition probability from state ‘*B*’ to state ‘*G*’. In each state, the channel is represented by a BSC with bit error probabilities *Pe(G)* and *Pe(B)*.

Error bursts are generated with an average length Burst-Length = E(Burst-length) and are separated by long gaps.

The channel evolution is completely specified by the channel state transition matrix.

$$P = \begin{pmatrix} G \\ \\ B \end{pmatrix} \begin{bmatrix} 1-b & b \\ & \\ 1-a & a \end{bmatrix} \quad (2.1)$$

(columns labelled G, B)

The steady-state probabilities are denoted:

$$\Pi = [p_G \;\; p_B] \;\; with \;\; \begin{cases} p_G = \dfrac{a}{a+b} \\ p_B = \dfrac{b}{a+b} \end{cases} \quad (2.2)$$

The average BER is given by:

$$BER = p_G Pe(G) + p_B Pe(B) = \frac{aPe(G) + bPe(B)}{a+b} \quad (2.3)$$

Under assumption that the channel fades vary slowly during bit interval, GE parameters can be related to physical statistics of fading channel [6][9].

We consider that Rayleigh fading follows an exponential distribution. The probability density function of Signal-to-Noise Ratio (SNR) g is given by [10].

$$f(g) = \frac{1}{\bar{g}} e^{-\frac{g}{\bar{g}}} \qquad g \geq 0 \quad (2.4)$$

where $\bar{g}$ is the average SNR.

The channel is considered in the good state when the SNR is above a threshold $g_{threshold}$. When SNR is below $g_{threshold}$, the channel is in the bad state.

The transition probabilities can be calculated as follows [6]:

$$\begin{cases} a = \dfrac{r f_D T_s \sqrt{2p}}{e^{r^2} - 1} \\ b = r f_D T_S \sqrt{2p} \end{cases} \quad (2.5)$$

$$with \quad r^2 = \frac{g_{threshold}}{\bar{g}}$$

T_s is the symbol interval. $f_D T_S$ is the Doppler normalized frequency where $f_D = \frac{v}{w} = \frac{vf}{C}$. v is the vehicle speed, f is the carrier frequency and C the speed of light (3*10^8 m/s).

The bit error rates in each state are determined for a given modulation scheme. In our study, we used a Binary Phase Shift Keying (BPSK) modulation.

III. Performance analysis

T-error correcting non-binary Reed-Solomon codes $RS(n,k,t,d)$ are block codes defined over GF (2^m) and are characterized by four parameters: *n*’as the codeword length, ‘*k*’ as the number of information symbols, ‘*t*’ as the error-correcting capability and ‘*d*’ as the minimum Hamming distance. Bits are grouped into '*m*' bit-symbols. This characteristic makes this kind of code particularly powerful to combat transmission bursts errors.

A *RS(n,k,t,d)* code is a non-binary *BCH(n,k,t,d)* code built over GF(q) with [11]:

Information block size	*n=q-1*
Redundancy bits number	*r=n-k*
Minimum Hamming distance	*d=n-k+1 =2*t+1*

This code is capable of correcting any error pattern that affects t or fewer *m-bit* symbols with $t = \lfloor 1/2*(d-1) \rfloor$. Each error concerns one symbol containing one or many erroneous bits.

For a RS code with block length n, code rate k/n and error correction capability t, the average residual bit error rate can be calculated by:

$$R_{er} = \sum_{i=d}^{n} \frac{iA_i P_{cw}}{n} \quad (3.1)$$

A_i represents the number of weight-h codewords and is given by (3.2) [12] [13].

$$A_i = \begin{cases} \binom{n}{i} \sum_{j=0}^{i-d} (-1)^j \binom{i}{j} \left(q^{i-d+1-j} - 1\right) & d \le i \le n \\ 0 & 0 < i < d \\ 1 & i = 0 \end{cases} \quad (3.2)$$

Using GE channel, the P_{cw} of a BCH code as a function of the channel parameters and of the interleaving depth has been evaluated in [14][15] based on the above expressions.

The probability that a block of n symbols contains m errors is calculated using (3.3) and (3.4) equations.

$$P(m,n) = \sum_k f(k) P_E(m|k) \quad (3.3)$$

$$P_E(m|k) = \sum_{j=0}^{m} \binom{k}{j} P_e(B)^j \left(1-P_e(B)\right)^{k-j} \binom{n-k}{m-j} P_e(G)^{m-j} \left(1-P_e(G)\right)^{n-k-m+j} \quad (3.4)$$

$f(k)$ represents the unconditional probability of k visits to bad state *B*.

Consequently, the probability to have more than t errors, is given by :

$$P_{cw} = \sum_{m=t+1}^{n} P(m,n) \quad (3.5)$$

Assuming that there are L blocks in a cell, C_{LR} is easily calculated by the expression:

$$C_{LR} = 1 - (1 - P_{cw})^L \quad (3.6)$$

We propose to extend theoretical results [15] to RS codes. We notice two cases.

<u>First case</u>

We consider two assumptions: 1) the channel state doesn't change during a symbol transmission, 2) bit errors within a symbol are independent and are uniformly distributed.

The bit error probabilities are then simply substituted by symbol error probabilities as follows:

$$\begin{cases} P_S(B) = 1 - (1 - Pe(B))^m \\ P_S(G) = 1 - (1 - Pe(G))^m \end{cases} \quad (3.7)$$

<u>Second case</u>

If the channel changes state throughout a symbol's duration, then we have to take dependencies between information bits within the same symbol into account. Given that there are i bits in good state *G* and *j* bits in bad state *B*, we will consider all possible combination (2^m) of error sequences between states *G/B* illustrated by probabilities $\left(1-(1-Pe(G))^i\right)*\left(1-Pe(B)^j\right)$.

Only the first case is concerned by the work accomplished in this paper.

The impact of interleaving on transmission delay is also studied. For a given code, the delay D is calculated by formula : $Delay = D = 2nlt$ (3.8)

where l : interleaving depth
n : size of block
t : transmission time for information unit

IV. Numerical results

We concentrate our work on handling BPSK modulation technique. Related simplified expressions for *Pe(B)* and *Pe(G)* are provided in [10]. A data rate of 240 kbit/s and a delay constraint of 20 ms are chosen.

Following the analysis described in Section III, we investigate the performance of a set of RS error correcting codes operating at approximately half rate but having different block lengths (see Table 4.1).

Code Size	Original Code	Min. dist.	Code Rate
7	(7,4,1) GF(8)	4	57%
15	(15,7,4) GF(16)	9	46%
31	(31,16,7) GF(32)	16	51%
63	(63,31,16) GF(64)	33	47%
127	(127,64,31) GF(128)	64	50%
108	(108,53,27) GF(256)	59	48%

Table 4.1: Analyzed RS codes

Figure 4.1 illustrates the variation of codeword error probability as a function of mean SNR of the received signal for different Doppler normalized frequencies $f_D T_S$ in the case where a perfect interleaving is considered. A threshold of SNR is set to 10 dB. Improvement in performance by increasing speed of mobile is clearly seen. This phenomenon can be explained by the fact that fast fading cause short burst errors which are easy to correct while slow fading cause long burst errors.

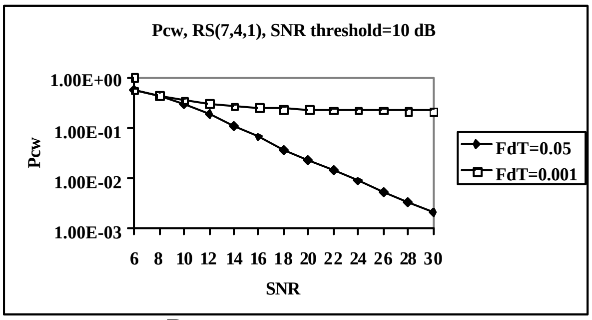

Figure 4.1: P_{cw} vs. mean SNR g for different Doppler normalized frequencies

Then, we compare the performance of different codes. Figure 4.2 shows visibly the improvement of the performance proportionally to code power. The results agree with those obtained by using binary symmetric channel model.

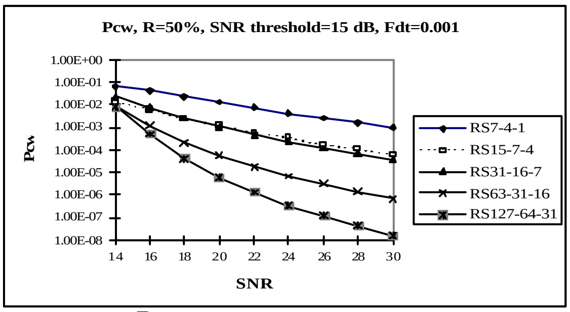

Figure 4.2: P_{cw} vs. mean SNR g for five RS codes with BPSK modulation and $f_D T_S$ =0.001

To study the impact of the relevant parameter $f_D T_S$ on coding performance in the case of imperfect interleaving, Figure 4.3 and Figure 4.4 contain a plot of P_{cw} as a function of mean SNR with SNR threshold of 10 dB and 15 dB respectively. A set of $f_D T_S$ values (0.001, 0.0001, 0.0005, 0.00005) is considered representing a large class of mobile environments.

We notice that increasing $f_D T_S$ implies performance improvement. In fact, interleaving works better thanks to the nature of errors that become more and more random.

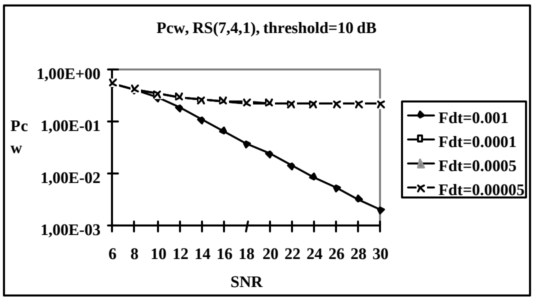

Figure 4.3: P_{cw} vs. mean SNR $\bar{g}$ with SNR treshold=10 dB

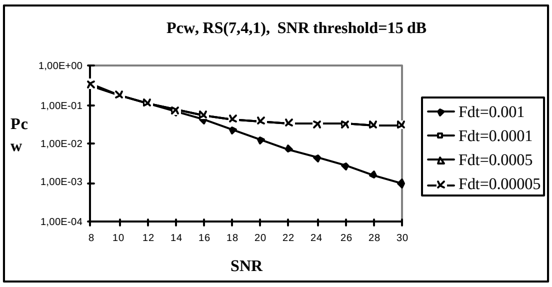

Figure 4.4: P_{cw} vs. mean SNR $\bar{g}$ with SNR treshold=15 dB

Also, we observe that curves for the three values $f_D T_S$ =0.0001, $f_D T_S$ =0.0005 and $f_D T_S$ =0.00005 are quite identical.

Figure 4.5 shows P_{cw} as a function of average SNR for codes with different error correcting capabilities (1,4,7,16,27,31). We can see the degradation of powerful codes comparatively to short ones.

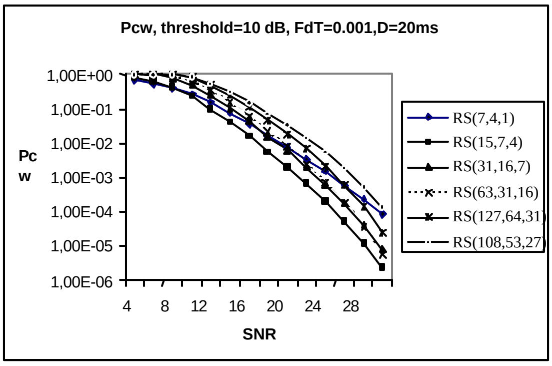

Figure 4.5: P_{cw} vs. mean SNR g for six RS code with code rate R=50%

Finally, we investigate the validity of analytical expressions derived in section III. Figure 4.6 illustrates a comparison between simulation and numerical results. We consider RS (15,7,4) code, SNR threshold of 10 dB and $f_D T_S$ of 0.001. Therefore, for a given set of parameters and delay constraint of 20 ms, the theoretical results agree with simulation results. Further, as depicted in the same figure, better performance is obtained in the case of memoryless channel.

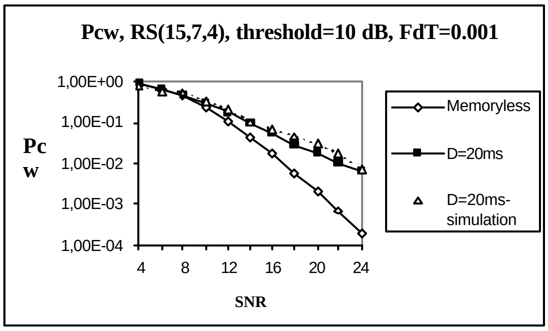

Figure 4.6: Comparison between numerical results and simulation results

Figure 4.7 reveals that the code BCH (7,4,1) is more powerful than RS (7,4,1) code. This result can be explained by the fact that when transition probabilities a and b increase, errors tend to become independent. In this case, applying interleaving leads BCH code to take maximum of benefit from correlation diminution existing between errors. RS code shows then a lowest reaction and provides less performance. We can conclude that BCH codes exhibit a higher robustness in relation to channel parameter variations in the case of wireless systems.

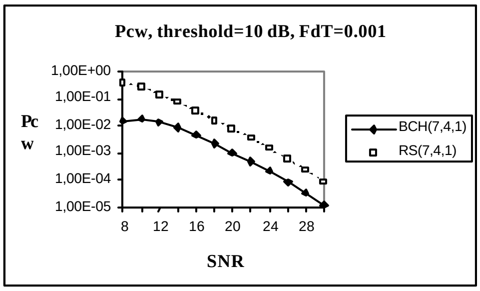

Figure 4.7: Performance comparison between BCH and RS codes

V. Conclusion

This paper presents an analysis of RS error-correcting codes over a fading channel modeled by the well-known Gilbert-Elliott model. We make an extension of theoretical results to burst error RS codes by deriving expressions for codeword error probability, residual error rate and cell loss ratio taking into account two main assumptions. We have applied this method to analyze P_{cw}. Similar conclusions are expected for other performance parameters such as R_{er} and C_{LR}. The GE channel model has been used to compare the performance of a large set of codes including those that lengths are 7, 15, 31, 63 and 127. We studied the impact of many parameters (Doppler normalized frequency, SNR,..). The main conclusions of the study are the following. Contrarily to situations where binary symmetric channel is applied, the performance obtained is improved using short codes and imperfect interleaving. SNR and Doppler normalized frequency affect particularly the performance of such FEC mechanisms. Also, we notice that a dynamic variation of SNR leads to the necessity of designing of adaptive error control schemes suited to wireless systems. Based on some channel parameters values, simulation results verify the good accuracy of our analytical results. For further work, we intend to examine heavily the second case cited in section III to take into account the dependence between bit errors within each information symbol.

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