Space-Time Codes that are Approximately Universal for the Parallel, Multi-Block and Cooperative DDF Channels

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Abstract—**Explicit codes are constructed that achieve the diversity-multiplexing gain tradeoff (DMT) of the cooperative-relay channel under the dynamic decode-and-forward protocol for any network size and for all numbers of transmit and receive antennas at the relays. Along the way, we prove that space-time codes previously constructed in the literature for the block-fading and parallel channels are approximately universal, i.e., they achieve the DMT for any fading distribution. It is shown how approximate universality of these codes leads to the first DMT-optimum code construction for the general, MIMO-OFDM channel.**

I. INTRODUCTION

Cooperative relay communication is a promising means of wireless communication in which cooperation is used to create a virtual transmit array between the source and the destination, thereby providing the much-needed diversity to combat the fading channel. Consider a communication system in which there are a total of $N+1$ nodes that cooperate in the communication between source node S and destination node D. The remaining $(N-1)$ nodes thus act as relays. We assume quasi-static fading, synchronous nodes, half-duplex operation at each node and i.i.d., $\mathbb{CN}(0, \sigma^2)$-distributed receiver noise.

II. THE DDF PROTOCOL

Under the DDF protocol, the source transmits for a total time duration of BT channel uses. This collection of BT channel uses is partitioned into B blocks with each block composed of T channel uses. Communication is slotted in the sense that each relay is constrained to commence transmission only at block boundaries. A relay will begin transmitting after listening for a time duration equal to b blocks only if the channel "seen" by the relay is good enough to enable it to decode the signal from the source with negligible error probability. We explain below. An expanded version of this manuscript can be found in [1].

A. Notation and Expressions for the Received Signal

It will be convenient at times to regard the source as the first relay, i.e., $S \equiv R_1$ and the destination as the $(N+1)$th relay, i.e., $D \equiv R_{N+1}$. The notation below is with respect to a channel realization that for simplicity, stays fixed for the B-block duration. The extension to the case where it stays fixed for a single block differs only in the notation.

Let $\underline{x}_b(n)$, $1 \leq b \leq B$, $n = 1, 2, \cdots, N$ denote the T-tuple transmitted by the nth node during the bth block. Since all nodes do not transmit in all blocks, we will make the assignment $\underline{x}_b(n) = \underline{\varphi}$, where we regard $\underline{\varphi}$ as the "empty" vector to handle the case of no transmission. In particular, the vectors $\underline{x}_b(1)$, $b = 1, 2, \cdots, B$ denote the B successive transmissions by the source.

Let us assume that up until the end of the $(b-1)$th block, we know which relays began transmitting and when. We will assume that once a relay has begun transmitting, it will keep on transmitting thereafter until the end of the Bth block. Let $\mathcal{I}_k$ denote the set of indices of the relays that transmit during the kth block, $k = 1, 2, \cdots, B$. We will refer to $\mathcal{I}_k$ as the kth activation set. Clearly

$$\mathcal{I}_1 = \{1\} \quad \text{and} \quad \mathcal{I}_k \subseteq \mathcal{I}_{k+1}, \quad 1 \leq k \leq (B-1).$$

We next proceed to determine $\mathcal{I}_b$ given $\{\mathcal{I}_k\}_{k=1}^{b-1}$. Since $\mathcal{I}_1$ is known, this procedure will allow us to recursively determine the activation sets $\mathcal{I}_k$ for all $1 \leq k \leq B$.

We will begin by first identifying the signal received by such a relay during the $(b-1)$th block. Let ζ_b, $1 \leq b \leq B$, denote the size of $\mathcal{I}_b$ i.e., $|\mathcal{I}_b| = \zeta_b$. Clearly, $1 = \zeta_1 \leq \zeta_2 \leq \cdots \leq \zeta_{b-1} \leq N$. Let the elements of $\mathcal{I}_k$, $1 \leq k \leq (b-1)$, be given by $\mathcal{I}_k = \{1 = m_1, m_2, \cdots, m_{\zeta_k}\}$. We use $h(m, n)$ to denote the fading coefficient between the mth and nth nodes. Let $n \notin \mathcal{I}_{b-1}$ and

$$
\begin{aligned}
\underline{h}_k^t(n) &= [h(m_1, n), h(m_2, n), \cdots, h(m_{\zeta_k}, n)] \\
X_k &= \begin{bmatrix} \underline{x}_k^t(m_1) \\ \vdots \\ \underline{x}_k^t(m_{\zeta_k}) \end{bmatrix}.
\end{aligned}
$$

Let

$$
\begin{aligned}
\underline{y}_k^t(n) &= [y_{(k,1)}(n) \; y_{(k,2)}(n) \; \cdots \; y_{(k,T)}(n)] \\
\underline{w}_k^t(n) &= [w_{(k,1)}(n) \; w_{(k,2)}(n) \; \cdots \; w_{(k,T)}(n)]
\end{aligned}
$$

denote the received signal and noise vector at the nth node during the kth block. Then we have

$$\underline{y}_k^t(n) \;=\; \underline{h}_k^t(n)X_k \;+\; \underline{w}_k^t(n).$$

Therefore the totality of the received signal at the nth node up until the end of the $(b-1)$th block is given by

$$[\underline{y}_1^t(n) \;\cdots\; \underline{y}_{b-1}^t(n)] = [\underline{h}_1^t(n) \;\cdots\; \underline{h}_{b-1}^t(n)]$$
$$\begin{bmatrix} X_1 & & \\ & \ddots & \\ & & X_{b-1} \end{bmatrix} + [\underline{w}_1^t(n) \;\cdots\; \underline{w}_{b-1}^t(n)]. \quad (1)$$

1) Signal at Destination: Since $D \equiv R_{N+1}$, by replacing n by $(N+1)$ and $b-1$ by B in equation (1) above, we recover the expression for the received signal at the destination during the Bth block:

$$[\underline{y}_1^t(N+1) \;\cdots\; \underline{y}_B^t(N+1)]$$
$$= [\underline{h}_1^t(N+1) \;\cdots\; \underline{h}_B^t(N+1)] \begin{bmatrix} X_1 & & \\ & \ddots & \\ & & X_B \end{bmatrix}$$
$$+ [\underline{w}_1^t(N+1) \;\cdots\; \underline{w}_B^t(N+1)]. \quad (2)$$

2) Outage of Relay Node: From (1), we note that the channel "seen" by the nth relay node over the course of the first $b-1$ blocks is the MISO (multiple-input single output) channel characterized by the matrix equation

$$y \;=\; [\underline{h}_1^t(n) \;\cdots\; \underline{h}_{b-1}^t(n)]\underline{x} \;+\; w. \quad (3)$$

The nth relay node can only hope to decode reliably at the end of the $(b-1)$th block if at that point, it has sufficient mutual information to recover the transmitted signal whose information content equals $rBT\log(\rho)$ bits. Here r denotes the multiplexing gain, ρ the signal to noise ratio, and $r\log(\rho)$ the rate of communication between source and destination [4]. If it does not have sufficient information, then we say that the relay is in outage. Thus the probability of outage $\mathcal{P}_{\text{out},n,b-1}(r)$ of the nth relay node at the end of the $(b-1)$th block equals

$$\Pr\left(\left(1 + \rho\sum_{l=1}^{b-1} \mid \underline{h}_l^t(n) \mid^2 \right) < \frac{rB}{(b-1)}\log(\rho) \right).$$

Under the DDF protocol, the nth relay node at the end of block $b-1$ uses this expression to decide whether or not it is ready to decode. If it is ready to decode, then it will proceed to do so and then begin transmitting from block b onwards, i.e., $n \in \mathcal{I}_b$.

B. Performance under the DDF Protocol

A lower bound on the probability of error of the DDF scheme can be derived by making the assumption that when the channel seen by a relay node is not in outage and the relay proceeds to decode the signal transmitted by the source, it will do so without error. Under this condition, the error probability of the DDF scheme, will be lower bounded by the probability of outage of the channel (2), seen by the

destination. In Section III-B, we will construct codes whose error performance at large SNR is equal to this lower bound, thereby establishing that this lower bound is indeed the error probability associated with the DMG tradeoff of the DDF protocol.

Let $\underline{\gamma}$ denote the vector composed of the $\binom{N+1}{2}$ fading coefficients

$$\left\{ h(m,n) \mid \; n > m, \quad \begin{array}{l} 1 \le m \le N, \\ 2 \le n \le (N+1), \end{array} \right\}$$

ordered lexicographically. We will use Γ to denote the random vector of which $\underline{\gamma}$ is a realization. The activations sets $\mathcal{I}_k$ are clearly a function of the channel realization $\underline{\gamma}$. Writing $\mathcal{I}_k(\underline{\gamma})$ in place of $\mathcal{I}_k$ to emphasize this, let us define

$$\mathcal{I}(\underline{\gamma}) \;=\; (\mathcal{I}_1(\underline{\gamma}), \cdots, \mathcal{I}_B(\underline{\gamma})).$$

Let $\mathcal{A}$ denote the collection of all possible activation sets. It follows that the error probability of the DDF scheme satisfies

$$\mathcal{P}_e(r) \;\ge\; \sum_{\mathcal{I} \in \mathcal{A}} \int_{\underline{\gamma} \in \mathcal{R}(\mathcal{I})} p_\Gamma(\underline{\gamma}) \, d\underline{\gamma}$$

where

$$\mathcal{R}(\mathcal{I}) = \left\{ \underline{\gamma} \;\middle|\; \begin{array}{c} \mathcal{I}(\underline{\gamma}) \;=\mathcal{I} \\ \left(1 + \rho\sum_{l=1}^{B} \mid \underline{h}_l^t(N+1) \mid^2 \right) < r\log(\rho) \end{array} \right\}.$$

C. Notation to Aid in Code Analysis

Returning to the expression for the signal at the nth relay node up until the $(b-1)$th block in (1), we extend the vectors $\underline{h}_k(n)$ and the matrices X_k to be of equal size with a view towards the ST code construction to be presented in Section III-B.

The vectors

$$\{\underline{h}_k(n) \mid 1 \le k \le b-1, \;\; 1 \le n \le (N+1)\}$$

will be extended by zero padding, while the matrices X_k, $1 \le k \le b-1$ will be padded with arbitrary row vectors. The extra row vectors can be chosen arbitrarily since the extended matrix $\hat{X}_k$ will be left multiplied by row vectors $\hat{\underline{h}}_k^t(n)$ having zeros in the locations corresponding to the indices of the row vectors where padding of the matrix X_k takes place.

We thus define, for $1 \le k \le b-1$,

$$\hat{\underline{h}}_k^t(n) \;=\; [\hat{h}_k(1,n) \; \hat{h}_k(2,n) \; \cdots \hat{h}_k(N,n)]$$

where

$$\hat{h}_k(m,n) \;=\; \begin{cases} h(m,n) & m \in \mathcal{I}_k \\ 0 & \text{else.} \end{cases}$$

Also, let

$$\hat{X}_k \;=\; [\hat{\underline{x}}_k(1) \;\; \ldots \;\; \hat{\underline{x}}_k(N)]^t$$

where

$$\hat{\underline{x}}_k(m) \;=\; \begin{cases} \underline{x}_k(m) & m \in \mathcal{I}_k \\ \text{arbitrary } n\text{-length vector} & \text{else.} \end{cases}$$

In terms of the extended vector and extended matrix notation, the received signal at the nth relay node, $n \notin \mathcal{I}_{b-1}$ and the destination can respectively be re-expressed in the form

$$[\underline{y}_1^t(n) \;\cdots\; \underline{y}_{b-1}^t(n)] \;=\; [\underline{\hat{h}}_1^t(n) \;\cdots\; \underline{\hat{h}}_{b-1}^t(n)]$$
$$\begin{bmatrix} \hat{X}_1 & & \\ & \ddots & \\ & & \hat{X}_{b-1} \end{bmatrix}$$
$$+ [\underline{w}_1^t(n) \;\cdots\; \underline{w}_{b-1}^t(n)], \quad (4)$$

$$[\underline{y}_1^t(N+1) \;\cdots\; \underline{y}_B^t(N+1)]$$
$$= [\underline{\hat{h}}_1^t(N+1) \;\cdots\; \underline{\hat{h}}_B^t(N+1)] \begin{bmatrix} \hat{X}_1 & & \\ & \ddots & \\ & & \hat{X}_B \end{bmatrix}$$
$$+ [\underline{w}_1^t(N+1) \;\cdots\; \underline{w}_B^t(N+1)]. \quad (5)$$

In this representation, all vectors $\underline{\hat{h}}_l^t(n)$ are of the same size, $(1 \times T)$. The same comment also applies to the matrices $\hat{X}_l$, $1 \le l \le b-1$, which are of size $(N \times T)$.

As will be shown in Section IV below, ST codes that are approximately universal for an appropriate class of block-fading channels will be the building blocks of codes for the DDF protocol that attain the DMG performance of this channel.

III. THE BLOCK-FADING CHANNEL

A. Outage Probability

Consider the block-fading MIMO channel with n_t transmit and n_r receive antennas and B blocks, characterized by

$$\underline{y}_b \;=\; H_b \underline{x}_b + \underline{w}_b, \quad 1 \le b \le B. \quad (6)$$

Thus each matrix H_b is of size $(n_r \times n_t)$. The probability of outage of this channel is given by

$$P_{\text{out}}(r) \;\doteq\; \Pr(\sum_{b=1}^{B} \log \det(I_{n_r} + \rho H_b H_b^\dagger) < rB \log(\rho))$$
$$= \; \Pr(\log \det(I_{Bn_t} + \rho \Lambda_H^\dagger \Lambda_H) < rB \log(\rho))$$

where ρ is the SNR and where Λ_H is the $(Bn_r \times Bn_t)$ block diagonal matrix $\Lambda = \text{diag}(H_1, H_2, \cdots, H_B)$. In the above, $\doteq$ and $\dot{\le}, \dot{\ge}$ corresponds to exponential equality and inequality. For example, $y \doteq \rho^x$ is used to indicate that $\lim_{\rho \to \infty} \frac{\log(y)}{\log(\rho)} = x$. Let $q = n_t B$ and let

$$\lambda_1 \le \lambda_2 \le \cdots \le \lambda_q \quad (7)$$

be an ordering of the q eigenvalues of $\Lambda_H^\dagger \Lambda_H$. Note that if $n_r < n_t$, then $\lambda_1 = \lambda_2 = \cdots = \lambda_{(n_t-n_r)B} = 0$. Let $\delta = ([n_t - n_r]B)^+$ where $(x)^+$ denotes $\max\{x, 0\}$, and let the α_i be defined by

$$\lambda_i \;=\; \rho^{-\alpha_i}, \quad \delta+1 \le i \le q.$$

Then

$$P_{\text{out}}(r) \;=\; \Pr(\sum_{i=\delta+1}^{q} (1 - \alpha_i)^+ < rB).$$

We will now proceed to identify a ST code in the next section, Section III-B, that is approximately universal for the class of block-fading channels, i.e., a code that achieves the D-MG tradeoff of the channel model in (6) for every statistical distribution of the fading coefficients $\{[H_b]_{i,j}\}$.

Similar construction of codes for such a setting have previously been identified in [7], [13] and independently in [9], [10]. We adopt the code-construction technique of these papers for the most part, although the construction presented here is slightly more general, for example, we permit the individual block codes to be rectangular and offer flexibility with respect to number of conjugate blocks employed. Most importantly though, our proof will establish the result that these codes are approximately universal for the block-fading channel and parallel channels. The results of the present submission also answer a question raised in [14] and relating to the existence of approximately universal codes for the parallel MIMO channel.

B. Approximately-Universal Codes for the Block-Fading Channel

1) Constructing the Appropriate Cyclic Division Algebra: Let T be an integer satisfying $T \ge n_t$. Let $m \ge B$ be the smallest integer such that the gcd of m, T equals 1, i.e., $(m, T) = 1$. Let $\mathbb{K}, \mathbb{M}$ be cyclic Galois extensions of $\mathbb{Q}(\imath)$ of degrees m, T whose Galois groups are generated respectively by the automorphisms ϕ_1, σ_1, i.e.,

$$\text{Gal}(\mathbb{K}/\mathbb{Q}(\imath)) = <\phi_1>, \quad \text{Gal}(\mathbb{M}/\mathbb{Q}(\imath)) = <\sigma_1>.$$

Let $\mathbb{L}$ be the composite of $\mathbb{K}, \mathbb{M}$. Then it is known that $\mathbb{L}/\mathbb{Q}(\imath)$ is cyclic and that further,

$$\text{Gal}(\mathbb{L}/\mathbb{Q}(\imath)) \;\cong\; \text{Gal}(\mathbb{K}/\mathbb{Q}(\imath)) \times \text{Gal}(\mathbb{M}/\mathbb{Q}(\imath)).$$

Thus every element of $\text{Gal}(\mathbb{L}/\mathbb{Q}(\imath))$ can be associated with a pair (ϕ_1^i, σ_1^j) belonging to $\text{Gal}(\mathbb{K}/\mathbb{Q}(\imath)) \times \text{Gal}(\mathbb{M}/\mathbb{Q}(\imath))$. Let ϕ, σ be the automorphisms associated to the pairs (ϕ_1, id), (id, σ_1) respectively. Then ϕ, σ are the generators of the Galois groups $\text{Gal}(\mathbb{L}/\mathbb{M})$, $\text{Gal}(\mathbb{L}/\mathbb{K})$ respectively.

Let $\gamma \in \mathbb{K}$ be a non-norm element of the extension $\mathbb{L}/\mathbb{K}$, i.e., the smallest exponent e for which γ^e is the norm of an element of $\mathbb{L}$ is T. Let z be an indeterminate satisfying $z^T = \gamma$. Consider the T-dimensional vector space

$$D = \{z^{T-1}\ell_{T-1} \oplus z^{T-2}\ell_{T-2} \oplus \cdots \ell_0 \mid \ell_i \in \mathbb{L}\}.$$

We define multiplication on D by setting $\ell_i z = z\sigma(\ell_i)$ and extending in a natural fashion. This turns D into a cyclic division algebra (CDA) whose center is $\mathbb{K}$ and having $\mathbb{L}$ as a maximal subfield. See [3], [5] for an exposition of the relevant background on division algebras. Every element $x = z^{T-1}\ell_{T-1} + z^{T-2}\ell_{T-2} + \cdots + \ell_0$ in D has the regular representation

$$X = \begin{bmatrix} \ell_0 & \gamma\sigma(\ell_{T-1}) & \dots & \gamma\sigma^{T-1}(\ell_1) \\ \ell_1 & \sigma(\ell_0) & \dots & \gamma\sigma^{T-1}(\ell_2) \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{T-1} & \sigma(\ell_{T-2}) & \dots & \sigma^{T-1}(\ell_0) \end{bmatrix}. \quad (8)$$

The determinant of such a matrix is known to lie in $\mathbb{K}$. Given a matrix X with components $X_{i,j} \in \mathbb{L}$, we define $\phi(X)$ to be the matrix over $\mathbb{L}$ whose $(i,j)^{th}$ component is given by $[\phi(X)]_{i,j} = \phi([X]_{i,j})$. Note that in this case,

$$\prod_{i=0}^{m-1} \det(\phi^i(X)) = \prod_{i=0}^{m-1} \phi_1^i(\det(X)) \in \mathbb{Q}(\imath).$$

Hence if the elements ℓ_i underlying the matrix X are in addition, restricted to lie in the ring $\mathcal{O}_\mathbb{L}$ of algebraic integers of $\mathbb{L}$, then we have

$$\prod_{i=0}^{m-1} \det(\phi^i(X)) \in \mathbb{Z}(\imath) \text{ so that } |\prod_{i=0}^{m-1} \det(\phi^i(X))|^2 \geq 1.$$

2) Space-time Code Construction on the CDA: Let $\mathcal{X}$ be the rectangular $(n_t \times T)$ ST code comprised of the first n_t rows of the regular representations of the elements $\sum_{i=0}^{T-1} z^i \ell_i$, where ℓ_i are restricted to be of the form:

$$\ell_i = \sum_{j=1}^{T} \ell_{i,j} \gamma_j, \quad \ell_{i,j} \in \mathcal{O}_\mathbb{K}$$

and where $\{\gamma_1, \cdots, \gamma_T\}$ are a basis for $\mathbb{L}/\mathbb{K}$. Note that as a result, we have ensured that $\ell_i \in \mathcal{O}_\mathbb{L}$. Also note that each code matrix in $\mathcal{X}$ is of the row-deleted form

$$X = \begin{bmatrix} \ell_0 & \gamma\sigma(\ell_{T-1}) & \dots & \dots & \gamma\sigma^{T-1}(\ell_1) \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \ell_{n_t-1} & \sigma(\ell_{n_t-2}) & \dots & \dots & \gamma\sigma^{T-1}(\ell_{n_t}) \end{bmatrix} \quad (9)$$

Let $\mathcal{S}$ be the $(Bn_t \times BT)$ ST code comprised of code matrices having the block diagonal form:

$$S = \left\{ \theta \begin{bmatrix} X & & \\ & \ddots & \\ & & \phi^{B-1}(X) \end{bmatrix}, \quad X \in \mathcal{X} \right\}$$

where θ accounts for SNR normalization. When this code matrix is in use, the received signal over the block-fading channel is given by

$$[Y_1\ Y_2\ \cdots\ Y_B] = [H_1\ H_2\ \cdots\ H_B]S + \quad (10)$$
$$[W_1\ W_2\ \cdots\ W_B]. \quad (11)$$

This can also be expressed in the form

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_B \end{bmatrix} = \theta \begin{bmatrix} H_1 & & \\ & \ddots & \\ & & H_B \end{bmatrix} \begin{bmatrix} X \\ \vdots \\ \phi^{B-1}(X) \end{bmatrix} + \begin{bmatrix} W_1 \\ \vdots \\ W_B \end{bmatrix}$$

in which the channel matrix is of block-diagonal form. This latter form is convenient when comparing the block-fading channel with the parallel channel. The proof of approximate

universality, i.e., proof of DMT optimality for every statistical characterization of the fading channel is skipped here for lack of space, the interested reader is referred to [1] for the proof.

C. Analogous Results Hold for the Parallel Channel

By parallel channel we will mean the channel given by

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_B \end{bmatrix} = \begin{bmatrix} H_1 & & \\ & \ddots & \\ & \cdots & H_B \end{bmatrix} S + \begin{bmatrix} W_1 \\ \vdots \\ W_B \end{bmatrix},$$

in which the channel matrix is of block-diagonal form. Consider the $(Bn_t \times T)$ space-time code $\mathcal{S}_{\text{par}}$ given by

$$\mathcal{S}_{\text{par}} = \left\{ \theta \begin{bmatrix} X \\ \vdots \\ \phi^{B-1}(X) \end{bmatrix}, \quad X \in \mathcal{X} \right\}$$

which when used over the parallel channel leads to the equation below for the received signal at the receiver,

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_B \end{bmatrix} = \theta \begin{bmatrix} H_1 & & \\ & \ddots & \\ & \cdots & H_B \end{bmatrix} \begin{bmatrix} X \\ \vdots \\ \phi^{B-1}(X) \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_B \end{bmatrix}.$$

Comparing this equation with the alternate expression for the block-fading channel given above we see that the expressions are identical. There is one important difference though. In the case of the block-fading channel, a rate requirement of R bits per channel use translates into a space-time code $\mathcal{S}$ of size $2^{RBT} = \rho^{rBT}$, whereas in the case of the parallel channel, the size of the corresponding ST code $\mathcal{S}_{\text{par}}$ is required to be $2^{RT} = \rho^{rT}$. It follows from this that by replacing rB by r, one can similarly prove approximate universality of the code $\mathcal{S}_{\text{par}}$ for the class of parallel channels. We omit the details.

D. DMT-optimal Codes for the General MIMO-OFDM Channel

The MIMO-OFDM channel can be regarded as a parallel channel in which each parallel block corresponds to a different subcarrier and can thus be represented in the form:

$$\underline{y}_i = \theta H_l \underline{x}_l + \underline{w}_l, \quad 1 \leq l \leq Q,$$

where where Q is the number of OFDM tones or subcarriers [8]. The matrices H_l are correlated in general, with a correlation derived from the time-dispersion of the original ISI channel. Since the code $\mathcal{S}_{\text{par}}$ is approximately universal, this means that the code $\mathcal{S}_{\text{par}}$ is DMG optimal when used over the MIMO fading channel. When the code $\mathcal{S}_{\text{par}}$ is used over the MIMO-OFDM channel, the received-signal equation will take on the form $[Y_1^t\ Y_2^t \cdots Y_Q^t]^t =$

$$\theta \begin{bmatrix} H_1 & & \\ & \ddots & \\ & \cdots & H_Q \end{bmatrix} \begin{bmatrix} X \\ \vdots \\ \phi^{Q-1}(X) \end{bmatrix} + \begin{bmatrix} W_1 \\ \vdots \\ W_Q \end{bmatrix}.$$

DMT-optimal codes for the OFDM channel have previously been constructed in [7] and [11]. In [7], the authors provide a

proof only for the case when the matrices H_n appearing along the diagonal are i.i.d. Rayleigh. The DMT-optimal construction in [11] is for the SIMO-OFDM case. We thus believe the results in [1] of which the present paper represents the first conference submission, provides the first construction of DMT-optimal codes for the general OFDM-MIMO channel.

IV. CODES ATTAINING THE DMG OF THE DDF PROTOCOL

We now show how ST codes constructed for the block-fading channel can be used to construct optimal codes under the DDF protocol. We consider the DDF protocol as it applies to a communication system in which there are a total of $N+1$ nodes that cooperate in the communication between source node S and destination node D.

As in Sections I,II, under the DDF protocol, the source transmits for a total time duration of BT channel uses. This collection of BT channel uses is partitioned into B blocks with each block composed of T channel uses. Communication is slotted in the sense that each relay is constrained to commence transmission only at block boundaries. A relay will begin transmitting after listening for a time duration equal to b blocks only if the channel "seen" by the relay is good enough to enable it to decode the signal from the source with negligible error probability.

Our coding strategy runs as follows. The role played by n_t in the block-fading scenario is now played by the number N which is the number of nodes in the network capable of transmitting to the destination. Let $\mathcal{X}$ be the rectangular $(N \times T)$ ST code comprised of the first N rows of the regular representations of the elements $\sum_{i=0}^{T-1} z^i \ell_i$, where ℓ_i are restricted to be of the form:

$$\ell_i = \sum_{j=1}^{T} \ell_{i,j} \gamma_j, \quad \ell_{i,j} \in \mathcal{O}_{\mathbb{K}}.$$

Let $\mathcal{D}$ be the $(BN \times BT)$ ST code comprised of code matrices having the block diagonal form:

$$\mathcal{D} = \left\{ \theta \begin{bmatrix} X & & \\ & \ddots & \\ & \cdots & \phi^{B-1}(X) \end{bmatrix}, \quad X \in \mathcal{X} \right\} \quad (12)$$

where θ accounts for SNR normalization. The code to be used then has the following simple description. The source S sends the first row of each of the matrices $X, \phi(X), \cdots, \phi^{B-1}(X)$ in successive blocks. Let us assume that relay node R_n, $2 \leq n \leq N$, is not in outage for the first time at the conclusion of the $(b-1)$th block. Then R_n is ready to decode at the end of the $b-1$th block. Thereafter, it proceeds to send in succession, the nth rows of the matrices $\phi^b(X), \phi^{b+1}(X), \cdots, \phi^{B-1}(X)$. Thus the padded matrices $\hat{X}_i$ appearing in (4), (5), correspond to the matrices $\phi^{i-1}(X)$ in (12).

It is easy to show using the results stated earlier relating to the block-fading channel that this coding strategy ensures that whenever a relay node decodes, it does so with negligible probability of error. The destination error probability is also

similarly guaranteed to have error probability that is SNR-equivalent to the outage probability, thus proving DMG-optimality of the constructed ST code.

This follows since each relay node R_n, $n \notin \mathcal{I}_{b-1}$ "sees" a block-fading channel (see (4)) and the coding strategy we have adopted ensures that the code matrix carrying data from the nodes in $\mathcal{I}_{b-1}$ is DMT optimal for the corresponding block-fading channel. A similar statement is true for the relay R_{N+1} that corresponds to the destination, since the corresponding channel equation is of the same block-fading form see (5).

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