An Optimized Unitary Beamforming Technique for MIMO Broadcast Channels

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***Abstract*—This paper addresses the problem of linear beamforming design in MIMO broadcast channels. An iterative optimization method for unitary beamforming is proposed, based on successive optimization of Givens rotations. Under the assumption of perfect channel state information at the transmitter (CSIT) and for practical average signal-to-noise ratios (SNR), the proposed technique provides higher sum rates than zero-forcing (ZF) beamforming while performing close to minimum-mean-squared-error (MMSE) beamforming when the number of transmit antennas equals the number of scheduled users. Moreover, it is shown to achieve linear sum-rate growth with the number of transmit antennas. Interestingly, the proposed unitary beamforming approach proves to be very robust to channel estimation errors. In the simulated scenarios, it provides better sum rates than ZF beamforming and even MMSE beamforming as the variance of the estimation error increases. When combined with simple vector quantization techniques for CSIT feedback in systems with multiuser scheduling, the proposed technique proves to be well suited for limited feedback scenarios with practical number of users, exhibiting performance gains over existing techniques.**

***Index Terms*—MIMO systems, broadcast channel, linear beamforming, unitary beamforming, scheduling.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) systems can significantly increase the spectral efficiency by exploiting the spatial degrees of freedom created by multiple antennas. The capacity can be boosted by exploiting the spatial multiplexing capability of transmit antennas, transmitting to multiple users simultaneously by means of space division multiple access (SDMA), rather than maximizing the capacity of a single-user link, as shown in [1], [2]. It has recently been proven in [3] that the capacity region of the MIMO broadcast channel coincides with the rate region of dirty paper coding (DPC) [4]. However, the applicability of DPC is limited due to its computational complexity and high sensitivity to channel estimation errors.

Unitary beamforming (UBF) techniques have recently become a focus of interest in MIMO broadcast channels, especially in scenarios where the amount of feedback available at the base station is limited. Particularly, random beamforming (RBF) [5] has been proposed as a simple technique that achieves optimal capacity scaling in MIMO broadcast channels.

In [6], unitary basis stream user and rate control (PU2RC) is proposed as a transmission technique in which the transmitter has a codebook containing an arbitrary number of unitary bases. In this approach, the users quantize the channel shape (channel direction) to the closest codeword in the codebook, feeding back the quantization index and expected signal-to-interference-plus-noise ratio (SINR). PU2RC supports SDMA and multiuser scheduling, as well as adaptive modulation and coding. An extension to scenarios with a sum feedback rate constraint is provided in [7], coined as orthogonal SDMA with threshold feedback (TF-OSDMA).

Codebook-based unitary precoding is a solid candidate for MIMO downlink transmission in future mobile communication standards, currently under study in 3GPP [8], [9], [10]. In fact, unitary precoding has been selected both in single user and multiuser modes of operation for evolved universal terrestrial radio access (E-UTRA) [11]. As reported in [6], feedback from the mobile users in the form of a quantization index and channel quality indicator are used for user scheduling and beamforming design. Simple unitary codebooks have been proposed, which yield smooth switching between single user point-to-point MIMO operation and multiuser SDMA. Besides its simplicity, another important advantage of unitary beamforming is its robustness to channel estimation errors, as we discuss later on in this paper. As it was shown in [12], [13], the use of unitary beamforming enables exact calculation of the SINR at the receiver, as long as the user knows its assigned beamforming vector and channel perfectly. Another advantage of unitary beamforming is the fact that each antenna transmits with the same peak power. This results in simpler design constraints for each RF chain, with efficient power amplifiers working near saturation. In addition, unitary beamforming is beneficial in multi-cellular systems, since the interference experienced by other cells is spatially white.

In order to obtain good sum rates, the precoding matrices, quantization codebooks and feedback strategies need to be jointly designed. When constraining the precoding matrices to be unitary, the performance of suboptimal schemes should be evaluated by comparison with optimal unitary beamforming in order to measure the degree of suboptimality introduced. Conversely, limited feedback schemes relying on unitary beamforming should be designed with low complexity and reduced feedback, while approaching the performance of the optimal unitary beamforming solution. However, optimal unitary beamforming in MIMO broadcast channels - in the sense of system sum-rate maximization - is not yet known. Thus,

Manuscript received February 13, 2008; revised December 28, 2008 and June 19, 2009; accepted November 28, 2009. The associate editor coordinating the review of this paper and approving it for publication was S. Affes.

Parts of this work were presented at the Allerton Conference on Communication, Control and Computing, Monticello, IL, USA, September 2007.

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Digital Object Identifier 10.1109/TWC.2010.080202

most limited feedback schemes with unitary beamforming use low complexity as main design criterion, evaluating their performances through simulations. Multiuser MIMO schemes based on full channel knowledge at the transmitter and unitary beamforming have been proposed in [14], exhibiting performance gains over ZF beamforming approaches particularly at low SNR. However, the beamforming matrices in [14] are generated by following low-complexity design criteria with the aim of simplifying the scheduling algorithms in scenarios where the number of users is larger than the number of transmit antennas.

In this paper, an iterative optimization method for unitary beamforming in MIMO broadcast channels is proposed, based on successive optimization of Givens rotations. Previous work in the literature using Givens rotations in point-to-point MIMO systems has been reported in [15], where Givens rotations are used for the quantization of unitary matrices and the MIMO channel is split into independent sub-channels by means of singular value decomposition (SVD). In [15], the distortion measure considered is the mean square error between quantized and optimal unitary matrices. In our work, we focus instead on the multiuser MIMO problem, using as optimization criterion the sum rate. Initially, we consider a system with perfect CSIT. As we show, the proposed technique provides higher sum rates than ZF beamforming while performing close to MMSE beamforming for practical average SNR values. However, as the average SNR becomes large, the slope of the sum-rate versus SNR curve converges to that of a system with time division multiple access (TDMA) that selects the best user, thus incurring a loss of multiplexing gain. Moreover, it is shown to achieve linear sum-rate growth with the number of transmit antennas.

The main advantage of the proposed unitary beamforming approach is its robustness to channel estimation errors. As shown through numerical simulations, it provides better sum rates than ZF beamforming and even MMSE beamforming as the variance of the estimation error increases. Hence, the proposed beamforming technique can be seen as an interesting alternative to other existing linear beamforming schemes, such as ZF and MMSE. Note that the performance of DPC is also very sensitive to imperfect channel knowledge at the transmitter, as it was shown in [16], which occurs in realistic systems with limited feedback from the users to the base station. The proposed technique exploits the robustness to estimation errors provided by unitary beamforming, but optimizes the unitary basis as opposed to simpler unitary beamforming approaches, such as RBF. In the last part of this paper, the proposed technique is investigated in MIMO broadcast channels with multiuser scheduling and limited feedback, evaluating the performance of unitary beamforming approaches with limited feedback, namely RBF and PU2RC. A simple vector quantization technique is used, based on random vector quantization (RVQ) with pruning. Our results highlight the importance of linear beamforming optimization in MIMO broadcast channels with limited feedback.

II. System Model

We consider a multiple antenna broadcast channel consisting of a transmitter equipped with M antennas and $K \geq M$ single-antenna receivers. Given a set of M users scheduled for transmission, the signal received at the k-th mobile is given by

$$y_k = \sqrt{\frac{P}{M}}\mathbf{h}_k^H \mathbf{w}_k s_k + \sqrt{\frac{P}{M}} \sum_{i=1, i\neq k}^{M} \mathbf{h}_k^H \mathbf{w}_i s_i + n_k \quad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$, $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$, $s_k \in \mathbb{C}$, $n_k \in \mathbb{C}$ and P are the channel vector, beamforming vector, transmitted signal, additive Gaussian noise at receiver k and power, respectively. The first term in the above equation is the useful signal, while the second term corresponds to the interference. We assume that the channels are i.i.d. block Rayleigh flat fading, the variance of the transmitted signal s_k is normalized to one and n_k is circularly symmetric complex Gaussian with zero mean and variance σ^2. Hence, the SINR of user k is given by

$$SINR_k = \frac{\frac{P}{M}|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{i=1, i\neq k}^{M} \frac{P}{M}|\mathbf{h}_k^H \mathbf{w}_i|^2 + \sigma^2} \quad (2)$$

A unitary beamforming matrix is considered at the transmitter $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2 \dots \mathbf{w}_M] \in \mathbb{C}^{M\times M}$, and thus the average transmitted power is equal to P. In order to design the bemforming matrix, perfect knowledge of the user channels at the transmitter is assumed unless otherwise stated.

A. Imperfect CSIT Model

The robustness of the proposed approach to channel estimation errors is evaluated through numerical simulations. When imperfect knowledge of the user channel vectors is available at the transmitter side, the estimation error is modeled as an additive spatially white complex Gaussian noise. Hence, the channel estimate of user k is given by

$$\mathbf{h}_k = \widehat{\mathbf{h}}_k + \widetilde{\mathbf{h}}_k \quad (3)$$

where $\widetilde{\mathbf{h}}_k$ has a distribution $\mathcal{CN}(0, \sigma_e^2)$. Imperfect CSIT can be the result of a combination of channel estimation noise, quantization errors, prediction errors, etc.

Notation: We use bold upper and lower case letters for matrices and column vectors, respectively. $(\cdot)^H$ stands for Hermitian transpose. $\mathbb{E}(\cdot)$ denotes the expectation operator and $tr(\cdot)$ is the trace operator. The notation $\|\mathbf{x}\|$ refers to the Euclidean norm of the vector $\mathbf{x}$ and $\|\mathbf{X}\|_F$ refers to the Frobenius norm of the matrix $\mathbf{X}$, defined as $\|\mathbf{X}\|_F = \sqrt{tr(\mathbf{X}\mathbf{X}^H)}$. The amplitude and phase of a complex scalar are denoted as $|\cdot|$ and $\angle(\cdot)$, respectively.

III. Problem Formulation

The optimization criterion considered in our problem is sum rate maximization, constrained to using linear unitary beamforming at the transmitter. Hence, the optimization problem can be formulated as follows

$$\begin{aligned} &\arg\max_{\mathbf{W}} \sum_{k=1}^{M} \log_2(1 + SINR_k) \\ &s.t. \quad \mathbf{W}^H\mathbf{W} = \mathbf{I}_M \end{aligned} \quad (4)$$

where $SINR_k$ represents the SINR of user k. This optimization problem is rather difficult to solve using this formulation, since the problem is nonconvex and the constraints are nonlinear. The problem can be reformulated by exploiting the particularities of the $SINR_k$ expression when unitary beamforming is used. Let ρ_k be the alignment between the k-th user instantaneous normalized channel vector $\overline{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ (channel direction) and the corresponding beamforming vector $\mathbf{w}_k$, defined as $\rho_k = \left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|$. The problem can be reformulated by exploiting the particularities of the $SINR_k$ expression when unitary beamforming is used, which can be simplified as [12], [17]

$$SINR_k = \frac{\|\mathbf{h}_k\|^2 \rho_k^2}{\|\mathbf{h}_k\|^2 (1-\rho_k^2) + \frac{M\sigma^2}{P}} \tag{5}$$

Define the vector $\boldsymbol{\rho} = [\rho_1\, \rho_2 \ldots \rho_M]$. Note that, when subtituting the $SINR_k$ expression shown in (5) into equation (4), the k-th term in the sum of logarithms becomes only a function of the variable ρ_k. The difficulty now lies in determining the feasible set of solutions for $\boldsymbol{\rho}$, i.e. the set of values for which a $\mathbf{W}$ matrix exists given that the user channels are known and fixed. This can be done by incorporating the geometrical structure of the problem into new constraints on $\boldsymbol{\rho}$, which is also a difficult task. Instead, in next section, we propose a simple method to iteratively improve $\boldsymbol{\rho}$, while ensuring its feasibility by algorithm construction.

Another way to simplify the constrained optimization problem in equation (4) is to transform it into an unconstrained problem. Define the initial matrix $\mathbf{W}^0$ as an arbitrary unitary matrix. Let $\mathbf{R}_{mn}$ be the Givens rotation matrix in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane, which performs an orthogonal rotation of the m-th and n-th columns of a unitary matrix while keeping the others fixed, thus preserving unitarity. Assume without loss of generality $n > m$. The Givens rotation matrix in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane is given by

$$\mathbf{R}_{mn}(\alpha,\delta) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & \cos\alpha & \cdots & \sin\alpha e^{j\delta} & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -\sin\alpha e^{-j\delta} & \cdots & \cos\alpha & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{6}$$

where the non trivial entries appear at the intersections of m-th and n-th rows and columns. Hence, any unitary matrix $\mathbf{W}$ can be expressed using the following parameterization

$$\mathbf{W} = \mathbf{W}^0 \prod_{m=1}^{M} \prod_{n=m+1}^{M} \mathbf{R}_{mn} \tag{7}$$

up to a global $e^{j\theta}$ factor. Note that such global factor has no importance for transmission purposes in the described system model, since it does not have an impact on the resulting SINR of the scheduled users. Each rotation matrix $\mathbf{R}_{mn}$ in (7) is function of 2 rotation parameters, α and δ. Hence, by imposing this structure, the optimization problem in equation (4) becomes unconstrained and it boils down to finding the optimal $2\binom{M}{2}$ rotation parameters of the corresponding $\binom{M}{2}$ rotation matrices. Since the resulting ρ_k values, $k = 1, \ldots, M$, are complicated non-linear functions of the rotation parameters, we propose an iterative algorithm to compute the optimal rotation matrix for a given plane, iterating along different planes until convergence is reached. Hence, the algorithm we propose is based on a divide-and-conquer type of approach. The matrix $\mathbf{W}$ is divided into smaller instances that are solved recursively in order to provide a solution to the optimization problem in (4). However, convergence to a global optimum can not be ensured for an arbitrary channel.

TABLE I
OUTLINE OF THE UNITARY BEAMFORMING OPTIMIZATION PROCEDURE

Initialization

- Initialize the UBF matrix $\mathbf{W}^0$

***i*-th iteration step**, $i = 1, \ldots, N_{PR}$

- Select an index pair $\{m, n\}$ from $\mathcal{G}$
- Find optimal rotation parameters for the $(\mathbf{w}_m, \mathbf{w}_n)$-plane $\{\alpha^*, \delta^*\} = \arg\min_{\alpha,\delta} F_{mn}(\alpha, \delta)$
- Update UBF matrix $\mathbf{W}^i = \mathbf{W}^{i-1}\mathbf{R}_{mn}(\alpha^*, \delta^*)$

IV. ALGORITHM DESCRIPTION

The proposed unitary beamformer is designed on the basis of the available user channels $\mathbf{h}_k, k = 1, \ldots, M$ and balances the amount of power and interference received by each user. Given an initial unitary beamforming matrix $\mathbf{W}^0$ available at the transmitter, we propose an iterative algorithm which consists of rotating the beamforming matrix by performing successive optimization of Givens rotations until convergence is reached. At the i-th iteration, a refined unitary beamforming matrix is computed by rotating the matrix $\mathbf{W}^{i-1}$ - computed at the previous iteration - in the plane defined by the complex vectors $(\mathbf{w}_m, \mathbf{w}_n)$, performing right multiplication with the rotation matrix defined in equation (6). For each plane rotation, the optimal α^* and δ^* rotation parameters are found. Let $\mathcal{G}$ be the set of all possible index pairs among the complete index set $\{1, \ldots, M\}$, in which each $\{m, n\}$ index pair satisfies $n > m$. Define N_{PR} as the total number of plane rotations performed by the proposed approach. An outline of the proposed algorithm is provided in Table I.

It can be seen from the structure of the matrix in (6) that rotation in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane does not change the directions of the remaining beamforming vectors. Equivalently, since $SINR_k$ is only function of $\rho_k = \left|\overline{\mathbf{h}}_k^H \mathbf{w}_k\right|$, a rotation in the $(\mathbf{w}_m, \mathbf{w}_n)$-plane only modifies $SINR_m$ and $SINR_n$. Hence, the optimal rotation parameters are found by solving the following optimization problem

$$\{\alpha^*, \delta^*\} = \arg\max_{\alpha,\delta} \left\{ \log_2\left(1 + \frac{\|\mathbf{h}_m\|^2 \rho_m^2(\alpha,\delta)}{\|\mathbf{h}_m\|^2 (1-\rho_m^2(\alpha,\delta)) + \frac{M\sigma^2}{P}}\right) + \log_2\left(1 + \frac{\|\mathbf{h}_n\|^2 \rho_n^2(\alpha,\delta)}{\|\mathbf{h}_n\|^2 (1-\rho_n^2(\alpha,\delta)) + \frac{M\sigma^2}{P}}\right) \right\} \tag{8}$$

where $\rho_m(\alpha, \delta), \rho_n(\alpha, \delta)$ are the modified alignments between channels and beamfoming vectors after rotation, given by

$$\rho_m(\alpha,\delta) = \left|\overline{\mathbf{h}}_m^H \left(\mathbf{w}_m \cos\alpha - \mathbf{w}_n \sin\alpha e^{-j\delta}\right)\right| \tag{9}$$
$$\rho_n(\alpha,\delta) = \left|\overline{\mathbf{h}}_n^H \left(\mathbf{w}_m \sin\alpha e^{j\delta} + \mathbf{w}_n \cos\alpha\right)\right|.$$

Defining the following variables

$$\begin{aligned} r_{mm} &= \left|\overline{\mathbf{h}}_m^H \mathbf{w}_m\right| & r_{mn} &= \left|\overline{\mathbf{h}}_m^H \mathbf{w}_n\right| \\ r_{nm} &= \left|\overline{\mathbf{h}}_n^H \mathbf{w}_m\right| & r_{nn} &= \left|\overline{\mathbf{h}}_n^H \mathbf{w}_n\right| \\ \Delta_{mn} &= \angle\overline{\mathbf{h}}_m^H \mathbf{w}_m - \angle\overline{\mathbf{h}}_m^H \mathbf{w}_n & \Delta_{nm} &= \angle\overline{\mathbf{h}}_n^H \mathbf{w}_n - \angle\overline{\mathbf{h}}_n^H \mathbf{w}_m \end{aligned} \tag{10}$$

we have that

$$\rho_m^2(\alpha,\delta) = r_{mm}^2 \cos^2\alpha + r_{mn}^2 \sin^2\alpha - r_{mm} r_{mn} \cos(\Delta_{mn}+\delta) \sin 2\alpha$$
$$\rho_n^2(\alpha,\delta) = r_{nm}^2 \sin^2\alpha + r_{nn}^2 \cos^2\alpha + r_{nm} r_{nn} \cos(\delta-\Delta_{nm}) \sin 2\alpha. \tag{11}$$

Define the parameter $\beta_k = \frac{M\sigma^2}{P\|\mathbf{h}_k\|^2}, k = m, n$. Since the logarithm is a monotonically increasing function, the optimization problem in equation (8) can be transformed into

$$\{\alpha^*, \delta^*\} = \arg\min_{\alpha,\delta} F_{mn}(\alpha,\delta) \tag{12}$$

where the function F_{mn} is defined as follows

$$F_{mn}(\alpha,\delta) = \left(1 - \rho_m^2(\alpha,\delta) + \beta_m\right)\left(1 - \rho_n^2(\alpha,\delta) + \beta_n\right). \tag{13}$$

The solution is found by equating the gradient of F_{mn} to zero

$$\frac{\partial F_{mn}(\alpha,\delta)}{\partial\alpha} = 0 \tag{14}$$
$$\frac{\partial F_{mn}(\alpha,\delta)}{\partial\delta} = 0 \tag{15}$$

In order to solve the above equations, we introduce the change of variable $t = \tan\alpha$ to solve equation (14) and $s = \tan\delta/2$ to solve equation (15). After some algebraic manipulations the problem is reduced to finding the roots of polynomials of the form

$$P_\alpha(t) = f_4 t^4 + f_3 t^3 + f_2 t^2 + f_1 t + f_0 \tag{16}$$
$$P_\delta(s) = g_4 s^4 + g_3 s^3 + g_2 s^2 + g_1 s + g_0 \tag{17}$$

where $f_i, g_i, i = 0, \ldots, 4$ are real coefficients involving simple arithmetic and trigonometric operations, defined in Appendix A. The roots of these 4-th degree polynomials can be found by solving the respective quartic equations, for which closed form solutions exist [18]. Once the real roots are found, we invert the changes of variable introduced. The roots of P_α correspond to the extremes of the function $F_{mn}(\alpha,\delta)$ for fixed δ, while those of P_δ are the extremes of $F_{mn}(\alpha,\delta)$ for fixed α. Since up to 4 real roots may be found, the function $F_{mn}(\alpha,\delta)$ needs to be evaluated in the obtained roots in order to find the minimizing value α^*. An equivalent operation is performed for obtaining δ^*. Since computing α^* requires a constant value for δ and computing δ^* requires a constant value for α, the optimal values are found iteratively. Hence, α^* is computed initially by considering a certain initial value for δ (e.g. $\delta = 0$) and the resulting α^* is kept constant for computation of δ^*. This operation is iterated I_R times until convergence, which in practice occurs after 1 or 2 iterations.

Hence, although the unconstrained optimization problem in (8) is nonconvex, it can be solved by finding the roots of the polynomials P_α and P_δ, selecting among these roots the maximizing values α^* and δ^*. The optimization procedure proposed to obtain α^* and δ^* for each plane rotation is summarized in Table II.

TABLE II
PROCEDURE TO OBTAIN THE ROTATION PARAMETERS FOR THE $(\mathbf{w}_m, \mathbf{w}_n)$-PLANE

Input vectors: $\mathbf{h}_m$, $\mathbf{h}_n$, $\mathbf{w}_m$, $\mathbf{w}_n$

Initialization

- Compute auxiliary variables (Table III)
- Initialize rotation parameters: $\alpha^0 = 0$, $\delta^0 = 0$

i**-th iteration step**, $i = 1, \ldots, I_R$

- **Computation of** α^i
  - Compute $\phi_1(\delta^{i-1})$, $\phi_2(\delta^{i-1})$
  - Compute polynomial coefficients of $P_\alpha(t)$: f_0, f_1, f_2, f_3, f_4
  - Compute roots of $P_\alpha(t)$: t_1, t_2, t_3, t_4
  - Change of variable: $\alpha_j = \tan^{-1}(t_j)$, $j = 1, \ldots, 4$
  - Evaluate F_{mn} for α_j, $j = 1, \ldots, 4$, and δ^{i-1}
    $F_{mn}(\alpha_j, \delta^{i-1}) = \left(1 - \rho_m^2(\alpha_j, \delta^{i-1}) + \beta_m\right)\left(1 - \rho_n^2(\alpha_j, \delta^{i-1}) + \beta_n\right)$
  - Choose minimizing value among the 4 obtained solutions
    $\alpha^i = \arg\min_{\alpha_j,\, j=1,\ldots,4} F_{mn}(\alpha_j, \delta^{i-1})$
- **Computation of** δ^i
  - Compute $\varphi_1(\alpha^i)$, $\varphi_2(\alpha^i)$, $\varphi_3(\alpha^i)$
  - Compute polynomial coefficients of $P_\delta(s)$: g_0, g_1, g_2, g_3, g_4
  - Compute roots of $P_\delta(s)$: s_1, s_2, s_3, s_4
  - Change of variable: $\delta_j = 2\tan^{-1}(s_j)$, $j = 1, \ldots, 4$
  - Evaluate F_{mn} for δ_j, $j = 1, \ldots, 4$, and α^i
    $F_{mn}(\alpha^i, \delta_j) = \left(1 - \rho_m^2(\alpha^i, \delta_j) + \beta_m\right)\left(1 - \rho_n^2(\alpha^i, \delta_j) + \beta_n\right)$
  - Choose minimizing value among the 4 obtained solutions
    $\delta^i = \arg\min_{\delta_j,\, j=1,\ldots,4} F_{mn}(\alpha^i, \delta_j)$

TABLE III
AUXILIARY VARIABLES FOR THE COMPUTATION OF POLYNOMIAL COEFFICIENTS

$r_{mm} = \left\|\overline{\mathbf{h}}_m^H \mathbf{w}_m\right\|$	$r_{nm} = \left\|\overline{\mathbf{h}}_n^H \mathbf{w}_m\right\|$
$r_{mn} = \left\|\overline{\mathbf{h}}_m^H \mathbf{w}_n\right\|$	$r_{nn} = \left\|\overline{\mathbf{h}}_n^H \mathbf{w}_n\right\|$
$\Delta_{mn} = \angle\overline{\mathbf{h}}_m^H \mathbf{w}_m - \angle\overline{\mathbf{h}}_m^H \mathbf{w}_n$	$\Delta_{nm} = \angle\overline{\mathbf{h}}_n^H \mathbf{w}_n - \angle\overline{\mathbf{h}}_n^H \mathbf{w}_m$
$\beta_m = \frac{M\sigma^2}{P\\|\mathbf{h}_m\\|^2}$	$\beta_n = \frac{M\sigma^2}{P\\|\mathbf{h}_n\\|^2}$
$b_{mn} = -(r_{mm}^2 + r_{mn}^2)$	$b_{nm} = -(r_{nn}^2 + r_{nm}^2)$
$c_{mn} = -(r_{mm}^2 - r_{mn}^2)$	$c_{nm} = -(r_{nn}^2 - r_{nm}^2)$
$d_1 = 2r_{mm}r_{mn}$	$a_m = 1 + \beta_m$
$d_2 = 2r_{nm}r_{nn}$	$a_n = 1 + \beta_n$
$d_3 = -2d_1d_2\cos(\Delta_{nm} - \Delta_{mn})$	$e_1 = 2c_{mn}c_{nm}$
$d_4 = -\frac{d_1d_2}{2}\sin(\Delta_{nm} - \Delta_{mn})$	$e_2 = \frac{a_m}{2} + \frac{b_{mn}}{4}$
$d_5 = 2d_1\cos\Delta_{mn}$	$e_3 = \frac{a_n}{2} + \frac{b_{nm}}{4}$
$d_6 = d_1\sin\Delta_{mn}$	$e_4 = \frac{c_{nm}}{4}$
$d_7 = 2d_2\cos\Delta_{nm}$	$e_5 = \frac{c_{mn}}{4}$
$d_8 = -d_2\sin\Delta_{nm}$	$e_6 = 4(c_{nm}e_2 + c_{mn}e_3)$

A. Practical Considerations

Although closed form solutions exist for quartic equations, fast converging algorithms can be applied involving much lower complexity. Since only real roots are sought, the quotient-difference (QD) algorithm can be used to identify the roots followed by a fast converging algorithm like Newton-Raphson (NR) [19]. The initial unitary beamforming matrix $\mathbf{W}^0$ can be generated randomly, although more complex initializations may yield faster convergence. For instance, $\mathbf{W}^0$ can be constrained to have one of its vectors well aligned with the user channel that has the largest channel norm, as proposed in [14] as a suboptimal beamforming approach. In practice, this can be implemented by storing a number of unitary matrices (codebook), selecting the most appropriate one for initialization at each slot. For simplicity, in the remainder of the paper, we consider that the proposed algorithm is initialized by choosing $\mathbf{W}^0$ randomly unless stated otherwise. Note that the proposed algorithm provides computational flexibility, since the number of plane rotations N_{PR} can be modified. In the most general case, all possible combinations of plane rotations should be performed, i.e. $\binom{M}{2}$ combinations. Moreover, the order in which these plane rotations are performed has an impact on the convergence. Hence, the total number of plane rotations can be expressed as $N_{PR} = I_T\binom{M}{2}$, where I_T is a natural number.

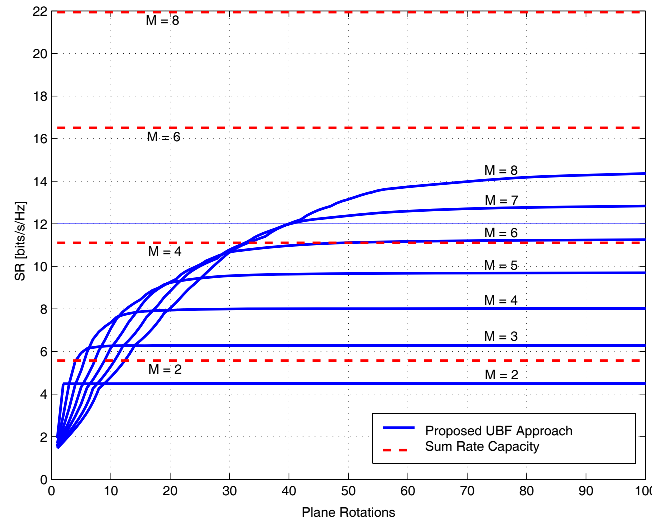

Fig. 1. Sum rate as a function of the number of plane rotations (algorithm iterations) for the proposed algorithm, using different number of transmit antennas, $K = M$ users and average SNR = 10 dB. The sum rate capacity for different values of M is shown for comparison.

V. Convergence

When optimizing the rotation along the $(\mathbf{w}_m, \mathbf{w}_n)$-plane, the sum of the rates provided by the m-th and n-th beamforming vectors is maximized with respect to the rotation parameters. Thus, defining $SR_{mn} = \log_2(1 + SINR_m) + \log_2(1 + SINR_n)$, at each plane rotation optimization we have that $SR_{mn}(\alpha^*, \delta^*) \geq SR_{mn}(\alpha, \delta)$. In addition, as discussed in the previous section, the SINR values associated to the remaining beamforming vectors do not change. Hence, at each iteration the resulting sum rate does not decrease, i.e. $SR(\mathbf{W}^i) \geq SR(\mathbf{W}^{i-1})$. On the other hand, since the transmitted power is finite, the sum rate - which is the objective function that the algorithm tries to maximize - is bounded from above. Thus, local convergence is guaranteed in the proposed optimization problem. The convergence behavior of the proposed iterative algorithm is exemplified in Figure 1 for different number of transmit antennas. In this simulation, Givens rotations are performed in all possible $(\mathbf{w}_m, \mathbf{w}_n)$-planes, and a large number of plane rotations $N_{PR} \to \infty$ is considered. The sum rate capacity [20][21] for different number of transmit antennas is also shown for comparison.

A. Comparison with the Quasi-Optimal Unitary Beamformer

In order to evaluate the convergence of the proposed algorithm, we present here a comparison with the quasi-optimal UBF solution through numerical simulations. The quasi-optimal solution is a brute-force approach, in which a finite set of unitary matrices with very high cardinality is generated, selecting the beamforming matrix that yields the highest sum rate. In order to bound the error between the optimal unitary matrix and the quasi-optimal unitary matrix, the set of unitary matrices is generated using the matrix parameterization of equation (7), setting $\mathbf{W}^0 = \mathbf{I}_M$. Each rotation parameter in this parameterization takes the following values $(0, \frac{2\pi}{L-1}, 2\cdot\frac{2\pi}{L-1}, 3\cdot\frac{2\pi}{L-1}, \ldots, 2\pi)$, L being the number of finite values considered for each rotation parameter. Hence, each unitary matrix in the set is generated from a given combination of the rotation parameter values involved in the parameterization. For instance, in the case of $M = 3$ transmit antennas, 6 rotation parameters are used to represent a given unitary matrix. Hence, in this case, the total number of unitary matrices generated to obtain the quasi-optimal unitary beamformer is L^6. In general, for an arbitrary number of transmit antennas, the total number of matrices generated is $L^{2\binom{M}{2}}$.

Since this approach involves very high complexity, it is not realizable in practice. In order to have a quasi-optimal solution close to the theoretical optimum, we focus on the case in which the transmitter has a reduced number of antennas, $M = 2$ and $M = 3$. For the case of $M = 2$ antennas, $L = 4000$ values per rotation parameter have been considered. This yields a total of $1.6 \cdot 10^7$ unitary matrices. Note that in a hypothetical system where these matrices would correspond to a quantization codebook, a total of 24 bits would be necessary for indexing all matrices. For the case of $M = 3$ antennas, $L = 22$ values per rotation parameter have been considered, which results approximately in a total of $1.1 \cdot 10^8$ unitary matrices. In this case, 27 bits would be necessary for indexing all matrices.

A comparison between the proposed UBF approach and the quasi-optimal UBF approach is given in Figure 2. As the figure shows, the proposed algorithm converges to the quasi-optimal solution after a few iterations, both for $M = 2$ and $M = 3$ transmit antennas. The sum-rate was averaged over 1000 channel realizations, and convergence was reached for each of them. Note that convergence of the sum rate does not imply that the resulting optimized UBF matrix converges to the matrix obtained in the quasi-optimal solution.

B. Simplified Scenario

In order to better illustrate the convergence speed of the proposed algorithm for different number of transmit antennas, we study a simple case in the remainder of this section for

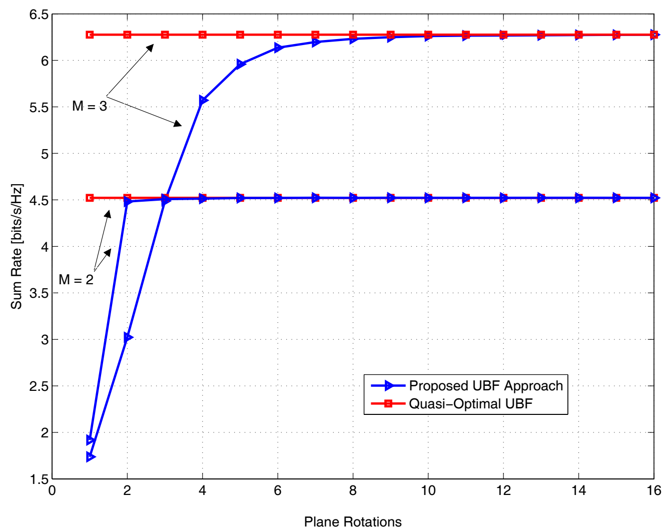

Fig. 2. Sum rate as a function of the number of plane rotations for the proposed algorithm and the quasi-optimal solution, $K = M$ users and average SNR $= 10$ dB.

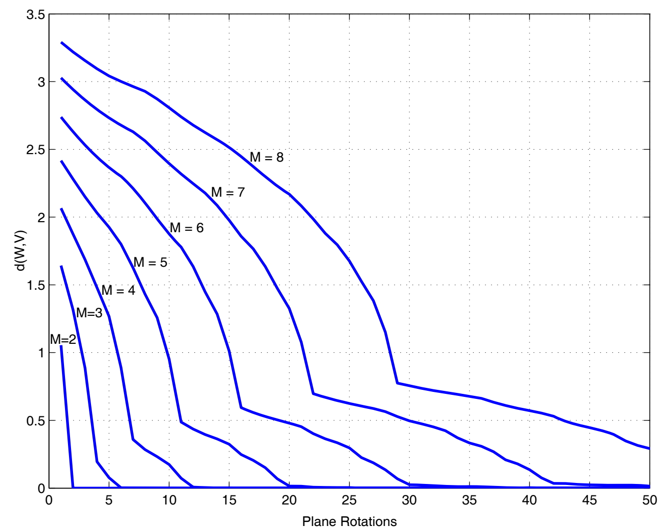

Fig. 3. Convergence of unitary beamforming matrix for different number of transmit antennas.

which the optimal solution is known. Let $\mathbf{H}$ be the concatenation of the user channels $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_M]^H$. Consider a simple channel model in which the concatenated channel can be factorized as $\mathbf{H} = \mathbf{\Lambda V}^H$, where $\mathbf{\Lambda}$ is a diagonal matrix with real entries ordered in descending order and $\mathbf{V}$ is a unitary matrix. This is equivalent to a point-to-point MIMO channel $\overline{\mathbf{H}}$ in which, given its singular value decomposition $\overline{\mathbf{H}} = \mathbf{U\Lambda V}^H$, the receiver filters the received signal with the matrix $\mathbf{U}^H$. If perfect channel state information is available at the transmitter and equal power allocation per beam is assumed, the optimal linear beamformer is known to be $\mathbf{W} = \mathbf{V}$, yielding M virtual parallel channels [22], [23]. In order to evaluate the convergence of the proposed algorithm to the optimal solution, we compute the following Frobenius distance at each iteration

$$d(\mathbf{W}, \mathbf{V}) = \left\|\mathbf{W}^H\mathbf{V} - \mathbf{I}\right\|_F. \tag{18}$$

Figure 3 shows the convergence behavior of the proposed algorithm for different number of transmit antennas. In this scenario, the proposed algorithm converges iteratively to the optimal solution. Note that for each value of M there are 2 differentiated regions with different convergence speed. The 1-st part converges faster, which corresponds to the 1-st $\binom{M}{2}$ iterations while the 2-nd part converges slower. This is due to the fact that the order in which plane rotations are performed matters, becoming more important as the size of the unitary beamforming matrix increases.

VI. Performance Evaluation

In this section, we evaluate the performance of the proposed unitary beamforming approach and compare it to other existing approaches. The proposed algorithm is initialized by choosing $\mathbf{W}^0$ randomly. Plane rotations are performed in all possible combinations, resulting in $N_{PR} = I_T\binom{M}{2}$ rotations, with $I_T = 3$. In the simulated scenarios, the algorithm approximately converges for this choice. The MATLAB function *roots* is used to compute the polynomial roots, which involves computing the eigenvalues of the companion matrix for each polynomial. In Subsections VI-A and VI-B, a system with $K = M$ is studied, hence assuming a given set of M users has been scheduled for transmission. While in VI-A perfect CSIT is assumed to be available, a system with imperfect CSIT is considered in VI-B. In the last subsection, a system with multiuser scheduling is considered, comparing the proposed approach to limited feedback techniques based on unitary beamforming.

A. Case $K = M$, Perfect CSIT

The performances of the proposed unitary beamforming technique, ZF beamforming and MMSE beamforming are compared in a system in which perfect CSIT is available, given a set of $K = M$ users scheduled for transmission. Both ZF and MMSE beamforming matrices are computed on the basis of the concatenated user channels $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_M]^H$. The ZF beamformer is computed as follows

$$\mathbf{W}_{ZF} = \frac{1}{\lambda}\mathbf{H}^H(\mathbf{HH}^H)^{-1} \tag{19}$$

where $\lambda = \frac{1}{\sqrt{P}} tr\left[(\mathbf{HH}^H)^{-1}\right]$. The MMSE beamformer is given by

$$\mathbf{W}_{MMSE} = \gamma\mathbf{H}^H(\mu\mathbf{I} + \mathbf{HH}^H)^{-1} \tag{20}$$

where γ is chosen such that $tr\left(\mathbf{W}_{MMSE}\mathbf{W}_{MMSE}^H\right) = P$. By setting $\mu = \frac{M\sigma^2}{P}$, the resulting SINR with MMSE beamforming is maximized for large K, as shown in [24]. In addition, the performance of a system that performs TDMA is also plotted for reference, selecting the user with largest channel norm out of M available users.

Figure 4 shows a performance comparison in terms of sum rate versus number of transmit antennas M, for SNR $= 10$ dB. As expected, the MMSE solution provides linear sum-rate growth with the number of transmit antennas, while ZF beamforming flattens out [24]. The proposed algorithm also provides linear growth with M, performing close to MMSE beamforming.

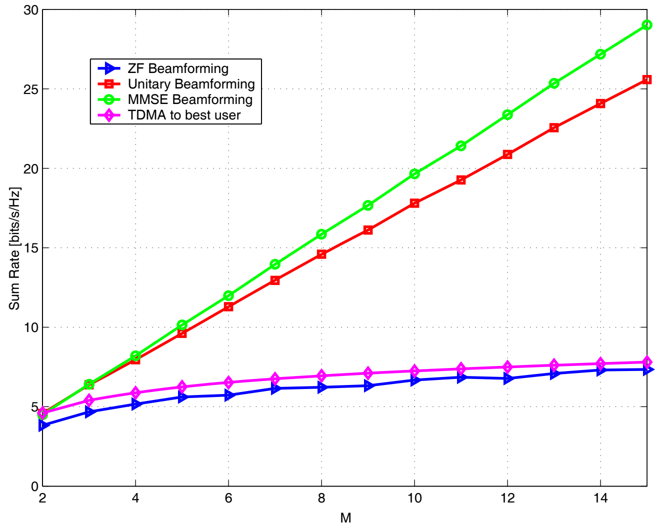

Fig. 4. Sum rate as a function of the number of antennas M for $K = M$ users and average SNR $= 10$ dB.

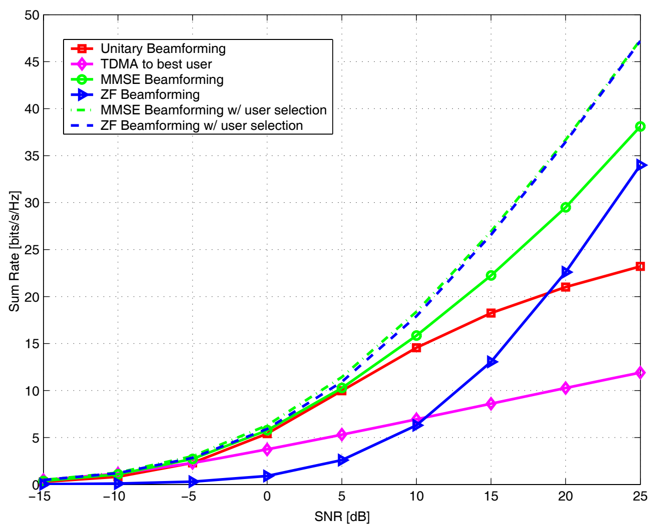

Fig. 5. Sum rate as a function of the average SNR for $M = 8$ transmit antennas and $K = M$ users.

In Figure 5, we compare the sum rate as function of the average SNR in a system with $M = 8$ transmit antennas. As the SNR increases, the MMSE solution converges to ZF, removing all multi-user interference. The proposed technique provides considerable gains over ZF in the regular SNR range, performing close to the MMSE solution. On the other hand, the proposed algorithm does not completely eliminate interference, since instead it balances the useful power and undesired interference in the SINR expression. Suboptimal techniques based on unitary beamforming have shown to become interference limited at high SNR, thus providing zero multiplexing gain [5], [25]. The multiplexing gain is defined as follows

$$m = \lim_{P\to\infty} \frac{\sum_{k=1}^{M} \mathbb{E}\left[\log_2\left(1 + SINR_k\right)\right]}{log_2(P)}. \tag{21}$$

However, as it can be observed from Figure 5, the multiplexing gain of the proposed scheme converges to the one of TDMA (same slope). A particular case of the proposed approach corresponds to the case in which one of the unitary beamforming vectors is aligned with the channel vector that has largest norm. In that case, at least one of the users does not see any interference from the other users and hence at least $m = 1$ is achieved. Thus, for the proposed approach we obtain

$$\begin{aligned} m^{UBF} \geq \lim_{P\to\infty} & \frac{\mathbb{E}\left[\log_2\left(1 + \frac{P}{M\sigma^2} \max_{i\in 1,\ldots,M} \|\mathbf{h}_i\|^2\right)\right]}{log_2(P)} \\ & + \frac{\sum_{k=1,k\neq i}^{M} \mathbb{E}\left[\log_2\left(1 + SINR_k\right)\right]}{log_2(P)} \geq 1 \end{aligned} \tag{22}$$

where the first term in the summation corresponds to aligning a unit-norm beamforming vector along the channel direction of the user with largest channel gain and the second term corresponds to the remaining $M-1$ beamforming vectors. The second inequality in the above equation follows from the fact that if none of the $M-1$ beamforming vectors in the second term is aligned with the remaining $M-1$ channels, they exhibit zero multiplexing gain.

Figure 5 also shows a comparison with ZF and MMSE with user selection. In these schemes, although the number of active users in the cell equals the number of transmit antennas, the base station schedules the subset of M^* users that maximizes the sum rate, with $1 \leq M^* \leq M$. As it can be observed, these approaches provide higher rates than the proposed UBF technique, due to the fact that in the proposed scheme the unitarity constraint forces the number of scheduled users to coincide with the number of transmit antennas.

B. Case $K = M$, Imperfect CSIT

The impact of imperfect channel knowledge at the transmitter in a system with $K = M$ users is investigated. The beamforming matrices are computed on the basis of noisy channel estimates, modeled as described in equation (3), which produces a performance degradation in terms of system sum rate. Figure 6 shows a sum-rate comparison between the proposed approach, ZF beamforming, MMSE beamforming and TDMA as a function of the variance of the channel estimation error, for $M = 4, 8$ antennas and average SNR of 10 dB. In the simulated scenarios, the proposed unitary beamforming approach appears to be more robust to CSIT errors than ZF or MMSE beamforming. Indeed, in the simulations shown here, a small error variance suffices for unitary beamforming to outperform MMSE beamforming, even for large number of transmit antennas. However, TDMA provides higher rates in scenarios with reduced number of transmit antennas and very low quality of CSIT.

C. Case $K \geq M$, Limited Feedback

The proposed technique is evaluated in a MIMO broadcast channel where limited feedback is available from the user terminals to the base station. Most existing techniques with joint linear beamforming and multiuser scheduling designed for limited feedback scenarios are based on simple beamforming

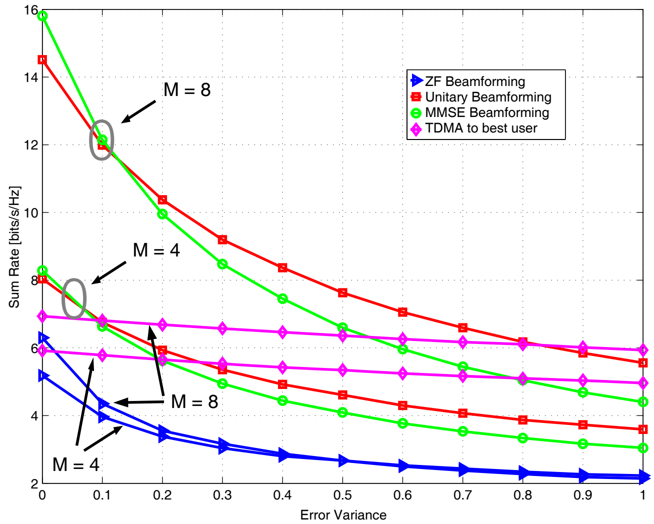

Fig. 6. Sum rate as a function of the channel estimation error variance for $M = 4, 8$ transmit antennas, $K = M$ users and average SNR $= 10$ dB.

designs. While the design of feedback measures is carefully taken into account, suboptimal beamforming techniques are often considered, such as the well known RBF [5] and PU2RC [6] techniques. In these techniques, the base station has precise SINR information from the users, but on the other hand, non-optimized linear beamformers are used. In the remainder of this section, we highlight the importance of beamforming design in limited feedback scenarios. By exploiting the robustness to channel estimation errors exhibited by the proposed approach, we show that optimization of the linear beamformers is crucial in MIMO broadcast channels with imperfect CSIT. Since we focus on the optimization of the linear beamformers rather than on the design of the quatization codebooks, a simple quantization technique based on random vector quantization (RVQ) is used. In a practical system, the proposed unitary beamforming technique could be combined with other quantization codebooks previously proposed in the literature for MIMO systems with limited feedback, such as [26][27][28].

A scenario with $K \geq M$ is considered and thus the need for multiuser scheduling arises. For simplicity, exhaustive user search is performed, i.e. the base station evaluates the estimated sum rate of all possible user sets with cardinality M and selects the one that provides higher estimated sum rate. Thus, the user set scheduled for transmission is found as follows

$$\mathcal{S}^* = \arg\max_{\mathcal{S}\in\mathcal{G}} \sum_{k\in\mathcal{S}} \log_2 \left[1 + \widehat{SINR}_k(\mathcal{S})\right] \quad (23)$$

where $\widehat{SINR}_k$ is the estimated SINR for user k computed at the base station and $\mathcal{G}$ is the set of all possible subsets of cardinality M of disjoint indices among the complete set of user indices $\mathcal{K} = \{1, \cdots, K\}$. In a more practical system, less complex user selection algorithms could be employed. An example of such algorithms is the greedy semi-orthogonal user selection introduced in [29]. In this algorithm, the first selected user is chosen such that a certain metric is maximized, for instance the estimated channel norm. Each user added to the selected set of users must be semi-orthogonal to the users already selected, i.e. the correlation between their channels cannot exceed a predetermined threshold. This can greatly reduce the pool of users considered for transmission in the set $\mathcal{G}$. Another way to further decrease complexity would be to use a less complex linear beamforming technique to perform user selection, such as ZF beamforming, and use the proposed unitary beamforming technique for transmission, since it provides better performance in systems with imperfect CSIT, as shown previously. In this case, the estimated SINR measure for each user in the set, $\widehat{SINR}_k(\mathcal{S})$, would be computed on the basis of ZF beamforming vectors. In the reminder of this section, we assume that exhaustive user search is performed, since user selection algorithms are beyond the scope of this paper.

In the proposed scheme, each user quantizes its channel vector based on a quantization codebook $\mathcal{V}$ that is common to all users in the system. The vector quantizer maps the user channel to the codeword in $\mathcal{V}$ with the smallest Euclidean distance. Each user sends the corresponding quantization index back to the transmitter through an assumed error-free, and zero-delay feedback channel using B bits. At the transmitter side, the estimated channel norm and channel direction are computed on the basis of the quantized user channels, which in turn are used to compute the proposed unitary linear beamformer. An RVQ channel quantization codebook is considered, complemented with simple codeword pruning as described in [30, pp. 359]. Pruning consists of starting with an initial training set of candidate codewords (randomly generated), and selectively eliminating (pruning) training vectors until obtaining a final set of 2^B vectors. The codebook is generated recursively, adding a new codeword to the codebook at each step. When a codeword is added, it must satisfy that the distortion measure - in our case given by the Euclidean distance - between the newly added codeword and the nearest neighbor in the codebook is greater than some threshold. In our case, this threshold has been set empirically in order to provide good performances.

The limited feedback approaches we consider for comparison are RBF and PU2RC, both based on unitary beamforming. These techniques involve much lower computational complexity than the proposed algorithm, since the beamformers are generated randomly. In addition, an advantage of RBF and PU2RC is that the exact SINR can be computed at the receiver side without additional training. In the case of PU2RC, a codebook with N random unitary matrices is considered (each with M unit-norm vectors), known both to the base station and mobile users. In the PU2RC scheme proposed in [6], the users feed back a codeword index using $B = log_2(MN)$ bits together with the expected SINR, which in the case of unitary beamforming can be precisely determined without knowledge of the beamforming vectors intended to other users. In the scenario under study, the data rate on the feedback link is limited to $B = 10$ bits/transmission. In order to make a fair comparison between the schemes, the SINR feedback of the PU2RC algorithm is also quantized. Thus, the PU2RC algorithm has to share the available B bits between the CDI, i.e., the index of the preferred beamforming vector, and the CQI, i.e., the SINR of the preferred beamforming vector. In our sim-

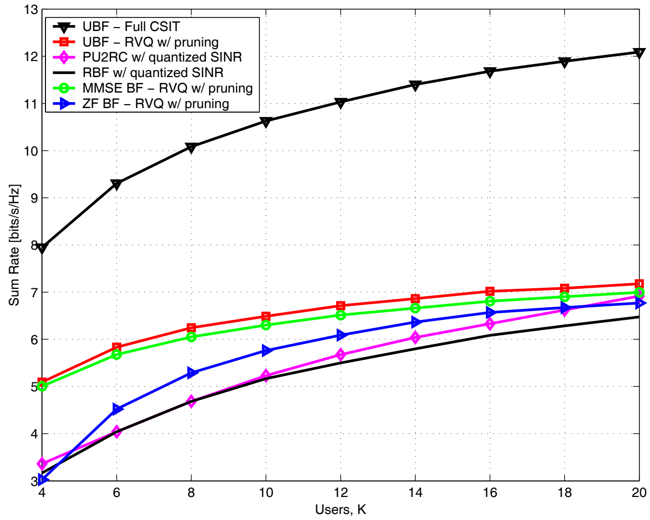

Fig. 7. Sum rate as a function of the number of users in a system with joint beamforming and user scheduling, $M = 4$ transmit antennas, SNR = 10 dB, and $B = 10$ feedback bits.

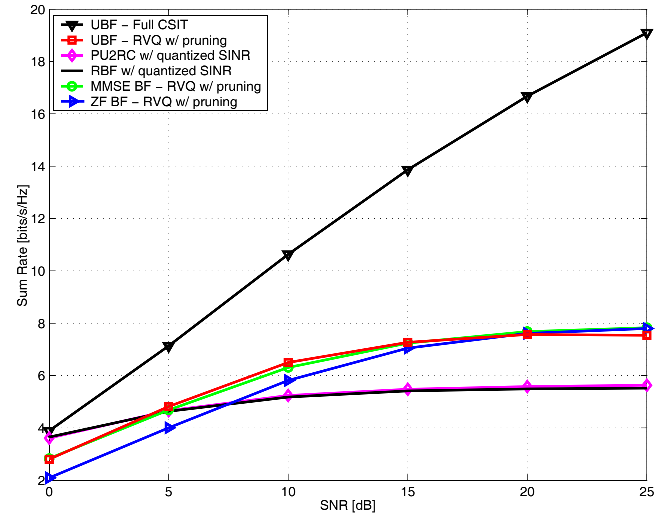

Fig. 8. Sum rate as a function of the average SNR in a system with joint beamforming and user scheduling, $M = 4$ transmit antennas, $K = 10$ users, and $B = 10$ feedback bits.

ulations, we simulate the performance of all possible CDI/CQI bit allocations, and finally select the allocation that results in the highest sum rate. The codebook to quantize the scalar CQI is designed with the generalized Lloyd algorithm [30], using the mean square error as distortion function. While CDI quantization incurs in loss of multiplexing gain, CQI quantization leads to a degradation of the multiuser diversity benefit. This means that as the average SNR increases, more feedback bits are allocated on channel direction information. On the other hand, as the number of users increases, more feedback bits are allocated on channel quality information. In the case of RBF, the amount of bits for CDI feedback is given by $\log_2 M$, since only a single beamforming matrix is generated at each time slot. The remaining bits are used for SINR quantization, following the same quantization criterion as for PU2RC.

Figure 7 depicts the performance for different numbers of users with a fixed SNR of 10 dB, in a system with $M = 4$ transmit antennas and $B = 10$ bits available for feedback. The performance of the proposed UBF approach with perfect CSIT is also provided. The performance of MMSE and ZF beamforming with limited feedback is provided for comparison, using the same RVQ channel quantization codebooks as the proposed UBF approach. The proposed UBF technique clearly outperforms ZF beamforming in the simulated scenario, providing moderate gains over MMSE beamforming. The proposed approach performs very similarly to MMSE beamforming in this particular scenario with $B = 10$ quantization bits, outperforming ZF beamforming in the low-to-moderate SNR region. The proposed approach combined with simple RVQ and pruning outperforms RBF and PU2RC, especially in systems with reduced number of users, providing sum-rate gains over 1.5 bps/Hz. Note that the performance of RBF, which was shown to achieve the optimal capacity scaling in [5], is a pessimistic lower bound on the performance of PU2RC when random unitary codebook bases are used. As the number of users increases, the simplicity of the quantization codebook used in the proposed unitary beamforming approach does not allow to capture all multiuser diversity gain and the sum rate curve flattens out. On the other hand, PU2RC exhibits optimal sum-rate growth in the simulated range, thanks to an optimal bit allocation for CDI/CQI information.

In Figure 8, a sum-rate comparison as function of the average SNR is shown in a system with $M = 4$ transmit antennas, $K = 10$ users and $B = 10$ bits. As expected, the limited feedback approaches become interference limited at high SNR. The proposed approach performs very similarly to MMSE beamforming in this particular scenario with $B = 10$ quantization bits, outperforming ZF beamforming in the low-to-moderate SNR region. In the simulated scenario, the proposed technique provides performance gains of up 2-bps/Hz over RBF and PU2RC for a given SNR. Note that the simulation parameters here used reflect realistic scenarios of practical importance, often encountered in indoor wireless systems.

In order to further clarify the comparison between PU2RC and the proposed UBF approach, an additional scenario is simulated. In this scenario, we consider a system in which the CQI information is transmitted unquantized over the feedback channel, and thus all the available feedback bits are used for CDI quantization. In the proposed UBF approach, RVQ with pruning is used for CDI quantization, while each user feeds back its channel norm unquantized. In the PU2RC scheme, unquantized SINR feedback is sent along with quantized CDI information. Figure 9 shows a performance comparison between these schemes for the described scenario. The proposed approach outperforms PU2RC as the number of feedback bits increases. Hence, in the moderate-large B regime the proposed technique provides higher sum rates than PU2RC. On the other hand, in the low resolution regime, PU2RC provides higher rates. In addition, in the simulated system with $M = 4$ transmit antennas and $K = 10$ users, increasing the amount of CDI bits does not increase the throughput of PU2RC. This is due to the fact that an increase in the codebook size reduces the probability of finding users with large channel gains that

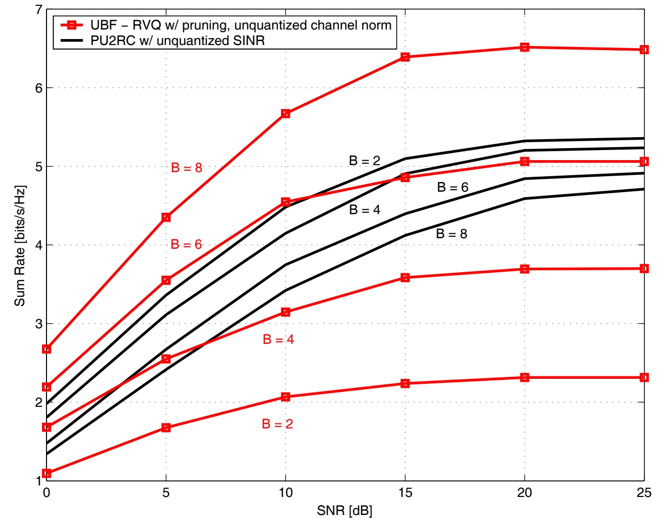

Fig. 9. Sum rate as a function of the average SNR for different number of CDI quantization bits, and unquantized CQI. The schemes perform joint beamforming and user scheduling, in a system with $M = 4$ transmit antennas and $K = 10$ users.

are also associated with the same orthonormal basis in the codebook. This adverse effect in PU2RC, as discussed in [25], diminishes as the number of users increases.

VII. Conclusions

An iterative optimization method for unitary beamforming in MIMO broadcast channels has been proposed, based on successive optimization of Givens rotations. In a scenario with perfect CSIT and for practical average SNR values, the proposed technique provides higher sum rates than ZF beamforming and performs close to MMSE beamforming when the number of transmit antennas equals the number of scheduled users, achieving linear sum-rate growth with the number of transmit antennas. The proposed unitary beamforming approach exhibits robustness to channel estimation errors, providing better sum rates than ZF beamforming and even MMSE beamforming as the variance of the estimation error increases. In addition, the proposed technique has been evaluated in scenarios with multiuser scheduling and limited feedback. As simulations have shown, our approach provides gains when compared to other existing techniques based on unitary beamforming and the same amount of feedback. A simple vector quantization technique has been used, based on RVQ with pruning. Hence, our work highlights the importance of linear beamforming optimization in limited feedback scenarios. While the random beamforming technique introduced in [5] enables perfect SINR knowledge of all users at the base station, the generation of the beamforming vectors is clearly suboptimal. Instead, as we have shown in this work, the system performance can be improved by performing simpler feedback design through direct channel vector quantization and optimization of the linear beamformers.

Appendix A
Computation of Polynomial Coefficients

This appendix describes a procedure to obtain the coefficients of the polynomials P_α and P_δ of equations (16) and (17), respectively. Although the procedure to obtain these coefficients can be described in different ways, here we present it in a simple and sequential fashion for straightforward software implementation. For each plane rotation, the auxiliary variables defined in Table III are computed and used for the computation of the coefficients of both P_α and P_δ. These auxiliary variables are functions of $\mathbf{h}_m$, $\mathbf{h}_n$, $\mathbf{w}_m$, and $\mathbf{w}_n$.

A. Computation of the polynomial coefficients of P_α

The coefficients of P_α are functions of the rotation parameter δ. For clarity of exposition, the following functions are defined

$$\begin{aligned}\phi_1(\delta) &= d_1\cos(\Delta_{mn}+\delta)\\ \phi_2(\delta) &= -d_2\cos(\delta-\Delta_{nm})\end{aligned} \tag{24}$$

The coefficients of the polynomial P_α are given by

$$\begin{aligned}f_4 &= -2\phi_1(\delta)(e_3-e_4)-2\phi_2(\delta)(e_2-e_5)\\ f_3 &= -2\phi_1(\delta)\phi_2(\delta)+e_1-e_6\\ f_2 &= -12\left[\phi_1(\delta)e_4+\phi_2(\delta)e_5\right]\\ f_1 &= 2\phi_1(\delta)\phi_2(\delta)-e_1-e_6\\ f_0 &= 2\phi_1(\delta)(e_3+e_4)+2\phi_2(\delta)(e_2+e_5)\end{aligned} \tag{25}$$

B. Computation of the polynomial coefficients of P_δ

The coefficients of P_δ are functions of the rotation parameter α. The following functions are defined

$$\begin{aligned}\varphi_1(\alpha) &= e_2\sin 2\alpha+\tfrac{e_5\sin 4\alpha}{2}\\ \varphi_2(\alpha) &= e_3\sin 2\alpha+\tfrac{e_4\sin 4\alpha}{2}\\ \varphi_3(\alpha) &= \tfrac{1-\cos 4\alpha}{4}\end{aligned} \tag{26}$$

The coefficients of the polynomial P_δ are given by

$$\begin{aligned}g_4 &= -d_8\varphi_1(\alpha)+d_6\varphi_2(\alpha)+d_4\varphi_3(\alpha)\\ g_3 &= d_7\varphi_1(\alpha)-d_5\varphi_2(\alpha)+d_3\varphi_3(\alpha)\\ g_2 &= -6d_4\varphi_3(\alpha)\\ g_1 &= d_7\varphi_1(\alpha)-d_5\varphi_2(\alpha)-d_3\varphi_3(\alpha)\\ g_0 &= d_8\varphi_1(\alpha)-d_6\varphi_2(\alpha)+d_4\varphi_3(\alpha)\end{aligned} \tag{27}$$

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