Double Iterative Precoder & Receiver Design for MU-MIMO Broadcast Channel

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***Abstract*—This paper proposes a new double iterative procedure for sum-rate maximization in a Multiuser MIMO system (MU-MIMO). The proposed algorithm is based on joint precoder and decoder optimization involving two different decoding schemes. For that we considered a precoding algorithm namely the iterative SJNR (Signal to Jamming and Noise Ratio) precoder combined with two iterative receivers. The first receiver is the MF (Matched Filter) determining the best direction maximizing the received power for each user. The resulting receiving vector from the first algorithm will be used as an initialization for the second one. The second receiver is the MSR (Maximum Sum Rate) receiver. The selection of the switching point between these two receivers is determined and performed by a dynamic algorithm introducing very low extra complexity.**

To link the precoder and the selected receiver through the iterations, we use an iterative procedure based on a virtual channel calculation evolving with the system towards convergence. Finally to validate our proposed solution we compare it with an existing MMSE based iterative optimization algorithm. This algorithm is based on MMSE approach for both the transmitting and receiving side. The obtained results demonstrate significant gains without introducing supplementary complexity.

***Index Terms*—Multi-user; MIMO; Broadcast channel; Capacity; SJNR; Iterative; MMSE.**

I. INTRODUCTION

Multiuser MIMO (MU-MIMO) downlink system known in the information theory as the broadcast channel system represents today one of the most important research fields in wireless communications because of the high potential it offers for improving both reliability and capacity of the system. Some theoretical analysis of the capacity demonstrated that the capacity of a broadcast MU-MIMO channel can be achieved by applying a Dirty-Paper Coding (DPC) [1], [2] algorithm as a precoder. Nevertheless, a DPC precoding is difficult to compute and is high resource consuming. Some suboptimal linear algorithms with low implementation complexity exist and can be divided into two families: the iterative [3]–[5] and the closed form solutions [6]–[9].

In the case of a MU-MIMO system, the precoder completely defines the system performance when only one receive antenna is used at each receiver side. The performance of a MU-MIMO system is measured by the total Sum-Rate and will be given in Section III. On the contrary, when multiple antennas are used at the receiver, the system performance depends also on the receiver structure. The optimum precoder depends on the structure of the receiver and vice versa the optimum receiver depends on the structure of the precoder applied at the transmission. That is why extracting the full performance of a MU-MIMO system requires the use of some iterative algorithms. More over, one of the main problems faced by the iterative solutions is the convergence towards local maxima or even some times divergence as mentioned in [8].

In this paper we are going to focus on the iterative linear solutions to be able to fully exploit the degrees of freedom at the transmission and the reception. In fact using a non iterative linear solution that is a one formula based algorithm provides a fast solution, but makes it difficult to cancel out all the interference created by the other users especially when the number of total transmitted streams is getting closer to the number of transmitting antennas.

Different iterative solutions exist and use different precoding and receiving structures in an iterative way to reduce the inter-user interference and enhance the system performances. In this paper, an SJNR precoder combined with two different receiving schemes the MF (Matched Filter) an the MSR (Maximum Sum-Rate) receiver is proposed. This scheme is compared to the MMSE/MMSE iterative algorithm given in [4] and to the iterative SJNR/MSR proposed in [3].

In next section, the model for the considered system is presented, followed by a detailed description of the proposed iterative algorithm and the employed receiver structure. In section IV, the simulation conditions and the obtained results are detailed and discussed. Finally some conclusions are given in the last section.

II. SYSTEM MODEL

Lets consider in our study a multi-user MIMO communication system with N_T transmission antennas at the base station and K different users with N_{R_k} receiving antennas for each user k. Such a system is represented on figure 1.
We assume that the base station has a perfect knowledge of the channel state information (CSI) of all K users. Let S_k a $Q_k \times 1$ vector representing the transmitted data symbols for user k where Q_k is the number of transmission streams for the same user. In our paper we are interested in the case of one stream per user Q_k=1.

The total transmit power at the base station is supposed to be constant and equal to P_T. The noise variance N_0 is equal to 1. For the channel part, H_k denotes the MIMO channel for user k which is a $N_{R_k} \times N_T$ matrix.

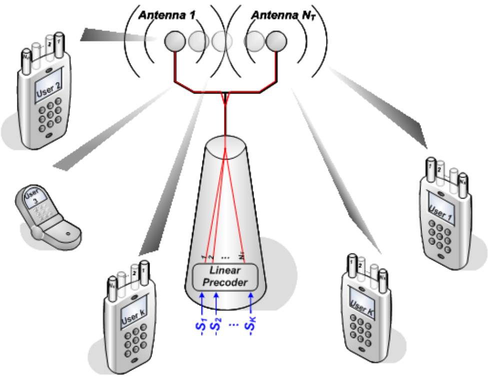

Fig. 1. System model.

III. SJNR/MFMSR DYNAMIC ITERATIVE ALGORITHM

In this section the proposed iterative algorithm is presented. The first subsection presents the employed precoder. The second one presents the involved receivers. After that a dynamic switching algorithm is proposed. And finally, the overall iterative joint optimization procedure is detailed.

A. SJNR Precoder

The objective is to design the transmit filters T_k under the total transmit power constraint $\sum_{k=1}^{K} P_k = P_T$. Here $P_k = trace\left(T_k R_{S_k} T_k^H\right)$ denotes the transmitted power aimed to user k. R_{Sk} is the covariance matrix of the symbols transmitted to user k.

We consider the Signal to Jamming plus Noise Ratio (SJNR) defined as the signal power over the noise plus total interfering power caused by the user k and received by the other mobiles. This concept has been introduced in [9] and is given by expression (1):

$$SJNR_k = \frac{T_k^H H_k^H H_k T_k}{T_k^H \sum_{j=1,j\neq k}^{K} H_j^H H_j T_k + N_0 I} \tag{1}$$

A solution to maximize the SJNR for the different users has been proposed in [9]. They demonstrate that the generalized eigenvalue of the SJNR expression is the optimal solution. The precoder for user k is therefore given by the expression of equation (2).

$$T_k = \sqrt{P_k}\zeta_m \left[\left(\sum_{j=1,j\neq k}^{K} H_j^H H_j + \frac{N_0}{P_k} I \right)^{-1} H_k^H H_k \right] \tag{2}$$

where $\zeta_m(X)$ represents the largest eigenvector of X. The largest eigenvector is defined as the eigenvector corresponding to the largest eigenvalue of X.

Iterative versions of the precoder is obtained by injecting the iterative virtual channel (10) into expression (2). The obtained iterative precoder is (3)

$$T_k^{iter} = \sqrt{P_k}\zeta_m \left[\left(\sum_{j=1,j\neq k}^{K} (H_j^{iter})^H H_j^{iter} + \frac{N_0^{iter}}{P_k} I \right)^{-1} (H_k^{iter})^H H_k^{iter} \right] \tag{3}$$

where $N_0^{iter} = N_0 \mathbf{D}_k^{iter-1} \left(\mathbf{D}_k^{iter-1}\right)^H$ and $\mathbf{D}_k^{iter}$ is the used receiver for user k and H^{iter} represents the virtual channel given by (10).

B. Receivers Design

Different structures have been proposed in the literature for the receiver design for MIMO systems. Among the existing proposed solutions there is the matched filter (MF) as proposed in [9] and given by equation (4)

$$D_{MF,k} = \frac{(H_k T_k)^H}{\|H_k T_k\|}, \tag{4}$$

where $\|X\|$ is the norm of vector X.

Another receiving structure that maximizes the total sum rate of the system is the one derived in [3] and named $D_{MSR,k}$.

In fact, maximizing the total sum rate for one stream per user system reduces the problem to maximizing all the throughputs for all K users given by (5).

$$r_k = log_2 \left(1 + \frac{D_k H_k T_k R_{S_k} T_k^H H_k^H D_k^H}{D_k \left(\Upsilon_k + N_0 I\right) D_k^H} \right) \tag{5}$$

where $\Upsilon_k = H_k \sum_{j=1,j\neq k}^{K} T_j R_{S_j} T_j^H H_k^H$ represents the interference generated by the other users and collected by user k.
The solution of this problem is the generalized eigenvector of matrices $\left(H_k T_k R_{S_k} T_k^H H_k^H, H_k \sum_{j=1,j\neq k}^{K} T_j R_{S_j} T_j^H H_k^H + N_0 I \right)$.
So finally the optimal receiver maximizing the system total sum-rate is given by equation (6).

$$D_{MSR,k}^H = \zeta_m(\psi), \tag{6}$$

where $\psi = \left(\sum_{j=1,j\neq k}^{K} H_k T_j R_{S_j} T_j^H H_k^H + N_0 I \right)^{-1} H_k T_k R_{S_k} T_k^H H_k^H$ and $\zeta_m(X)$ is, as previously defined, the largest eigenvector of matrix X.
The iterative proposed version of this receiver is then given by (7)

$$(D_{MSR,k}^{iter})^H = \zeta_m\left(\psi^{iter}\right) \tag{7}$$

where $\psi^{iter} = \left(\sum_{j=1,j\neq k}^{K} H_k^{iter} T_j^{iter} R_{S_j} (T_j^{iter})^H (H_k^{iter})^H + N_0 I \right)^{-1} H_k^{iter} T_k^{iter} R_{S_k} (T_k^{iter})^H (H_k^{iter})^H$

We also denote the total sum-rate of the MU-MIMO system by SR. The expression of the throughput is the sum over all users of the individual achieved throughputs and can be given by equation (8) according to [10]–[12].

$$SR = \sum_{k=1}^{K} log_2 \left(1 + \frac{D_k H_k T_k R_{S_k} T_k^H H_k^H D_k^H}{D_k \left(\Upsilon_k + N_0 I \right) D_k^H} \right) \qquad (8)$$

here, R_{S_k} is the covariance matrix (in this case a scalar) of the transmitted data S_k.

C. Dynamic Flip procedure

The main idea of the iterative procedure proposed in this paper is to combine two versions of the iterative algorithm derived from [3] using different receiving structures to be able to cover the largest part of the space containing the possible receivers. This minimizes the probability of entering a local maximum.

But combining two versions of the algorithm, implies a flipping point where the used algorithm (receiver) is changed. Further more, some statistical analysis of the throughputs given by the cascade of the two versions SJNR/MF and SJNR/MSR described in [3] demonstrated that the optimal flipping point is not only a function of the SNR (Signal to Noise Ratio), the system configuration (Number of transmitting and receiving antennas) but also of the channel realizations namely the matrices $H_k, k = 1..K$.

Figure 2 is a representation of the optimal flip point in function of the total transmitted power through two different stochastic analysis. The first curve gives the optimal flipping point maximizing the mean sum rate of the system by averaging over the channel realizations. The second curve plots the optimal mean flipping point maximizing the instantaneous sum rate of the system. These two curves and other statistical analysis demonstrated that the best flipping point is a function of all the parameters of the system. Therefore a standardized flipping point can not be defined. The solution would be to perform some lookup table in function of the system configuration. But the dimensions of these tables are exponential and can rapidly explode.

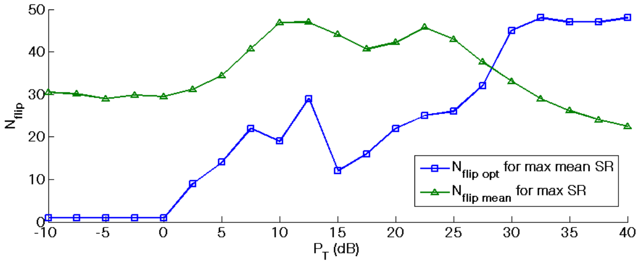

Fig. 2. optimal $iter_{flip}$ in function of the SNR and two simple statistical criteria for N_T=N_R=K=4.

To get rid of these constraints and be still able to get a significant gain, we proposed a dynamic selection procedure based on an instantaneous convergence analysis.

The selection procedure is then based on the monitoring of the obtained throughput over a fixed number of iterations that we are going to call sliding window. The number of considered iterations considered in this window is noted WIN_{MF} as the first considered receiver is the MF one.

To be able to run the selection procedure, a minimum of WIN_{MF} observations of the iterative algorithm must be available. Therefore, in the first phase, the algorithm derived from [3] is run for WIN_{MF} iterations. Starting from this point, the monitoring procedure is launched and at each iteration $iter > WIN_{MF}$ the variance of the obtained sumrate over the last WIN_{MF} considered iterations is computed according to equation (9). This quantity is noted SR_{Var}.

$$Var\left(\left[SR_{iter-WIN_{MF}}, \ldots, SR_{iter}\right]\right) = \frac{1}{WIN_{MF}} \sum_{i=0}^{WIN_{MF}} \left(SR_{iter-i} - \frac{1}{WIN_{MF}} \sum_{i=0}^{WIN_{MF}} SR_{iter-i} \right)^2 \qquad (9)$$

This variance is compared to a prefixed threshold ε_{MF} defining the convergence of the algorithm.

So the average evolution of the SR over the last WIN_{MF} iterations is observed. If the SR_{Var} is decreasing or if increase is below the prefixed threshold, the MF receiver giving the best SR in this window is retained.

A last control parameter is introduced to avoid the divergence problem. It consists in limiting the number of possible iterations for the SJNR/MF algorithm to $iter = N_{\max}^{iter} - \Delta_{MSR}$ where $\Delta_{MSR} \in \mathbb{N}^*$ and $1 \leq \Delta_{MSR} \leq N_{\max}^{iter} - WIN_{MF}$. Here $N_{\max}^{iter}$ is the total number of iterations allowed for the processing of a given transmission and $\mathbb{N}^* = \{1, 2, \ldots\}$.

The goal of this limitation in the number of the global iterations is to avoid that the algorithm gets blocked in case of divergence or of non convergence.

D. Iterative procedure

In this last subsection, the entire iterative procedure is presented. In a first phase, the SJNR/MF algorithm,derived from [3] is computed followed by the SJNR/MSR given in [3]. The decision is taken based on the Dynamic decision procedure presented in the past subsection. The evolution of the receiver and the precoder through the iterations is performed thanks to the virtual channel given by (10)

$$\mathbf{H}_k^{iter} = \mathbf{D}_k^{iter-1} \mathbf{H}_k \qquad (10)$$

The overall iterative algorithm is then given by the following steps:

Algorithm

Step 0/ Initialize $N_{\max}^{iter}$, WIN_{MF}, ε, ε_{MF} and Δ_{MSR}

Step 1/ A first iteration based on a closed form solution is done to initialize the algorithm.

Step 1.1 For that the SJNR linear precoder given by equation (2) is used. This first calculated precoder is called $\mathbf{T}_k^0$.

Step 1.2 Calculate the first optimal receiver $\mathbf{D}_k^0 = \mathbf{D}_{MF,k}$ for all users $k = 1, \ldots, K$ based on formula (4).

Step 1.3 Initialize $iter = 1$.

Step 2/ Transform the original transmission channel $\mathbf{H}_k$ to get the virtual channel linking the precoder and the decoder. The virtual channel is noted $\mathbf{H}_k^{iter}$ and is computed using expression (10).

Step 3/ The new precoder $\mathbf{T}_k^{iter}$ is computed using the new channel using equation (3).

Step 4/ Compute the new optimal receiver $\mathbf{D}_k^{iter} = \mathbf{D}_{MF,k}$ in function of the precoder $\mathbf{T}_k^{iter}$ defined by equation (4).

Step 5/ Compute the total obtained throughput SR_{iter} by injecting $\mathbf{T}_k^{iter}$ and $\mathbf{D}_k^{iter}$ in equation (8).

Step 6/ Repeat steps 2/ to 5/ WIN_{MF} times and increment each time $iter = iter + 1$

Step 7/ Verify the convergence of the MF algorithm by verifying the flip point condition $Var\left(\left[SR_{iter-WIN_{MF}}, \dots, SR_{iter}\right]\right) \leq \varepsilon_{MF}$.

If this condition is fulfilled or if the number of iterations exceeds the second condition $iter = N_{\max}^{iter} - \Delta_{MSR}$

then Consider the best set of decoding vectors $\mathbf{D}_k^{iter_{best}}, k = 1 \dots K, iter_{best} \in \{iter - WIN_{MF}, \dots, iter\}$ that gives the highest SR among those present in the actual window and jump to 8/.

else increment $iter = iter + 1$ and repeat steps 2/, 3/, 4/, 5/ and 7/.

Step 8/ Increment the counter $iter = iter + 1$ and compute the virtual channel given by the cascade on the channel and the receiver $\mathbf{H}_k^{iter} = \mathbf{D}_k^{iter-1}\mathbf{H}_k$. It must be noted that for the first time (when the algorithm comes from ***Step 7***), $\mathbf{D}_k^{iter-1}$ is an MF receiver such as calculated in ***Step 4*** in the iteration $iter_{best}$.

Step 9/ The new precoder $\mathbf{T}_k^{iter}$ is computed using equation (3) with the new obtained virtual channel.

Step 10/ Compute the new optimal receiver $\mathbf{D}_k^{iter} = \mathbf{D}_{MSR,k}$ in function of the precoder $\mathbf{T}_k^{iter}$ based on equation (7).

Step 11/ Evaluate the total sumrate SR^{iter} using $\mathbf{T}_k^{iter}$, $\mathbf{D}_k^{iter}$ and equation (8).

Step 12/ Repeat steps 8/ to 11/ until the algorithm converges. The convergence is either detected by the stabilisation of the sum rate $|SR_{iter} - SR_{iter+1}| < \varepsilon$; or by achieving the maximal number of iterations $N_{\max}^{iter}$

IV. SIMULATIONS AND RESULTS

In all our simulations, we consider that we have only one stream per user $Q_k = 1$ and that the number of receiving antennas is the same for all users N_{R_k}=N_R=4 or 2. We choose a Rayleigh fading channel $H_k = (h_{i,j}^k)_{1 \leq i \leq N_R, 1 \leq j \leq N_T}$ such as $E\|h_{i,j}^k\|^2 = 1$. The simulation generates 10000 independent channel realizations for each user. To generate the total throughput of the system, we perform an average over all channel realizations on the quantity SR given in equation (8). For the SJNR precoder, we distribute the energy equally over all considered users according to $P_k = P_T / K$. The two convergence control parameters for both algorithms ε_{MF} and ε are fixed and equal to to 0.001. In all the following, the maximal number of iterations N_{max}^{iter} is fixed to 50. The number of iterations is calculated by summing the number of iterations performed by each of the two iterative optimization procedures.

Furthermore, we consider $\varepsilon = 10^{-3}$, $WIN_{MF} = 5$, $\Delta_{MSR} = 5$ and $\varepsilon_{MF} = 10^{-3}$ For all the figures, we are going to use the same following notations for the curve names. The $SJNR/MF$ denotes the throughput curve for the SJNR/MF algorithm derived from [3]. The $SJNR/MSR$ describes the SJNR/MSR algorithm from [3] and the $SJNR/MFMSR_D$ corresponds to the curve for this proposed algorithm.

Figures 3 and 4 present for a fixed number of total iterations namely $N_{max}^{iter} = 50$ the total throughput for two configurations of the systems $SJNR/MSR$, $SJNR/MF$ and the proposed $SJNR/MFMSR_D$. The first configuration is a fully loaded system with $N_T = N_R = K = 4$, the second one is also fully loaded but has N_R=2 receiving antennas per user.

These two figures show that the $SJNR/MF$ algorithm outperforms the $SJNR/MSR$ especially at high SNRs and the obtained throughput curve increases linearly in function of the total transmitted power. This behaviour can be explained by the fact that at high SNRs, the streams can be well separated just by using a matched filter at the reception and through the iterative procedure, the optimal precoder is calculated to maximize the received power for each user. At low SNRs, on the other hand, the MF filter fails to recover the streams in an optimal way inducing suboptimal precoders derivations. But, the MSR receiver is capable of providing a better separation of the users and used with the iterative procedure generates better precoders. This explains why the $SJNR/MSR$ algorithm outperforms the $SJNR/MF$ in the low SNRs region.

Based on these observation, we proposed a combination of the two algorithms described in this paper. The proposed algorithms gives the curve $SJNR/MFMSR_D$. Comparing this curve to the $SJNR/MF$ and $SJNR/MSR$ ones demonstrates a better throughput in all the considered SNR range. The obtained throughputs are even higher that the maximum obtainable by selecting the best among the two considered algorithms. In fact, analysing figures 3.b, and 4.b, shows that at high SNRs, the proposed algorithm gives slightly better throughput performances. At low SNRs, as shown on the curves 3.c and 4.c, the proposed procedure is capable of recovering the best of the two used algorithms offering by the way the best obtainable throughputs.

To verify, the stability of our proposed algorithm we have conducted simulations for various system configurations (especially the LTE defined ones). Figure 5 represents simulation results obtained in one of the most constrained configurations as there are $N_T = 2$ transmitting antennas with $N_R = 2$ receiving antennas and deserving 2 streams to $K = 2$ users. The figures show in fact that the $SJNR/MF$ can no more follow and resolve the best directions for the streams and is therefore worse than the $SJNR/MSR$ one even at very high SNRs.

On the other hand, the proposed $SJNR/MFMSR_D$ algorithm is fully capable of getting the best out of the system based on the two previous algorithms. It even gives some slight ameliorations (in the order of 10^{-4}) compared to the $SJNR/MSR$ (The best algorithms for all considered SNR range). This shows the stability of our algorithm and its convergence for any system configuration. These performances are obtained just by introducing a dynamic flipping procedure that

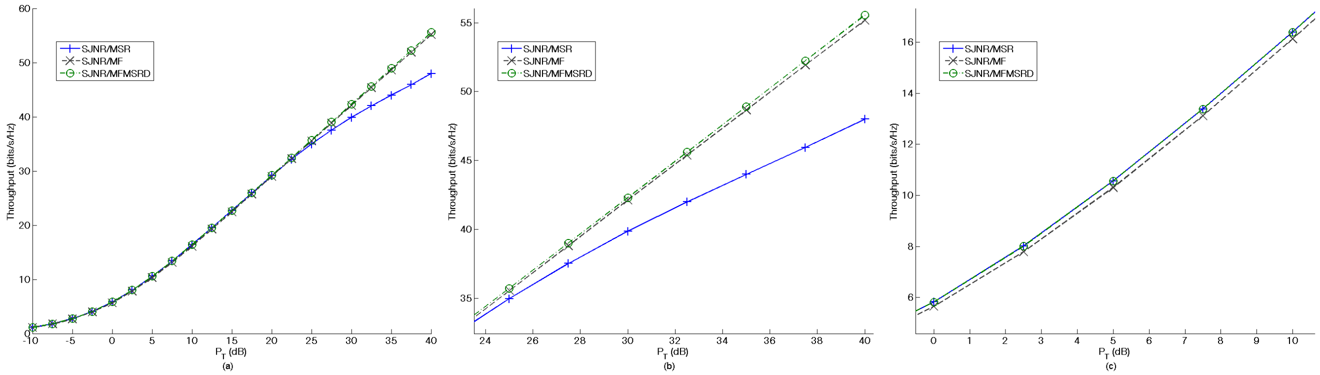

Fig. 3. Throughput in function of total transmit power P_T for $N_T = N_R = K = 4$.

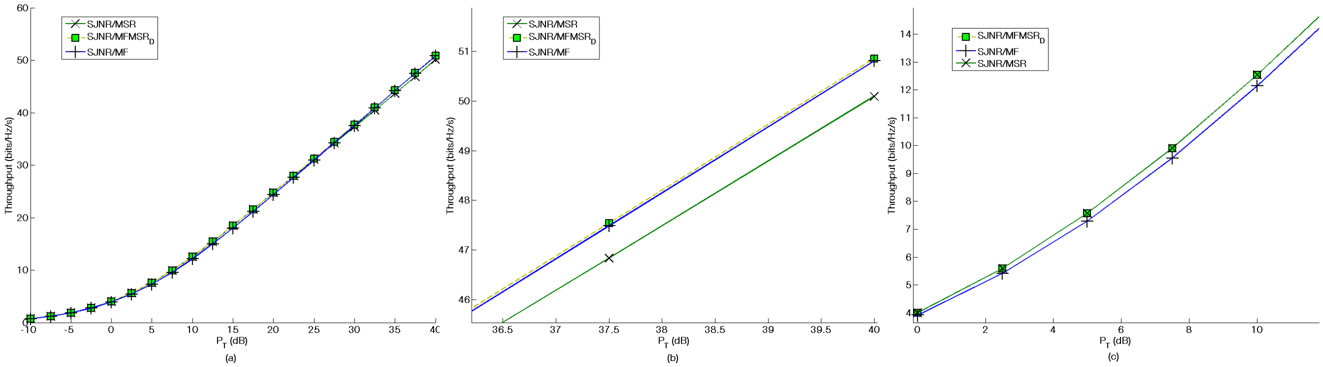

Fig. 4. Throughput for $N_T = K = 4$ and $N_R = 2$.

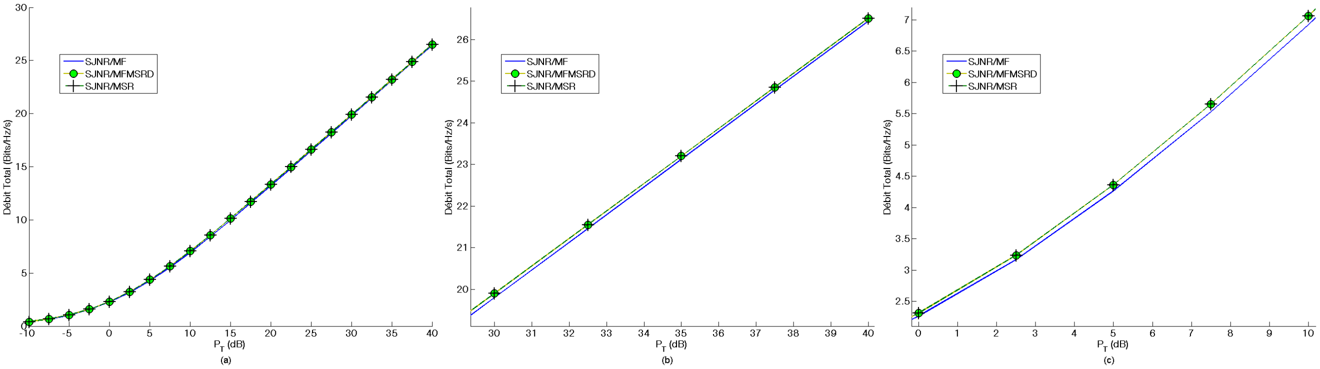

Fig. 5. Throughput for $N_T = N_R = K = 2$.

do not introduce any supplementary computational complexity and no extra delay or increase in the number of iterations.

Figure 6 represents the throughput achieved by the system when we use the flipping point calculated through statistical analysis given in figure 2. The first curve corresponds to a flipping point ($N_{Flipopt}$) calculated to maximize the mean sum rate of the system. The second curve is the total throughput that we get by applying at a given level of the transmission power the mean ($N_{Flipmean}$) of the obtained flipping point maximizing instantaneous throughput. It is to be noted that for these two curves we used a lookup table obtained from the statistical analysis to determine the flipping point for each transmit power. These performance curves are confronted to the results obtained with our proposed double iterative algorithm using a dynamic flipping point. The results demonstrate that the proposed algorithm used to switch from one iterative procedure to another gives higher mean sum rates.

The last figure, figure 7 gives a comparison of the performances obtained by the proposed algorithm and the existing ones in the literature.

Here, we added the curves representing the cooperative algorithm and two variant of the MMSE/MMSE iterative algorithm. The cooperative (i.e. single user MIMO on the overall channel $H^T = \left[H_1^T \cdots H_K^T\right]$) curve is considered as a benchmark of the system. The cooperative curve is indeed the highest upper bound of the considered system as

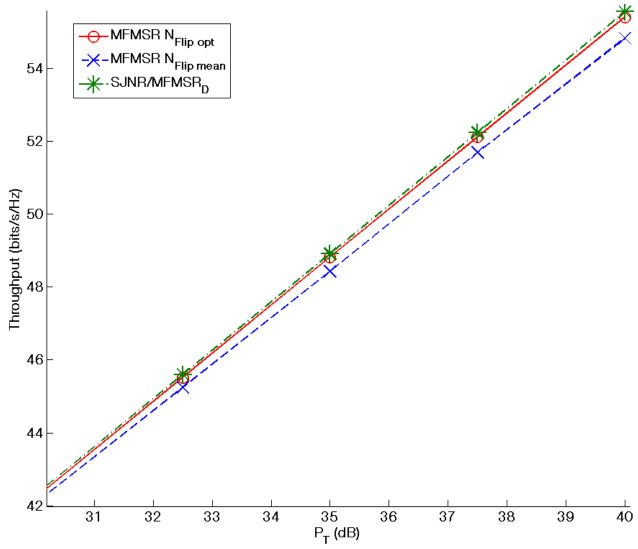

Fig. 6. Throughput for $N_T = N_R = K = 4$.

it considers perfect cooperation between all users. The first MMSE/MMSE curve named $MMSE/NormalizedMMSE$ is a modified version of the algorithm proposed in [4], [13] where the considered receiver is a normalized MMSE. The second MMSE/MMSE algorithm is the original one proposed in [4], [13] and the corresponding curve is entitled $MMSE/MMSEOriginal$.

These curves show that for the same computational complexity, the proposed solution offers an important gain. Furthermore, compared to the performances given by a cooperative system, the $SJNR/MFMSR_D$ algorithm gives a throughput curve with a slope tending towards the cooperative (Ideal system) and the DPC performances. The DPC curve has been generated using the algorithm described in [14].

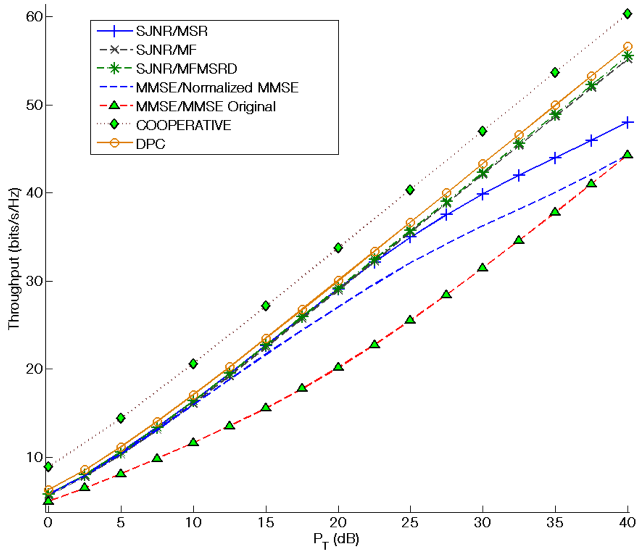

Fig. 7. Throughput for MMSE/MMSE, cooperative and proposed iterative algorithm in case N_T=N_R=K=4.

V. CONCLUSION

In this paper, a novel iterative joint optimization procedure for sum-rate maximization is proposed. We introduce a new iterative procedure which combines two iterative sum-rate maximization algorithms based on joint precoder and receiver optimization namely the SJNR/MF derived from the algorithm in [3] and the SJNR/MSR proposed in [3]. To be able to cascade these two algorithms in a way to get the best of both without introducing further complexity a dynamic switching solution has been proposed eliminating by the way constraints of optimal flipping point. We also showed throughout the realized simulations that the presented algorithm is converging toward the best system throughput given by the two used algorithms and that it even gives further gains getting closer to the cooperative optimal performances. Comparisons done with an existing MMSE/MMSE iterative solution given in [4], [13] and with an ameliorated version of it, showed better performances with the same complexity levels.

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