# **BLIND MAXIMUM SINR RECEIVER FOR THE DS-CDMA DOWNLINK**

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## ABSTRACT

We address the problem of downlink interference rejection in a DS-CDMA system. Periodic orthogonal Walsh-Hadamard sequences spread different users' symbols followed by scrambling by a symbol aperiodic base-station specific overlay sequence. The point-to-point propagation channel from the cell-site to a certain mobile station is the same for all downlink signals (desired user as well as the intracell interference). Orthogonality of the underlying Walsh-Hadamard sequences is destroyed by multipath propagation, resulting in multiuser interference if a coherent combiner (the RAKE receiver) is employed. In this paper, we propose a blind linear equalization algorithm which equalizes for the common downlink channel, thus rendering the user signals orthogonal again. A simple code matched filter subsequently suffices to cancel the multiple access interference (MAI) from intracell users. It is shown that the receiver maximizes the signal-to-interference plus noise ratio (SINR) at its output.

## 1. INTRODUCTION

We introduced a chip-rate zero-forcing (ZF) receiver for the multichannel DS-CDMA downlink followed by the desired user correlator in [1]. This formulation is motivated by the particular structure of the downlink channel from a fixed cell-site to a mobile receiver, where the propagation channel for all downlink signals is the same, and the primary spreading sequences are chosen from the orthogonal Walsh Hadamard set. Downlink equalization prior to despreading has also been suggested in [2] [3]. However, the multichannel aspect by means of oversampling/multiple sensors has been taken up in [1] at the mobile station to facilitate and render more robust, the equalization. There has been exploding interest in this area ever since and exactly the same receiver as [1] has been reinvented a year later in [4].

A blind downlink equalization algorithm was presented in [5] where the blind cost function to be minimized was the energy associated with the projection of the equalizer output on the unused spreading codes subject to fixed energy constraint for the signal of interest. The cost-function becomes quadratic and needs to be solved for a quadratic constraint, leading to an extreme generalized eigenvector as solution. However, there are some subtle issues (addressed below) in the formulation of the problem related to the equalizer and the propagation channel length, that the authors do not realize. Especially, it was stated in [5] that the presence or absence of masking code (scrambler) does not have any effect on the

functioning of the receiver. As shown below, this is not true in general. This paper presents the maximum SINR downlink receiver obtained through a blind criterion and examines the implications and properties of scrambling.

## 2. DOWNLINK DATA MODEL

Fig. 1 illustrates the downlink channel model. The K intracell users are assumed to transmit linearly modulated signals over a linear multipath channel with additive Gaussian noise. It is assumed that the signal is received at the mobile station through multiple (diversity) discrete-time channels, obtained from oversampling the received signal multiple times per chip or through multiple sensors (or a combination of the two schemes). We shall consider the signal to be received through precisely M channels where, M = no. of sensors  $\times$  oversampling factor. The signal received through the *m*th channel can be written in baseband notation as

$$y_m(t) = \sum_{k=1}^{K} \sum_{n} b_{k,n} h_{km}(t - nT_c) + v_m(t), \qquad (1)$$

where the subscript k denotes the user index;  $T_c$  is the chip period; the chip sequences  $\{b_{k,n}\}_{k=1}^K$  are assumed to be independent of the additive noise  $\{v_m(t)\}$ ; and  $h_{km}(t)$  characterizes the channel impulse response between the k th user signal and the m th sensor or the oversampled phase of the received signal. Let us



Figure 1: The downlink signal model.

denote by  $w_k^l = [w_{k,P_k-1}^l, w_{k,P_k-2}^l, \dots, w_{k,0}^l]^T$ , the structured aperiodic spreading sequence vector for the *l*th symbol of the *k*th user. The aperiodic spreading sequences consist of a periodic Walsh-Hadamard spreading sequence  $c_k = [c_{k,P_k-1}, \dots, c_{k,0}]$ , overlaid by a base-station specific scrambling sequence  $s_{i,l}$ . Then,

<sup>\*</sup> Eurécom's research is partially supported by its industrial partners: Ascom, Cegetel, France Telecom, Hitachi, IBM France, Motorola, Swisscom, Texas Instruments, and Thomson CSF

 $w_{k,i}^{l} = c_{k,i}s_{i,l}, i \in \{0, \dots, P_{k} - 1\}$ . The chip sequence  $\{b_{k,n}\}$  corresponding to the data symbol,  $a_{k,l}$ , of the kth user becomes

$$b_{k,n} = a_{k,l} w_{k,n \mod P_k}^l. \tag{2}$$

The chip period  $T_c$  is a constant, while the symbol period  $T_k$ ,  $\forall k$ , is a function of the transmission rate of the kth user. The symbol and chip periods are related through the processing gain  $P_k$ :  $T_k = P_k T_c$ . We shall also consider the chip sequence to have normalized energy:  $|c_k|^2 = 1$ .

Let us assume a common spreading factor, P. As the downlink propagation channel is the same for all k, we shall suppress the subscript k from  $h_{km}(t)$  in the following development.

The oversampled cyclostationary received signal at M times the chip rate can be stacked together to obtain the  $M \times 1$  stationary vector signal  $y_n$  at the chip rate, which can be expressed as

$$y_{n} = \sum_{k=1}^{K} \sum_{i=0}^{N-1} h_{i} b_{k,n-i} + v_{n}, \qquad (3)$$

where

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Stacking together a block of  $l_1P + l_2 + l_6$  data vectors  $y_n$ , and denoting it by  $Y_n$  we obtain,

$$\boldsymbol{Y}_{n} = \mathcal{T}(h) \widetilde{\boldsymbol{S}}_{n} \sum_{k=1}^{K} \widetilde{\boldsymbol{C}}_{k} \boldsymbol{A}_{k,n} + \boldsymbol{V}_{n}, \qquad (4)$$

where,

$$\boldsymbol{Y}_{n} = \begin{bmatrix} \boldsymbol{y}_{n,l_{6}-1} \\ \vdots \\ \boldsymbol{y}_{n,0} \\ \boldsymbol{y}_{n-1,P-1} \\ \vdots \\ \boldsymbol{y}_{n-l_{1},0} \\ \boldsymbol{y}_{n-l_{1}-1,P-1} \\ \vdots \\ \boldsymbol{y}_{n-l_{1}-1,P-l_{2}} \end{bmatrix} \boldsymbol{h} = \begin{bmatrix} \boldsymbol{h}_{N-1}^{H}, \cdots, \boldsymbol{h}_{0}^{H} \end{bmatrix}^{H} \\ \begin{bmatrix} \boldsymbol{\underline{c}}_{k} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{c}_{k} & \boldsymbol{0} & \cdots \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{c}_{k} \end{bmatrix} \\ \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{c}_{k} \end{bmatrix}$$

 $\mathcal{T}(h)$  is the  $M(L+P-1) \times (L+P+N-2)$  block Toeplitz channel convolution matrix filled up with the channel coefficients grouped together in h, and has full column rank, and the periodic code matrix  $\tilde{C}_k$  is the  $(l_3P + l_4 + l_6) \times (l_3 + 2)$  matrix accounting for the contribution of  $l_3 + 2$  symbols in the received signal  $Y_n$ .  $\underline{c}_k$  and  $\overline{c}_k$  denote the partial contribution of the end symbols of the data block. We shall denote the  $l_3 + 2$  columns of  $\tilde{C}_k$  as  $C_{k,l}$ , for  $l \in \{0, \ldots, l_3 + 1\}$ .  $A_{k,n} = [a_{k,n}, \ldots, a_{k,n-l_3-1}]^T$  is the symbol sequence vector, and  $\tilde{S}_n$  denotes the  $L + P + N - 2 = l_3P + l_4 + l_6$  diagonal scrambling code matrix with the diagonal element given by  $[s_{n,l_6-1}, \ldots, s_{n,0}, s_{n-1,P-1}, \cdots, s_{n-l_3,0}, s_{n-l_3-1,P-1}, \ldots, s_{n-l_3-1,P-l_4}]$ .

## 3. DOWNLINK RECEIVER STRUCTURE

As shown in fig. 2, the downlink receiver has a constrained structure composed of an equalizer followed by a descrambler and a desired user code correlator. Let us write as  $\mathbf{h}(z) = \sum_{i=0}^{N-1} h(i)z^{-i}$ ,

the  $M \times 1$  z-domain FIR transfer function of the channel. It is well known, that a  $1 \times M$  FIR equalizer  $\mathbf{f}(z) = \sum_{i=0}^{L-1} \mathbf{f}(i) z^{-i}$  qualifies as a zero-forcing equalizer with a delay d if  $\mathbf{f}(z)\mathbf{h}(z) = z^{-d}$ . Let us further note that  $d = l_5 P + l_6$ . An arbitrary  $\mathbf{f}$  gives  $\mathbf{f}(z)\mathbf{h}(z) = \sum_{i=0}^{L+N-2} \alpha_i z^{-i}$ . In the time domain we can write this set of equations as

$$\mathcal{T}(\boldsymbol{f})\mathcal{T}(\boldsymbol{h}) = \mathcal{T}(\underline{\boldsymbol{\alpha}}) = \mathcal{T}(\underline{\boldsymbol{\alpha}}_d) + \mathcal{T}(\overline{\underline{\boldsymbol{\alpha}}}_d),$$
(5)

where,  $\mathcal{T}(\mathbf{f})$  is a  $P \times M(L + P - 1)$  block Toeplitz convolution matrix filled up with the equalizer coefficients.  $\mathcal{T}(\underline{\alpha})$  denotes a Toeplitz matrix with the first row  $[\underline{\alpha} \quad \mathbf{0}_{P-1}]$ . Same holds for  $\mathcal{T}(\underline{\alpha}_d)$  and  $\mathcal{T}(\overline{\underline{\alpha}}_d)$ 

$$\underline{\boldsymbol{\alpha}} = [\alpha_0 \ \alpha_1 \ \dots \ \alpha_{L+N-2}], \ \underline{\boldsymbol{\alpha}}_d = [0 \dots 0 \ \alpha_d \ 0 \dots 0]$$
$$\underline{\overline{\boldsymbol{\alpha}}}_d = [\alpha_0 \ \dots \ \alpha_{d-1} \ 0 \ \alpha_{d+1} \ \dots \ \alpha_{L+N-2}]. \tag{6}$$

The  $P\times 1$  vector of successive equalizer outputs can now be written as

$$\boldsymbol{Z}_{n} = \mathcal{T}(\boldsymbol{f})\boldsymbol{Y}_{n} = \mathcal{Y}_{n}\boldsymbol{f}^{T}, \qquad (7)$$

where, the last equality follows from the commutativity property of convolution.  $\mathcal{Y}_n$  is a block Hankel matrix with  $M \times 1$  blocks (received signal components,  $\boldsymbol{y}_n$ ). The equalized signal,  $\boldsymbol{Z}_n$  needs to be descrambled as  $\boldsymbol{X}_n = \boldsymbol{S}_{n-l_5-1}^H \boldsymbol{Z}_n$ , where

$$\boldsymbol{S}_n = ext{diag}\left\{s_{n,P-1},\ldots,s_{n,1},s_{n,0}\right\}$$

Note that if the equalizer is ZF ( $\overline{\underline{\alpha}}_d = \mathbf{0}$ ), then in the noiseless case ( $v(t) \equiv 0$ ), the correlator by itself suffices to suppress the interference contributions in  $X_n$  [1]. Let us denote by

 $\underline{\underline{C}} = [c_1 \ldots c_K], \text{ and } \underline{\underline{C}}^{\perp} = [c_{K+1} \ldots c_P],$ 

the matrix constituting of used and unused Walsh-Hadamard sequences respectively for the system ( $\underline{\underline{C}}^{\perp H} \underline{\underline{C}} = \mathbf{0}$ ).

$$\boldsymbol{y}_n \longrightarrow \boldsymbol{f}_n$$
  $\hat{\boldsymbol{b}}_n \longrightarrow \boldsymbol{c}_k$   $\boldsymbol{c}_k$   $\hat{\boldsymbol{a}}_{k,l-d}$ 

Figure 2: The downlink receiver.

### 3.1. Blind Maximum SINR Receiver

In [5], the equalizer was obtained from the above problem formulation by imposing that the descrambled output of the equalizer be orthogonal to the codes  $\underline{C}^{\perp}$  in the absence of noise. In other words, the equalizer was obtained as the argument of the following cost function:

$$\arg\min_{\boldsymbol{f}} E \| \underline{\underline{C}}^{\perp H} \boldsymbol{X}_n \|^2.$$
(8)

A fixed response constraint must be applied to the descrambled output of the equalizer for the desired signal (user 1) to avoid signal cancelation, i.e.,

$$E|\boldsymbol{c}_1^H \boldsymbol{X}_n|^2 = \text{constant.}$$
(9)

The solution to this constrained optimization problem can be written as the following generalized eigenvalue problem

$$\boldsymbol{f}^{T} = \arg \min_{\boldsymbol{f}} \; \frac{\boldsymbol{f}^{*} \hat{\boldsymbol{R}}_{0} \boldsymbol{f}^{T}}{\boldsymbol{f}^{*} \hat{\boldsymbol{R}}_{1} \boldsymbol{f}^{T}}, \tag{10}$$

where,  $\hat{R}_0 = \operatorname{avg}\{\mathcal{Y}_n S_n \underline{C}^{\perp} \underline{C}^{\perp H} S_n^H \mathcal{Y}_n^H\}$ , and  $\hat{R}_1 = \operatorname{avg}\{\mathcal{Y}_n S_n c_1 c_1^H S_n^H \mathcal{Y}_n^H\}$ , and avg denotes the temporal averaging operation, and can be replaced by an expectation operator if the scrambler is inactive, i.e.,  $S_n \equiv I_P$  and  $\tilde{S}_n \equiv I$ .

In [5], the above receiver is presented without a name or any analysis of what it corresponds to. As we show in the sequel, the overall receiver turns out to be a maximum SINR receiver.

### 3.2. Asymptotic Analysis

Note that,  $S_n^H \mathcal{T}(\underline{\alpha}_d) S_n \equiv \mathcal{T}(\underline{\alpha}_d)$ . The scrambler is modeled as *i.i.d.*, and hence asymptotic results need to be averaged over it. We shall assume symbol period cyclostationarity and that the input sequence is zero mean *i.i.d* with variance  $\sigma_a^2$ . User powers  $\beta_k$  are included in the input sequence variance,  $\sigma_k^2 = \beta_k \sigma_a^2$ . We shall replace  $S_{n-l_5-1}$  by  $S_n$  in the definition of  $X_n$  to simplify notation.

#### 3.2.1. RX Output Energy - The Constraint

The output energy (variance) of the receiver which also is the constraint term, can be written as

$$E|\boldsymbol{c}_{1}^{H}\boldsymbol{X}_{n}|^{2} = E\left\{\boldsymbol{c}_{1}^{H}\boldsymbol{S}_{n}^{H}\mathcal{T}(\boldsymbol{f})\boldsymbol{R}_{VV}\mathcal{T}^{H}(\boldsymbol{f})\boldsymbol{S}_{n}\boldsymbol{c}_{1}\right\}$$
$$+\sum_{k=1}^{K}\sigma_{k}^{2}E\left\{\boldsymbol{c}_{1}^{H}\boldsymbol{S}_{n}^{H}\mathcal{T}(\underline{\alpha})\widetilde{\boldsymbol{S}}_{n}\widetilde{\boldsymbol{C}}_{k}\widetilde{\boldsymbol{C}}_{k}^{H}\widetilde{\boldsymbol{S}}_{n}^{H}\mathcal{T}^{H}(\underline{\alpha})\boldsymbol{S}_{n}\boldsymbol{c}_{1}\right\},$$
(11)

where,  $\mathbf{R}_{VV} = E \mathbf{V}_n \mathbf{V}_n^H$  is the noise covariance matrix. Then we shall consider the following two cases:

#### a. no scrambler

The output energy is given by

$$E|\boldsymbol{c}_{1}^{H}\boldsymbol{X}_{n}|^{2} = \boldsymbol{f}\boldsymbol{R}_{1}\boldsymbol{f}^{H} + \sigma_{1}^{2}\|\underline{\boldsymbol{\alpha}}_{d}^{'}\|^{2} + 2\sigma_{1}^{2}\operatorname{Re}\left\{\underline{\boldsymbol{\alpha}}_{d}^{''}\widetilde{\boldsymbol{C}}_{1}^{H}\boldsymbol{\mathcal{T}}^{H}(\underline{\overline{\boldsymbol{\alpha}}}_{d}^{'})\boldsymbol{c}_{1}\right\}$$
$$+ \sum_{k=1}^{K}\sigma_{k}^{2}\|\widetilde{\boldsymbol{C}}_{k}^{H}\boldsymbol{\mathcal{T}}^{H}(\underline{\overline{\boldsymbol{\alpha}}}_{d}^{'})\boldsymbol{c}_{1}\|^{2}, \qquad (12)$$

and in the above expression,  $\mathbf{R}_1 = \mathcal{T}(\mathbf{c}_1^H) \mathbf{R}_{VV} \mathcal{T}^H(\mathbf{c}_1^H)$ ,  $\underline{\boldsymbol{\alpha}}_d' = [\dots \ 0 \ \alpha_{d-P} \ 0 \ \dots \ 0 \ \alpha_d \ 0 \ \dots \ 0 \ \alpha_{d+P} \ 0 \ \dots ]$ ,  $\overline{\underline{\boldsymbol{\alpha}}}_d' = \underline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}}_d'$ , and  $\underline{\boldsymbol{\alpha}}_d' = [\dots \ \alpha_{d-P} \ \alpha_d \ \alpha_{d+P} \ \dots ]$  is a  $1 \times l_3$  vector consisting of non-zero elements of  $\underline{\boldsymbol{\alpha}}_d'$ .

#### b. with scrambler

In this case,

$$E|c_{1}^{H}\boldsymbol{X}_{n}|^{2} = c_{1}^{H}\operatorname{diag}\left\{\mathcal{T}(\boldsymbol{f})\boldsymbol{R}_{VV}\mathcal{T}^{H}(\boldsymbol{f})\right\}c_{1} + \sum_{k=1}^{K}\sigma_{k}^{2}\left[c_{1}^{H}\mathcal{T}(\underline{\boldsymbol{\alpha}}_{d})\tilde{\boldsymbol{C}}_{k}\right]$$
$$\tilde{\boldsymbol{C}}_{k}^{H}\mathcal{T}^{H}(\underline{\boldsymbol{\alpha}}_{d})c_{1} + E\left\{c_{1}^{H}\mathcal{T}(\underline{\boldsymbol{\alpha}}_{d})\tilde{\boldsymbol{C}}_{k}\tilde{\boldsymbol{C}}_{k}^{H}\tilde{\boldsymbol{S}}_{n}^{H}\mathcal{T}^{H}(\underline{\boldsymbol{\alpha}}_{d})\boldsymbol{S}_{n}c_{1}\right\}$$
$$+ E\left\{c_{1}^{H}\boldsymbol{S}_{n}^{H}\mathcal{T}(\underline{\boldsymbol{\alpha}}_{d})\tilde{\boldsymbol{S}}_{n}\tilde{\boldsymbol{C}}_{k}\tilde{\boldsymbol{C}}_{k}^{H}\mathcal{T}^{H}(\underline{\boldsymbol{\alpha}}_{d})c_{1}\right\}$$
$$+ E\left\{c_{1}^{H}\boldsymbol{S}_{n}^{H}\mathcal{T}(\underline{\boldsymbol{\alpha}}_{d})\tilde{\boldsymbol{S}}_{n}\tilde{\boldsymbol{C}}_{k}\tilde{\boldsymbol{C}}_{k}^{H}\tilde{\boldsymbol{S}}_{n}^{H}\mathcal{T}^{H}(\underline{\boldsymbol{\alpha}}_{d})\boldsymbol{S}_{n}c_{1}\right\}\right].$$
(13)

Note that in (13) the second term gives  $\sigma_a^2 |\alpha_d|^2$  which is the desired signal energy, while in the case without scrambling (12),  $l_3$  terms (symbols) appear as the desired signal, i.e., the contributions to output energy by several symbols are the same. The fourth term in (13) can be written as  $\sum_{l=0}^{l_3+1} E\{c_1^H S_n^H T^H(\overline{\alpha}_d) \tilde{S}_n C_{k,l}\} C_{k,l}^H T^H(\underline{\alpha}_d) c_1$ , of which the term outside the expectation is non-zero only for k = 1 and  $l = l_5 + 1$  (corresponding to the correct positioning for the scrambler). The expectation term can be written as  $\operatorname{tr}\{\mathcal{T}(\overline{\mathbf{\Omega}}_l) E(\widetilde{\mathbf{S}}_n \mathbf{C}_{1,l} \mathbf{c}_1^H \mathbf{S}_n^H)\}, l \in \{0, \ldots, l_3 + 1\}$ , and is always zero. Similar treatment applies for the third term of (13). As for the last contribution of (13), which can be written as  $\sum_{l=0}^{l_3+1} E\{\mathbf{c}_1^H \mathbf{S}_n^H \mathcal{T}(\overline{\mathbf{\Omega}}_d) \widetilde{\mathbf{S}}_n \mathbf{C}_{k,l} \mathbf{C}_{k,l}^H \widetilde{\mathbf{S}}_n^H \mathcal{T}^H(\overline{\mathbf{\Omega}}_d) \mathbf{S}_n \mathbf{c}_1\}$ , it is a fourth-order expectation in matrices  $\mathbf{S}_n$  and  $\widetilde{\mathbf{S}}_n$  and can be written as a sum of three (two) second order terms in the case of real (complex) scrambling. It can be shown that the overall contribution can be written as  $\|\underline{\mathbf{\Omega}}_d\|^2/P$  per user in the complex scrambling case, to which a term  $\sum_{k=1}^{K} \sigma_k^2 \operatorname{tr}\{\mathbf{B} D_k D_1 B^* D_k D_1\}$  is added in the real scrambling case.  $D_k = \operatorname{diag}\{c_k\}$  and

$$\boldsymbol{B} = \begin{bmatrix} 0 & \alpha_{d+1} & \cdots & \alpha_{d+P-1} \\ \alpha_{d-1} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \alpha_{d+1} \\ \alpha_{d-P+1} & \cdots & \alpha_{d-1} & 0 \end{bmatrix}.$$

is a  $P \times P$  Toeplitz matrix. We can now write the RX output variance as

$$E|\boldsymbol{c}_{1}^{H}\boldsymbol{X}_{n}|^{2} = \boldsymbol{f}\boldsymbol{R}_{VV}\boldsymbol{f}^{H} + \sigma_{1}^{2}|\boldsymbol{\alpha}_{d}|^{2} + \frac{1}{P}\left(\sum_{k=1}^{K}\sigma_{k}^{2}\right)\|\underline{\boldsymbol{\alpha}}_{d}\|^{2} + \sum_{k=1}^{K}\sigma_{k}^{2}\mathrm{tr}\left\{\boldsymbol{B}\boldsymbol{D}_{k}\boldsymbol{D}_{1}\boldsymbol{B}^{*}\boldsymbol{D}_{k}\boldsymbol{D}_{1}\right\},$$
(14)

where tr stands for the *trace* operator, and \* is the complex conjugation operation. The output SINR of the receiver can be finally be written as

$$\Gamma = \frac{\sigma_1^2 |\alpha_d|^2}{\boldsymbol{f} \boldsymbol{R}_{VV} \boldsymbol{f}^H + \frac{1}{P} \left( \sum_{k=1}^K \sigma_k^2 \right) \| \boldsymbol{\overline{\alpha}}_d \|^2 + \sum_{k=1}^K \sigma_k^2 \operatorname{tr} \left\{ \boldsymbol{B} \boldsymbol{D}_k \boldsymbol{D}_1 \boldsymbol{B}^* \boldsymbol{D}_k \boldsymbol{D}_1 \right\}}$$

in the case of real scrambling.

### 3.2.2. The Criterion

The criterion can be written as

$$E \|\underline{\underline{C}}^{\perp H} \boldsymbol{X}_{n}\|^{2} = \operatorname{tr} \left[ E \left\{ \underline{\underline{C}}^{\perp H} \boldsymbol{S}_{n}^{H} \mathcal{T}(\boldsymbol{f}) \boldsymbol{R}_{VV} \mathcal{T}^{H}(\boldsymbol{f}) \boldsymbol{S}_{n} \underline{\underline{C}}^{\perp} \right\} \right] + \sum_{k=1}^{K} \sigma_{k}^{2} \operatorname{tr} \left[ E \left\{ \underline{\underline{C}}^{\perp H} \boldsymbol{S}_{n}^{H} \mathcal{T}(\underline{\alpha}) \widetilde{\boldsymbol{S}}_{n} \widetilde{\boldsymbol{C}}_{k} \widetilde{\boldsymbol{C}}_{k}^{H} \widetilde{\boldsymbol{S}}_{n}^{H} \mathcal{T}^{H}(\underline{\alpha}) \boldsymbol{S}_{n} \underline{\underline{C}}^{\perp} \right\} \right].$$
(15)

Again let us consider the two cases:

## a. no scrambler

Observing that  $\underline{\underline{C}}^{\perp H} \mathcal{T}(\underline{\alpha}_{d}') \widetilde{C}_{k} = 0$ , for  $k = 1, \ldots, K$ , (15) reduces to

$$E\|\underline{\underline{C}}^{\perp H} X_n\|^2 = \boldsymbol{f} \boldsymbol{R} \boldsymbol{f}^H + \sum_{i=k+1}^{P} \sum_{l=0}^{l_3+1} \sum_{k=1}^{K} \sigma_k^2 |\boldsymbol{c}_i^H \mathcal{T}(\underline{\overline{\alpha}}_d') \boldsymbol{C}_{k,l}|^2,$$
(16)

where,  $\mathbf{R} = \sum_{i=K+1}^{P} \mathcal{T}(\mathbf{c}_{i}^{H}) \mathbf{R}_{VV} \mathcal{T}^{H}(\mathbf{c}_{i}^{H})$ . This criterion becomes zero and the fixed output energy constraint in (12) can be satisfied in the high SNR region  $(v(t) \to 0)$  at zero-forcing. Note that  $\overline{\underline{\alpha}}'_{d} = \mathbf{0}$ . However,  $\underline{\alpha}'_{d} \neq \mathbf{0}$ . Several terms contribute in the solution to the criterion. In other words, if there is no scrambler,

nothing distinguishes one symbol period from another. Thus, there is an ISI term and can be removed by symbol rate equalization at the correlator output. However, a serious handicap in the realm of blind symbol-rate equalization will be the monochannel aspect of the correlator output.

#### b. with scrambler

It can be shown that for this case,

$$E \|\underline{\underline{C}}^{\perp H} \boldsymbol{X}_{n} \|^{2} = (P - K) \boldsymbol{f} \boldsymbol{R}_{VV} \boldsymbol{f}^{H} + (P - K) \frac{1}{P} \sum_{k=1}^{K} \sigma_{k}^{2} \|\underline{\overline{\alpha}}_{d}\|^{2} + \sum_{i=K+1}^{P} \sum_{k=1}^{K} \sigma_{k}^{2} \operatorname{tr} \{ \boldsymbol{B} \boldsymbol{D}_{k} \boldsymbol{D}_{i} \boldsymbol{B}^{*} \boldsymbol{D}_{k} \boldsymbol{D}_{i} \},$$
(17)

The final contribution in (17) only arises in the case of real scrambling. The overall problem for the complex scrambling case then becomes  $\min\{\boldsymbol{f}\boldsymbol{R}_{VV}\boldsymbol{f}^{H} + (\sum_{k=1}^{K}\sigma_{k}^{2})|\boldsymbol{\overline{\alpha}}_{d}\|^{2}/P\}$  subject to  $\boldsymbol{f}\boldsymbol{R}_{VV}\boldsymbol{f}^{H} + (\sum_{k=1}^{K}\sigma_{k}^{2})|\boldsymbol{\overline{\alpha}}_{d}\|^{2}/P + \sigma_{1}^{2}|\alpha_{d}|^{2} = \text{constant. Apart from a scale factor, this is equivalent to max SINR.}$ 

## 3.3. Alternative Criteria and Constraints

The constraint can also involve all used codes. This results in

$$E \| \underline{\underline{C}}^{H} \boldsymbol{X}_{n} \|^{2} = \sum_{i=1}^{K} |\boldsymbol{c}_{i}^{H} \boldsymbol{X}_{n}|^{2} = K \boldsymbol{f} \boldsymbol{R}_{VV} \boldsymbol{f}^{H} + \left(\sum_{k=1}^{K} \sigma_{k}^{2}\right) |\boldsymbol{\alpha}_{d}|^{2} + \left(\sum_{k=1}^{K} \sigma_{k}^{2}\right) \frac{K}{P} \| \underline{\overline{\alpha}}_{d} \|^{2} + \sum_{i=1}^{K} \sum_{k=1}^{K} \sigma_{k}^{2} \operatorname{tr} \left\{ \boldsymbol{B} \boldsymbol{D}_{k} \boldsymbol{D}_{i} \boldsymbol{B}^{*} \boldsymbol{D}_{k} \boldsymbol{D}_{i} \right\},$$
(18)

The criterion in (17) subject to the above constraint still gives the max SINR receiver. The sum of (17) and (18), i.e.,

$$E \|\boldsymbol{X}_{n}\|^{2} = \left(\sum_{k=1}^{K} \sigma_{k}^{2}\right) |\alpha_{d}|^{2} + P \left\{ \boldsymbol{f} \boldsymbol{R}_{VV} \boldsymbol{f}^{H} + \left(\sum_{k=1}^{K} \sigma_{k}^{2}\right) \frac{1}{P} \|\boldsymbol{\overline{\alpha}}_{d}\|^{2} \right\}.$$
(19)

which holds for both real and complex scrambling owing to the disappearance of the last term in the sum  $(\sum_{i=1}^{P} D_i B D_i = 0)$ , can also be maximized subject to the constraint  $E \| \underline{\underline{C}}^{\perp H} X_n \|^2 = \text{constant, or}$ 

$$\max_{f} \frac{E \|\boldsymbol{X}_{n}\|^{2}}{E \|\boldsymbol{\underline{C}}^{\perp H} \boldsymbol{X}_{n}\|^{2}}$$
(20)

Other alternatives are max  $E \|\underline{\underline{C}}^H X_n\|^2$  subject to  $E \|X_n\|^2 = \text{constant}$ . It can easily be shown that all these criterion and constraint sets lead to the max SINR solution.

#### 4. NUMERICAL EXAMPLES

For the simulation framework a common spreading factor of 16 is assumed. The user spreading sequences are Walsh Hadamard functions overlaid by a common (cell-site specific) real scrambler randomizing the periodic user code sequences. We consider a channel longer than the symbol period (about 160% of T, the symbol period, assumed to be the same for all users. There is no change in the model if users with different rates are present, since the basic signature waveforms are orthogonal. The input signal constellation is QPSK with the primary spreading sequences from



Figure 3: Output SINR comparison of the RAKE, the ZF, and the max. SINR receiver K = 10 equal power intracell users, P = 16, with an input symbol constellation of QPSK and real scrambling

the binary Walsh-Hadamard set, followed by the randomly selected scrambler with an alphabet  $s_{i,l} \in \{+1, -1\}$ . A root-raised cosine pulse with a roll-off factor of 22% is used in these simulations conform with the UMTS WCDMA norm [6]. We choose a relatively long (64 chip periods) equalizer in these simulations in order to run into the ISI situation discussed in section 3.2) in all cases. It is easy to see that in the absence of noise, several peaks are obtained since  $\underline{\alpha}'_{d} \neq 0$ . Simulation conditions in [5] do not show such scenarios. The performance of the max. SINR receiver for the case of real scrambling is shown in fig. 3. Note that the receiver performs better than the ZF receiver in low SNR regions.

### 5. CONCLUSIONS

We presented the blind maximum SINR receiver for the DS-CDMA downlink. The receiver can be adapted blindly only if scrambler is active. Otherwise the receiver length is constrained to be short so as not to involve more than one symbol period in the received signal processing window. If not, ISI results, and a symbol rate equalizer will be needed at the correlator output. We also show that complex scrambling is a better alternative in terms of the output SINR performance of the receiver.

#### 6. REFERENCES

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