BLIND CHANNEL ESTIMATION FOR DS-CDMA WITH LONG CODES

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ABSTRACT

In this paper, blind channel estimation is considered for Code-Division Multiple-Access systems employing randomized spreading codes with periods longer than a symbol duration. The motivation for such a study is both current commercial as well as proposed military wireless systems, where such signature sequences are under consideration. This kind of spreading presents particular challenges that render impossible the use of existing techniques based on short signature waveforms. The proposed blind channel estimation method is based on the second order statistics of the received signal, in addition, the algorithm exploits knowledge of the desired spreading code and its properties. The new method uses the Toeplitz structure of the autocorrelation matrix. A theoretical analysis of the mean squared error is provided and the resulting approximations are shown to be tight via simulation.

1. INTRODUCTION

Code-Division Multiple-Access (CDMA) is a multiuser scheme that can improve the capacity of a cellular system relative to other multiple access strategies. CDMA enables desirable features such as soft capacity or soft handoff. CDMA systems use pseudorandom codes to expand the transmitted bandwidth of a user's signal. Current commercial and proposed military Direct - Sequence CDMA (DS-CDMA) systems employ spreading codes that have periods which are much longer than the symbol duration. For the problem of channel estimation, this kind of spreading can be viewed as linear time varying filtering.

Systems in which the period of the spreading sequence is the symbol duration are termed short code systems. Short code systems lead to an interference structure that remains the same from symbol to symbol. Thus, short code systems are amenable to adaptive multiuser algorithms for detection and estimation. However, when using long code schemes, the interference pattern will be varying randomly from symbol to symbol.

In this paper, we explore a new blind channel identification scheme for multi-user systems employing DS-CDMA with long codes. The new technique exploits the statistical properties of the autocorrelation matrix obtained from the received signal after matched filtering. For delay spreads that are much smaller than the symbol period, a moderate number of matched filter outputs form a set of statistics suitable for performing blind channel identification.

Past work in this arena required the knowledge of all of the spreading codes of each of the active users [1, 3, 2]. The current work exploits the asymptotic statistics of the signals (as done in [4]) and as such, only requires the spreading code and timing of the user of interest.

The paper is organized as follows. Section 2 presents the signal model for DS-CDMA using long codes. Section 3 introduces the proposed blind technique for channel identification. A theoretical mean-squared error analysis is presented in Section 4. Section 5 provides numerical results. Final conclusion are drawn in Section 6.

2. SIGNAL MODEL

We shall presume a synchronous system[1] where the active users transmit DS-CDMA signals with spreading gain N through a channel of length M. Considering a

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[1]We consider this assumption for simplicity reasons in the exposition of the signal model. The proposed algorithm is directly applicable to asynchronous systems and, in fact, the numerical results will be based on an asynchronous system.

baseband representation of the received signal after coherent reception, an observation vector with $aN+M-1$ samples is

$$\mathbf{x}(n) = \sum_{p=1}^{P} A_p \mathbf{C}_p(n)\mathbf{H}_p \mathbf{b}_p(n) + \mathbf{w}(n), \qquad (1)$$

where P is the number of users, $\mathbf{x}(n) = [x(n), \cdots, x(n+aN+L-2)]^T$, $\mathbf{b}(n) = [b\left(\lfloor \frac{n}{N} \rfloor - 1\right), \cdots, b\left(\lfloor \frac{n}{N} \rfloor + a\right)]^T$ is a $(a+2) \times 1$ vector of transmitted symbols and $\mathbf{w}(n) = [w(n), \cdots, w(n+aN+L-2)]^T$ is a $aN+L-1 \times 1$ vector of noise samples. The operator, $\lfloor \cdot \rfloor$ is the floor operator and returns the largest integer smaller than its argument. The channel matrix for user p, of dimension $(a+2)M \times a+2$, is given by

$$\mathbf{H}_p = \mathbf{h}_p \otimes \mathbf{I}_{a+2} \qquad (2)$$

where $\mathbf{h}_p = [h_p(0), \cdots, h_p(M-1)]$ is a vector with the channel impulse response, $\otimes$ denotes the Kronecker product operator and $\mathbf{I}_{a+2}$ is the identity matrix of dimension $(a+2) \times (a+2)$. The matrix $\mathbf{C}_p(n)$ of dimension $(aN+M-1) \times (a+2)M$, is a time-varying matrix that is a function of the spreading sequence of user p only and incorporates the effects of the user delay,

$$\mathbf{C}_p^T(n) = \qquad (3)$$

$$\begin{pmatrix} \mathbf{C}^2_{p,M}(n-N) & & & \mathbf{0} \\ \mathbf{C}^1_{p,M}(n) & \mathbf{C}^2_{p,M}(n) & & \\ & \mathbf{C}^1_{p,M}(n+N) & & \\ & \ddots & \ddots & \\ & & \ddots & \mathbf{C}^2_{p,M}(n+aN) \\ \mathbf{0} & & & \tilde{\mathbf{C}}^1_{p,M}(n+(a+1)N) \end{pmatrix}$$

We define $\mathbf{C}^1_{p,M}(n)$ and $\mathbf{C}^2_{p,M}(n)$ to be the first N rows and the last $M-1$ rows, respectively, of a matrix $\mathcal{C}(\mathbf{c}_p(n), M)$. $\mathcal{C}(\mathbf{c}_p(n), M)$ is a $(N+M-1) \times M$ Sylvester matrix representing the convolution of a vector whose elements are the spreading code chips, $\mathbf{c}_p(n) = [c_p(n), \cdots, c_p(n+N-1)]^T$, with another vector of length M. The matrix $\tilde{\mathbf{C}}^1_{p,M}(n+(a+1)N)$ is composed of the $M-1$ first rows of $\mathbf{C}^1_{p,M}(n+(a+1)N)$.

In order to more fully exploit the properties of DS-CDMA signals, the identification scheme presented in this work manipulates matched filter outputs synchronized to the desired user's signals rather than the received signal $\mathbf{x}(n)$ directly. Assume that the desired user is the first user. Given M matched filters per received symbol, the matched filter observation $aM \times 1$ vector, $\mathbf{y}(n)$, is given by

$$\begin{aligned} \mathbf{y}(n) &= \mathbf{S}_1(n)\mathbf{x}(n) \qquad (4) \\ &= \mathbf{S}_1(n)\left(\sum_{p=1}^{P} A_p \mathbf{C}_p(n)\mathbf{H}_p \mathbf{b}_p(n)\right) + \mathbf{S}_1(n)\mathbf{w}(n), \end{aligned}$$

where $\mathbf{S}_1(n)$ is the matched filtering matrix of dimension $aM \times (aN+M-1)$ and is related to $\mathbf{C}_1(n)$ in the following manner:

$$\mathbf{C}_1(n) = \begin{pmatrix} \mathbf{C}^2_{1,M}(n-N) & \vdots & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_1^T(n) & \mathbf{0} \\ \mathbf{0} & \vdots & \tilde{\mathbf{C}}^1_{1,M}(n+(a+1)N) \end{pmatrix}$$

3. IDENTIFICATION TECHNIQUE

The randomized spreading code model implies that the ideal autocorrelation function of the desired user's sequence is a Kronecker delta function. Similarly, the cross-correlation function between two distinct users is ideally the zero function. Let us consider the $aM \times 1$ matched filter output vector $\mathbf{y}(n)$ of (4). With our assumptions on the data symbols and the noise, we take expectation with respect to the data symbols and the noise to form the output vector covariance matrix,

$$\begin{aligned} \mathbf{R}_y &= \frac{1}{N_s}\sum_{n=1}^{N_s} \mathbf{E}_{\mathbf{b},\mathbf{w}}\left[\mathbf{y}(n)\mathbf{y}^H(n)\right] \\ &= \frac{1}{N_s}\sum_{n=1}^{N_s} \{\sigma_1^2 \mathbf{S}_1(n)\mathbf{C}_1(n)\mathbf{H}_1\mathbf{H}_1^H \mathbf{C}_1(n)^H \mathbf{S}_1^H(n) \\ &\quad + \mathbf{R}_I(n) + \mathbf{R}_w(n)\} \\ &\simeq \tilde{\mathbf{H}}_1\tilde{\mathbf{H}}_1^H + \overline{\mathbf{R}}_I(n) + \overline{\mathbf{R}}_w(n) \qquad (5) \end{aligned}$$

where σ_p^2 is the p-th user power, $\mathbf{R}_w = \sigma_w^2 \mathbf{S}_1(n)\mathbf{S}_1(n)^H$ is the noise autocorrelation matrix and $\tilde{\mathbf{H}}_1 = \mathbf{h}_1 \otimes \mathbf{I}_a$. The contribution of the interfering users at time n is thus

$$\mathbf{R}_I(n) = \sum_{p=2}^{P} \sigma_p^2 \mathbf{S}_1(n)\mathbf{C}_\mathbf{p}(n)\mathbf{H}_p\mathbf{H}_p^H \mathbf{C}_p(n)^H \mathbf{S}_1^H(n)$$

Asymptotically, as the spreading length and the number of symbols used to average increase $(N, N_s \uparrow)$, $\overline{\mathbf{R}}_I$ and $\overline{\mathbf{R}}_w$ converge to Toeplitz matrices. As $\tilde{\mathbf{H}}_1$ is a rank a matrix, the contribution of the desired user's channel to $\overline{\mathbf{R}}_y$ is rank a. Channel identification can be performed by noting that the Toeplitz contribution of the channel noise and interference can be removed by considering the following displacement operation:

$$\begin{aligned} \mathbf{R}_h &= \mathbf{R}_y(2:aM, 2:aM) \\ &\quad - \mathbf{R}_y(1:aM-1, 1:aM-1) \\ &= \mathbf{R}_y^+ - \mathbf{R}_y^- \simeq \tilde{\mathbf{H}}_1^+ \tilde{\mathbf{H}}_1^{+^H} - \tilde{\mathbf{H}}_1^- \tilde{\mathbf{H}}_1^{-^H} \qquad (6) \end{aligned}$$

where the matrix notation $\mathbf{B}(i:j, i:j)$ corresponds to the submatrix of $\mathbf{B}$ formed by the appropriately truncated i through j columns and i through j rows. $\tilde{\mathbf{H}}_1^+$ and $\tilde{\mathbf{H}}_1^-$ are the truncated versions of the matrix $\tilde{\mathbf{H}}_1$

formed by removing the last and the first row, respectively. Note that $\mathbf{R}_h$ is a $(aM-1) \times (aM-1)$ matrix. The eigenvalues of this matrix have the following properties: a are positive, a are negative and the rest are equal to zero. The eigenvectors associated with the non-zero eigenvalues define a subspace fitting problem to determine the unknown channel,

$$\begin{aligned} \hat{\mathbf{h}}_1 &= \arg \min_{||\mathbf{h}||_1 = 1} \text{Trace}\left\{\mathcal{H}^H \mathbf{P} \mathcal{H}\right\} \\ &= \arg \min_{||\mathbf{h}_1|| = 1} \mathbf{h}_1^H \mathbf{Q} \mathbf{h}_1 \end{aligned} \tag{7}$$

where $\mathbf{P} = \mathbf{I} - \mathbf{V}\mathbf{V}^H$, $\mathbf{V}$ is a matrix whose columns are the eigenvectors associated to non-zero eigenvalues, $\mathcal{H} = [\tilde{\mathbf{H}}_1^+ \tilde{\mathbf{H}}_1^-]$ and $\mathbf{Q}$ is defined as

$$\mathbf{Q} = \sum_{i=1}^{2a} \mathbf{D}_i \mathbf{P} \mathbf{D}_i^H \tag{8}$$

where $\mathbf{D}_i$ are $M \times aM-1$ permutation matrices defined as follows

$$\mathbf{D}_i \Delta l, m\Delta = \begin{cases} \Delta & m-l = \Delta i - \Delta\Delta M - \Delta \\ & \text{for} \quad \Delta \leq l \leq M \\ & \text{and} \quad \Delta \leq m \leq aM - \Delta \\ \Delta & \text{otherwise} \end{cases}$$

$$\mathbf{D}_{i+a} \Delta l, m\Delta = \begin{cases} \Delta & m-l = \Delta i - \Delta\Delta M \\ & \text{for} \quad \Delta \leq l \leq M \\ & \text{and} \quad \Delta \leq m \leq aM - \Delta \\ \Delta & \text{otherwise} \end{cases}$$

$$\text{for} \quad i = \Delta, \cdots, a$$

In practice, the matrix in (6) is estimated ($\hat{\mathbf{R}}_h$) from the sampled averaged matrix $\hat{\mathbf{R}}_y = \frac{1}{N_s} \sum_{n=1}^{N_s} \mathbf{y}(n)\mathbf{y}^H(n)$. Usually, the exact channel order is not known *a priori*. However, the algorithm still works when the channel order is overestimated since the channel vector to be estimated will be a larger vector with null elements. With respect to the choice of the number of symbols considered in the observation vector, a, in the subspace fitting problem in (7) there are $2a(a(M-2)-1)$ equations, since the noise subspace dimension is equal to $a(M-2)-1$, and M unknowns. Therefore, in order to obtain a solution, the following condition must be satisfied

$$2a^2(M-2) - 2a - M > 0. \tag{9}$$

Note that there is no positive value of a that satisfied (9) when $M = 1, 2$. However, for $M = 3, 4, 5$ and $M \geq 6$ values of $a \geq 2$ and $a \geq 1$, respectively, are sufficient.

The recognition of the Toeplitz interference matrix was also made in [4]. However, instead of working with the Toeplitz displacement of $\mathbf{R}_y$, the difference between the covariance matrices before and after code matched filtering was manipulated. Under ideal spreading code assumptions, it can be shown that this difference is, in principle, $\mathbf{h}_1\mathbf{h}_1^H$. Thus the principal eigenvector of the difference between the matched filtered covariance matrix and the non-matched filtered covariance matrix will yield the desired channel estimate. While the work in [4] focused on equalization, there is the allusion to a corresponding identification scheme as just noted. We shall compare the proposed channel identification scheme to that of [4] in the simulations section.

4. ANALYSIS OF MSE

A theoretical approximation of the mean squared error (MSE) can be found by using a perturbation technique [5] that exploits the property $\hat{\mathbf{Q}}\hat{\mathbf{h}} \approx \mathbf{0}$, where $\hat{\mathbf{Q}} = \mathbf{Q} + \Delta\mathbf{Q}$ and $\hat{\mathbf{h}} = \mathbf{h} + \Delta\mathbf{h}$. Note that we have removed the subindex $_1$ of the desired user vector channel . Therefore

$$\hat{\mathbf{Q}}\hat{\mathbf{h}} = (\mathbf{Q} + \Delta\mathbf{Q})(\mathbf{h} + \Delta\mathbf{h}) \approx \mathbf{0}. \tag{10}$$

Since $\mathbf{Q}\mathbf{h} = \mathbf{0}$ and assuming that second order terms are negligible ($\Delta\mathbf{Q}\Delta\mathbf{h} \approx \mathbf{0}$) we obtain the following approximation

$$\mathbf{Q}\Delta\hat{\mathbf{h}} \approx -\Delta\mathbf{Q}\mathbf{h} \tag{11}$$

and therefore

$$\Delta\hat{\mathbf{h}} \approx -\mathbf{Q}^\dagger \hat{\mathbf{Q}}\mathbf{h} \tag{12}$$

where $\mathbf{Q}^\dagger$ is the left pseudoinverse of $\mathbf{Q}$. By exploiting properties of the estimate of (8), it can be shown that the k-th component of $\Delta\mathbf{h}$ is given by

$$\Delta h(k) = -\text{Trace}\{\mathbf{Q}_k^H \hat{\mathbf{P}} \mathcal{H}\} \tag{13}$$

where $\hat{\mathbf{P}}$ is the estimated noise subspace projection matrix, $\mathbf{Q}_k = [\mathbf{D}_1\mathbf{q}_k, \cdots, \mathbf{D}_{2a}\mathbf{q}_k]$ and $\mathbf{q}_k$ is the k-th column of ${\mathbf{Q}^\dagger}^H$.

Based on the results of [6] and the fact that the eigenvalues associated to the noise subspace are ideally zero, we obtain

$$\begin{aligned} \Delta h(k) &= \hat{h}(k) - h(k) \\ &= \text{Trace}\{\mathbf{Q}_k^H \mathbf{P} \hat{\mathbf{R}}_h \mathbf{V} \mathbf{\Lambda}^{-1} \mathbf{V}^H \mathcal{H}\} \end{aligned} \tag{14}$$

where $\mathbf{\Lambda}$ is a diagonal matrix whose elements are the $2a$ eigenvalues of (6) associated to the signal subspace (*i.e.*, the eigenvalues different from zero).

Taking into account that[2] $\hat{\mathbf{R}}_h = \hat{\mathbf{R}}_y^+ - \hat{\mathbf{R}}_y^-$, (14) can be expressed as follows:

$$\Delta h \Delta k \Delta =$$

[2] $\hat{\mathbf{R}}_y^+ = \hat{\mathbf{R}}_y(2 : aM, 2 : aM)$ and $\hat{\mathbf{R}}_y^- = \hat{\mathbf{R}}_y(1 : aM-1, 1 : aM-1)$

$$
\begin{aligned}
&= \mathrm{Trace}\{\mathbf{Q}_k^H \mathbf{P} \Delta \hat{\mathbf{R}}_y^{+} - \hat{\mathbf{R}}_y^{-} \Delta \mathbf{\Lambda}\mathbf{\Lambda}^{-1} \mathbf{\Lambda}^H \mathcal{H}\} \\
&= \sum_{i=1}^{2a} \sum_{\beta=-}^{+} \beta \mathbf{\Lambda}_k^H \mathbf{D}_i \mathbf{P} \hat{\mathbf{R}}_y^{\beta} \mathbf{\Lambda}\mathbf{\Lambda}^{-1} \mathbf{\Lambda}^H \mathbf{D}_i^H \mathbf{\Lambda} \\
&= \sum_{i=1}^{2a} \sum_{\beta=-}^{+} \beta \mathbf{u}_{ki}^H \hat{\mathbf{R}}_y^{\beta} \mathbf{\Lambda}_i \qquad \Delta\Delta\Delta\Delta
\end{aligned}
$$

where the super-index $^{\beta}$, with $\beta = -, +$, represents the kind of displacement in the matrix $\hat{\mathbf{R}}_y$, $\mathbf{u}_{ki} = (\mathbf{q}_k^H \mathbf{D}_i \mathbf{P})^H$ and $\mathbf{g}_i = \mathbf{V}\mathbf{\Lambda}^{-1}\mathbf{V}^H \mathbf{D}_i^H \mathbf{h}$. The mean-squared error (MSE) for the k-th element is

$$
E[\Delta h(k) \Delta h^*(k)] = \sum_{i=1}^{2a} \sum_{\beta=-}^{+} \sum_{j=1}^{2a} \sum_{\gamma=-}^{+} \beta\gamma E[\mathbf{u}_{ki}^H \hat{\mathbf{R}}_y^{\beta} \mathbf{g}_i \mathbf{g}_j^H \hat{\mathbf{R}}_y^{\gamma} \mathbf{u}_{kj}], \tag{16}
$$

where $\beta\gamma$ represents a sign that is positive when $\beta = \gamma$ and negative when $\beta \neq \gamma$. The autocorrelation matrix given β is defined as follows

$$
\hat{\mathbf{R}}_y^{\beta} = \frac{1}{N_s} \sum_{l=1}^{N_s} \mathbf{y}^{\beta}(l) \mathbf{y}^{\beta^H}(l) \tag{17}
$$

where $\mathbf{y}^{+}(l) = \mathbf{y}(l+2 : l+aM)$ and $\mathbf{y}^{-}(l) = \mathbf{y}(l+1 : l+aM-1)$.

By exploiting the fourth order statistics of binary and Gaussian random variables and the independence between users and noise, and after a cumbersome development [7], we can obtain a final expression for the MSE defined as $E[\mathbf{\Delta h}^H \mathbf{\Delta h}] = \sum_{k=1}^{M} E[\Delta h(k) \Delta h^*(k)]$, where

$$
\begin{aligned}
E[\Delta h(k)\Delta h^*(k)] = & \qquad (18) \\
= \frac{1}{N_s^2} \sum_{\beta=-}^{+} \sum_{\gamma=-}^{+} \beta\gamma \Big\{ & \mathbf{\Psi}_T^{\beta} \mathbf{\Psi}_T^{\gamma} + \\
& \sum_{l=1}^{N_s} \sum_{\substack{m=l-(a+1) \\ 1 \le m \le N_s}}^{l+(a+1)} \mathrm{Trace} \Big\{ \mathbf{U}_k^H \mathbf{S}_l^{\beta} \mathbf{X}_{lm} \mathbf{S}_m^{\gamma^H} \mathbf{U}_k \mathbf{G}^H \\
& \mathbf{S}_m^{\gamma} \mathbf{X}_{lm}^H \mathbf{S}_l^{\beta^H} \mathbf{G} \Big\} - \sum_{p=1}^{P} \mathbf{\Phi}_p(l,l) \mathbf{J}_{m-l}^{(a+2)M} \mathbf{\Phi}_p(m,m) \Big\}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{\Phi}_p(l,l) &= \mathrm{diag}(\mathbf{H}_p^H \mathbf{C}_p(l)^H \mathbf{S}_l^{\beta^H} \mathbf{U}_k \mathbf{G}^H \mathbf{S}_l^{\beta} \mathbf{C}_p(l) \mathbf{H}_p) \\
\mathbf{U}_k &= [\mathbf{u}_{k1}, \cdots, \mathbf{u}_{k2a}] \\
\mathbf{G} &= [\mathbf{g}_1, \cdots, \mathbf{g}_{2a}] \\
\mathbf{\Psi}_T^{\beta} &= \sum_{l=1}^{N_s} \mathrm{Trace} \Big\{ \mathbf{U}_k^H \mathbf{S}_l^{\beta} \Big(\sum_{p=1}^{P} \sigma_p^2 \mathbf{C}_p(l) \mathbf{H}_p \mathbf{H}_p^H \mathbf{C}_p^H(l) + \\
& \qquad \sigma_w^2 \mathbf{I} \Big) \mathbf{S}_l^{\beta^H} \mathbf{G} \Big\} \\
\mathbf{X}_{lm} &= \Big(\sum_{p=1}^{P} \sigma_p^2 \mathbf{C}_p(l) \mathbf{H}_p \mathbf{J}_{m-l}^{(a+2)M} \mathbf{H}_p^H \mathbf{C}_p^H(m) \\
& \qquad + \sigma_w^2 \mathbf{J}_{N(m-l)}^{aN+M-1} \Big) \\
\mathbf{S}_l &\equiv \mathbf{S}_1(l) \\
\mathbf{J}_k^M &= M \times M \text{ matrix whose element } (i,j) \\
&= \begin{cases} 1 & \text{if } (i-j) = k \text{ and } |k| \le M-1 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

5. SIMULATIONS

In order to show the performance of the proposed algorithm, the calculated and the simulated MSE are studied. The MSE found via simulation for the desired user, user 1, is determined as follows:

$$
MSE = \frac{1}{N_r} \sum_{i=1}^{N_r} \left|\left| \alpha \hat{\mathbf{h}}_1(i) - \mathbf{h}_1 \right|\right|^2, \tag{19}
$$

where N_r is the number of Monte Carlo runs simulations and $\alpha = \frac{\mathbf{h}_1^T \mathbf{h}_1}{\mathbf{h}_1^T \hat{\mathbf{h}}_1(i)}$ is a complex scalar used to remove the effect of the phase ambiguity inherent in estimates stemming from second order statistics [8].

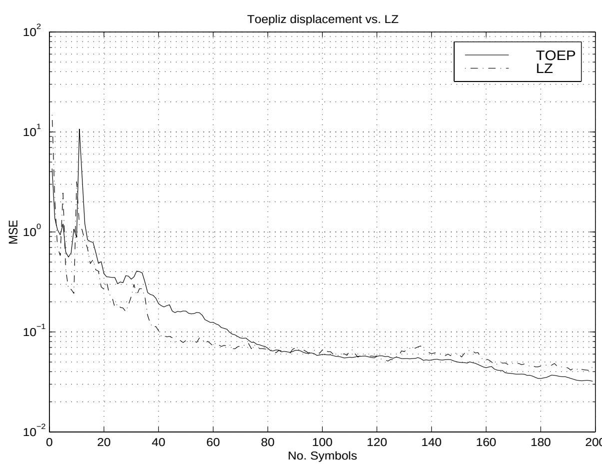

Figure 1: Performance comparison of LZ algorithm versus TOEP algorithm as a function of the number of symbols.

We first compare the performance of the new algorithm with the one alluded to in [4] (labeled as LZ) in Figure 1. This figure shows the time evolution of the MSE for both algorithms. A 10 asynchronous user environment with a $SNR = 15$dB is considered . Plots are made considering 20 simulations averaged over different random sets of channels and codes. We have used random codes with a spreading gain of 50 and the multipath channels have length $M = 5$. For the new algorithm, $a = 2$ is employed. It is clear that both algorithms provide near equivalent performance.

Figure 2 shows the accuracy of the theoretical MSE analysis of (18). An environment with $K = 8$ asynchronous DS-CDMA users employing spreading gain

$N = 30$ is considered. The channel length is $M = 5$ while the SNR was 20dB. It is observed that the analytical approximation is quite tight even for a small number of symbols.

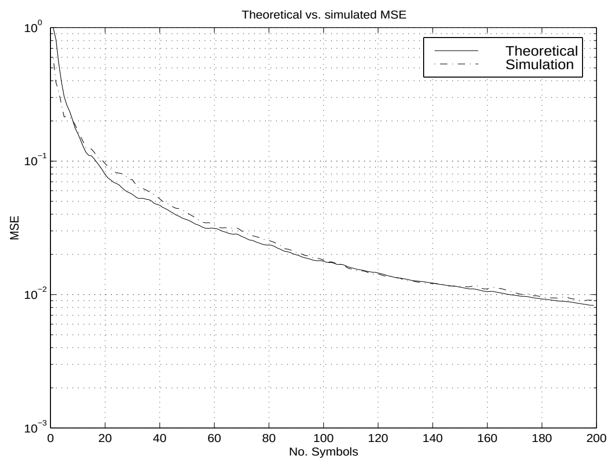

Figure 2: Comparison of simulated and analytically approximated MSE as a function of the number of symbols.

Finally, we consider performance as a function of the number of symbols in the observation vector, a. Figure 3 plots the theoretical MSE versus a for three different channel lengths in an environment with $K = 8$ asynchronous users, $SNR = 20$dB, spreading gain $N = 30$. 200 Monte Carlo simulations were run. Note that the results support condition (9) since for a channel length $M = 4$ a value of $a \geq 2$ is necessary, but for $M = 6$ the MSE is almost constant for $a \geq 1$.

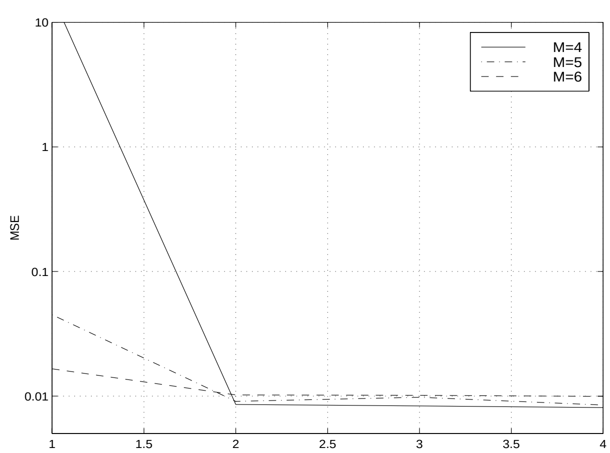

Figure 3: MSE versus the number of symbols in the observation vector, a.

6. CONCLUSIONS

We have introduced a new blind multipath channel identification scheme suitable for DS-CDMA systems where long randomized spreading codes are employed. The estimation method exploits the asymptotic statistics of the spreading codes and is based on the Toeplitz structure of the autocorrelation matrix. A tight analytical approximation of the mean squared error was derived employing perturbation techniques. Simulations show that the proposed method offers performance which is comparable to a previously proposed algorithm [4], and they corroborate the theoretical approximation of the MSE.

7. REFERENCES

[1] M. Torlak, B. Evans and G. Xu, "Blind Estimation of FIR Channels in CDMA Systems with Aperiodic Spreading Sequences", *Proceedings of the Asilomar Conference*, pp. 495-499, October 1997.

[2] A. Weiss and B. Friedlander, "Channel Estimation for DS-CDMA Downlink with Aperiodic Spreading Codes", *Proceedings of the Asilomar Conference*, October 1998.

[3] Z. Xu and M. Tsatsanis, "Blind Channel Estimation for Multiuser CDMA Systems with Long Spreading Codes", *Proceedings of ICASSP*, vol. 5, pp. 2531–2534, March 1999.

[4] H. Liu and M. Zoltowski, "Blind equalization in antenna array CDMA systems", *IEEE Transactions on Signal Processing*, vol. 45, no. 1, pp. 161-172, January 1997.

[5] W. Qiu and Y. Hua, "Performance Analysis of the Subspace Method for Blind Channel Identification", *Signal Processing*, no. 50, pp. 71-81, 1996.

[6] P. Stoica and T. Söderström, "Statistical Analysis and Subspace Rotation Estimates of Sinusoidal Frequencies", *IEEE Transactions on Signal Processing*, vol. 39, no. 8, pp.1836-1847, August 1991.

[7] C. J. Escudero, U. Mitra and D. T. Slock, "A Toeplitz Method for Blind Multipath Estimation for Long Code DS/CDMA Signals", *Submitted to the IEEE Transactions on Signal Processing*, May 1999.

[8] E. De Carvalho and D. T. Slock, "Cramer-Rao Bounds for Semi-Blind, Blind and Training Sequence Based Channel Estimation", *IEEE Signal Processing Advances in Wireless Communications Workshop*, pp. 129-132, Paris, April 1997.