BEAMSPACE LOW COMPLEXITY PARTIALLY ADAPTIVE BEAMFORMING AND ANGLE OF ARRIVAL ESTIMATION

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ABSTRACT

Several applications such as RADAR, SONAR, underwater, space-satellite communication systems involve the use of very large arrays. In these cases the computational complexity related to array processing algorithms can become prohibitive for both adaptive interference cancellation and directions of arrival (DOA) estimation (or in general, parameters estimation). Herein we deal with the problem of reducing the degrees of freedom of the array processor for achieving a low complexity adaptive beamforming-DOA estimator system. We are mainly focussed into the problem of the interference cancellation. Hence the most part of the paper is devoted to outline a technique for a beamspace based partially adaptive beamformer (PAB) design. Then we show how effective beamspace-multirate DOA estimation algorithms can be implemented jointly to the proposed PAB.

1. INTRODUCTION

The computational complexity inherent to adaptive algorithms for arrays composed of large number of sensors is often prohibitive, forcing one to employ partially adaptive processing [1, 2, 3]. A partially adaptive beamformer uses only a subset of the available degrees of freedom. This reduces the real-time computational load associated with adaptive algorithms and improves their convergence rate.

Several previous approaches found suboptimal fixed transformations providing the reduction of degrees of freedom, by exploiting some information on the likely scenarios wherein the beamformer operates (by considering signal and interference powers, numbers, directions of arrival, and bandwidths distribution), The SVD based partially adaptive beamformer [3] represents one of the best solutions, especially when the statistics of the likely scenarios needs a large number of trials to be well described. Van Veen [1] discussed several partially adaptive beamforming design methods, including eigenvectors based and a beamspace based methods [4]. The proposed approach is different from Gabriel's one [4], whose beamspace beamformer taking into account the incoming data via an eigenvector decomposition of the adapted weight vector. The partially adaptive beamformer results from this decomposition.

Obviously a real-time designed PAB would make useless the previous statistical analysis, but it would have the drawback of requiring a real-time eigenvector decomposition of the input data, that could be computationally expensive or even prohibitive.

Statistically-optimum PAB methods previously cited avoid such a real-time computational complexity but their performances are strongly dependent on the model used for describing the statistics of the likely scenarios.

In this paper we aim at relaxing, even only partially, the dependence on statistical scenario modeling reducing the computational load inherent to a data dependent PAB. Hence we propose a novel beamspace based beamforming for uniform linear arrays (ULA), based on linear discrete orthogonal transforms (e.g., space Fourier transform (DSFT) and wavelet transform (DWT)).

A novel beamspace approach to angle of arrival estimation in beamspace for uniform linear arrays (ULA) was recently proposed by Kautz and Zoltowski [5]. The advantages of the beamspace processing in the estimation of the signals directions of arrival (DOA) arise in terms of reduced complexity and lower SNR resolution thresholds. In fact the angles of arrival result by rooting a polynomial of order equal to the twice of the number of beams, as opposed to a costly spectral search, despite of the non-Vandermonde structure of the beamspace manifold. This technique allows the use of multirate processing and numerical filtering techniques, providing a great flexibility in the algorithm design.

The global complexity reduction and high performance level, in terms of interference rejection, are obtainable by jointly using beamspace DOA estimation and partially adaptive beamforming.

2. ARRAY SIGNAL MODEL

The array geometry assumed is that of ULA with N identical elements. For the sake of simplicity, the interelement spacing is set equal to one half wavelength associated with the highest frequency in the band of operation. Narrowband signal conditions are assumed. In this setting, let $\mathbf{x} = [x[0], \ldots, x[N-1]]^T$ denote the $N \times 1$ spatial sequence associated with the input array data vector at a given time instant. The discrete space Fourier transform (DSFT) of that sequence is a function of the direction sine, $u = \pi \sin\theta$, where θ is the bearing angle of a source relative to the array broadside. It is given by

$$f(u) = \sum_{n=0}^{N-1} x[n] \exp(-\mathrm{j}nu)$$

Defining the $N \times 1$ DFT beamforming weight vector as

$$\mathbf{v}_N = [1, \exp(\mathrm{j}u), \ldots, \exp(\mathrm{j}(N-1)u)]^T$$

it follows that the quantity $\mathbf{v}_N^H(u_0)\mathbf{x}$ is the DSFT of $\mathbf{x}$, at a given instant of time, evaluated at $u = u_0$. By discretizing the spatial frequency u, setting $u_k = k2\pi/N$, $k = 0, \ldots N-1$, we can form a DSFT matrix $\mathbf{W}$ such that

$$\mathbf{W} = \frac{1}{\sqrt{N}} [\mathbf{v}_N(0), \mathbf{v}_N(2\pi/N), \ldots, \mathbf{v}_N((N-1)2\pi/N)]$$

Hence, being the columns of the matrix $\mathbf{W}$ mutually orthogonal, $\mathbf{z} = \mathbf{W}^H\mathbf{x}$ denotes the beamspace input data vector, where each component $z[k]$ gives the amount of signal which passes through that kth elementary subband.

Then we can roughly estimate in which spatial subbands is located the significant part of the input power and in which is not. By using a well chosen power estimator, it is possible to evaluate which subbands carry the most part of information. Then we can use only them for beamforming, interference cancellation, and DOA estimation, rejecting the other noisy ones.

2.1. Real-time PAB design

It can be noted that with respect to the eigenvalue decomposition of the input data covariance matrix, for estimating signal and noise subspace, the DSFT is independent on the data. Moreover, it can be very easier implemented. The cost to be payed is in the roughness of the estimate, because of the poor resolution of the DSFT beams.

We have implemented a DSFT transformation at the input of an adaptive beamforming. In many practical cases the noise power is negligible with respect to the interference and signal powers. By choosing an advisable power threshold it is possible to identify and reject the noise subbands, where the power is low, and retain only the signal and interferences subbands, where the power is generally really higher. The noise subbands rejection is performed by a partial inverse DSFT (P-IDSFT), that accounts only the estimated signal and interference subbands. This operation yields to a reduced dimension of the input data vector at the adaptive interference canceler as well as at the DOA estimator.

The resulting PAB exhibits reduced degrees of freedom and therefore lower computational complexity and higher convergence rate.

The choice of the power threshold is an issue related to the probability of *false alarm* and to the probability of *missing detection* typical of RADAR and SONAR problems. However it can be adapted by jointly using an estimate of the number of present signals. At this aim a procedure based on information theoretic criteria was proposed in [6].

Ideally the contributions to a given subband due to out-band sources should be negligible. Nevertheless the presence of common peak sidelobe locations among the beams can make the out-band sources be not sufficiently deemphasized. In these cases the system takes into account more subbands than the strictly necessary, that is the number of subbands is over-estimated. This behaviour is characteristic of the rectangular weighting of the DSFT matrix, and it becomes particularly evident if the in-band sources are closely spaced and/or highly correlated. Hence the use of tapering windows could be recommended as a mean for reducing out-band sidelobes, in order to diminish the effects of strong out-band sources lying at or near a common sidelobe peak location. For this purpose Cosine, Hamming and Hanning orthogonalized windows have been used in order to diminish the necessary degrees of freedom of the adaptive beamforming processor. Note that the same problem arise in beamspace root-MUSIC algorithm [7].

Signal-interference and noise subbands can be defined and identified not only via a DSFT but also via any general orthogonal spectral transformation. For instance, we can use a discrete wavelet transform (DWT) [9] matrix $\mathbf{W}$ to process the input sequence. Note that DWT provides orthogonal beams with non-uniform spatial bandwidth: in fact increasing the spatial frequency (the angular distance from the array broadside) the beam bandwidth increases as well. In general we can modulate the spatial filter associated with the chosen orthogonal transformation, so as to achieve high spatial resolution in correspondence of signals of interest and low resolution elsewhere. This fact adds large flexibility to the proposed method, and avoids eventual over-estimation in the number of useful subbands, as it results using DSFT beams. Note that non uniform subband schemes can be planned through use of wavelet packet transforms [9]. Both wavelet as wavelet packets transform can be efficiently implemented with tree filter banks.

2.2. Beamspace DOA estimation

Finally the use of beamspace DOA estimation featuring multirate eigenvector processing can be implemented [5]. In fact, beamspace noise eigenvectors may be telescoped to vectors in the element-space noise subspace. The telescoped noise eigenvectors are bandpass, facilitating multirate processing involving modulation to baseband, filtering, and decimation. As these operations are linear (excepted decimation) a matrix transformation applied to the eigenvectors may be constructed *a priori*. This technique can be incorporated in either Root-MUSIC or ESPRIT algorithms providing a computationally efficient procedure.

Note that the PAB and the DOA estimation systems can be arranged so as to provide a reciprocal feedback. Hence the interference cancellation performed by the PAB becomes useful to the DOA estimator for achieving better estimates of the angles of arrival related to the signals of interest, and such estimates can be useful for better defining the constraints of the PAB.

3. IMPLEMENTATION

In this section we describe the implementation outline of our beamspace based array processor. For this purpose we need to consider some fundamental issues related to the applications that we have previously addressed.

3.1. Subbands selection

The real-time beamspace based PAB implementation in a non stationary scenario requires a clever estimate of the power associated with a specified subband. The simplest power estimator for wide sense stationary processes is given by

$$\hat{P}_k = \frac{1}{N_s} \sum_{i=0}^{N_s-1} |z_i[k]|^2$$

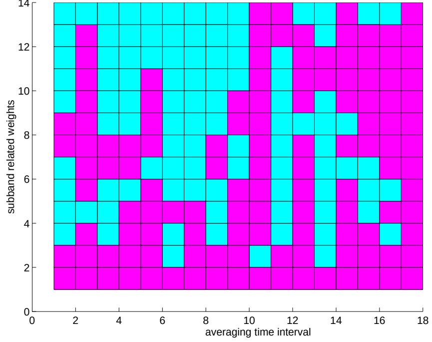
where $\hat{P}_k$ denotes the power estimate related to the kth subband, i is the discrete time variable, $z_i[k]$ denotes the signal associated with the kth spatial subband at the ith time instant, and N_s is the number of collected temporal samples. Note that, a part of the estimator bias, the estimate accuracy increases as N_s increases (i.e., the estimation variance decreases), and a specified accuracy in the estimate can be achieved with a lower N_s for higher SNR.

Our selection criterion is based on the following assumptions:

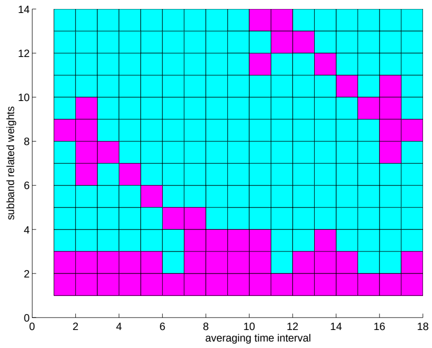
- The noise power can be assumed negligible with respect to signal powers.
- An *a priori* estimation of the total amount of received power is provided.

Hence if we are interested in making as small as possible the probability of missing detection, we sequentially select the useful subbands starting from those associated with the largest power estimates P_k, until an high percentage (90% in our case) of the total estimated power is reached.

Note that short time averaging (small N_s values) generally leads to sub-estimate the power associated with a specified subband but allows short temporal sample collections, and consequently provides processing time saving. However if the subband power is sub-estimate we need a larger number of subbands for reaching the *a priori* estimated total received power. So that the cost to be payed for reducing the probability of missing detection is in a longer time averaging, not always allowed because of real-time



(a)



(b)

Figure 1. ENABLED SUBBANDS VS. AVERAGE TIME INTERVAL (ENABLED= DARK GRAY, DISABLED=LIGHT GRAY) WITH POWER ESTIMATE ON $N_s = 10$ (a) AND $N_s = 50$ (b) SAMPLES.

processing requirements, or in a larger number of subbands to be considered, i.e., in a higher computational load than necessary. That is the same issue of RADAR *false alarm* event. The opposite effect is obtained by decreasing the percentage of the total estimated power to be reached for selecting the useful subbands.

Note that estimating the total amount of reached power is not a big issue. In fact a dedicated power estimator can collect temporal samples and update the estimate at each time, while the PAB and DOA estimator attend to their tasks.

For instance we consider a scenario with a useful signal incoming from the broadside and one interference moving in the range (-80, 80) deg with steps of 10 deg for each time interval during which the subband powers are estimated. In fig. 1 is represented the behaviour of the subband selection procedure, in case of short time (a) and long time (b) averaging. That figure reports the enabled (dark grey) and disabled (light grey) subbands versus the time index corresponding to the time interval wherein the average power is estimated.

An averaging time interval of 50 samples suffices in this case to achieve the best performances of the beamformer, i.e., the minimum complexity without information loss.

Note that the first subband relates to the array broadside and therefore to the useful signal, in our simulation setting. In fig. $1(b)$ it can be observed the interference DOA tracking, moving from an averaging time interval to the following one. By considering that the beamspace has a periodic structure due to the discrete nature of the transforms, a rough estimation of the time-space localization of the interferences can be desumed from the diagram as that of fig. 1. In this example the theoretical minimum number of subbands, i.e., the minimum number of adaptive weights that have to be adapted is 2. However the

N_s	mean no. of enabled weights
10	7.7
30	3.7
50	3.5
100	3.5

Table 1. MEAN NUMBER OF ENABLED WEIGHTS (SUBBANDS) IN SIMULATION OF FIG. 1 VERSUS THE NUMBER OF OBSERVED SAMPLES N_s FOR THE POWER ESTIMATE

performances of the beamspace PAB are upperbounded by the resolution of the array (related to the number of elements) and by the chosen space-band subdivision in the beamspace (i.e., the overlap between the subbands). So that the PAB performances are not further improved by a better power estimation exploiting a longer averaging time. In table 1 are reported the average number of weights (subbands) adapted versus the number of the collected samples for the power estimate.

3.2. Other implementation issues

In order to complete this preliminary feasibility analysis, we have here considered two simulation cases of real-time PAB design, by using a DSFT, selecting the subbands of interest and using a P-IDFT, followed by a well known GSC [8], which operates on a reduced input data vector. In the first case we have an array with $N = 16$ element, one signal incoming from the array broadside and three co-channel interferences, with DOA's equal to -25, -30 and 30 deg, with respect to the array broadside. The signal power is equal to 30 dB and the interference powers are 34 dB, 27 dB and 30 dB respectively, with respect to the noise power. The PAB employs only 5 subbands, or degrees of freedom, achieving the same performances of the full adaptive beamformer. The useful subbands estimation is made by setting a power threshold at the output of the DSFT matrix $\mathbf{W}$. The estimates converge with only 30 samples. In figure 2 are shown adapted radiation patterns and the output SINR's related to the considered case. In order to reduce the effect of high sidelobe level an orthogonalized Hamming tapering window has been used. The example shows how the proposed beamspace based adaptive beamforming provides not only a complexity reduction, but in some interference scenarios outperforms a full adaptive beamformer. That fact is easily justified since the noise subbands rejection leads to a lower noise power at the input of the GSC so that a higher steady state output SINR can be achieved. In fig. 3 the full adaptive beamformer and the designed PAB performances are compared, for an interference scenario where 3 jammers randomly change their positions and their amplitudes at every 250 samples time interval. In each of those time intervals the first 50 samples are collected, squared and averaged for estimating the power corresponding to each subband. It can be observed the higher convergence rate and robustness of the designed PAB with respect to the full adaptive beamformer.

4. CONCLUSIONS

In this paper we proposed a real-time PAB design method based on beamspace orthogonal transformation. We showed the effectiveness of the proposed method, in terms of high computational load reduction, noise reduction capability, and high convergence rate in non-stationary operating scenario, provided that the beamspace subbands are well designed and a suitable power estimator is used for their selection. Moreover the possibility of jointly implementing a beamspace DOA estimator was addressed about which further details will be reported in a future paper.

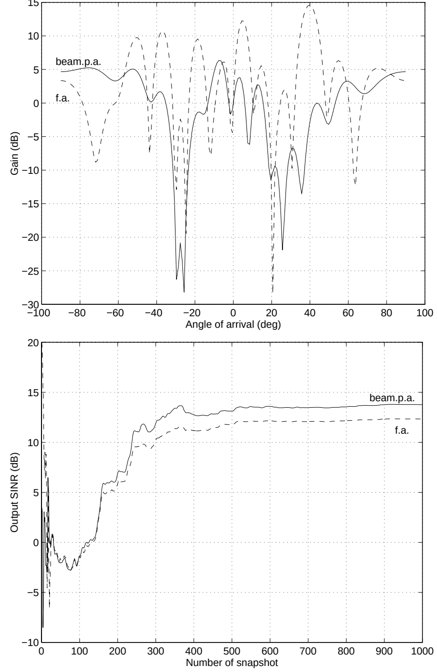


Figure 2. ADAPTED RADIATION PATTERNS AND OUTPUT SINRS FOR THE FULLY ADAPTIVE (f.a.) AND BEAMSPACE PARTIALLY ADAPTIVE (beamspace p.a.) BEAMFORMERS

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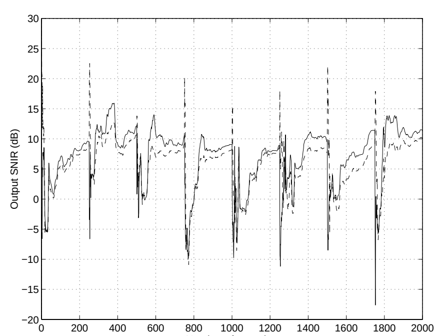

Figure 3. OUTPUT SNIRS FOR THE FULLY ADAPTIVE (- -) AND BEAMSPACE PARTIALLY ADAPTIVE (–) BEAMFORMERS