BLIND MULTICHANNEL ESTIMATION EXPLOITING THE FINITE SYMBOL ALPHABET

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ABSTRACT

Unlike the recent works dealing with purely blind channel estimation algorithms that are based on the second-order statistics of the received signal, in this paper we address the exploitation of the finite alphabet of the transmitted symbols to improve the blind estimation performance of the channel. The incorporation of the finite alphabet nature leads the symbols present in the problem to act as a training sequence for the channel estimation. Hence, a blind approach that exploits the symbol alphabet outperforms its purely blind version. We propose to incorporate the prior knowledge of the finite alphabet by combining a purely blind channel estimation criterion with a decision-directed linear MMSE equalization criterion. This combined criterion corresponds to an optimally weighted least-squares approach. Simulation results demonstrate that significant improvement can be obtained by exploiting the finite symbol alphabet.

1 INTRODUCTION

In a mobile radio transmission context, channels are specific in that they may vary rapidly. Due to bandwidth limitations and multipath propagation, the transmission channel distorts the signal being transmitted, leading to Inter-Symbol Interference (ISI). In order to recover the emitted data, the receiver needs to identify this channel distortion and equalize it. Classical system identification techniques require the use of both system input and output, which leads to the transmission of a training sequence, i.e. a set of fixed data (that do not carry information) that are known to both transmitter and receiver. The use of a training sequence reduces the transmission rate, especially when the training sequence has to be retransmitted often, due to the possibly fast channel variations that occur in mobile communications and consequently decreases the bandwidth efficiency. The goal of blind identification is to identify the unknown channel using the received signal only. Blind single-user multichannel estimation techniques exploit a multichannel formulation corresponding to a Single Input Multiple Output (SIMO) vector channel. The channel is assumed to have a finite delay spread NT. The multiple FIR channels can be obtained by oversampling a single received signal, but can also be obtained from multiple received signals from an array of antennas (in the context of mobile digital communications [1],[2]) or from a combination of both. For m channels the discrete-time input-output relationship can be written as:

$$\boldsymbol{y}(k) = \sum_{i=0}^{N-1} \boldsymbol{h}(i)a(k-i) + \boldsymbol{v}(k) = \boldsymbol{H} A_N(k) + \boldsymbol{v}(k) \quad (1)$$

where $\boldsymbol{y}(k) = [y_1^H(k)\cdots y_m^H(k)]^H, \boldsymbol{h}(i) = \left[h_1^H(i)\cdots h_m^H(i)\right]^H$, $\boldsymbol{v}(k) = [v_1^H(k)\cdots v_m^H(k)]^H$, $\boldsymbol{H} = [\boldsymbol{h}(N-1)\cdots \boldsymbol{h}(0)]$, $A_N(k) = \left[a(k-N+1)^H \cdots a(k)^H\right]^H$ and superscript ^{H} denotes Hermitian transpose. Let $\mathbf{H}(z) = \sum_{i=0}^{N-1}\boldsymbol{h}(i)z^{-i} = [\mathrm{H}_1^H(z)\cdots \mathrm{H}_m^H(z)]^H$ be the SIMO channel transfer function, and $\boldsymbol{h} = \left[\ \boldsymbol{h}^H(N-1)\cdots \boldsymbol{h}^H(0)\ \right]^H$. Consider the symbols i.i.d. if required and additive independent white Gaussian circular noise $\boldsymbol{v}(k)$ with $r_{\boldsymbol{vv}}(k-i) = \mathrm{E}\,\boldsymbol{v}(k)\boldsymbol{v}(i)^H = \sigma_v^2 I_m\,\delta_{ki}$. Assume we receive M samples:

$$\boldsymbol{Y}_M(k) \ = \ \mathcal{T}_M(\boldsymbol{H})\,A_{M+N-1}(k) + \boldsymbol{V}_M(k) \quad (2)$$

where $\boldsymbol{Y}_M(k) = [\boldsymbol{y}^H(k-M+1)\cdots \boldsymbol{y}^H(k)]^H$ and similarly for $\boldsymbol{V}_M(k)$. $\mathcal{T}_M(\boldsymbol{X})$ is a block Toeplitz matrix with M block rows and $[\boldsymbol{X}\ \ 0_{p\times(M-1)q}]$ as first block row, $\boldsymbol{X}$ being considered as a block row vector with $p\times q$ blocks. We shall simplify the notation in (2) with $k = M-1$ to

$$\boldsymbol{Y} \ = \ \mathcal{T}(\boldsymbol{H})\,A + \boldsymbol{V}\ . \quad (3)$$

We assume that $mM > M+N-1$ in which case the channel convolution matrix $\mathcal{T}(\boldsymbol{H})$ has more rows than columns. If the $\mathrm{H}_i(z)$, $i = 1,\ldots,m$ have no zeros in common, then $\mathcal{T}(\boldsymbol{H})$ has full column rank (which we will henceforth assume). For obvious reasons, the column space of $\mathcal{T}(\boldsymbol{H})$ is called the signal subspace and its orthogonal complement the noise subspace. The signal subspace is parameterized linearly by $\boldsymbol{h}$.

2 EXPLOITATION OF THE FINITE SYMBOL ALPHABET

We propose to incorporate the prior knowledge of the finite alphabet by combining a purely blind channel estimation criterion with a decision-directed linear MMSE equalization criterion.

2.1 The Blind Channel Estimation Criterion

Consider a noise subspace parameterization in term of subchannels impulse reponse $\mathbf{H}^{\perp\dagger}(z)$ which satisfies $\mathbf{H}^{\perp\dagger}(z)\mathbf{H}^{\perp}(z) = 0$. In [3], we proposed different choices for $\mathbf{H}^{\perp\dagger}(z)$ and we discussed blind channel estimation methods using these parameterizations. To begin with, consider the case of two channels: $m = 2$. One can observe that for noise-free signals, we have $\mathrm{H}_2(z)y_1(k) - \mathrm{H}_1(z)y_2(k) = 0$, which can be written in a matrix form as $[\mathrm{H}_2(z) \;\; -\mathrm{H}_1(z)]\, \boldsymbol{y}(k) = \mathbf{H}^{\perp\dagger}(z)\boldsymbol{y}(k) = 0$. The matrix $\mathbf{H}^{\perp\dagger}(z)$ is parametrized by the channel impulse response and satisfies $\mathbf{H}^{\perp\dagger}(z)\mathbf{H}(z) = 0$. The FIR $\mathbf{H}^{\perp\dagger}(z)$ filter is called blocking equalizer. For $m > 2$, blocking equalizers $\mathbf{H}^{\perp\dagger}(z)$ can be constructed by considering the channels in pairs. The choice of $\mathbf{H}^{\perp\dagger}(z)$ is far from unique. To begin with, the number of pairs to be considered, which is the number of rows in $\mathbf{H}^{\perp\dagger}(z)$, is not unique. The minimum number is $m-1$ whereas the maximum number is $\frac{m(m-1)}{2}$. We shall call $\mathbf{H}^{\perp\dagger}(z)$ balanced if $\mathrm{tr}\,\{\mathbf{H}^{\perp\dagger}(z)\mathbf{H}^{\perp}(z)\} = \alpha\mathbf{H}^{\dagger}(z)\mathbf{H}(z)$ for some real scalar α and $\mathbf{H}^{\dagger}(z) = \mathbf{H}^{H}(1/z^*)$. People usually take the maximum number of rows, which corresponds to a balanced $\mathbf{H}^{\perp\dagger}(z)$: $\mathbf{H}^{\perp\dagger}_{bal,max}(z)$. The minimum number of rows in $\mathbf{H}^{\perp\dagger}(z)$ to be balanced is m. We get for instance

$$\mathbf{H}^{\perp\dagger}_{min}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\mathrm{H}_m(z) & 0 & \cdots & \mathrm{H}_1(z) \end{bmatrix}, \quad (4)$$

$$\mathbf{H}^{\perp\dagger}_{bal,min}(z) = \begin{bmatrix} -\mathrm{H}_2(z) & \mathrm{H}_1(z) & 0 & \cdots & 0 \\ 0 & -\mathrm{H}_3(z) & \mathrm{H}_2(z) & \cdots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \mathrm{H}_m(z) & 0 & \cdots & 0 & -\mathrm{H}_1(z) \end{bmatrix} \quad (5)$$

Note that using $\mathbf{H}^{\perp\dagger}_{min}(z)$ doesn't lead to span the overall noise subspace ($(m-2)(N-1)$ independant vectors are missing to accomplish this purpose); whereas using $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ spans the totality of the noise subspace. Ofcourse, using the maximum number of rows in $\mathbf{H}^{\perp\dagger}(z)$ leads also to span the noise subspace since $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ is a subset of this $\mathbf{H}^{\perp\dagger}(z)$. Using $\mathbf{H}^{\perp\dagger}_{bal,min}(z)$ leads to the orthogonal complement of the Toeplitz channel matrix $\mathcal{T}(\boldsymbol{h}^{\perp})$ which satisfies:

$$\mathcal{T}(\boldsymbol{h}^{\perp})\mathcal{T}(\boldsymbol{H}) = 0 \quad (6)$$

Hence, it is clear that in the noiseless case, $\mathcal{T}(\boldsymbol{h}^{\perp})\boldsymbol{Y} = \mathcal{T}(\boldsymbol{h}^{\perp})\mathcal{T}(\boldsymbol{H})A = 0$. A channel estimation method based on this noise subspace parameterization is called Subchannel Response Matching (SRM) [4] and consists in minimizing the criterion $\|\mathcal{T}(\boldsymbol{h}^{\perp})\mathbf{Y}\|$.

2.2 The Decision-Directed Linear MMSE Equalization Criterion

In order to exploit the finite alphabet nature of the symbols in the channel estimation problem, we consider an equalizer that combines the blind information of a channel estimation method (SRM) and the finite alphabet aspect. The finite alphabet aspect is illustarted through the detection of the symbols according to a decision-directed criterion. The detected symbols are given by (the subscripts are omitted for simplicity of notation) $\widehat{\widehat{A}} = \mathrm{dec}(\widehat{A})$ ($\mathrm{dec}(.)$ is the decision operation that chooses the element in the symbol alphabet closest to its argument), where $\widehat{A}$ is the output of a linear MMSE equalizer:

$$\widehat{A} = R_{AY}R_{YY}^{-1}\boldsymbol{Y} = \sigma_a^2\mathcal{T}^H(\boldsymbol{h})R_{YY}^{-1}\boldsymbol{Y}. \quad (7)$$

If we consider $\widehat{\widehat{A}} = A$, then the error in the linear symbol estimates $\widehat{A}$ is

$$\begin{aligned} \tilde{A} &= A - \widehat{A} \\ &= \left(I - \sigma_a^2\mathcal{T}^H(\boldsymbol{h})R_{YY}^{-1}\right)A - \sigma_a^2\mathcal{T}^H(\boldsymbol{h})R_{YY}^{-1}\boldsymbol{V}. \end{aligned} \quad (8)$$

Hence, we construct a criterion by combining blind and decision-directed equalizer based error terms:

$$E = \begin{bmatrix} \mathcal{T}(\boldsymbol{h}^{\perp})\boldsymbol{Y} \\ A - \sigma_a^2\mathcal{T}^H(\boldsymbol{h})R_{YY}^{-1}\boldsymbol{Y} \end{bmatrix} \quad (9)$$

The randomness of E can be decoupled in the symbols contribution and the noise contribution as:

$$\begin{bmatrix} 0 \\ I - \sigma_a^2\mathcal{T}^H(\boldsymbol{h})R_{YY}^{-1}\mathcal{T}(\boldsymbol{h}) \end{bmatrix} A + \begin{bmatrix} \mathcal{T}(\boldsymbol{h}^{\perp}) \\ -\sigma_a^2\mathcal{T}^H(\boldsymbol{h})R_{YY}^{-1} \end{bmatrix} \boldsymbol{V} \quad (10)$$

It can be shown that the minimization of $E^H R_{EE}^{-1} E$ w.r.t. $\boldsymbol{h}$ leads to the following decoupled minimization criterion: $\min_{\boldsymbol{h}}$ of

$$\mathbf{Y}^H P_{\mathcal{T}^H(\boldsymbol{h}^{\perp})}\boldsymbol{Y} + \|A - \sigma_a^2\mathcal{T}(\boldsymbol{h})R_{YY}^{-1}\mathbf{Y}\|^2_{\left(\mathcal{T}^H(\boldsymbol{h})\mathcal{T}(\boldsymbol{h}) + \frac{\sigma_v^2}{\sigma_a^2}I\right)}. \quad (11)$$

The first term in (11) is the Deteministic Maximum Likelihood (DML) criterion, and the second term is an optimally Weighted Least Squares (WLS) decision-directed equalization criterion. An optimal strategy to solve the DML part is presented in [5] and is called Pseudo-Quadratic Maximum Likelihood. In the following we explain the principle of this method.

2.3 The Pseudo-Quadratic ML (PQML)

In the DML context, both channel coefficients and input symbols are considered as deterministic quantities, which are jointly estimated through the criterion:

$$\max_{A,\boldsymbol{h}} f(\boldsymbol{Y}|\boldsymbol{h}) \quad \Leftrightarrow \quad \min_{A,\boldsymbol{h}} \|\boldsymbol{Y} - \mathcal{T}(\boldsymbol{h})A\|^2 \quad (12)$$

$f(\boldsymbol{Y}|\boldsymbol{h})$ is Gaussian conditional distribution of $\boldsymbol{Y}$ given $\boldsymbol{h}$ (assuming $\boldsymbol{V}$ is circular white Gaussian noise). We consider here that the blind DML identifiability conditions are verified: the channel is irreducible, the input symbols are persistently exciting and the burst sufficiently long. The channel is then identifiable up to a scale factor and we assume the regularizing constraint $\|\boldsymbol{h}\| = 1$. Optimizing (12) w.r.t. A and replacing in (12), leads to

$$\min_{\|h\|=1} \boldsymbol{Y}^H P^{\perp}_{\mathcal{T}(\boldsymbol{h})} \boldsymbol{Y} \tag{13}$$

$P^{\perp}_{\mathcal{T}(\boldsymbol{h})}$ is the orthogonal projection on the noise subspace. Since $P^{\perp}_{\mathcal{T}(\boldsymbol{h})} = P_{\mathcal{T}^H(\boldsymbol{h}^{\perp})}$, (13) can be written as:

$$\min_{\|h\|=1} \boldsymbol{Y}^H \mathcal{T}^H(\boldsymbol{h}^{\perp}) \mathcal{R}^+ \mathcal{T}(\boldsymbol{h}^{\perp}) \boldsymbol{Y} \tag{14}$$

where $\mathcal{R} = \mathcal{T}(\boldsymbol{h}^{\perp})\mathcal{T}^H(\boldsymbol{h}^{\perp})$ and $^+$ denotes the Moore-Penrose pseudo-inverse ($\mathcal{T}(\boldsymbol{h}^{\perp})$ may not be full-row rank). $\mathcal{T}(\boldsymbol{h}^{\perp})$ being linear in $\boldsymbol{h}$, a matrix $\mathcal{Y}$ filled out with the elements of the observation vector $\boldsymbol{Y}$ can be found such that $\mathcal{Y}\boldsymbol{h} = \mathcal{T}(\boldsymbol{h}^{\perp})\boldsymbol{Y}$. Then (5) becomes:

$$\min_{\|\boldsymbol{h}\|=1} \boldsymbol{h}^H \mathcal{Y}^H \mathcal{R}^+ \mathcal{Y} \boldsymbol{h} \tag{15}$$

Usually, this criterion is solved in the Iterative Quadratic ML (IQML) fashion in which $\mathcal{Y}^H\mathcal{R}^+\mathcal{Y}$ is considered constant.

The principle of PQML has been first applied to sinusoids in noise estimation [6] and then to blind channel estimation in [7]. The gradient of the DML cost function may be arranged as $\mathcal{P}(\boldsymbol{h})\boldsymbol{h}$, where $\mathcal{P}(\boldsymbol{h})$ is (ideally) positive semi-definite. The ML solution verifies $\mathcal{P}(\boldsymbol{h})\boldsymbol{h} = 0$, which is solved under the constraint $\|\boldsymbol{h}\| = 1$ by the PQML strategy as follows: $\mathcal{P}(\boldsymbol{h})\boldsymbol{h}$ is also the gradient of the quadratic cost function $\boldsymbol{h}^H\mathcal{P}(\boldsymbol{h})\boldsymbol{h}$ in which $\mathcal{P}(\boldsymbol{h})$ is considerd constant, and as $\mathcal{P}(\boldsymbol{h})$ is positive semi-definite, $\boldsymbol{h}$ is chosen in [7] as the eigenvector corresponding to the smallest absolute eigenvalue of $\mathcal{P}(\boldsymbol{h})$. This solution is used to reevaluate $\mathcal{P}(\boldsymbol{h})$ and other iterations may be done. The difficulty consists in finding the right $\mathcal{P}(\boldsymbol{h})$ and especially with the positive semi-definite constraint. In our problem:

$$\mathcal{P}(\boldsymbol{h}) = \mathcal{Y}^H\mathcal{R}^+\mathcal{Y} - \mathcal{B}^H\mathcal{B} \tag{16}$$

where $\mathcal{B}$ is the matrix such that $\mathcal{T}^H(\boldsymbol{h}^{\perp})\boldsymbol{B} = \mathcal{B}^*\boldsymbol{h}^*$ with $\boldsymbol{B} = \left[\mathcal{T}(\boldsymbol{h}^{\perp})\mathcal{T}^H(\boldsymbol{h}^{\perp})\right]^+ \mathcal{T}(\boldsymbol{h}^{\perp})\boldsymbol{Y}$ (superscript * denotes complex conjugate). Asymptotically, the effect of the second term is to remove the noise contribution present in the first one. The criterion is asymptotically globally convergent: any initialization of $\mathcal{P}(\boldsymbol{h})$ results in a consistent PQML channel estimate and the second iteration finds the global minimizer.

The matrix $\mathcal{P}(\boldsymbol{h})$ is indefinite for finite M, and applying directly the PQML strategy will not work as stated in [7], except for high SNR. We introduce an arbitrary λ so that the PQML criterion becomes the following minimization problem:

$$\min_{\|\boldsymbol{h}\|=1,\lambda} \boldsymbol{h}^H \left\{\mathcal{Y}^H\mathcal{R}^+\mathcal{Y} - \lambda\mathcal{B}^H\mathcal{B}\right\} \boldsymbol{h} \tag{17}$$

with semi-definite positivity constraint on the central matrix. $\boldsymbol{h}$ is the minimal generalized eigenvector of $\mathcal{Y}^H\mathcal{R}^+\mathcal{Y}$ and $\mathcal{B}^H\mathcal{B}$, and λ the minimal generalized eigenvalue. Asymptotically, there is global convergence for $\boldsymbol{h}$, and λ tends to one. The stationary points of PQML are the same as those of DML, this is why PQML has the same performance as DML. Asymptotically PQML gives the global ML minimizer.

2.4 PQML-WLS

We use PQML to solve the first term of the criterion (11) and we solve the second term in a WLS sense in which the weighting matrix $W = \left(\mathcal{T}^H(\boldsymbol{h})\mathcal{T}(\boldsymbol{h}) + \frac{\sigma_v^2}{\sigma_a^2} I\right)$ and R_{YY} are considered constant. Since PQML solves the DML problem in an optimal sense and the WLS part is optimally weighted, the previous strategy to solve the whole criterion given in (11) is expected to perform well. The problem is solved iteratively: at each iteration, the solution for $\boldsymbol{h}$ is:

$$\boldsymbol{h} = \sigma_a^2 \left(\mathcal{Y}^H\mathcal{R}^+\mathcal{Y} - \lambda\mathcal{B}^H\mathcal{B} + \sigma_a^4 \mathcal{X}^T W^T \mathcal{X}^*\right)^{-1} \mathcal{X}^T W^T A^*, \tag{18}$$

where the matrix $\mathcal{X}$ is defined such that $\mathcal{X}\boldsymbol{h}^* = \mathcal{T}^H(\boldsymbol{h})X$ and $X = R_{YY}^{-1}\boldsymbol{Y}$ (superscript ^{T} denotes the transpose operator). The initialization of the PQML-WLS algorithm can be done by a purely blind estimation method: one can apply the PQML algorithm (initialized in turn by the SRM channel estimate).

The performance of the algorithm can be improved by using soft decisions on the symbols instead of the hard ones. In this case, we obtain a maximum a posteriori (MAP) estimator that exploits the gaussian distribution of the $\tilde{a}_k$:

$$\hat{\hat{a}}_k = \frac{\sum_{i=1}^{K} e^{-\frac{\left|\hat{a}_k - a_i\right|^2}{\sigma^2}} a_i}{\sum_{i=1}^{K} e^{-\frac{\left|\hat{a}_k - a_i\right|^2}{\sigma^2}}}, \tag{19}$$

where $\{a_i, i = 1, \cdots, K\}$ is the symbol alphabet and σ^2 is the diagonal element of $\sigma_a^2\left(I - \sigma_a^2\mathcal{T}^H(\boldsymbol{h})R_{YY}^{-1}\mathcal{T}(\boldsymbol{h})\right)$.

2.5 Decision-Directed Least-Squares (DD-LS)

The criterion given in (12) is equivalent to the following minimization problem

$$\min_{\boldsymbol{h}} \|\boldsymbol{Y} - \mathcal{A}\boldsymbol{h}\|^2, \tag{20}$$

where the matrix $\mathcal{A}$ is defined such that $\mathcal{A}\boldsymbol{h} = \mathcal{T}(\boldsymbol{h})A$. Solving this least-squares problem w.r.t. $\boldsymbol{h}$ leads to find the channel as:

$$\boldsymbol{h} = \left(\mathcal{A}^H\mathcal{A}\right)^{-1}\mathcal{A}^H\boldsymbol{Y} \tag{21}$$

3 SIMULATION RESULTS

We consider a burst length of $M = 200$, an irreducible complex randomly generated channel $\boldsymbol{H}$ of length $N = 3$ with $m = 2$ subchannels. The input symbols are drawn from an i.i.d. QPSK symbols sequence. The considered SNR is the average SNR per subchannel at the channel ouput. It is defined as

$$\text{SNR} \; = \frac{\|\boldsymbol{h}\|^2 \sigma_a^2}{m\sigma_v^2}. \tag{22}$$

Blind estimation methods give a channel estimate $\hat{\hat{\boldsymbol{h}}}$ with $\|\hat{\hat{\boldsymbol{h}}}\| = 1$, we adjust the right scale factor α so that $\boldsymbol{h}_o^H(\alpha\hat{\hat{\boldsymbol{h}}}) = \boldsymbol{h}_o^H \boldsymbol{h}_o$ where $\boldsymbol{h}_o$ is the true channel (see [8]): the final estimate is $\widehat{\boldsymbol{h}} = \alpha\hat{\hat{\boldsymbol{h}}}$. The performance measure is the Normalized MSE: NMSE, averaged over 100 Monte-Carlo runs and defined as

$$\text{NMSE} \; = \; \text{E}\|\boldsymbol{h} - \widehat{\boldsymbol{h}}\|^2 / \|\boldsymbol{h}\|^2. \tag{23}$$

We simulated the previously described methods and evaluated their performance. In Fig. 1, we consider a SNR=5dB and we perform one iteration of the PQML algorithm initialized by the SRM method. The obtained channel estimate is used to initialize the PQML-WLS method and the WLS part of the criterion (11) considered as a multichannel estimation method by decision direct equalization. We perform two iterations for both these two strategies. We observe that the PQML-WLS algorithm outperforms the WLS method especially in the first iteration. This means that the combined criterion (SRM criterion and DD equalization criterion) leads to significant performance improvement compared to the performance of each of the two methods to be combined. We ploted also the curves corresponding to the DD-LS method in the following two cases: in the first case, the symbols are detected and in the second case the symbols are considered known (the performance obtained in this second case can be seen as a lower perfromance bound for the PQML-WLS method). It can be noted that the DD-LS method in which the symbols are detected performs well (the obtained NMSE is close to the one obtained in the case of DD-LS with known symbols). This good performance is due to the good initalization given by the PQML method.

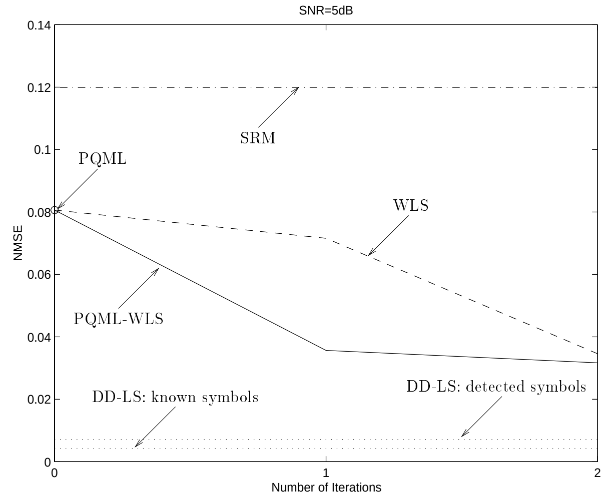

Figure 1: Performance of: PQML-WLS, WLS, DD-LS (detected symbols) and DD-LS (known symbols).

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