On Duality in the MISO Interference Channel

Francesco Negro*, Irfan Ghauri†, Dirk T.M. Slock*
†Infineon Technologies France SAS, GAIA, 2600 Route des Crêtes, 06560 Sophia Antipolis Cedex, France
Email: irfan.ghauri@infineon.com
*Mobile Communications Department, EURECOM, 2229 Route des Crêtes, BP 193, 06904 Sophia Antipolis Cedex, France
Email: francesco.negro@eurecom.fr, dirk.slock@eurecom.fr

***Abstract*— SINR duality is shown in a multi-input single-output (MISO) interference channel (IFC) and its dual SIMO with linear transmit (Tx) beamformers (BF). While uplink (UL) downlink (DL) duality for the SINR balancing (max min SINR) beamforming problem under the sum power constraint is well-established between the Broadcast channel (BC) and its (easier to solve) UL Multiple Access (MAC) dual channel, such duality does not at first seem relevant for the IFC. We show that SINR duality under the sum power constraint nevertheless holds in the MISO IFC leading to BF design through similar considerations as the BC-MAC case. We next impose further per-Tx power constraints meaningful for the IFC structure and show continued existence of SINR duality in the MISO IFC and the corresponding UL SIMO dual channel, but this time with a different UL noise. The beamformers, Tx powers and noise variances are found through an iterative algorithm.**

I. Introduction

In modern cellular systems a frequency reuse factor of 1 is used to increase the spectral efficiency. The throughput of such systems are seriously affected by the inter-cell interference that is commonly identified as the major bottleneck of modern wireless communication systems. This has led the major standardization bodies to include interference management strategies in modern wireless communication systems.
In the scientific community the problem of inter-cell interference is mathematically described as a interference channel where K pairs of users want to communicate between each other without exchanging information with the non intended receivers.

In this paper we focus on UL-DL duality in a Multiple Input Single Output (MISO) interference channel and how to use this framework to solve the beamformer design problem. UL-DL duality is a well-established tool for the study of the traditional Broadcast (BC) channel [1], for example it is used recently [2] [3] to solve the BC beamforming and power allocation problem. Using this duality, the BF designed in the virtual (dual) uplink mode can be used in the actual downlink problem to achieve the same SINR values by choosing appropriate downlink power allocations. Initially we consider the power minimization problem in the IFC imposing a set of quality of service (QoS) constraints then we describe the SINR balancing problem for an IFC with general power constraints. To solve these problem we extend the Uplink (UL) Downlink (DL) duality to the MISO IFC. The power minimization problem has been extensively studied for the BC case in [3] and [4]. There is one fundamental difference between linear BF design and power allocation problems in BC and IFC, namely there are individual power constraints in the latter as opposed to a total power constraint in the former. Minimizing total Tx power in the IFC still makes sense for example in green wireless communication systems.

We initially focus on the duality regime in the MISO-SIMO IFC in order to identify if any structure similar to UL-DL duality of the BC exists in this case. Zero-forcing duality and the more specific interference alignment duality are known to hold in the K-user IFC [5] [6]. In this work, we show that UL-DL SINR duality holds for the MISO IFC. We also show that interestingly, the mechanics of this duality are quite similar to the UL-DL duality in the BC setting. This observation allows beamformer design in the MISO IFC using the same techniques as the ones well-known in the BC channel.

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II. General IFC Signal Model

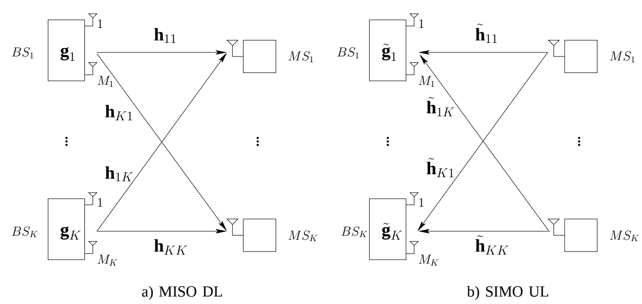

Fig. 1: System Model

Fig. 1 depicts a K-user MISO IFC with K transmitter-receiver pairs. The k-th Base Station (BS) is equipped with M_k transmitter antennas and k-th mobile station (MS) is a single antenna node. The k-th transmitter generates interference at all $l \neq k$ receivers. Assuming the communication channel to be frequency-flat, the received signal y_k at the k-th receiver, can be represented as

$$y_k = \mathbf{h}_{kk}\mathbf{x}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{h}_{kl}\mathbf{x}_l + n_k \tag{1}$$

where $\mathbf{h}_{kl} \in \mathbb{C}^{1 \times M_l}$ represents the channel vector between the l-th transmitter and k-th receiver, $\mathbf{x}_k$ is the $\mathbb{C}^{M_k \times 1}$ transmit signal vector of the k-th transmitter and n_k represents (temporally white) AWGN with zero mean and variance σ_k^2. Each entry of the channel matrix is a complex random variable drawn from a continuous distribution. It is assumed that each transmitter has complete knowledge of all channel vectors.

We denote by $\mathbf{g}_k$, the $\mathbb{C}^{M_k \times 1}$ precoding matrix of the k-th transmitter. Thus $\mathbf{x}_k = \mathbf{g}_k s_k$, where s_k represents the independent symbol for the k-th user pair. We assume s_k to have a temporally white Gaussian distribution with zero mean and unit variance. In the SIMO UL channel the k-th BS applies a receiver $\tilde{\mathbf{f}}_k$ to suppress interference and retrieve its desired symbol. The output of such a receive filter is then given by

$$\tilde{r}_k = \tilde{\mathbf{f}}_k \tilde{\mathbf{h}}_{kk} \tilde{s}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \tilde{\mathbf{f}}_k \tilde{\mathbf{h}}_{kl} \tilde{s}_l + \tilde{\mathbf{f}}_k \tilde{n}_k$$

where we denoted with $(\tilde{.})$ all the quantities that appear in the UL in order to differentiate with the same quantities in the DL.

III. UL-DL duality in MISO/SIMO Interference Channel Under Sum Power Constraint

In this section we will derive UL-DL duality for a MISO IFC under a total power constraint. To simplify the following analysis henceforth we assume that each receiver is characterized by the same noise variance, so $\sigma_k^2 = \sigma^2$, $\forall k$ and the beamforming vectors $\mathbf{g}_k, \forall k$ are unit norm. The received signal for the MISO DL IFC at the k-th mobile station is written in (1) and the corresponding SINR is defined as:

$$SINR_k^{DL} = \frac{p_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l \neq k} p_l \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma^2} \tag{2}$$

where p_k is the TX power at the BS for the stream intended to the k-th user. Imposing a set of DL SINR constraints at each mobile station: $SINR_k^{DL} = \gamma_k$ it is possible to rewrite equation (2) in matrix notation:

$$\mathbf{\Phi p} + \boldsymbol{\sigma} = \mathbf{D}^{-1}\mathbf{p} \tag{3}$$

where the two matrices $\mathbf{\Phi}$ and $\mathbf{D}$ are defined in (4) and (5), $\mathbf{p} = [p_1, \ldots, p_K]^T$ and $\boldsymbol{\sigma} = \sigma^2 \mathbf{1}$ are two vectors that contain all the TX powers and and the noise variances respectively. $\mathbf{1}$ is a column vector of dimensions $K \times 1$ that contains all ones.

$$[\mathbf{\Phi}]_{ij} = \begin{cases} \mathbf{g}_j^H \mathbf{h}_{ij}^H \mathbf{h}_{ij} \mathbf{g}_j, & j \neq i \\ 0, & j = i \end{cases} \tag{4}$$

$$\mathbf{D} = \text{diag}\{\frac{\gamma_1}{\mathbf{g}_1^H \mathbf{h}_{11}^H \mathbf{h}_{11} \mathbf{g}_1}, \ldots, \frac{\gamma_K}{\mathbf{g}_K^H \mathbf{h}_{KK}^H \mathbf{h}_{KK} \mathbf{g}_K}\}. \tag{5}$$

We can determine the TX power solving (3) w.r.t. $\mathbf{p}$ obtaining:

$$\mathbf{p} = (\mathbf{D}^{-1} - \mathbf{\Phi})^{-1} \boldsymbol{\sigma} \tag{6}$$

Now we analyze the SINR in the SIMO UL IFC. Due to channel reciprocity we have that $\tilde{\mathbf{h}}_{kl} = \mathbf{h}_{lk}^H$ $\forall k, l$ and the receiver filter in the UL is the reciprocal of the transmitter filter of the DL $\tilde{\mathbf{f}}_k = \mathbf{g}_k^H$, $\forall k$. The SINR for the UL channel can be written as:

$$SINR_k^{UL} = \frac{q_k \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\mathbf{g}_k^H (\sum_{l \neq k} q_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \sigma^2 \mathbf{I}) \mathbf{g}_k} \tag{7}$$

where q_k represent the Tx power from the k-th MS. Imposing a set of SINR constraints also in the UL: $SINR_k^{UL} = \gamma_k$ it is possible to rewrite that constraints as:

$$\tilde{\mathbf{\Phi}} \mathbf{q} + \boldsymbol{\sigma} = \mathbf{D}^{-1} \mathbf{q} \tag{8}$$

where $\mathbf{D}$ is defined as in (5), $\mathbf{q} = [q_1, \ldots, q_K]^T$ and

$$[\tilde{\mathbf{\Phi}}]_{ij} = \begin{cases} \mathbf{g}_i^H \mathbf{h}_{ji}^H \mathbf{h}_{ji} \mathbf{g}_i, & j \neq i \\ 0, & j = i \end{cases} \tag{9}$$

the power vector can be found as:

$$\mathbf{q} = (\mathbf{D}^{-1} - \tilde{\mathbf{\Phi}})^{-1} \boldsymbol{\sigma} \tag{10}$$

Comparing the definition in (4) and (9), we can see that $\tilde{\mathbf{\Phi}} = \mathbf{\Phi}^T$. This implies that there exists a duality relationship between the DL MISO and UL SIMO interference channels.

It is also interesting to note that there is a strong parallel between the equations reported above to show the duality in the MISO interference channel and the ones used to prove duality in a BC-MAC in [2]. Observing that it is possible to extend the results obtained for the UL-DL duality in the BC-MAC to the IFC under a sum power constraint.

A set of SINRs $\gamma_1, \ldots, \gamma_K$ is feasible whenever there exists a positive power allocation such that (3) for the DL ((8) for the UL) is fulfilled. In [2] the following is proved for the BC-MAC duality but it is also valid for the IFC:

Targets $\gamma_1, \ldots, \gamma_K$ are jointly feasible in UL and DL if and only if the spectral radius ρ of the weighted coupling matrix satisfies $\rho(\mathbf{D\Phi}) < 1$.

Because $\rho(\mathbf{D\Phi}) = \rho(\mathbf{D\Phi}^T)$ target SINRs are feasible in the UL if and only if the same targets are feasible in the DL. The power allocation vectors that satisfy that constraints can be found using (6), for the DL, and (10), for the UL.

In addition the total required UL power $q_{tot} = \sum_i q_i$ is the same as the DL power $p_{tot} = \sum_i p_i$, this can be simply shown as follows:

$$\begin{aligned} \textstyle\sum_i q_i = \quad & \mathbf{1}^T \mathbf{q} = \sigma \mathbf{1}^T (\mathbf{D}^{-1} - \mathbf{\Phi}^T)^{-T} \mathbf{1} \\ & = \sigma \mathbf{1}^T (\mathbf{D}^{-1} - \mathbf{\Phi})^{-1} \mathbf{1} = \textstyle\sum_i p_i \end{aligned} \tag{11}$$

According to the relationship (11) it is possible to state that both UL and DL have the same SINR feasible region under the same sum-power constraint, i.e., target SINRs are feasible in the DL if and only if the same targets are feasible in the UL.

Using the results obtained before it is possible to extend some beamforming design techniques that use the BC-MAC duality to the beamforming design for a MISO IFC.

IV. UL-DL duality in MISO/SIMO Interference Channel Under per User Power Constraint

In the MISO interference channel if the problem of BF design is formulated under the sum power constraint we have shown that there exist an UL-DL duality in this kind of channels that can be used to solve the problem. Even though the sum power constraint is analytically attractive such constraint is not enough in a practical interference channel. In reality each user is subject to a per user power constraint that the transmit power can not violate. For this reason in this section we will introduce an alternative BF design problem that still minimizes the total Tx power but imposing also per user power constraints. Here we will introduce a different UL-DL relation for the MISO IFC based on Lagrangian duality

[7] that was previously introduced for the BC channel in [4]. During the definition of the final version of this paper the authors came across an independent work [8], where a similar problem has been studied but the possibility of having per-user power constraints has not been taken into account.

For the rest of the paper we assume that the SINR constraints are such that there exist at least a feasible solution to the problem. The problem now becomes:

$$\begin{aligned} &\min_{\mathbf{g}_1,\dots,\mathbf{g}_K} \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{g}_k \\ &\text{s.t.} \quad \mathbf{g}_k^H \mathbf{g}_k \le P_k;\ k=1,\dots,K \\ &SINR_k^{DL} = \frac{\mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k}{\sum_{l\neq k} \mathbf{g}_l^H \mathbf{h}_{kl}^H \mathbf{h}_{kl} \mathbf{g}_l + \sigma_k^2} \ge \gamma_k;\ k=1,\dots,K \end{aligned} \tag{12}$$

where P_k represents the maximum Tx power for user k. The Lagrangian of the optimization problem reported above is:

$$\begin{aligned} \mathcal{L}(\lambda_i,\mu_i,\mathbf{g}_i) =& \sum_{i=1}^{K} \mathbf{g}_i^H \mathbf{g}_i + \sum_{i=1}^{K} \mu_i [\mathbf{g}_i^H \mathbf{g}_i - P_i] \\ &+ \sum_{i=1}^{K} \lambda_i [-\frac{1}{\gamma_i} \mathbf{g}_i^H \mathbf{h}_{ii}^H \mathbf{h}_{ii} \mathbf{g}_i + \sum_{l\neq i} \mathbf{g}_l^H \mathbf{h}_{il}^H \mathbf{h}_{il} \mathbf{g}_l + \sigma_i^2] \end{aligned} \tag{13}$$

where λ_k represents the Lagrange multiplier of the k-th SINR constraint and μ_k is the Lagrange multiplier associated to the Tx power constraint at user k.

The Lagrange dual of original DL problem (12) can be stated as follows:

$$\begin{aligned} &\max_{\lambda_1,\dots,\lambda_K,\mu_1,\dots,\mu_K,} \sum_{k=1}^{K} \lambda_k \sigma_k^2 - \sum_{k=1}^{K} \mu_k P_k \\ &\text{s.t.} - \frac{\lambda_k}{\gamma_k} \mathbf{h}_{kk}^H \mathbf{h}_{kk} + \sum_{l\neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + (1+\mu_k)\mathbf{I} \succeq 0;\ k=1,\dots,K \\ &\lambda_k \ge 0;\ k=1,\dots,K \\ &\mu_k \ge 0;\ k=1,\dots,K \end{aligned} \tag{14}$$

Because strong duality holds between the original problem (12) and its dual (14) the optimal solution of the dual problem is also optimal for the original one. The proof that the duality gap is zero between the two optimization problems is essentially based on converting the non convex original problem into a convex problem like in [4].

Proof: We first observe that a phase rotation to the optimal beamforming vectors, $\{\mathbf{g}_k e^{j\phi_k}\}_{k=1}^{K}$, does not influence the SINRs values. Therefore we can choose the phase rotation such that $\mathbf{h}_{kk}\mathbf{g}_k$ is real. The SINR constraints in (12) can be rewritten as:

$$\begin{aligned} (1+\frac{1}{\gamma_k}) \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k \ge \sum_{i=1}^{K} \mathbf{g}_i^H \mathbf{h}_{ki}^H \mathbf{h}_{ki} \mathbf{g}_i + \sigma_k^2 \\ (1+\frac{1}{\gamma_k}) |\mathbf{h}_{kk}\mathbf{g}_k|^2 \ge \|\mathbf{H}_k \mathbf{G}\ \sigma_k\|^2 \end{aligned} \tag{15}$$

where the compound channel matrix for the k-th user is defined as $\mathbf{H}_k = [\mathbf{h}_{k1},\dots,\mathbf{h}_{kK}]$ and the block diagonal matrix $\mathbf{G}$ contains the k-th beamformer in the k-th diagonal block. Due to the phase rotation introduced before we take the square root of both terms in the inequation above and we can rewrite the original problem (12) as:

$$\begin{aligned} &\min_{\mathbf{g}_1,\dots,\mathbf{g}_K} \sum_{k=1}^{K} \mathbf{g}_k^H \mathbf{g}_k \\ &\text{s.t.} \quad \mathbf{g}_k^H \mathbf{g}_k \le P_k;\ k=1,\dots,K \\ &\sqrt{1+\tfrac{1}{\gamma_k}}\, \mathbf{h}_{kk}\mathbf{g}_k \ge \|\mathbf{H}_k \mathbf{G}\ \sigma_k\|;\ k=1,\dots,K. \end{aligned} \tag{16}$$

The modified SINR constraint becomes a second-order cone programming constraint that is convex. The modified optimization problem (16) now is a convex problem and is equivalent to problem (12). ∎

The Lagrange dual of the DL beamforming problem (12) can be rewritten as an equivalent UL optimization problem for the Rx filter:

$$\tilde{\mathbf{g}}_k = (\sum_{l\neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I})^{-1} \mathbf{h}_{kk}^H \tag{17}$$

in which the Tx power λ_k and the noise power $\eta_k = 1+\mu_k$ are to be optimized. In the UL problem, in (18), each user transmits with power $\lambda_k,\ \forall k$, and the value of the dual UL noise at the receiver is represented by $\eta_k,\ \forall k$:

$$\begin{aligned} &\max_{\lambda_1,\dots,\lambda_K,\mu_1,\dots,\mu_K,} \sum_{k=1}^{K} \lambda_k \sigma_k^2 - \sum_{k=1}^{K} \mu_k P_k \\ &SINR_k^{UL} = \frac{\lambda_k \tilde{\mathbf{g}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \tilde{\mathbf{g}}_k}{\tilde{\mathbf{g}}_k^H (\sum_{l\neq k} \lambda_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k \mathbf{I}) \tilde{\mathbf{g}}_k} \le \gamma_k;\ k=1,\dots,K \\ &\lambda_k \ge 0;\ k=1,\dots,K \\ &\mu_k \ge 0;\ k=1,\dots,K \end{aligned} \tag{18}$$

At the optimum the SINR constraints in the UL and the DL problems must be satisfied with equality. Using this relationship it is possible to derive the DL BF from the UL receiver filter. Because a scaling factor in the receiver filter at the BS does not affect the SINR it is possible to show that the optimal DL BFs are given by:

$$\mathbf{g}_k = \sqrt{p_k}\, \tilde{\mathbf{g}}_k \tag{19}$$

where p_k is such that the SINRs in the DL are satisfied with equality so:

$$\boldsymbol{p} = (\mathbf{D}^{-1} - \boldsymbol{\Phi})^{-1} \boldsymbol{\sigma} \tag{20}$$

where matrices $\mathbf{D}$ and $\boldsymbol{\Phi}$ are defined in (5) and (4) respectively.

V. DESIGN ALGORITHM

In this section we report two numerical algorithms to solve the problem of optimal downlink beamformer design with per user power constraints. The first algorithm, Table 1, is an iterative algorithm that solves the dual UL problem (18). The second algorithm that we propose solves the Lagrange dual problem (14) using the the off-the-shelf optimization package [9].

Algorithm 1 Beamformer Design via UL-DL duality

Initialize: $i=0$, $\lambda_k^{(0)} = 1, \forall k=1,\dots,K$, $\mu_k^{(0)} = 1, \forall k = 1,\dots,K$

repeat

$i = i+1$

For $k=1,\dots,K$ find the UL receiver filter as

$$\tilde{\mathbf{g}}_k^{(i)} = (\sum_{l\neq k}^{K} \lambda_l^{(i-1)} \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k^{(i-1)} \mathbf{I})^{-1} \mathbf{h}_{kk}^H$$

and determine $\lambda_k^{(i)}$ as:

$$\lambda_k^{(i)} = \gamma_k \frac{\tilde{\mathbf{g}}_k^{(i)H} (\sum_{l\neq k} \lambda_l^{(i-1)} \mathbf{h}_{lk}^H \mathbf{h}_{lk} + \eta_k^{(i-1)} \mathbf{I}) \tilde{\mathbf{g}}_k^{(i)}}{\tilde{\mathbf{g}}_k^{(i)H} \mathbf{h}_{kk}^H \mathbf{h}_{kk} \tilde{\mathbf{g}}_k^{(i)}}$$

Determine the optimal DL BF $\mathbf{g}_k^{(i)}$ using (19)

Update the matrix $\mathbf{M}^{(i)} = diag\{\mu_1^{(i)},\dots,\mu_K^{(i)}\}$ using the subgradient projection method with step size $\epsilon^{(i)}$

$$\mathbf{M}^{(i)} = [\mathbf{M}^{(i-1)} + \epsilon^{(i)} \mathbf{Q}^{(i)}] \tag{21}$$

where

$$\mathbf{Q}^{(i)} = diag\{\mathbf{g}_1^{(i)H}\mathbf{g}_1^{(i)},\dots,\mathbf{g}_K^{(i)H}\mathbf{g}_K^{(i)}\} - diag\{P_1,\dots,P_K\}$$

until convergence

VI. Max-Min SINR in the MISO IFC with generic power Constraints

In this section we consider a MISO IFC where each receiver has a fixed target SINR $\gamma_i, \forall i = 1, \ldots, K$ and each TX has a general power constraint to satisfy. In order to satisfy simultaneously all the SINR thresholds we need to determine the power allocation vector and the TX beamformers to maximize the minimum of the weighted SINRs. The optimization problem reads:

$$\begin{array}{cl} \max\limits_{\mathbf{g}_1,\ldots,\mathbf{g}_K,\mathbf{p}} \min\limits_{k=1,\ldots,K} \frac{SINR_k}{\gamma_k} & \\ \text{s.t.} \quad \mathbf{c}_l^T \mathbf{p} \leq P_l, \;\; \forall l = 1,\ldots,K \end{array} \quad (22)$$

where vector $\mathbf{c}_l^T = [c_{l1}, \ldots, c_{lK}]$ represent a general power constraint for user l (e.g. a per user power constraint becomes $\mathbf{c}_l^T = \mathbf{e}_l^T$) and $\mathbf{p} = [p_1, \ldots, p_K]^T$. This problem, under a sum power constraint, was already discussed in [10].The optimal solution of this problem leads to a situation where all the weighted SINRs are equal, for this reason this problem is also called SINR Balancing. In addition we can also state that at the optimum only one power constraint is satisfied with equality. This is clear at two extremal SNR points. In very low SNR regime the optimal transmission strategy for each user is to maximize the useful signal part, hence matched filters (MF) to the direct link channel is used at each TX. In this case the user with the worse direct link channel transmits with full power. A similar reasoning can be applied to the high SNR regime where zero-forcing (ZF) transmitters are optimal. In this case the user with the worse direct channel will TX at full power. We conjecture that this reasoning can be extended to all SNR points.

In this problem it is possible to show that different optimal points may exist. Consider the system where all the users can ZF the interference to the non intended receivers. In this case the users that are not constrained in TX power can increase their power from the level that equate all the SINRs to their maximum TX power because this will not affect the other SINR values. Now since the TX power can vary also the BF of the correspondent user can vary. In the SISO case the power distribution that solves the SINR balancing problem is unique, with one user transmitting at full power, but in the MISO case the variability of the solution increases and hence several optimal points may exist. This reasoning can be also extended from the MISO to the MIMO case.

The optimization problem (22) consider in this section is very similar to the SINR balancing problem solved in [3], where now among all the power constraints only one is active. Hence the global optimum can be efficiently found by alternating minimization: fixing the BF vectors we solve w.r.t. the power allocation then assuming fixed the powers we solve for the BFs. Not all details are reported here due to lack of space.

Before introducing the two steps procedure we rewrite problem (22) in an equivalent form adding an additional constrain on the BF that modifies also the power constraints:

$$\begin{array}{cl} & \max\limits_{\mathbf{g}_1,\ldots,\mathbf{g}_K,\mathbf{p}} \min\limits_{k=1,\ldots,K} \frac{SINR_k}{\gamma_k} \\ \text{s.t.} & \mathbf{g}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \mathbf{g}_k = 1, \;\; \forall k = 1,\ldots,K \\ & \mathbf{x}_l^T \mathbf{p} \leq 1, \;\; \forall l = 1,\ldots,K \end{array} \quad (23)$$

where $[\mathbf{x}_l]_k = w_{lk} \mathbf{g}_k^H \mathbf{g}_k$ and $w_{lk} = c_{lk}/P_l$.

A. Power allocation optimization

The optimum of problem (23) and the corresponding power allocation vector can be found solving an eigenvalue problem. As explained before, at the optimum all the weighted SINRs are equal. Denoting with τ the optimal value, using the matrix notation introduced in section III we can write:

$$\frac{1}{\tau}\mathbf{p} = \mathbf{D}\mathbf{\Phi}\mathbf{p} + \mathbf{D}\boldsymbol{\sigma} \quad (24)$$

where now $\mathbf{D} = diag\{\gamma_1, \ldots, \gamma_K\}$. Assuming now that the $j-$th power constraint is the only one satisfied with equality and multiplying both sides of the previous equation by $\mathbf{x}_j^T$ we get:

$$\frac{1}{\tau} = \mathbf{x}_j^T \mathbf{D}\mathbf{\Phi}\mathbf{p} + \mathbf{x}_j^T \mathbf{D}\boldsymbol{\sigma} \quad (25)$$

Introducing the compound matrix:

$$\mathbf{\Delta} = \begin{bmatrix} \mathbf{D}\mathbf{\Phi} & \mathbf{D}\boldsymbol{\sigma} \\ \mathbf{x}_j^T \mathbf{D}\mathbf{\Phi} & \mathbf{x}_j^T \mathbf{D}\boldsymbol{\sigma} \end{bmatrix} \quad (26)$$

and the extended vector $\overline{\mathbf{p}} = [\mathbf{p} \;\; 1]^T$, using the results from the nonnegative matrix framework [11] the solution of the power optimization problem (23), where only the $j-$th power constraint is satisfied with equality is given by:$\tau = \frac{1}{\lambda_{max}(\mathbf{\Delta})}$ and the power vector is the corresponding positive eigenvector with the $(K+1)-$th entry normalized to one. This approach that allows to extend the known result from SIR balancing SINR balancing is called *Bordering Method*, it was introduced by [11] and then used in [3]. A different approach is to consider a rank one modification of the matrix $\mathbf{D}\mathbf{\Phi}$ that leads to the same solution. In particular the fact the the $j-$th power constraint is active: $\mathbf{x}_l^T \mathbf{p} = 1$ allows us to write the following:

$$\frac{1}{\tau}\mathbf{p} = (\mathbf{D}\mathbf{\Phi} + \mathbf{D}\boldsymbol{\sigma}\mathbf{x}_j^T)\mathbf{p}. \quad (27)$$

Also in this case the solution of the problem is given by the positive eigenvalue $\tau = \frac{1}{\lambda_{max}(\mathbf{D}\mathbf{\Phi} + \mathbf{D}\boldsymbol{\sigma}\mathbf{x}_j^T)}$ and the associated positive eigenvector is the optimal power vector. At this point a question arises: Which power constraint is the only one satisfied with equality? It is possible to show that the only feasible constraint is given by $\mathbf{x} = \arg\max_{\mathbf{x}_l} \lambda_{max}(\mathbf{B})$, where $\mathbf{B}$ can be the rank 1 modified matrix or matrix in (26).

B. Beamformer Optimization

Before discussing how to find the optimal BF vectors it is important to introduce also for this problem an equivalent dual UL problem. Fixing the $j-$th power constraint $\mathbf{x}_j$ in the DL problem to be active it is possible to define a dual UL SINR of the form:

$$\frac{SINR_k^{UL}}{\gamma_k} = \frac{\frac{q_k}{\gamma_k} \tilde{\mathbf{g}}_k^H \mathbf{h}_{kk}^H \mathbf{h}_{kk} \tilde{\mathbf{g}}_k}{\tilde{\mathbf{g}}_k^H (\sum_{l \neq k} q_l \mathbf{h}_{lk}^H \mathbf{h}_{lk} + w_{jk}\mathbf{I}) \tilde{\mathbf{g}}_k}; \; k = 1, \ldots, K \quad (28)$$

with this observation it is possible to obtain a dual UL SINR balancing problem where now the dual power constraint is: $\boldsymbol{\sigma}^T \mathbf{q} = \mathbf{x}_j^T \mathbf{P}$. Normalizing the noise vector as $\mathbf{n} = \frac{\boldsymbol{\sigma}}{\mathbf{x}_j^T \mathbf{P}}$ the UL power allocation vector can be found using the same method of the DL problem in the previous section. For fixed BF vectors the solution of the UL SINR balancing problem is $\tau_{UL} = \frac{1}{\lambda_{max}(\mathbf{D}\mathbf{\Phi}^T + \mathbf{D}\mathbf{x}_j \mathbf{n}^T)}$, and the dual power vector is the corresponding positive eigenvector.

Once we fix the original DL powers $\mathbf{p}$ we can optimize w.r.t. the BF vectors. The optimization problem now becomes:

$\min_{\{\mathbf{g}_k\}} \lambda_{max}(\mathbf{B})$, where $\mathbf{B} = \mathbf{D}\mathbf{\Phi} + \mathbf{D}\boldsymbol{\sigma}\mathbf{x}_s^T$, where we have imposed that the $s-$th power constraint is active. This problem can be solved extending the solution proposed in [10] to our case. In particular now the problem reads as:

$$\begin{aligned} &\min_{\{\mathbf{g}_k\}} \mathbf{q}^T\mathbf{B}\mathbf{p} \\ \text{s.t.} \quad &\mathbf{g}_k^H\mathbf{h}_{kk}^H\mathbf{h}_{kk}\mathbf{g}_k = 1, \ \ \forall k = 1, \ldots, K \\ &\mathbf{x}_s^T\mathbf{p} = 1 \end{aligned} \tag{29}$$

where $\mathbf{q}$ is the correspondent optimal UL power vector. Because we want to solve a cost function that is quadratic in the optimization variables under quadratic constraints the expression of the BF that solves the problem above is given a maximum generalized eigenvector solution. In particular the optimal BF vector is given by:

$$\mathbf{g}_k = \frac{\tilde{\mathbf{g}}_k}{\sqrt{\tilde{\mathbf{g}}_k^H\mathbf{h}_{kk}^H\mathbf{h}_{kk}\tilde{\mathbf{g}}_k}} \tag{30}$$

where $\tilde{\mathbf{g}}_k\mathbf{v}_{max}(\mathbf{h}_{kk}^H\mathbf{h}_{kk}, \sum_{l\neq k}\mathbf{h}_{lk}^H\mathbf{h}_{lk}q_l + w_{sk}\mathbf{I})$. For the fact that we are working with rank one channel this solution can be expressed as an MMSE like expression:

$$\tilde{\mathbf{g}}_k = [\sum_{l\neq k}\mathbf{h}_{lk}^H\mathbf{h}_{lk}q_l + w_{sk}\mathbf{I}]^{-1}\mathbf{h}_{kk}^H \tag{31}$$

Finally the only problem left is related to define which power constraint is satisfied with equality in this phase of the optimization procedure. What we propose here is to solve, for fixed DL powers, w.r.t. the BF vectors trying all the constraint one at time and check which one leads to the minimum value of $\lambda_{max}(B)$, and that satisfy the remaining power constraints with inequality. The SINR balancing problem and the power minimization in section IV are very closely related to each other. In particular the SINR balancing problem can tell us if the SINR constraints imposed in the power minimization problem are feasible or not. In case of positive answer, hence $\tau \geq 1$, there is room to minimize the total Tx power. Otherwise a feasible solution does not exist.

VII. Numerical Results

In this section we present some numerical results in which we study the convergence behaviour of the proposed iterative algorithm for the power minimization problem. In particular we present the Normalized Root Mean Square Error (NRMSE) between the Euclidean norm of the beamformer found using the iterative algorithm and the Euclidean norm of the beamformer obtained using the interior point method. In Fig. 2 is plotted the NRMSE, defined by the following expression

$$NRMSE = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^N\sum_{k=1}^K(\|\mathbf{g}_k^{(i)}(n)\|_2 - \|\mathbf{g}_k^*(n)\|_2)^2}}{\sqrt{\frac{1}{N}\sum_{n=1}^N\sum_{k=1}^K(\|\mathbf{g}_k^*(n)\|_2)^2}}$$

where $\|\mathbf{g}_k^{(i)}(n)\|_2$ represents the Euclidean norm of the DL BF determined using the iterative algorithm at iteration (i) for the n-th Monte Carlo run and $\mathbf{g}_k^*(n)$ is the DL BF obtained using the interior point method. The considered system is given by interference channel with $K = 3$ users with $M = 3$ Tx antennas each. The target SINR are $\gamma_k = 10, \ \forall k$, and the noise variance is equal to $\sigma^2 = -20dB$.

VIII. Conclusions

In this paper we have shown that the concept of uplink-downlink duality holds in the IFC in the form of a MISO-SIMO SINR duality. In particular the dual of an IFC is still

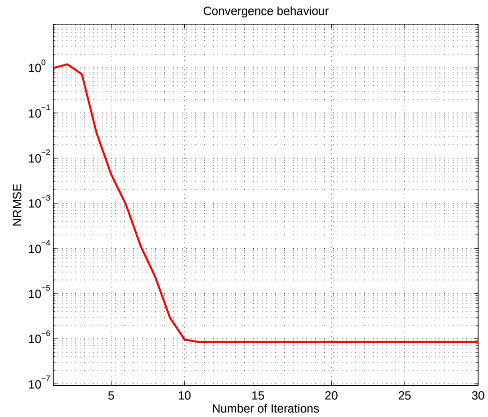

Fig. 2: NRMSE for $K = 3, M = 3$

an IFC, the advantage of duality is that the beamformer design problem is simplified in the dual UL SIMO IFC because it now corresponds to a receiver design problem. We also show that the underlying mechanics of this duality regime are similar to the UL-DL duality in the BC setting. This observation allows beamformer design in the MISO IFC using the same techniques as the ones well-known in the BC channel.

In addition we have shown that introducing the more realistic per-user power constraint in the interference channel it is still possible to define a dual UL problem where another set of optimization variables appears. These quantities now play the role of dual noise variances that need to be optimized.

References

[1] P. Viswanath and D.N.C. Tse, "Sum capacity of the vector gaussian broadcast channel and uplink-downlink duality," *Information Theory, IEEE Transactions on*, vol. 49, no. 8, pp. 1912 – 1921, aug. 2003.

[2] H. Boche and M. Schubert, "A general duality theory for uplink and downlink beamforming," 2002, vol. 1, pp. 87 – 91 vol.1.

[3] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *Vehicular Technology, IEEE Transactions on*, vol. 53, no. 1, pp. 18 – 28, jan. 2004.

[4] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. on Signal Processing*, vol. 55, no. 6, June 2007.

[5] K. Gomadam, V.R. Cadambe, and S.A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM)*, Dec 2008.

[6] F. Negro, S. Shenoy, D.T.M. Slock, and I. Ghauri, "Interference alignment limits for K-user frequency-flat MIMO interference channels," in *Proc. European Signal Proc. Conf. (Eusipco)*, Glasgow, Scotland, Aug. 2009.

[7] S. Boyd and L. Vandenberghe, Eds., *Convex Optimization*, Cambridge Univ. Press,, Cambridge U.K., 2004.

[8] H. Dahrouj and Wei Yu, "Coordinated beamforming for the multi-cell multi-antenna wireless system," in *Information Sciences and Systems, 2008. CISS 2008. 42nd Annual Conference on*, 2008, pp. 429 –434.

[9] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 1.21," http://cvxr.com/cvx, May 2010.

[10] G. Montalbano and D.T.M. Slock, "Matched filter bound optimization for multiuser downlink transmit beamforming," in *Universal Personal Communications, 1998. ICUPC '98. IEEE 1998 International Conference on*, Oct. 1998, vol. 1, pp. 677 –681 vol.1.

[11] Weidong Yang and Guanghan Xu, "Optimal downlink power assignment for smart antenna systems," in *Acoustics, Speech and Signal Processing, 1998. Proceedings of the 1998 IEEE International Conference on*, May 1998, vol. 6, pp. 3337 –3340 vol.6.