Performance of Closed Form and Iterative MU-MIMO Precoders for Different Broadcast Channel Configurations

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***Abstract*—We investigate in this paper the performance achieved by some SDMA (Space Division Multiple Access) access techniques in a broadcast channel configuration. The study includes the two basic linear precoding schemes present in the literature namely the CF (Closed Form) and the iterative precoding techniques. These techniques have already been widely studied for the i.i.d Rayleigh channel. In fact, these precoding methods are essentially based on the exploitation of diversity created by the existing multipaths in such a channel to enhance the beamforming design and consequently increase the SINR (Signal to Interference and Noise Ratio) for each user. In this work the impact of a reduction of these degrees of freedom has been investigated through simulations under other channel configurations. The obtained results demonstrate that these precoding techniques remain efficient only for systems offering high number of degrees of freedom and fail when a direct LOS (Line Of Sight) is present and thus limits the achievable Sum Rate (SR) of the system. The obtained results have also been compared to the TDMA (Time Division Multiple Access) communication mode.**

***Index Terms*—Multi-user, MIMO, SDMA, broadcast channel, capacity, SJNR, iterative, MMSE, rician, rayleigh, TDMA.**

I. INTRODUCTION

Multiuser MIMO (MU-MIMO) downlink system known in the information theory as the broadcast channel system represents today one of the most important research fields in wireless communications because of the high potential it offers in improving both reliability and capacity of the system. Some theoretical analysis of the capacity demonstrated that the capacity of a broadcast MU-MIMO channel can be achieved by applying a Dirty-Paper Coding (DPC) [1]–[3] algorithm as a precoder. Nevertheless, a DPC precoding is difficult to compute and is high resource consuming. Some suboptimal linear algorithms with lower implementation costs exist and can be divided into two main families: the iterative [4]–[8] and the closed form solutions [9]–[12].

For a MU-MIMO system, as shown in [11] the performance depends on both the receiver and the precoder that are also interdependent. This demonstrates the importance of iterative algorithms for MU-MIMO. Nevertheless these solutions present a major convergence problems [10] as iterative algorithms may converge to local maxima or even diverge. A solution solving this convergence problem has been proposed in [6], [13] thanks to a good initialization process and an optimized decision criteria guiding the convergence procedure.

On the other hand, the closed form precoder, are much simpler algorithms as they use one equation to define the precoding matrix. They are thus very fast and do not present the convergence problem. But, this kind of algorithms remain very suboptimal and present a high level of saturations in a fully charged system [6] as they are unable to fully exploit the offered diversity.

Different iterative solutions exist and use different precoding and receiving structures in an iterative way to reduce the inter-user interference and enhance the system performances. In this paper, an SJNR (Signal to Jamming and Nose Ratio) precoder combined with an MSR (Maximum Sum-Rate) receiver is considered. This scheme is compared to the MMSE/MMSE iterative algorithm given in [4], [8].Furthermore, we will consider the SVH(Stojnic, Vikalo, and Hassibi) algorithm proposed in [12] as it is an iterative algorithm for MU-MISO sumrate maximization based on a mathematical derivation of the optimization problem; and we generalized it for MU-MIMO in [7].

Among the closed form linear solutions presented in the literature, two seem to be interesting. The first algorithm is a Per User Minimum Mean Square Error (PU-MMSE) based linear pre-coder [9] and the second one is a maximum Signal to Jamming and Noise Ratio (SJNR) [14] linear precoder.

Nevertheless, all these performances and presented results have basically been obtained for the Rayleigh i.i.d channel where the present multipath gives an essential supplementary degree of freedom increasing the performances of such precoders.

In this paper we are going to focus on the study of the impact of the channel geometry and characteristics on both the iterative linear solutions and the linear closed form ones. In fact using a Rician channel that increases the LOS (Line Of Sight) contribution in the channel matrix shows a decrease in performances for the CF precoders. The impact of a θ distributed channel, that might be seen as the result of a good scheduling process, is also investigated.

In next Section, the model for the considered system is presented, followed by a detailed description of the existing iterative and CF algorithms. A detailed presentation of the employed receivers is done. In Section IV, the simulation conditions and the obtained results are detailed and discussed. Finally some conclusions are given in the last Section.

II. System Model

Let us consider in our study a multi-user MIMO communication system with N_T transmission antennas at the base station and K different users with N_{R_k} receiving antennas for each user k. Such a system is represented in figure 1.

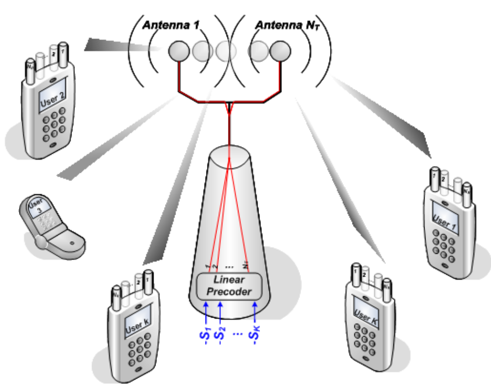

Fig. 1. MU-MIMO system model.

We assume that the base station has a perfect knowledge of the channel state information (CSI) of all K users. Let S_k a $Q_k \times 1$ vector representing the transmitted data symbols for user k where Q_k is the number of transmission streams for the same user. In our paper we are interested in the case of one stream per user Q_k=1.

The total transmit power at the base station is supposed to be constant and equal to P_T. The noise variance is noted N_0. For the channel part, H_k denotes the MIMO channel for user k which is a $N_{R_k} \times N_T$ matrix. Different channel configurations are taken into consideration. The first channel is a Rician channel. The expression of the channel coefficients are given by the expression

$$\mathbf{H}_{\mathrm{k}} = \sqrt{\frac{\mathrm{K_{R_k}}}{\mathrm{K_{R_k}}+1}}\mathbf{1}_{N_R \times N_T} + \sqrt{\frac{1}{\mathrm{K_{R_k}}+1}} \begin{pmatrix} f_{1,1}^k & \cdots & f_{1,N_T}^k \\ \vdots & \ddots & \vdots \\ f_{N_R,1}^k & \cdots & f_{N_R,N_T}^k \end{pmatrix}_{N_R \times N_T} \tag{1}$$

In this case the terms $f_{i,j}^k$ of the matrix represent i.i.d random Gaussian complex variable and K_{R_k} is the Racian factor. So it turns out that for $K_{R_k} = 0$ the Rayleigh channel is obtained.

The second considered channel is a channel that we constructed in such a manner to be able to study the impact of spacially separated users (This can be the case if a good scheduler is employed and that the number of users is important). Inspired from [15], [16], the expression of such a channel is given by

$$\mathbf{H}_{\mathrm{k}} = \sqrt{\frac{\mathrm{K_{R_k}}}{\mathrm{K_{R_k}}+1}}\mathbf{1}_{N_R \times N_T} \left(\mathbf{D}_{\mathbf{a}}^k\right)^H + \sqrt{\frac{1}{\mathrm{K_{R_k}}+1}} \begin{pmatrix} f_{1,1}^k & \cdots & f_{1,N_T}^k \\ \vdots & \ddots & \vdots \\ f_{N_R,1}^k & \cdots & f_{N_R,N_T}^k \end{pmatrix}_{N_R \times N_T} \mathbf{R}_k^{1/2} \tag{2}$$

where $\mathbf{R}_k = \mathbf{D}_{\mathbf{a}}^k \mathbf{B}_k \left(\mathbf{D}_{\mathbf{a}}^k\right)^H$ represents the covariance matrix at the transmitter taking into account the spacial distribution of the antennas. $\mathbf{B}_k = \exp\left(-\left(Toepliz\left(\mathbf{a}_{1\times N_T}\right)^2\right) v_{\theta_k}/2\right)$ with v_{θ_k} is a uniformly distributed random variable over $[-V_\theta/2, V_\theta/2]$. $\mathbf{D}_{\mathbf{a}}^k = \exp\left(\frac{j2\pi d}{\lambda}\sin\left(\theta_k\right) diag\left(\mathbf{a}_{1\times N_T}\right)\right)$ is the direction shaping matrix, and the terms $f_{i,j}^k$ represent i.i.d random Gaussian complex variable. The $\exp(.)$ is here applied on the terms of the matrix and $\mathbf{a} = [0, \ldots, N_T - 1]$.

III. Linear Precoders

In this section a description of the existing precoders presented in the literature is detailed. Rather than dissociating the two families: the iterative and the CF precoders, we present the precoders according to the criteria used to derive them. In fact, according to this classification, three classes appear. The first one is the basic MMSE precoders trying to minimize the MSE (Mean Square Error), the SJNR that minimizes the SJNR (Signal to Jamming and Noise Ratio) and finally the SVH (Stojnic, Vikalo, and Hassibi) that maximizes the sum capacity function.

A. MMSE Precoder

For the MU CF form of the MMSE precoder a block analysis had to be performed using a successive precoder construction method. This is done in [9] by simplifying the successive MMSE (SMMSE) to cancel out the interference generated by all the other users and received by the user of interest. The precoder of user k is constructed by

$$\hat{\mathbf{T}}_k = \left(\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k + \frac{N_{R_k}}{P_T} N_0 \mathbf{I}\right)^{-1} \hat{\mathbf{H}}_k^H \tag{3}$$

Where

$$\tilde{\mathbf{H}}_k^T = \left[\mathbf{H}_1^T \cdots \mathbf{H}_{k-1}^T \mathbf{H}_{k+1}^T \cdots \mathbf{H}_K^T\right] \tag{4}$$

and

$$\hat{\mathbf{H}}_k^T = \left[\mathbf{H}_k^T \tilde{\mathbf{H}}_k^T\right] \tag{5}$$

An SVD decomposition is applied to the virtual channel (composed of the cascade of the channel and the transmitter) $\mathbf{H}_k \hat{\mathbf{T}}_k$. The eigenvector $\mathbf{V}_k^{(1)}$ corresponding to the largest eigen value is considered as it is the direction maximizing the transmitted power (i.e. the best link of the channel). The final obtained precoder is then given by equation (6).

$$\mathbf{t}_k = \beta \hat{\mathbf{T}}_k \mathbf{V}_k^{(1)} \tag{6}$$

where

$$\mathbf{H}_k \hat{\mathbf{T}}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H \tag{7}$$

and

$$\beta = \sqrt{\frac{P_T}{tr\left(\sum_{j=1}^{K} \hat{\mathbf{T}}_j \mathbf{V}_j^{(1)} \left(\hat{\mathbf{T}}_j \mathbf{V}_j^{(1)}\right)^H\right)}} \tag{8}$$

An iterative form of the MMSE precoding scheme involving a MMSE decoder has been proposed in [4], [8]. The iterative form is based on the successive calculation of the MMSE receiver $\mathbf{d}_k$ based on the obtained MMSE precoder $\mathbf{t}_k$ and vice versa. The MMSE iterative precoder is given by

$$\mathbf{t}_k = \beta\left(\sum_{i=1}^{K}\mathbf{H}_i^H\mathbf{d}_i^H\mathbf{d}_i\mathbf{H}_i+\frac{1}{P_T}\sum_{j=1}^{K}N_0 tr\left(\mathbf{d}_j^H\mathbf{d}_j\right)\mathbf{I}\right)\mathbf{H}_k^H\mathbf{d}_k^H \tag{9}$$

where β is a normalization factor to respect the total transmit power constraint given by $tr\left(\sum_{j=1}^{K}\mathbf{t}_j\left(\mathbf{t}_j\right)^H\right) = P_T$ and the MMSE iterative receiver is given by

$$\mathbf{d}_k = \mathbf{t}_k^H\mathbf{H}_k^H\left(\mathbf{H}_k\sum_{i=1}^{K}\mathbf{t}_i^H\mathbf{t}_i\mathbf{H}_k^H + N_0\mathbf{I}\right)^{-1} \tag{10}$$

B. SJNR Precoder

This precoder is designed to increase the SJNR ratio. The SJNR for user k is defined as the total transmitted signal aimed to user k over the noise and the extra transmitted power received by the other users generated by the considered user k. This principle has been introduced in [14] and the CF precoder maximizing this quantity is given by

$$\mathbf{t}_k = \sqrt{P_k}\zeta_m\left[\left(\sum_{j=1,j\neq k}^{K}\mathbf{H}_j^H\mathbf{H}_j + \frac{N_0}{P_k}\mathbf{I}\right)^{-1}\mathbf{H}_k^H\mathbf{H}_k\right] \tag{11}$$

This solution is in fact the generalized eigenvector of the two matrices in the SJNR expression $\mathbf{H}_k^H\mathbf{H}_k$ and $\sum_{j=1,j\neq k}^{K}\mathbf{H}_j^H\mathbf{H}_j + N_0/P_T\mathbf{I}$. Here $\zeta_m\left[\mathbf{X}\right]$ represents the largest eigenvector of $\mathbf{X}$. The largest eigenvector is defined as the eigenvector corresponding to the largest eigenvalue of $\mathbf{X}$.

The iterative versions of the precoder is obtained by injecting the iterative virtual channel

$$\mathbf{h}_k^{iter} = \mathbf{d}_k^{iter-1}\mathbf{H}_k \tag{12}$$

into expression (11). The obtained iterative precoder becomes

$$\mathbf{t}_k^{iter}=\sqrt{P_k}\zeta_m\left[\left(\sum_{j=1,j\neq k}^{K}(\mathbf{h}_j^{iter})^H\mathbf{h}_j^{iter}+\frac{N_0^{iter}}{P_k}\mathbf{I}\right)^{-1}(\mathbf{h}_k^{iter})^H\mathbf{h}_k^{iter}\right] \tag{13}$$

where $N_0^{iter} = N_0\mathbf{d}_k^{iter-1}\left(\mathbf{d}_k^{iter-1}\right)^H$ and $\mathbf{d}_k^{iter}$ is the used receiver for user k and $\mathbf{h}^{iter}$ represents the virtual channel given by (12).

For the iterative procedure we are going to consider an MSR (Maximum Sum Rate) receiver derived in [7] given by

$$(\mathbf{d}_{MSR,k}^{iter})^H = \zeta_m\left(\mathbf{\Psi}_k^{iter}\right) \tag{14}$$

to calculate the virtual channel. $\mathbf{\Psi}_k^{iter}$ is given by

$$\mathbf{\Psi}_k^{iter}=\left(\sum_{j=1,j\neq k}^{K}\mathbf{H}_k\mathbf{t}_j^{iter}\mathbf{R}_{\mathbf{s}_j}(\mathbf{t}_j^{iter})^H(\mathbf{H}_k)^H+N_0\mathbf{I}\right)^{-1} \mathbf{H}_k\mathbf{t}_k^{iter}\mathbf{R}_{\mathbf{s}_k}(\mathbf{t}_k^{iter})^H(\mathbf{H}_k)^H \tag{15}$$

Moreover, and considering the description of the iterative algorithm given in [6], the algorithm iterates until the obtained SR stabilizes (i.e. $\left|SR^{iter} - SR^{iter-1}\right| \leq \varepsilon$) where ε is a predefined threshold or when the number of iterations exceeds a maximum number of iterations given by N_{max}^{iter}. The first iteration (initialization) is done by calculating the precoders using the real channels $\mathbf{H}_k$ and the corresponding MSR receivers that is used to calculate the first virtual channel according to (12). This principle is applied to all iterative algorithms treated in this work.

C. SVH Precoder

This last family of precoders is derived based on the SR (Sum-Rate) maximization. For that a Lagrangian optimization problem is solved. The obtained solution named ”method 2.1” is given in [12] and describes the optimal solution derived for the quasi-convex optimization problem. Solving the problem using the bisection method gives the following system of equations (16).

$$\begin{cases} \mathbf{F}^{iter_{SVH}}=diag\left(\frac{num_1}{den_1\left(den_1+num_1\right)},\cdots,\frac{num_K}{den_K\left(den_K+num_K\right)}\right) & \text{(16a)}\\ \mathbf{G}^{iter_{SVH}}=diag\left(\frac{\left(\mathbf{HT}^{iter_{SVH}-1}\right)_{11}}{den_1},\cdots,\frac{\left(\mathbf{HT}^{iter_{SVH}-1}\right)_{KK}}{den_K}\right) & \text{(16b)}\\ \mathbf{T}^{iter_{SVH}} =\frac{\mathbf{H}^H\mathbf{G}^{iter_{SVH}}}{\sigma^2 tr\ \mathbf{F}^{iter_{SVH}}\ \ \mathbf{I}\ \ \mathbf{H}^H\mathbf{F}^{iter_{SVH}}\mathbf{H}} & \text{(16c)} \end{cases}$$

where

$$num_k = \left|\left(\mathbf{HT}^{iter_{SVH}-1}\right)_{kk}\right|^2 \tag{17}$$

and

$$den_k = \sigma^2 tr\left(\mathbf{T}^{iter_{SVH}-1}\left(\mathbf{T}^{iter_{SVH}-1}\right)^H\right) + \sum_{n=1,n\neq k}^{K}\left|\left(\mathbf{HT}^{iter_{SVH}-1}\right)_{kn}\right|^2 \tag{18}$$

Here $\mathbf{T}^{iter_{SVH}} = \left[\left(\mathbf{t}_1^{iter_{SVH}}\right),\cdots,\left(\mathbf{t}_K^{iter_{SVH}}\right)\right]$ and $\mathbf{H} = \left[\mathbf{h}_1^T,\cdots,\mathbf{h}_K^T\right]^T$.

The iterative algorithm consists in initializing the $\mathbf{F}^{iter_{SVH}}$ and $\mathbf{G}^{iter_{SVH}}$ matrices with $\mathbf{I}$ and to calculate the corresponding precoder $\mathbf{T}$.

The algorithms then iterates by computing the new $\mathbf{F}$ and $\mathbf{G}$ corresponding to the last precoder. The new precoder is then calculated in function of these obtained $\mathbf{F}$ and $\mathbf{G}$. At each iteration, the SR is calculated according to (27) taking $\mathbf{d}_k = 1$.

The system converges when it is stabilized meaning that the obtained value for the precoder no longer changes $|SR^{iter_{SVH}} - SR^{iter_{SVH}-1}| < \varepsilon_{SVH}$. The end of the algorithm can also be controlled by fixing the number of iterations.

An iterative version based on a joint optimization of the precoder and the receiver has been proposed in [7]. In fact, the

MU-MIMO channel has been reduced to a MU-MISO channel through the virtual channel calculation computed through (12).

The iterative procedure considers an MSR (Maximum Sum Rate) receiver given by (14) and the precoder calculation becomes in this case as follows

$$\begin{cases} \mathbf{F}^{iter_{SVH}} = diag\left(\frac{num_1}{den_1\left(den_1+num_1\right)},..,\frac{num_K}{den_K\left(den_K+num_K\right)}\right) & \text{(19a)} \\ \mathbf{G}^{iter_{SVH}} = diag\left(\frac{\left(\mathbf{H}^{iter}\mathbf{T}^{iter_{SVH}-1}\right)_{11}}{den},..,\frac{\left(\mathbf{H}^{iter}\mathbf{T}^{iter_{SVH}-1}\right)_{KK}}{den}\right) & \text{(19b)} \\ \mathbf{T}^{iter_{SVH}} = \frac{\left(\mathbf{H}^{iter}\right)^H\mathbf{G}^{iter_{SVH}}}{\left(\left(\sigma^2 tr\left(\mathbf{F}^{iter_{SVH}}\right)\right)I+\left(\mathbf{H}^{iter}\right)^H\mathbf{F}^{iter_{SVH}}\mathbf{H}^{iter}\right)} & \text{(19c)} \end{cases}$$

with

$$num_k = \left|\left(\mathbf{H}^{iter}\mathbf{T}^{iter_{SVH}-1}\right)_{kk}\right|^2 \tag{20}$$

$$den_k = \sigma^2 tr\left(\mathbf{T}^{iter_{SVH}-1}\left(\mathbf{T}^{iter_{SVH}-1}\right)^H\right) + \sum_{n=1,n\neq k}^{K}\left|\left(\mathbf{H}^{iter}\mathbf{T}^{iter_{SVH}-1}\right)_{kn}\right|^2 \tag{21}$$

$\mathbf{T}$ and $\mathbf{H}$ are as previously defined.

IV. Simulations And Results

A. SR Calculation

To evaluate the performance of the algorithms, the sum-rate (SR) is evaluated for the different presented configurations. For a given system, the SR can be evaluated as the maximum of the mutual information between the received signal and the transmitted signal. Let us focus on user k and denote $\mathbf{x}_k$ the transmitted signal aimed to user k. Considering one stream per user, the transmitted signal $\mathbf{x}_k$ is given by

$$\mathbf{x}_k = \mathbf{t}_k \times \mathbf{s}_k \tag{22}$$

with $\mathbf{t}_k$ the precoder for user k and $\mathbf{s}_k$ a symbol vector of dimention Q_k. The received signal on the other hand (that we denote $\mathbf{y}_k$) is given by

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{H}_k\sum_{i=1;i\neq k}^{K}\mathbf{x}_i + \mathbf{n}_k \tag{23}$$

After decoding the received signal becomes

$$\hat{\mathbf{y}}_k = \mathbf{d}_k\mathbf{H}_k\mathbf{x}_k + \mathbf{d}_k\mathbf{H}_k\sum_{i=1;i\neq k}^{K}\mathbf{x}_i + \mathbf{d}_k\mathbf{n}_k \tag{24}$$

Under these assumptions and considering perfect CSIT (Channel State Information at the Transmitter), the mutual information can be written

$$\begin{aligned} & I\left(\mathbf{x}\left|(\hat{\mathbf{y}},\mathbf{H})\right.\right) = I\left(\mathbf{x};\hat{\mathbf{y}}\left|\mathbf{H}\right.\right) \\ & = h\left(\hat{\mathbf{y}}\right) - h\left(\hat{\mathbf{y}}\left|\mathbf{x}\right.\right) \\ & = \log_2\left(\det\left(\mathbf{I}+\mathbf{d}_k\mathbf{H}_k\mathbf{R}_{\mathbf{x}_k}\mathbf{H}_k^H\mathbf{d}_k^H\mathbf{K}_k^{-1}\right)\right) \end{aligned} \tag{25}$$

with $\mathbf{K}_k^{-1}$ is the covariance matrix of the interference and noise part.

$$\mathbf{K}_k = \mathbf{d}_k\left(\mathbf{H}_k\sum_{i=1;i\neq k}^{K}\mathbf{R}_{\mathbf{x}_i}\mathbf{H}_k^H + \mathbf{R}_{\mathbf{n}_k}\right)\mathbf{d}_k^H \tag{26}$$

$\mathbf{R}_{\mathbf{x}_i}$ and $\mathbf{R}_{\mathbf{n}_k}$ are the covariance matrix of respectively the symbols and the noise.

The final expression of the sum rate for an SDMA system in the broadcast case with one stream per user is then given by

$$SR_{SDMA} = \sum_{k=1}^{K} log_2\left(1+\frac{\mathbf{d}_k H_k\mathbf{t}_k\mathbf{R}_{\mathbf{s}_k}\mathbf{t}_k^H\mathbf{H}_k^H\mathbf{d}_k^H}{\mathbf{d}_k\left(\mathbf{\Upsilon}_k+N_0\mathbf{I}\right)\mathbf{d}_k^H}\right) \tag{27}$$

where $\mathbf{\Upsilon_k} = \mathbf{H_k}\sum_{\mathbf{j=1,j\neq k}}^{\mathbf{K}}\mathbf{t_j}\mathbf{R}_{\mathbf{s_j}}\mathbf{t_j^H}\mathbf{H_k^H}$ represents the interference generated by the other users and collected by user k. For a TDMA access mode the SR is given by

$$SR_{TDMA} = \sum_{\mathrm{k=1}}^{\mathrm{K}}\mathrm{Dis_K(k)}\log_2\left|\mathbf{I}_{N_{R_k}}+\mathbf{H}_k\mathbf{V}\mathbf{\Lambda}\mathbf{V}^H\mathbf{H}_k^H\right| \tag{28}$$

where $\mathrm{Dis_K(k)}$ is the temporal distribution given to the different users, $\mathbf{V}$ and $\mathbf{\Lambda}$ are respectively the precoding and the water-filling matrices.

B. Simlation Parameters

In all our simulations, we consider that we have only one stream per user $Q_k = 1$ and the number of receiving antennas is the same for all users $N_{R_k}{=}N_R$. Different channel configurations have been considered according to the schemes described in section II.

The simulation generates 10000 independent channel realizations for each user. To generate the total throughput of the system, we perform an average over all channel realizations on the quantity SR given in equation (27) or (28). The fading part of the channel coefficients $(h_{i,j}^k)_{1\leq i\leq N_R, 1\leq j\leq N_T}$ are generated such as $E\|h_{i,j}^k\|^2{=}1$.

The two convergence control parameters for the iterative algorithms ε_{SVH}, ε are fixed and equal to 0.001.

In all the following, the maximal number of iterations N_{max}^{iter} is fixed to 50.

For the SJNR precoder, we distribute the energy equally over all considered users according to $P_k{=}P_T/K$.

For the TDMA case, a uniform temporal distribution of the users is considered i.e. $Dis_K(k) = 1/K$; $\forall k \in [1 \ldots K]$.

Finally for the θ distributed channel we consider $\theta \in 2\pi/360 * [0, 45, 90, -45]$ for the 4 considered users and the value of $V_\theta = 10 * 2\pi/360$. The term d/λ is taken equal to $1/2$.

C. Simulation Results

In this subsection, the simulation results are presented. In all following figures we are going to adopt these notations: The names of the curves start with the corresponding access technique C_{SDMA} or C_{TDMA} followed by the used precoder and decoder. The third element is indicating the family of the precoding technique namely $Iter$ for iterative or CF for Closed Form and finally the value of the Rician factor K_{Rice}.

Figure 2 represent the evolution of the different considered algorithms for a variation of the Rician factor for $N_T = 4$ transmission antennas, $N_R = 4$ receiving antennas and $K = 4$ users. The first subcurve 2.a represents the CF

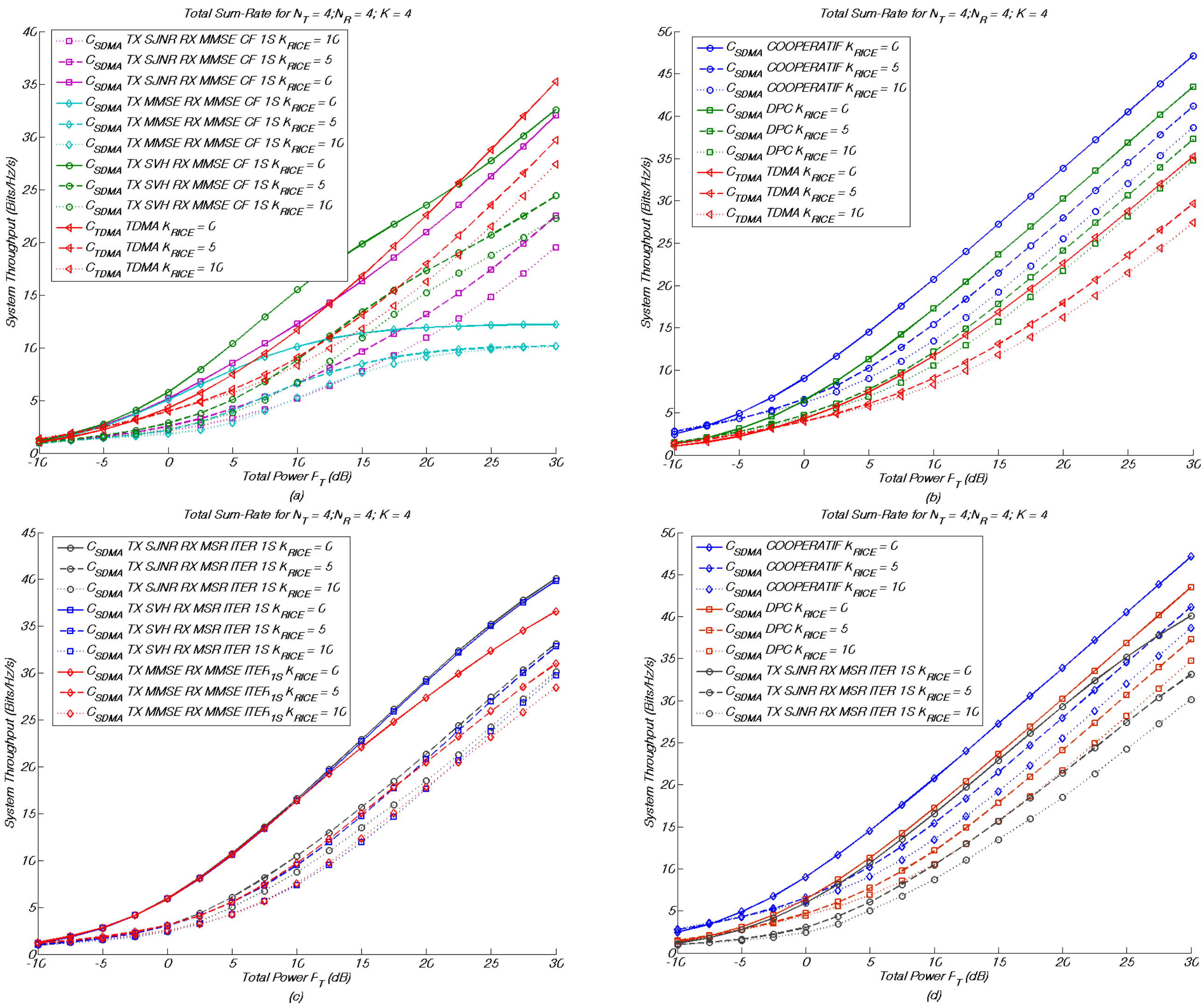

Fig. 2. Throughput as a function of total transmit power P_T for $N_T = 4, N_R = 4$ and $K = 4$.

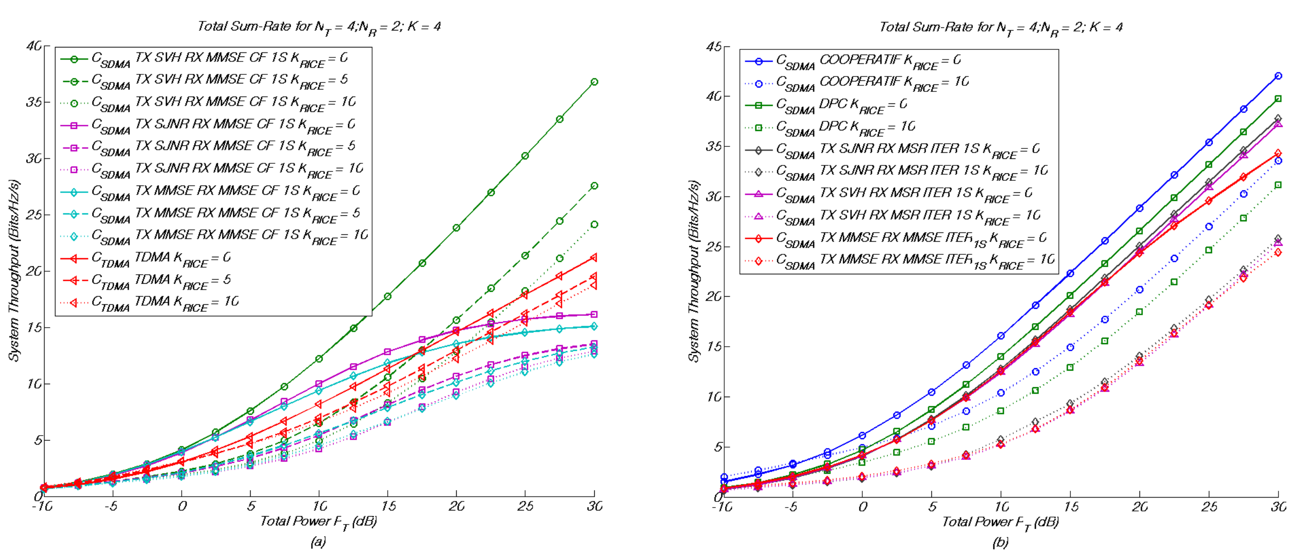

Fig. 3. Throughput as a function of total transmit power P_T for $N_T = 4, N_R = 2$ and $K = 4$.

algorithms namely the SJNR/MMSE, PU-MMSE/MMSE and the SVH/MMSE. This figure confirms the results obtained in [11] as the SJNR/MMSE performs better than the PU-MMSE algorithm. This result is here extended to all channel configurations and remains stable while changing the Rician factor. It must be noted, at this point, that despite the fact that the SVH algorithm is an iterative algorithm calculating the optimal precoder for the MU-MISO system, it has been proven in [7] that 2 iterations are sufficient for convergence. Moreover, taking a closer look at the algorithm, shows that the first iteration is playing the role of a simple initialization for the algorithm. In this case, the SVH/MSR iterative algorithm described in [7] and called $Algorithm2$ becomes equivalent to a CF algorithm when $iter_{SVH_{max}} = 2$ and $N_{iter} = 1$. In addition to that, it confirms the superiority of the SVH/MSR (equivalent to an SVH/MMSE) algorithm described in [7] as it is the optimal CF precoder associated with the optimal receiver maximizing mathematically the system sum-rates. We also remark in this curve the main problem encountered by the MMSE algorithm that always saturates at high SNRs when the system is fully charged. Comparing now these curves to the TDMA case, we can see that the waterfilling algorithm performed in this algorithm becomes stronger especially with an increase of the Rician factor. These observations combined with the results of figure 2.b suggest that there might be better ways of extracting the system diversity. In fact the TDMA curves remains alway far below the DPC and cooperative curves; and methods like the iterative ones could generate much better performances.

Analyzing at the next subcurves 2.c and 2.d we see that by introducing the iterative procedure, the obtained results get closer to the DPC performances. The sum-rates offered by the TDMA system remain always lower than those offered by the iterative algorithms. Moreover, the impact of increase of the Rician factor (implying a decrease of the degrees of freedom generated by the multipahts) is thus much more stable.

The next figure, figure 3 gives the performances for a system with lower number of receiving antennas. In this case, we remark almost the same phenomenons as the MMSE and SJNR CF algorithms saturate at high SNRs, with a domination of the SJNR. On the other hand, the SVH algorithm is capable of optimally eliminating the interference and no saturation is noted at high SNRs. Nevertheless, an overall decrease in performances must be noted except for SVH.

In the iterative mode, the algorithms get weaker and the difference compared to the DPC gets higher as the Rician factor gets higher. This demonstrates the importance of the number of receiving antennas and thus the impact of the receiving structure in eliminating the interference part.

Figure 4 and 5 include some curves representing the performance of the cooperative system. The cooperative system is a configuration where all users perfectly cooperate and can thus decode all the signals of the other users. The system becomes in this case equivalent to a single user MIMO system with the same number (N_T) of transmitting antennas and with $\sum_{k=1}^{K} N_{R_k}$ receiving antennas. The corresponding channel is then $\mathbf{H} = \left[\mathbf{H}_1^T, \ldots, \mathbf{H}_K^T\right]^T$. The performances of this system is obtained by an SVD decomposition over H followed by a water filling algorithm. We also plot out the performance achieved by the DPC algorithm implemented according to [3].

Figure 4 gives the results obtained with a fully charged 2 by 2 system ($N_T = 2$, $N_R = 2$ and $K = 2$). The performances of the iterative and the CF algorithms gets closer and are near the DPC limit. We also remark that the sequencing of the different algorithms remains the same. But, the most important observation to be noted here, is that even with the worst CF algorithm we can outperform the sum-rate offered by the TDMA system in the case of Rayleigh channel.

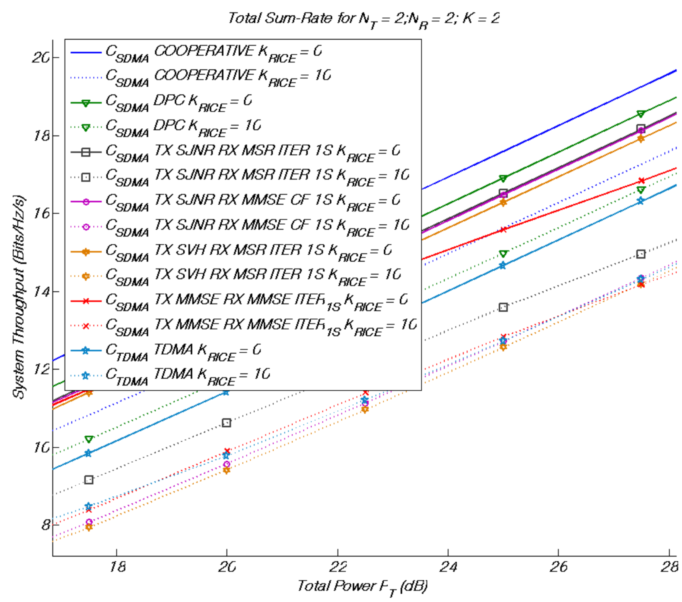

Fig. 4. Throughput as a function of total transmit power P_T for $N_T = 2, N_R = 2$ and $K = 2$.

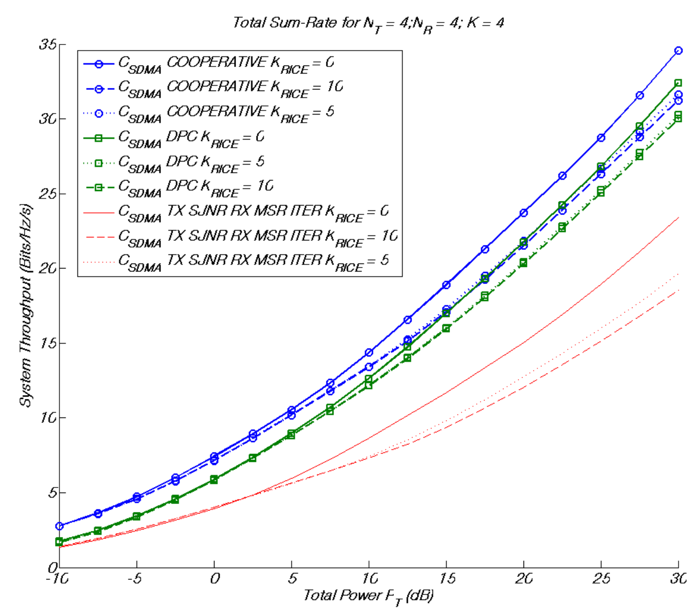

Fig. 5. Throughput as a function of total transmit power P_T for $N_T = 4, N_R = 4$ and $K = 4$ with θ channel.

The last curves plotted on figure 5 are given for a θ distributed channel. Meaning that we considered specially uncorrelated channels for the different users. This case can easily be obtained with a good user scheduling choosing the users among a big set. These curves have been plotted for the case of a fully charged system with 4 transmitting antennas, 4 receiving antennas for each of the 4 users.

The obtained curves show that in this case, the impact of the Rician factor diminishes a lot and no longueur degrades the performances on the Cooperative system, the DPC algorithm and even on the SJNR/MSR iterative algorithm. This can easily be achieved using a good scheduling procedure selecting the least interfering users. This decreases also the constraints on the precoder making it easier to find the optimal solution maximizing the sum-rate.

V. Conclusion

In this paper, we present a study of the impact of the channel geometry on some of the main precoding algorithms presented in the literature. We considered in fact, three main algorithms of the CF and iterative families. These algorithms are based on three different criteria to maximize the performance of the system. The MMSE minimizing the MSE, The SJNR minimizing the jamming signal and the SVH maximizing the throughput of the system. For our simulations we considered a Rician channel with variant K_R factor. The simulation results confirmed the fact that the main gain achieved by these MU-MIMO algorithms comes from the multi path structure of the channel. It shows also that the change in the channel configuration does not change the goodness of the algorithms.

On the other hand the second series of simulations using a θ distributed channel showed that a good scheduling algorithm and high number of users can limit the impact of a decrease of degrees of freedom of our channel.

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