

Utility/Pricing-based Resource Allocation Strategy for Cognitive Radio Systems

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Abstract—In this paper¹, we propose a resource allocation scheme based on a *utility/pricing* strategy with the objective to maximize a defined *utility function* subject to minimize the mutual interference caused by secondary users (SUs) with protection for primary users (PUs). Specifically, we formulate a *utility function* to reflect the needs of PUs by verifying the outage probability constraint, and the per-user capacity by satisfying the signal-to-noise and interference ratio (SNIR) constraint, as well as to limit interference to PUs. Furthermore, the existence of the Nash equilibrium of the proposed game is established, as well as its uniqueness under some sufficient conditions. Theoretical and simulation results based on a realistic network setting, and a comparison with a previously published binary power allocation method will be provided in this paper. The reported results demonstrate the efficiency of the proposed technique in terms of cognitive radio network (CRN) deployment while maintaining quality-of-service (QoS) for the primary system, and its superiority to the binary power allocation.

Keywords—Cognitive Radio, Resource Allocation, Game Theory, User Selection.

I. INTRODUCTION

Over the years, the licensed use of the radio spectrum has limited the number of technologies using it. Even though all its actors made a priority of using the spectrum at its maximum efficiency, the analysis of a portion of the spectrum showed three conclusions [1]:

- Some frequency band in the spectrum are largely unoccupied most of the time;
- Some other frequency bands are only partially occupied;
- The remaining frequency bands are heavy used.

Cognitive radio (CR) is an emerging technology in wireless technology that uses software-defined radio to aim to the efficient use of the spectrum by exploiting the unused frequency bands at the time and space [2].

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In this paper we will combine CR with game theory. Game theory was at first a mathematical tool used for economics, political and business studies. It helps understand situations in which decision-makers interact in a complex environment according to a set of rule [3]. Many different types of game exists which are used to reflect to analyzed situation for example potential games, repeated game, cooperative or non-cooperative games. In the cognitive radio network (CRN), the formal game model for the power control can be defined as follows:

- Players: are the cognitive users (secondary users (SUs)).
- Actions: called also as the decisions, and are defined by the transmission power allocation strategy.
- Utility function: represents the value of the observed quality-of-service (QoS) for a player, and is defined later in this section.

The central idea in game theory is how the decision from one player will affects the decision-making process from all other players and how to reach a state of equilibrium that would satisfy most of the players. A well known contributor in the field is Nash for the Nash equilibrium [4]. The theory shows that you can reach a state equilibrium for your system where all decisions are set, unchanging and is the best possible situation for the players.

CR need to perform sophisticated adaptation and dynamically learn from the environment. This situation makes the learning process a very complicated one comparable to situation found in economics. Game theory is already used in other field of communication to better understand for example congestion control, routing, power control, topology control and trust management [5]. Our interests rest in its use for power control as it can be considered a game with fixed number of players where each tries to optimize their power levels. There are a number of properties that makes this problem appropriate for a cognitive radio game model:

- The player's payoff is a function of her own transmit power level and her signal-to-noise and interference ratio (SINR). The player's SINR is a function of her own

transmit power and the transmit powers of the other players in the cell.

- When a player increases her power level, this will increase her own SINR, but will decrease the SINRs of all other players.
- For a fixed SINR, the players prefer lower power levels to higher ones. That is, players wish to conserve power and extend their battery life when possible.
- For a fixed power level, players prefer higher SINR to lower one. That is, players want the best possible channel conditions for a given expenditure of power.

There are many ways to cope with these issues such as to add restriction to the use of the power resource by charging it to users. This is done by adding a cost component to the *payoff function* to add fairness to the network. Another idea is to model the scenario as a repeated game [5].

In this paper we formulate the problem of resource allocation in the context of a CRN to reflect the needs of PUs and SUs. We consider the primary uplink of a single CRN, where cognitive transmitters transmit signals to a number of SUs, while the primary BS receives its desired signal from a primary transmitter and interference from all the cognitive transmitters.

To resolve the problem of resource allocation, we propose a *utility function* that meets the objective to maximize the SUs capacity, and the protection for PUs. Specifically, we define a *payoff function* that represents the SNIR constraint, and a *price function* specifies the outage probability constraint. The *utility function* is defined as:

$$utility\ function = payoff\ function - price\ function$$

We introduce a *payoff* to express the capacity need of SU m , and a *price function* to represent the protection for PUs by means of the outage probability. And each SU adjusts its transmitted power to maximize its *utility function*. Therefore, we will present in this paper a power allocation algorithm that maximize the defined *utility function* to compute the transmitted power of each SU.

The paper is organized as follows. In Section II we describe the channel model and introduce the proposed game theory strategy. In Section III, the power allocation algorithm is presented. The existence of the Nash equilibrium of the proposed game is established in Section IV, as well as its uniqueness under some sufficient conditions. Simulation results and a comparison with a previously published binary power allocation method are provided in Section V, and Section VI concludes the paper.

II. CHANNEL MODEL

Consider the uplink of a CRN that consists of a PU, a base station (BS), and M pairs of SUs randomly distributed over the system [2]. The channel gains are i.i.d random variable. Throughout this paper, we will use the following notation:

- the index of SUs m lies between 1 and M ,
- $h_{l,m}$ denotes the channel gain from SU l to the desired user m ,

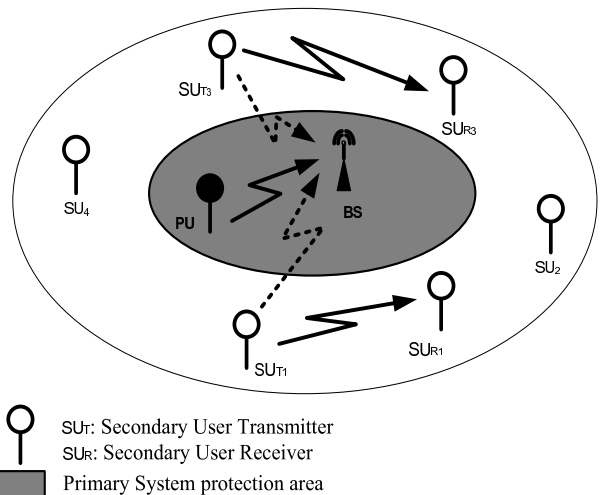


Fig. 1. The Cognitive Radio Network with one PU and $M = 4$ secondary transmitters attempting to communicate with their respective receivers in an ad-hoc manner during an uplink transmission of the PU, subject to mutual interference.

- the data destined from SU m is transmitted with power p_m and a maximum power P_{max} ,
- $h_{pu,m}$ denotes the channel gain from the PU indexed by pu to the desired user m ,
- the data destined from the primary system is transmitted with power p_{pu} .

In the coverage area of the primary system, there is an *interference boundary* within which no SUs can communicate in an ad-hoc manner. Thus, as can be seen in Fig. 1, for the impairment experienced by the primary system to be as small as possible, a SU must be able to detect very reliably whether it is far enough away from a primary base station, i.e., in the area of possible cognitive radio operation. The expression of the PU instantaneous capacity is

$$C_{pu} = \log_2 \left(1 + \frac{p_{pu} |h_{pu,pu}|^2}{\sum_{m=1}^M p_m |h_{m,pu}|^2 + \sigma^2} \right) \quad (1)$$

where σ^2 is the ambient noise variance. On the other hand, by making SUs access the primary system spectrum, the m^{th} SU experiences interference from the PU and all neighboring co-channel SU links that transmit on the same band. Accordingly, the m^{th} SU instantaneous capacity is given by:

$$C_m = \log_2 (1 + \text{SINR}_m) \quad (2)$$

where

$$\text{SINR}_m = \frac{p_m |h_{m,m}|^2}{\sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2 + p_{pu} |h_{pu,m}|^2 + \sigma^2} \quad (3)$$

SUs need to recognize their communication environment and adapt the parameters of their communication scheme in order

to maximize the per-user cognitive capacity, expressed as

$$C_{sum} = \sum_{m=1}^M C_m \quad (4)$$

while minimizing the interference to the PUs, in a *distributed* fashion. The sum here is made over the M SUs allowed to transmit [7] [8]. Moreover, we assume that the coherence time is sufficiently large so that the channel stays constant over each scheduling period length. We also assume that SUs know the channel state information (CSI) of their own links, but have no information on the channel conditions of other SUs. No interference cancelation capability is considered. Power control is used for SUs both in an effort to preserve power and to limit interference and fading effects. The interference power (Intf) is given by:

$$\text{Intf}_m = \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2 + p_{pu} |h_{pu,m}|^2 + \sigma^2 \quad (5)$$

Combining (3) and (5), we define the SINR as a function of Intf:

$$\text{SINR}_m = \frac{p_m |h_{m,m}|^2}{\text{Intf}_m} \quad (6)$$

and

$$p_m = \frac{\text{SINR}_m \text{Intf}_m}{|h_{m,m}|^2} \quad (7)$$

The protection for PU must be guaranteed in a CRN. This protection is guaranteed if the sum of all SUs transmitters' powers is not larger than the interference constraint P_T . Then, PU verifies his outage probability constraint. The interference constraint is given by:

$$\sum_{m=1}^M p_m |h_{pu,m}|^2 \leq P_T \quad (8)$$

and the notion of outage probability defined as the probability that the capacity of the user is below the transmitted code rate [6]. In the proposed framework, the outage probability can be expressed as [9]:

$$P_{out} \equiv \text{Prob}\{C_{pu} \leq R_{pu}\} \leq q, \quad (9)$$

where R_{pu} is the PU transmitted data rate and q is the maximum outage probability. The information about the outage failure can be carried out by a band manager that mediates between the primary and secondary users [10], or can be directly fed back from the PU to the secondary transmitters through collaboration and exchange of the CSI between the primary and secondary users as proposed in [11].

III. POWER ALLOCATION ALGORITHM

We derive in this section the *utility function*: we define a *payoff function* specifies the SU capacity constraint and a *price function* that represents the interference constraint as a function of the outage probability constraint. Therefore, the *price function* is given by (2), and we will derive here the equation of the interference constraint P_T .

The margin of $P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{pu,l}|^2$ is the maximum interference that SU m could generate under the description of (8). Divide $p_m |h_{pu,m}|^2$ by $P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{pu,l}|^2$, we found the interference level expression:

$$L_{\text{Intf}_m} = \frac{p_m |h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{pu,l}|^2} \quad (10)$$

which is a normalized value. As long as this ratio $\in [0, 1]$, the protection for PU is met. We compute now P_T as a function of the outage probability.

To proceed further with the analysis and for the sake of emphasis, we introduce the PU average channel gain estimate G_{pu} based on the following decomposition:

$$h_{pu,pu} \equiv G_{pu} * h'_{pu,pu} \quad (11)$$

where $h'_{pu,pu}$ is the random component of channel gain and represents the *normalized* channel impulse response tap. This gives us the following PU outage probability expression in an interference-limited context:

$$\begin{aligned} P_{out} &= \text{Prob} \left\{ \log_2 \left(1 + \frac{p_{pu} G_{pu}^2 |h'_{pu,pu}|^2}{\sum_{m=1}^M p_m |h_{m,pu}|^2} \right) \leq R_{pu} \right\} \\ &\simeq \text{Prob} \left\{ \frac{p_{pu} G_{pu}^2 |h'_{pu,pu}|^2}{\sum_{m=1}^M p_m |h_{m,pu}|^2} \leq 2^{R_{pu}} - 1 \right\} \\ &\simeq \text{Prob} \left\{ |h'_{pu,pu}|^2 \leq (2^{R_{pu}} - 1) \left(\frac{\sum_{m=1}^M p_m |h_{m,pu}|^2}{G_{pu}^2 p_{pu}} \right) \right\} \end{aligned} \quad (12)$$

From now on we assume for simplicity of analysis that the channel gains are i.i.d rayleigh distributed. However, the results can be immediately translated into results for any other channel model by replacing by the appropriate probability distribution function. Continuing from (12), we have:

$$P_{out} \simeq \int_0^{(2^{R_{pu}} - 1) \left(\frac{\sum_{m=1}^M p_m |h_{m,pu}|^2}{G_{pu}^2 p_{pu}} \right)} \exp(-t) dt \quad (13)$$

Finally, we get the following outage constraint:

$$P_{out} \simeq 1 - \exp \left[- (2^{R_{pu}} - 1) \left(\frac{\sum_{m=1}^M p_m |h_{m,pu}|^2}{G_{pu}^2 p_{pu}} \right) \right] \quad (14)$$

Replacing the interference constraint equation in (14), we can express the probability outage as:

$$P_{out} = 1 - \exp \left[- (2^{R_{pu}} - 1) \frac{P_T}{G_{pu}^2 p_{pu}} \right] \quad (15)$$

Then, the corresponding interference constraint is:

$$P_T = \frac{p_{pu} G_{pu}^2}{1 - 2^{R_{pu}}} \ln(1 - P_{out}) \quad (16)$$

We introduce now a *utility function* for which each SU adjusts its transmitted power in order to maximize it. It is composed of a *payoff function* expressed as the capacity C_m of the SU, and of a *price function* composed of the interference level to the PU and the power consumption.

Then, the *utility function* is expressed as follow:

$$U_m = C_m - \left(\frac{p_m |h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \quad (17)$$

The parameter a_m is adjustable to have a comparable values, i.e. the *payoff function* value and the *price function* value. This parameter gives the flexibility needed to adjust the SU capacity over the interference to the PU. We choose $a_m < 0$. It could be easily obtained that the *price function* decreases as the ratio L_{Intf_m} increases. This fact is caused by the negative property of a_m .

Mathematically, the game G can be expressed as:

$$\text{Find } p_m |_{m=1, \dots, M} = \arg \max_{p_m} U_m(p_m, \mathbf{P}_{-m}) \quad (18)$$

subject to:

$$\begin{cases} \sum_{m=1}^M p_m |h_{pu,m}|^2 \leq P_T \\ P_{out} \leq q \\ 0 \leq p_m \leq P_{max} \end{cases} \quad (19)$$

Recall that p_m denotes the strategy adopted by SU m and $\mathbf{P}_{-m} = (p_l)_{l \neq m, l \in \{1, \dots, M\}}$ denotes the strategy adopted by the other SUs. We replace the capacity by expression given by (2) and use (7) to obtain the following equation:

$$U_m = \log_2(1 + \text{SINR}_m) - \left(\frac{|h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \times \left(\frac{\text{SINR}_m \text{Intf}_m}{|h_{m,m}|^2} \right)^{a_m} \quad (20)$$

We are going to maximize the *utility function* in terms of the SINR, which is equivalent to the transmitted power. The solution of the system is found by calculating the derivatives of U_m with respect to the signal-to-noise and interference ratio parameters SINR_m :

$$\frac{\partial U_m}{\partial \text{SINR}_m} = \frac{1}{(1 + \text{SINR}_m) \ln 2} - \left(\frac{|h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \times a_m \left(\frac{\text{SINR}_m \text{Intf}_m}{|h_{m,m}|^2} \right)^{a_m - 1} \frac{\text{Intf}_m}{|h_{m,m}|^2} \quad (21)$$

We can express the solution of (21) as:

$$(1 + \text{SINR}_m) \text{SINR}_m^{a_m - 1} = \frac{1}{a_m \beta_m \ln 2} \quad (22)$$

where:

$$\beta_m = \left(\frac{|h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \left(\frac{\text{Intf}_m}{|h_{m,m}|^2} \right)^{a_m} \quad (23)$$

denoting the slope of the *price function*. Let $f(\text{SINR}_m) = (1 + \text{SINR}_m) \text{SINR}_m^{a_m - 1}$. Finally, we obtain the following set of equalities:

$$\text{SINR}_m = f^{-1} \left(\frac{1}{a_m \beta_m \ln 2} \right) \quad (24)$$

The maximization problem is dependent on a_m which is defined in the *utility function* as an adjustment parameter to the *price function*. For simulation results $a_m = -0.2$. It was chosen to stay with this value after different simulations to show its influence on the obtained results.

Our main contribution within this work is the QoS management of the CR system. The originality in the proposed method is that we guarantee a QoS to PU by maintaining the PU's outage probability unaffected in addition to a certain QoS to SUs and ensuring the continuity of service even when the spectrum sub-bands change from vacant to occupied. Thus by the outage probability control, if we have a vacant spectrum holes in the PU band, we set the outage probability $P_{out} = 1$ to exploit the available spectrum band by SUs, and if we have occupied sub-bands, the outage probability is set to $P_{out} = q$ depending on the PU's QoS.

IV. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

In the proposed game, each SU chooses an appropriate power to maximize its *utility function*. In this context, it is important to ensure the stability of the system. A concept which relates to this issue is the Nash equilibrium. As definition in [4], a pure strategy profile $\{p_l^*\}_{l \neq m, l \in \{1, \dots, M\}}$ is a

Nash equilibrium of the proposed game if, for every player m (i.e. SU m):

$$U_m(p_m^*, \mathbf{P}_{-m}^*) \geq U_m(p_m, \mathbf{P}_{-m}^*), \quad \forall m \in \{1, \dots, M\} \quad (25)$$

A Nash equilibrium can be regarded as a stable solution, at which none of the users has the incentive to change its power p_m .

A. Existence of the Nash Equilibrium

Theorem 1: Game G admits at least one Nash equilibrium.

proof: The conditions for the existence of Nash equilibrium in a strategic game are given in [12]:

- 1) The set P_m is a nonempty, convex, and compact subset of some Euclidean space for all m .
- 2) The utility function $U_m(p_m, \mathbf{P}_{-m})$ is continuous on P and quasi-concave on P_m .

According to the above description of the strategy space, it is straightforward to see that P_m is nonempty, convex and compact. Notice that $U_m(p_m, \mathbf{P}_{-m})$ is a linear function of either p_m , which means the second condition is satisfied. Hence, game G admits at least one Nash equilibrium.

B. Uniqueness of the Nash Equilibrium

Theorem 2: Game G always possesses a unique Nash equilibrium under the sufficient conditions.

proof: It's established in [13] that if the utility function $U_m(p_m) : (p_m)_{m \in \{1, \dots, M\}}$ is a standard function, then the Nash equilibrium in this game will be unique. A function $f(x)$ is said to be a standard function if it satisfies the following three properties [13]:

- 1) Positivity: $f(x) > 0$.
- 2) Monotonicity: If $x \geq x'$, then $f(x) \geq f(x')$.
- 3) Scalability: For all $\mu > 1$, $\mu f(x) \geq f(\mu x)$.

The positivity is obviously satisfied by adjusting parameter a_m .

Considering $p_m \geq p'_m$, we have

$$C_m(p_m) \geq C_m(p'_m) \quad (26)$$

Using the propriety that $a_m < 0$, we can obtain that

$$\left(\frac{p_m |h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \leq \left(\frac{p'_m |h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \quad (27)$$

According to (26) and (27), the monotonicity property is proved $\forall m \in \{1, \dots, M\}$.

For all $\mu > 1$, it's got that:

$$\begin{aligned} \mu C_m(p_m) &= \mu \log_2(1 + \text{SINR}_m) \\ &= \log_2(1 + \text{SINR}_m)^\mu \\ &\geq \log_2(1 + \mu \text{SINR}_m) = C_m(\mu p_m) \end{aligned} \quad (28)$$

Since $a_m < 0$, we have also:

$$\begin{aligned} \left(\frac{\mu p_m |h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} &= \mu^{a_m} \left(\frac{p_m |h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \\ &\leq \mu \left(\frac{p_m |h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^M p_l |h_{l,m}|^2} \right)^{a_m} \end{aligned} \quad (29)$$

Finally, according to (28) and (29) the scalability property is proved. Therefore, the proposed game G always possesses a unique Nash equilibrium.

V. NUMERICAL RESULTS

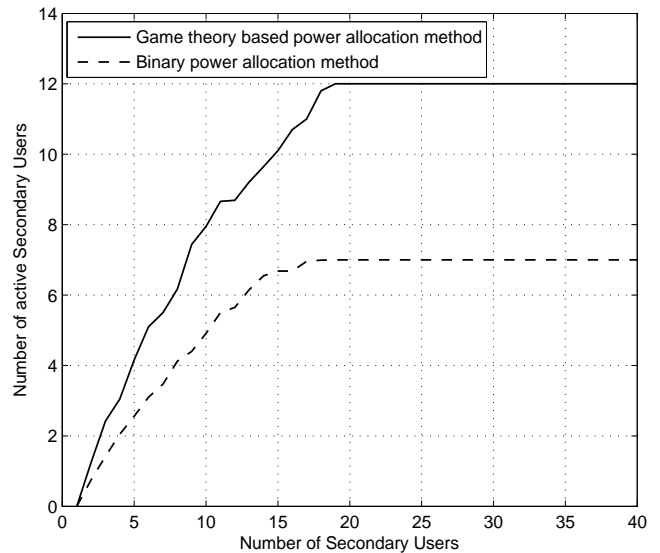


Fig. 2. Number of active SUs vs. number of SUs at rate = 0.3 bits/s/Hz, a tradeoff variable $a_m = -0.3$ and an outage probability = 1% in the uplink (the uplink distributed binary power allocation method and the proposed method).

To go further with the analysis, we resort to realistic network simulations. Specifically, we consider a CRN with one PU and M SUs attempting to communicate during a transmission, subject to mutual interference. A hexagonal cellular system functioning at 1.8 GHz with a primary cell of radius $R = 1000$ meters and a primary protection area of radius $R_p = 600$ meters is considered. Secondary transmitters may communicate with their respective receivers of distances $d < R_p$ from the BS. Channel gains are based on the COST-231 path loss model including log-normal shadowing with standard deviation of 10 dB, plus fast-fading assumed to be i.i.d. circularly symmetric with distribution $\mathcal{CN}(0, 1)$ [14].

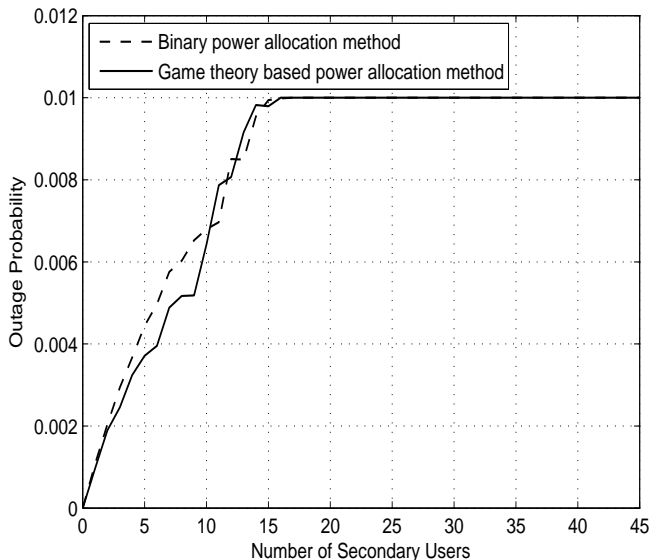


Fig. 3. The uplink outage probability as function of the number of SUs for a target outage probability = 1%, a tradeoff variable $a_m = -0.3$ and a rate = 0.3 bits/s/Hz (the uplink distributed binary power allocation method and the proposed method).

In Fig. 2, the number of active SU links under the proposed algorithm as a function of the total number of users, for a target outage probability = 1%, tradeoff variable $a_m = -0.3$ and a rate = 0.3, is depicted. It can be seen from the figure that increasing the number of SUs yields improvements in the number of active users. Asymptotically, i.e., as the number of SUs goes large, the number of active SUs keeps constant due to the influence of interference impairments on the PU's QoS. We also compare the results obtained by the proposed method to those obtained using the distributed binary power allocation [7]. It can be observed that the proposed scheme allows almost 5 additional active SUs more than the binary power allocation scheme. As an example, we get 12 and 7 active SUs for 25 potential SUs for the proposed method and the one presented in [7], respectively.

In order to validate our theoretical derivation, we also compare the outage probability defined in (15) for both the proposed method and the distributed binary power allocation method. As an example we carry out simulations at PU rate = 0.3 bits/s/Hz. First, it is clear from Fig. 3 that the outage probability using both schemes are similar. We also remark that, for the outage probability of interest, the number of allowed SUs to transmit is equal to 18 SUs. Now, how about the Nash equilibrium?

In general, a Nash equilibrium is a profile of strategies such that each player's strategy is a best response to the other players' strategy. Thus, no player (i.e. SU) has the incentive to leave the Nash equilibrium, as a deviating action would imply a reduction of its own *utility function*. Therefore, the Nash equilibrium is a value for the game's stability. Hence, it can be seen as a lower limit for the QoS that can be guaranteed. As depicted in Fig. 3, depending on QoS to the PU, a unique Nash equilibrium is found. This is shown in the saturation state.

VI. CONCLUSION

In this paper, we explored the idea of combining game theory with resource allocation in CRN to maximize the SU capacity while maintaining a QoS to the PU. Our contribution within this paper is to define a *utility/pricing* strategy that meets the objective to maximize the SUs capacity, and the protection for PUs by means of outage probability. Indeed, we discussed the existence of the Nash equilibrium of the proposed game, as well as its uniqueness. We demonstrated that the proposed game admits one and only one Nash equilibrium. Simulation results show that the proposed method exhibits a significant number of cognitive users able to transmit while minimizing interference to guarantee a QoS for the PU. We also compare the results obtained by the proposed method to those obtained using a binary power allocation method. The reported results demonstrate the efficiency of the proposed technique to maximize the SU rate while maintaining a QoS to PUs.

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