ON THE USAGE OF ANTENNAS IN MIMO AND MISO INTERFERENCE CHANNELS

Mariam Kaynia, Andrea J. Goldsmith†, David Gesbert‡, and Geir E. Øien**

*Dept. of Electronics and Telecommunications, Norwegian Univ. of Science and Technology
†Dept. of Electrical Engineering, Stanford University, Stanford
‡Mobile Communications Department, Eurecom Institute

ABSTRACT

We investigate the performance of an uncoordinated 2 link MIMO interference channel as a function of the number of antennas and the average channel gains. The channel coefficients are random and uncorrelated, all links undergo Rayleigh fading, and the transmitters have no channel state information. The analysis is done in terms of ergodic capacity and outage probability. For non-asymptotic networks (i.e., when the number of transmit and receive antennas is finite and small), we derive upper and lower bounds to both our performance metrics. Moreover, the particular case of a MISO channel is considered, where exact expressions for the ergodic capacity, distribution of the capacity, and outage probability are derived. It is concluded theoretically, and somewhat surprisingly, that using all transmit antennas is not always optimal. That is, depending on the average channel gains and the requested communication rate, a transmitter should apply its antennas in different ways in order to minimize the outage probability.

1. INTRODUCTION

Multiple input multiple output (MIMO) systems have assumed great popularity because of their ability to reach remarkably higher transmission rates and better signal qualities compared to single input single output (SISO) systems. While isolated MIMO systems have been evaluated extensively in the literature, MIMO point-to-point *interference channels (ICs)* have come in focus only in recent years. An IC is a model for studying networks with two or more source-destination pairs and where the source signals interfere with each other at the receivers. One of the main qualities of the IC is the fact that a change in some system parameters not only affects the performance of the link under observation, but also the impact of this link on the rest of the network. This complicates the prediction of how nodes in an uncoordinated network will perform and thus the performance of a link. However, understanding the behavior of ICs is of great importance in today's communication networks, as there is an increasing demand for allowing simultaneous transmissions between independent transmitter (TX) - receiver (RX) pairs.

Extensive work has been done on characterizing the degrees of freedom in MIMO ICs when some coordination is allowed between the transmitters [1], and understanding the impact of interference in MIMO ad hoc and cellular networks using simulators and test beds [2, 3]. In [4], the distribution of the capacity of a MISO broadcast channel with a random beamformer is derived. However, the impact of interference between users is ignored, as the broadcast channel considers only a single TX. In point-to-point MIMO ICs, analytical expressions are derived in [5] for the *asymptotic* ergodic capacity, i.e., when the number of TX and RX antennas go to infinity. For the non-asymptotic case, the performance of MIMO systems with interference has been evaluated in [6], however only for the case when the TXs have full channel state information (CSI).

Obtaining exact or even partial CSI at the TX is not always feasible, due to delay constraints or hardware limitations. Only a few works have studied the capacity of MIMO systems in the presence of co-channel interference when the TXs have no CSI. Most of the works considering ICs rely on simulation results or approximations. In particular, in [7], a cellular system is simulated utilizing 3×3 MIMO transmission techniques. The simulation results confirm that co-channel interference can profoundly degrade the capacity of MIMO links in cellular networks. Also in [8] and [9], the mutual information of MIMO systems in the presence of co-channel interference is evaluated. It is concluded that for certain signal-to-noise ratio (SNR) and signal-to-interference ratio (SIR), the use of multiple antennas in fact degrades the performance of the system compared to SISO networks. These results are constrained to 2 TX and 2 RX antennas, and the observations are made through simulations only. Also in [10] it is concluded that using fewer antennas can in some scenarios improve the system performance. In the very recent work of [11], Chiani *et. al.* evaluate the effect of interference on the capacity of MIMO ICs. Closed-form expressions are derived for the ergodic capacity of both single-user MIMO systems, and MIMO systems with multiple MIMO interferers. Our work serves as a parallel to this work, where we extend the research domain to also consider outage probability (OP), and evaluate the particular case of multiple input single output (MISO) channels, where interesting behavior is observed as the system parameters vary.

Despite the recent interest in applying MIMO in cellular and ad hoc wireless networks, the performance of such networks is not yet fully understood. The goal of this work is hence to establish an understanding of the ergodic capacity and OP performance of MIMO ICs with a finite number of antennas. Ergodic capacity is the average capacity for each link, whereas outage is defined as the event when the instantaneous capacity falls below the transmission rate required for correct packet reception. We consider a 2 link MIMO IC, but our results can be easily extnded to K links, as noted in Section 4.3. The key contributions of this paper are as follows:

- Upper and lower bounds to the ergodic capacity and OP of 2 link MIMO ICs with arbitrary number of TX and RX antennas are evaluated.
- The probability density function of the capacity of MISO ICs with arbitrary number of TX antennas are derived.
- Exact expressions for the ergodic capacity and OP of MISO ICs with arbitrary number of TX antennas are developed.
- Analysis is performed on the different behaviors observed in the OP of MISO networks as the system parameters vary. TDMA is introduced to improve the system performance.

2. SYSTEM MODEL

Consider a wireless network with two transmitting nodes, TX_1 and TX_2 (these could be base stations), each communicating with its own dedicated RX, RX_1 and RX_2 (e.g., mobile stations). Denote the number of TX and RX antennas at each of the TXs and RXs by N_t and N_r, respectively. The channel response between each TX and its RX is specified with an $N_r \times N_t$ random matrix $\mathbf{G}_i$, whose random process is presumed to be zero-mean and ergodic. The elements of $\mathbf{G}_i$ are independent and identically distributed complex Gaussian random variables, each with zero mean and constant variance g_i. Moreover, we define a normalized channel matrix $\mathbf{H}_i$ with unit-variance entries such that $\mathbf{G}_i = \sqrt{g_i}\mathbf{H}_i$. Similarly, the channel between a RX i and its interfering TX j is denoted by $\mathbf{G}_{ij} = \sqrt{g_{ij}}\mathbf{H}_{ij}$.

At the TXs, the angular spread tends to be small, and the antennas are assumed to be decorrelated. The transmitted signals are assumed to be independent and equi-powered[1] at the TX antennas. TX_1 transmits with power P_1 and TX_2 with P_2. Perfect channel estimation is assumed at the RXs, while the TXs have no CSI. The channel entails additive white Gaussian noise (AWGN), and the interference from the other link in the network adds to the impairment of the received signal. Denoting the data to be transmitted by TX_1 by $\mathbf{x}_1$, the data of TX_2 by $\mathbf{x}_2$, and the noise signal by $\mathbf{n}$, we have that the signal at RX_1 is:

$$\mathbf{y}_1 = \sqrt{g_1}\mathbf{H}_1\mathbf{x}_1 + \sqrt{g_{12}}\mathbf{H}_{12}\mathbf{x}_2 + \mathbf{n}. \quad (1)$$

In the following, we let TX_1-RX_1 be the link under observation, and we refer to g_1 and $\mathbf{H}_1$ by g and $\mathbf{H}$, respectively.

With a sufficiently long coding horizon, we can code over the short-term channel fluctuations. Assuming single user detection at the RX, the ergodic capacity becomes

$$C_{erg} = \mathbb{E}\left\{\log_2 \det\left(\mathbf{I}_{N_r} + \frac{P_1 g}{N_t}\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1}\right)\right\}, \quad (2)$$

where $\mathbf{Q}$ is the covariance of the channel impairment, consisting of noise (with power σ^2) plus interference;

$$\mathbf{Q} = \frac{P_2 g_{12}}{N_t}\mathbf{H}_{12}\mathbf{H}_{12}^\dagger + \sigma^2\mathbf{I}_{Nr}. \quad (3)$$

Outage occurs when the signal-to-interference-plus-noise ratio (SINR) falls below a required threshold, or equivalently when the rate required for correct reception is higher than the capacity of the network. Thus, denoting the required rate at each RX antenna by R, the OP is defined as:

$$P_{out} = \Pr\left\{\log_2 \det\left[\mathbf{I}_{N_r} + \frac{P_1 g}{N_t}\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1}\right] < N_r R\right\}, \quad (4)$$

where the sources of randomness in the probability expression are the matrices $\mathbf{H}$ and $\mathbf{Q}$.

3. MIMO INTERFERENCE CHANNELS

In this section, we evaluate the performance of the MIMO IC described above in terms of ergodic capacity and OP. In order to derive the OP, we need to know the distribution of the capacity. When there are no interferers in the channel (equivalent to when $g_{12} = 0$, resulting in $\mathbf{Q} = \sigma^2\mathbf{I}_{N_r}$), the distribution of the capacity is known [12]. However, with the addition of the interference term (i.e., as g_{12} increases), these distributions are no longer valid. By decomposing the capacity formula into terms with known distributions (e.g., the determinant or trace of a Wishart matrix), we can derive bounds or approximate expressions to the OP.

[1] This maximizes the mutual information when the TX has no CSI.

3.1. Trace Bound

In this section, we derive an upper bound to the ergodic capacity, and equivalently a lower bound to the OP, by using the trace of the SINR matrix. For this, we apply the arithmetic-geometric inequality

$$\left(\prod_{i=1}^{\min(N_t,N_r)} x_i\right)^{1/\min(N_t,N_r)} \leq \frac{1}{\min(N_t,N_r)}\sum_{i=1}^{\min(N_t,N_r)} x_i. \quad (5)$$

Furthermore, we know that the trace of a square matrix is equal to the sum of its distinct eigenvalues. Denoting the eigenvalues of $\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1}$ by λ_i, we have that $\sum_{i=1}^{\min(N_t,N_r)}\lambda_i = \mathrm{tr}(\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1})$. Let $x_i = (1 + \frac{P_1 g}{N_t}\lambda_i)$ in Eq. (5). The ergodic capacity can then be upper bounded as follows:

$$\begin{aligned} C_{erg} &= \mathbb{E}\left[\log_2 \prod_{i=1}^{\min(N_t,N_r)}\left(1 + \frac{P_1 g}{N_t}\lambda_i\right)\right] \quad (6) \\ &\leq \mathbb{E}\left[\log_2\left(\frac{1}{\min(N_t,N_r)}\sum_{i=1}^{\min(N_t,N_r)} 1 + \frac{P_1 g}{N_t}\lambda_i\right)^{\min(N_t,N_r)}\right] \\ &= \min(N_t,N_r)\mathbb{E}\left[\log_2\left(1 + \frac{P_1 g}{N_t\min(N_t,N_r)}\mathrm{tr}(\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1})\right)\right]. \end{aligned}$$

Consequently, given the required rate for correct reception of packets per RX antenna is R, the OP is lower bounded by

$$P_{out} \leq \Pr\left[\mathrm{tr}(\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1}) < \frac{N_t\min(N_t,N_r)}{P_1 g}\left(2^{\frac{N_r R}{\min(N_t,N_r)}} - 1\right)\right].$$

The distribution of $\mathrm{tr}(\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1})$ is unknown, meaning that for our simulations in Section 3.3, we rely on Monte-Carlo simulations.

Note that when there is no interference, i.e., $g_{12} = 0$, the trace bound yields a decent upper bound to the instantaneous capacity. Knowing the distribution of the trace of a Wishart matrix [12], a lower bound to the OP in the absence of interference is derived to be

$$P_{out}^{lb} = 1 - \frac{1}{(N_t N_r - 1)!}\Gamma\left[N_t N_r, l\right] = e^{-l}\cdot\sum_{k=0}^{N_t N_r - 1}\frac{l^k}{k!}, \quad (7)$$

where $l = \frac{\min(N_t,N_r)\eta}{P_1 g/N_t}\left(2^{\frac{N_r R}{\min(N_t,N_r)}} - 1\right)$. As the interference increases, this lower bound loses its tightness and is no longer valid. Hence, we now take a step further to consider the determinant bound to the ergodic capacity in Section 3.2.

3.2. Determinant Bound

Knowing that the determinant of a square matrix (here: $\mathbf{H}\mathbf{H}^\dagger\mathbf{Q}^{-1}$) is equal to the product of its non-zero eigenvalues λ_i, we have that the instantaneous capacity is:

$$C_i = \frac{1}{N_r}\log_2\prod_{i=1}^{\min(N_t,N_r)}\left(1 + \frac{P_1 g}{N_t}\lambda_i\right). \quad (8)$$

Assuming w.l.o.g. that $N_r \leq N_t$, and applying the inequality [12]

$$\prod_{i=1}^{N_r}(1 + x_i) \geq \left[1 + \left(\prod_{i=1}^{N_r} x_i\right)^{1/N_r}\right]^{N_r} \quad \forall\, x_i > 0, \quad (9)$$

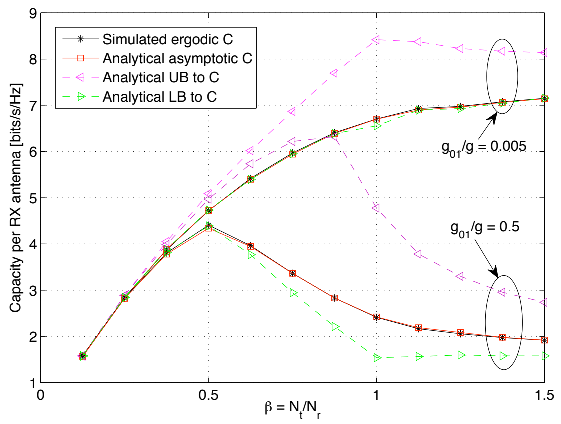

Fig. 1. Simulated ergodic capacity, along with derived lower and upper bounds, of a 2 link MIMO IC as a function of $\beta = N_t/N_r$, for both strong and weak interference.

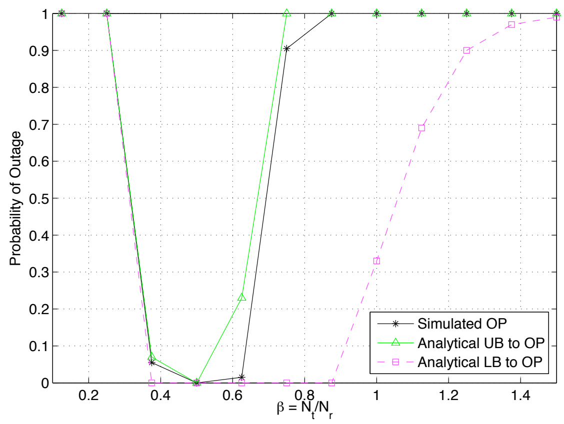

Fig. 2. Simulated ergodic OP, along with derived lower and upper bounds, of a 2 link MIMO IC when the interference is strong ($g_{01}/g = 0.5$) as a function of $\beta = N_t/N_r$.

we derive a lower bound on the instantaneous capacity:

$$\begin{aligned} C_i &\geq \log_2 \left\{ 1 + \frac{P_1 g}{N_t} \left(\prod_{i=1}^{N_r} \lambda_i \right)^{1/N_r} \right\}^{N_r} \\ &= N_r \log_2 \left\{ 1 + \frac{P_1 g}{N_t} \left(\det(\mathbf{H}\mathbf{H}^\dagger \mathbf{Q}^{-1}) \right)^{1/N_r} \right\} \\ &= N_r \log_2 \left\{ 1 + \frac{P_1 g}{N_t} \left(\frac{\det(\mathbf{H}\mathbf{H}^\dagger)}{\det(\mathbf{Q})} \right)^{1/N_r} \right\}. \qquad (10) \end{aligned}$$

The distribution $f_X(x)$ of the determinant of a Wishart matrix, $X = \det(\mathbf{H}\mathbf{H}^\dagger)$, is derived in [12] and [14]. Based on this, the distribution of $Y = \det(\mathbf{Q})$, denoted as $f_Y(y)$, can also be easily derived. Hence, the ergodic capacity is given as

$$C_{erg} = \int_0^\infty \int_0^\infty N_r \log_2 \left[1 + \frac{P_1 g}{N_t} \left(\frac{x}{y} \right)^{\frac{1}{N_r}} \right] f_X(x) f_Y(y)\, dx dy.$$

Furthermore, the OP of the MIMO IC may be upper bounded by

$$\begin{aligned} P_{out}^{ub} &\leq \Pr\left[\frac{\det(\mathbf{H}\mathbf{H}^\dagger)}{\det(\mathbf{Q})} < \left(\frac{N_t}{P_1 g} (2^R - 1) \right)^{N_r} \right] \\ &= \int_0^\infty \Pr\left[X < \left(\frac{N_t}{P_1 g} (2^R - 1) \right)^{N_r} Y | Y = y \right] \cdot f_Y(y) dy. \\ &= \int_0^\infty 1 - \sum_{n=0}^{|N_t - N_r|} \left(\prod_{n=1}^{N_t - 1} \frac{(N_r - n - k)!}{(N_r - n)!} \right) \\ &\quad \cdot \frac{(sy)^{kN_t}}{k!} e^{-\frac{(N_r - N_t - k)!}{(N_r - 1 - k)!} (sy)^{N_t}} dy, \qquad (11) \end{aligned}$$

where $s = \frac{N_t}{P_1 g} (2^{N_r R / \min(N_t, N_r)} - 1)$.

3.3. Numerical Results for MIMO Channels

In Fig. 1, the lower and upper bounds to the ergodic capacity of the MIMO ICs are plotted along with Monte Carlo simulation results, as a function of $\beta = N_t/N_r$, for $N_r = 8$. As can be seen from the figure, the determinant bound is a reasonable lower bound, although it loses its tightness as g_{12}/g increases. The trace bound, on the other hand, fails to give a decent upper bound, especially for higher values of g_{12}/g.

Futhermore, we observe that for $\beta \leq 1/2$, the capacity increases with β, because the total number of TX antennas in the network $(2 \cdot N_t)$ does not exceed the number of RX antennas, N_r, and so the RX approaches capacity by completely suppressing the interfering signals while simultaneously detecting the desired signals. For $1/2 < \beta \leq 1$, the number of RX antennas is less than the combined number of desired and interfering antennas, and the RX must hence compromise between assigning its degrees of freedom to interference suppression and to signal detection. This means that $\beta^{opt} = 1/2$. For $\beta > 1$, the number of interfering antennas exceeds N_r, meaning that the RX can no longer suppress the totality of the interference, and the capacity is thus determined mainly by the SIR.

In Fig. 2, the OP of the MIMO IC is plotted as a function of $\beta = N_t/N_r$, when the interference is strong. As expected, the OP is minimized when $N_t = N_r/2$. The determinant upper bound works reasonably, while the trace lower bound is clearly too loose to give a correct picture of the network performance. This holds true also for other values of g_{12}/g. When the interference is weak, the OP decreases abruptly in the same manner as the left part of the curve in Fig. 2, and it continues reducing monotonically with β. In this case, the determinant bound is extremely tight around the simulation results, while the trace bound still fails to give a decent representation of the actual OP.

4. MISO INTERFERENCE CHANNELS

In this section, we consider the particular case of the MISO channel by assigning only a single antenna to the RX (i.e., $N_r = 1$) in our network, while the TX still has $N_t < \infty$ antennas. The channel matrices $\mathbf{H}$ and $\mathbf{H}_{12}$ reduce to channel vectors $\mathbf{h}$ and $\mathbf{h}_{12}$. The instantaneous capacity of this MISO channel is then

$$C_i = \log_2 \left[1 + \frac{P_1 g}{N_t} \frac{\mathbf{h}\mathbf{h}^\dagger}{\sigma^2 + \frac{P_2 g_{12}}{N_t} \mathbf{h}_{12} \mathbf{h}_{12}^\dagger} \right]. \qquad (12)$$

In the following, we derive C_{erg} and P_{out} of the 2 link MISO IC.

4.1. Ergodic Capacity

To find C_{erg}, we first derive the distribution of the capacity. We consider an interference-limited network, and thus use that $\sigma^2 \approx 0$. Since the elements of $\mathbf{h}$ and $\mathbf{h}_{12}$ are complex Gaussian random variables, it follows that $|h_{ij}|$ is Rayleigh distributed. Thus, we have that $|\mathbf{h}|^2 = \sum_{i,j}^{N_t} |h_{ij}|^2$ is χ_k^2-distributed with $k = 2N_t$ degrees of freedom. Denote $X = |\mathbf{h}|^2$ and $Y = |\mathbf{h}_{12}|^2$, yielding $C_i = \log_2\left(1 + \frac{P_1 g X}{P_2 g_{12} Y}\right)$. Using that $\frac{dX}{dC} = \frac{P_2 g_{12}}{P_1 g} Y 2^C \ln(2)$, we have

$$
\begin{aligned}
f_C(c) &= \int_0^\infty f_{C|Y}(c|y) f_Y(y) dy = \int_0^\infty f_X(x) f_Y(y) \frac{dX}{dC} dy. \\
&= \frac{P_2 g_{12}}{P_1 g} \ln(2) 2^c \int_0^\infty y \frac{y^{N_t-1} e^{-y/2}}{2^{N_t}(N_t-1)!} \frac{x^{N_t-1} e^{-x/2}}{2^{N_t}(N_t-1)!} dy \\
&= k_c \int_0^\infty y^{2N_t-1} e^{-\frac{1+(2^c)P_2 g_{12}/P_1 g}{2}} dy, \qquad (13)
\end{aligned}
$$

where $k_c = \left(\frac{P_2 g_{12}}{P_1 g}\right)^{N_t} \frac{\ln(2) 2^c (2^c-1)^{N_t-1}}{\left(2^{N_t}(N_t-1)!\right)^2}$. Solving the integral yields

$$
\begin{aligned}
f_C(c) &= \left(\frac{P_2 g_{12}}{P_1 g}\right)^{N_t} \ln(2) 2^c (2^c-1)^{N_t-1} \frac{(2N_t-1)!}{(N_t-1)!^2} \\
&\quad \left(\frac{1}{1+(2^c-1)P_2 g_{12}/P_1 g}\right)^{2N_t}. \qquad (14)
\end{aligned}
$$

Having the distribution of the capacity of a MISO channel, we can easily derive the ergodic capacity as $C_{erg} = \int_0^\infty c\ f_C(c)\ dc$. The closed form expression for the ergodic capacity is rather complicated, and so, due to space constraint, we will not state this here.

4.2. Outage Probability

The OP of the 2 link MISO IC can be expressed as

$$
P_{out} = \int_0^\infty \Pr\left[|\mathbf{h}|^2 < \frac{P_2 g_{12}}{P_1 g}(2^R-1) Y | Y = y\right] f_Y(y) dy.
$$

Let $s = \frac{P_2 g_{12}}{P_1 g}(2^R - 1)$. The OP is then derived as

$$
P_{out} = \int_0^\infty \int_0^{sy/2} x^{N_t-1} e^{-x} dx \ \frac{y^{N_t-1} e^{-y/2}}{2^{N_t}\Gamma(N_t)^2}\ dy. \qquad (15)
$$

Expanding the inner integral of Eq. (15) into a series yields

$$
P_{out} = \int_0^\infty -\frac{e^{-y(1+s)/2}}{2^{N_t}(N_t-1)!} \sum_{k=0}^{N_t-1} \frac{y^{2N_t-k-2}}{(N_t-k-1)!} \left(\frac{s}{2}\right)^{N_t-k-1} dy.
$$

where we have used the result: $\int_0^{sy/2} x^{N_t} e^{-x} dx = -e^{-\frac{sy}{2}} \left[\left(\frac{sy}{2}\right)^{N_t} + \sum_{k=1}^{N_t} N_t(N_t-1)...(N_t-k+1)\left(\frac{sy}{2}\right)^{N_t-k}\right]$.

4.3. Numerical Results for MISO Channels

Fig. 3 shows the OP of the 2 link MISO channel. The analytical results follow the simulations tightly, confirming our derived expressions. As the figure illustrates, when $s = \frac{P_2 g_{12}}{P_1 g}(2^R - 1) < 1$, the OP decreases with N_t. When $s > 1$, however, we observe an opposite behavior, i.e., the OP increases as a function of N_t. This is because the destruction from the reduction in randomness in the interference from TX_2 is greater than the diversity improvement that a higher N_t provides in the desired signal. This is discussed further in Section 5. Note that our results can be easily extended to K links, by applying that the sum of K $\chi_{2N_t}^2$ random variables is $\chi_{2KN_t}^2$ distributed.

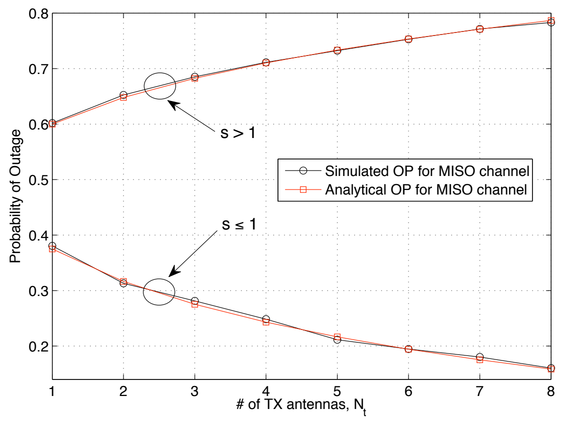

Fig. 3. OP of a 2 link MISO IC as a function of N_t, both for $s = \frac{P_2 g_{12}}{P_1 g}(2^R - 1) \leq 1$ and $s > 1$.

5. OPTIMAL USE OF ANTENNAS IN MISO CHANNELS

In Section 4.3, we observed different behaviors in the OP as a function of N_t depending on the parameter values. This gives rise to the following theorem.

Theorem: In an uncoordinated 2 link MISO system with Gaussian random uncorrelated channels,

- if $\frac{P_2 g_{12}}{P_1 g}(2^R - 1) \leq 1$, the maximum number of TX antennas available should be applied, i.e., $N_t^{opt} = N_t^{max}$.
- if $\frac{P_2 g_{12}}{P_1 g}(2^R - 1) > 1$, we have that $N_t^{opt} = 1$.

Proof: To find out for what ranges of the system parameters the OP increases or decreases as a function of N_t, we evaluate the following discrete differential

$$
\begin{aligned}
P_{out}(N_t+1) - P_{out}(N_t) &= \Delta P_{out} \qquad (16) \\
= -\frac{1}{2^{N_t}(N_t-1)!^2} \int_0^\infty & \frac{N_t!}{2N_t^2} \sum_{k=0}^{N_t} \left(\frac{s}{2}\right)^{N_t-k} x^{N_t-k} \frac{e^{-x(1+s)/2}}{(N_t-k)!} \\
-(N_t-1)! \sum_{k=0}^{N_t-1} & \left(\frac{s}{2}\right)^{N_t-k-1} x^{2N_t-k-2} \frac{e^{-x(1+s)/2}}{(N_t-k-1)!} dy,
\end{aligned}
$$

where $s = \frac{P_2 g_{12}}{P_1 g}(2^R - 1)$. Moreover, we have that $\int_0^\infty y^\nu e^{-\mu y} dy = \frac{1}{\mu^{\nu+1}\Gamma(\nu+1)}$. Inserting this into Eq. (16), setting the difference equal to 0, and canceling out the constant terms, results in

$$
\begin{aligned}
\frac{1}{N_t s} \sum_{k=0}^{N_t} \left(\frac{s}{1+s}\right)^{-k+1} & \frac{(2N_t-k)!}{(N_t-k)!} \\
&= \sum_{k=0}^{N_t-1} \left(\frac{s}{1+s}\right)^{-k-1} \frac{(2N_t-k-2)!}{(N_t-k-1)!}. \qquad (17)
\end{aligned}
$$

Now, this expression must remain valid for all N_t. Setting $N_t = 1$ and rearranging yields $s = 0$ or $s = 1$. Since $s = 0$ would mean no signals are being received, the result is that $\Delta P_{out} = 0$ for $s = 1$. That is, when $s = \frac{P_2 g_{12}}{P_1 g}(2^R - 1) \leq 1$, the OP decreases as a function of N_t, and when $s > 1$, it increases with N_t. ■

To better understand why this is the case, we note that the increase in N_t decreases the randomness in the received signal. When $N_t \to \infty$, the expected capacity per link is: $C_\infty = \log_2\left(1 + \frac{P_1 g}{P_2 g_{12}}\right)$.

For $N_t < \infty$, the capacity has a distribution around the mean C_∞. As N_t increases, the standard deviation shrinks, making the probability distribution more concentrated around C_∞. Hence, depending on whether the requested transmission rate R is greater or smaller than C_∞, the OP increases or decreases (respectively) with N_t.

The conclusion of $N_t^{opt} = 1$ when $s > 1$ indicates an inefficient use of the TX antennas. Hence, we must employ additional techniques in order to decrease the OP as a function of N_t. Firstly, note that when the interference is strong, i.e., $g_{12} > g$, traditional interference cancelation techniques can be applied, making the network interference-free. This leaves us with the region when $\frac{P_1 g}{P_2(2^R-1)} < g_{12} \leq g$. Since the OP increases as a function of N_t, it means that decreasing randomness when $s > 1$ is in fact hurting the network performance. Hence, by introducing a random beamformer at the TXs, we can improve the performance, as shown in Fig. 4. However, although at lower rate, the OP still increases with N_t, and our conclusion on $N_t^{opt} = 1$ remains intact. Also randomly choosing one single TX antenna does not add to the randomness and the OP does not reduce beyond the SISO channel OP.

Realizing that we cannot outperform the SISO channel by adding randomness, we introduce TDMA into the system; Each time frame is divided in two, and each link transmits over *one* of these subframes, thus precluding interference between the links. The OP (in the presence of noise) then becomes

$$P_{out} = -\frac{e^{-\frac{s_{tdma}}{2}}}{(N_t-1)!}\sum_{k=0}^{N_t-1}\frac{1}{(N_t-k-1)!}\left(\frac{s_{tdma}}{2}\right)^{N_t-k-1},$$

where $s_{tdma} = \frac{\sigma^2 N_t}{P_1 g}(2^{2R}-1)$.

TDMA improves the performance of our MISO network considerably, as seen in Fig. 4. This is because the channel of each link becomes interference-free and the decrease in randomness in the desired signal power yields a better performance. Comparing the OP expressions of the MISO channel with and without TDMA, we obtain that the addition of TDMA is advantageous when $\frac{2^{2R}-1}{2^R-1}\frac{\sigma^2}{P_2 g_{12}} < \sum_{j=1}^{N_t}|h_{1j}|^2$ holds true. As N_t increases, TDMA becomes more beneficial on average, as is also seen from the figure. TDMA can also be applied for the case when $g_{12} > g$, if the system has no interference cancelation abilities. Note that we obtain such great improvement with TDMA simply because we only have two links in the system. If the number of links increases, dividing the time slot between all users could become more hurtful than the amount of improvement it provides in the received signal.

6. CONCLUSIONS AND FUTURE WORK

The performance of the 2 link MIMO and MISO IC has been evaluated in terms of ergodic capacity and outage probability (OP). In the case of MIMO, upper and lower bounds to both metrics are derived. It is seen that the determinant bound provides a decent lower (resp. upper) bound on the capacity (resp. OP), while the trace bound fails to yield a reasonable upper (resp. lower) bound. For the particular case of the MISO IC, exact analytical expressions are derived for both the ergodic capacity and the OP, and we establish how our expressions can be extended to K links. Monte Carlo simulation results are generated to verify the analytical results. Interesting behavior is observed in the OP performance of MISO channels; when $s = \frac{P_2 g_{12}}{P_1 g}(2^R-1) \leq 1$, the OP decreases with the number of transmit antennas, while for $s > 1$, it increases. As a consequence, when $s \leq 1$, the capacity achieving approach is to utilize all antennas available, while for $s > 1$, the SISO channel provides the best performance. In the latter case, we introduce TDMA between the links. This reduces the OP below that of the SISO channel, and again using the maximum number of transmit antennas available is optimal.

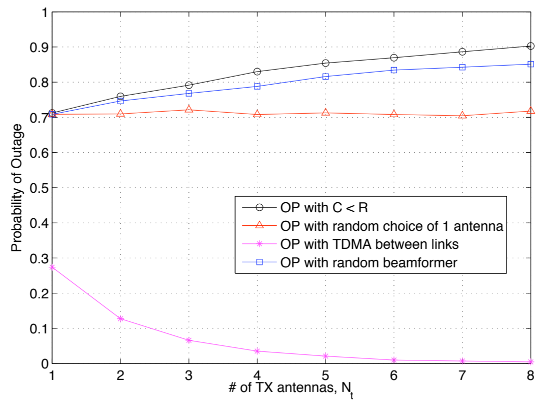

Fig. 4. OP of a 2 link MISO IC when $s > 1$, with various techniques applied. TDMA between the links yields the lowest OP.

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