

Blind Sensing Techniques based on Kullback-Leibler Distance for Cognitive Radio Systems

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A General Cognitive Radio Network

Such devices must be able to:

- 1 **sense** the spectral environment over a wide bandwidth,
- 2 **detect** the presence/absence of primary users (PUs),
- 3 **adapt** the parameters of their communication scheme only if the communication does not interfere with PUs.

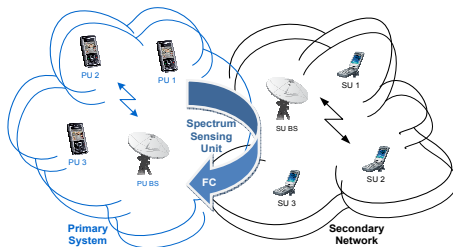


Figure 2: Dynamic spectrum access in cognitive radio network.

Recent trends

In addition to the classical Primary-secondary networks co-existence scenario, there are new applications where cognitive radio approach is not restricted to the classical scheme mentioned above

- 1 Competitive and opportunistic way to access an open band (790 – 862MHz band opened by ARCEP in France, see SACRA project)
- 2 Interference management for femto cells, small cells networks
- 3 ...

Challenges

Some challenges associated with the spectrum sensing for cognitive radio are:

- 1 Sensing time and complexity.
- 2 Blind detection.
- 3 Multi-path, shadowing, interference environment, etc: cooperation.
- 4 Performance in low signal to noise ratios (SNR) region.

Our approach is focused in desining low complexity blind spectrum sensing techniques to fit the requirements of most of the target scenarios in cognitive radio

Spectrum Sensing Goal

The received signal at a sensor node, denoted by \mathbf{x} , can be modeled as

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{n} \quad (1)$$

where \mathbf{A} is the channel matrix whose columns are determined by the unknown parameters associated with each signal. \mathbf{s} is a PU transmitted signal and \mathbf{n} is a complex, stationary, and Gaussian noise with zero mean and covariance matrix $E\{\mathbf{n}\mathbf{n}^H\} = \sigma^2\mathbf{I}$.

The goal of spectrum sensing is to decide between the following two hypotheses:

$$\mathbf{x} = \begin{cases} \mathbf{n} & H_0 \\ \mathbf{A}\mathbf{s} + \mathbf{n} & H_1 \end{cases} \quad (2)$$

Spectrum Sensing Goal

The probability of false alarm can be expressed as:

$$P_{FA} = Pr(H_1 | H_0) = Pr(\mathbf{x} \text{ is present} | H_0) \quad (3)$$

and the probability of detection is

$$P_D = 1 - Pr(H_0 | H_1) = 1 - Pr(\mathbf{x} \text{ is absent} | H_1) \quad (4)$$

The decision threshold is determined by using the required probability of false alarm P_{FA} given by (3). The threshold γ for a given false alarm probability is determined by solving the equation

$$P_{FA} = Pr(\Upsilon(\mathbf{x}) > \gamma | H_0) \quad (5)$$

where $\Upsilon(\mathbf{x})$ denotes the test statistic for the given detector.

Model Selection using Kullback-Leibler distance

- Model selection is based on the comparison of the properties of an analyzed process with a set of candidates models.
- Assuming that the received signal is distributed according to an original probability density function f , called the operating model.
- An approximating probability model must be specified using the observed data, in order to estimate the operating model. The approximating model is denoted as g_θ .
- The Kullback-Leibler distance describes the discrepancy between the two probability functions f and g_θ and is given by:

$$D(f\|g_\theta) = -h(X) - \int f_X(x) \log g_\theta(x) dx \quad (6)$$

where the random variable X is distributed according to the original but unknown probability density function f , and $h(\cdot)$ denotes differential entropy.

Kullback-Leibler distance vs Akaike Information Criterion (AIC)

- By averaging the log-likelihood values given the model over N independent observations $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N$, we obtain

$$-\int f_X(x) \log g_\theta(x) dx \approx -\frac{1}{N} \sum_{n=1}^N \log g_\theta(\mathbf{x}_n) \quad (7)$$

- The AIC criterion is an approximately unbiased estimator for (7) and is given by:

$$\text{AIC} = -2 \sum_{n=1}^N \log g_{\hat{\theta}}(\mathbf{x}_n) + 2U \quad (8)$$

The parameter vector θ for each family should be estimated using the minimum discrepancy estimator $\hat{\theta}$, which minimizes the empirical discrepancy.

Blind Sensing based on Signal Probability Distribution Analysis

- The distribution of a sum of independent random variables is the convolution of their distributions. \Rightarrow When the SNR is low, the noise distribution will dominate and the resulting distribution will tend to become close to Gaussian (the envelop distribution is close to Rayleigh distribution).
- In the presence of a communication signal, due to the contribution of the dominant propagation paths on the distribution of the communication signal, the envelop distribution of the received communication signal tends to become close to Rician distribution.

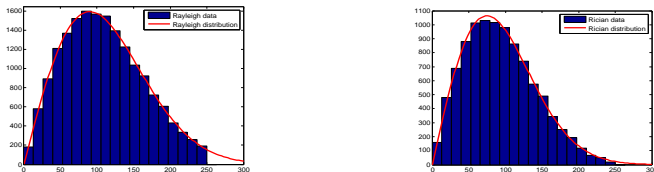


Figure 3: Histogram of the envelope of a captured noise block and data block using an UMTS signal.

Model Selection Using Akaike Weight

- The operating model f will be compared with Rice and Rayleigh probability density functions.
 - The log-likelihood function for the Rayleigh distribution:

$$L_{Rayleigh}^*(\sigma) = \sum_{i=1}^p \log x_i - p \log \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^p x_i^2 \quad (9)$$

where the parameter $\theta = (\sigma)$. The MLE of the parameter σ is given by:

$$\hat{\sigma}^2 = \frac{1}{2p} \sum_{i=1}^p x_i^2 \quad (10)$$

- The log-likelihood function for the Rice distribution:

$$L_{Rice}^*(\nu, \sigma) = \log \left(\frac{\prod_{i=1}^p x_i}{\sigma^{2p}} \exp \left(-\frac{\sum_{i=1}^p (x_i^2 + \nu^2)}{2\sigma^2} \right) \prod_{i=1}^p I_0 \left(\frac{x_i \nu}{\sigma^2} \right) \right) \quad (11)$$

Parameters ν and σ are given by the solution of the following set of equations:

$$\begin{cases} \nu - \frac{1}{p} \sum_{i=1}^p x_i \frac{I_1 \left(\frac{x_i \nu}{\sigma^2} \right)}{I_0 \left(\frac{x_i \nu}{\sigma^2} \right)} = 0 \\ 2\sigma^2 + \nu^2 - \frac{1}{p} \sum_{i=1}^p x_i^2 = 0 \end{cases} \quad (12)$$

Model Selection Using Akaike Weight

Akaike weights can be interpreted as estimate of the probabilities that the corresponding candidate distribution show the best modeling fit:

$$W_{Rice} = \frac{\exp\left(-\frac{1}{2}\Phi_{Rice}\right)}{\exp\left(-\frac{1}{2}\Phi_{Rice}\right) + \exp\left(-\frac{1}{2}\Phi_{Rayleigh}\right)} \quad (13)$$

$$W_{Rayleigh} = \frac{\exp\left(-\frac{1}{2}\Phi_{Rayleigh}\right)}{\exp\left(-\frac{1}{2}\Phi_{Rayleigh}\right) + \exp\left(-\frac{1}{2}\Phi_{Rice}\right)} \quad (14)$$

where

$$\Phi_{Rice} = AIC_{Rice} - \min(AIC_{Rice}, AIC_{Rayleigh}) \quad (15)$$

$$\Phi_{Rayleigh} = AIC_{Rayleigh} - \min(AIC_{Rayleigh}, AIC_{Rice}) \quad (16)$$

and

$$AIC_{Rice} = -2L_{Rice} + 2U_{Rice} \quad (17)$$

$$AIC_{Rayleigh} = -2L_{Rayleigh} + 2U_{Rayleigh} \quad (18)$$

Blind Sensing Based on Signal Space Dimension Estimation

- The dimension of the signal is represented with the rank of a different candidates matrix.
- Considering N observations $\mathbf{x}_n \in \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ received in a sequence, the covariance matrix can be defined as

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{x}_n \mathbf{x}_n^T \quad (19)$$

Let p be the length of one observation and q the length of the transmitted signal \mathbf{s} and the additive noise \mathbf{n} .

- The AIC and minimum description length (MDL) criteria are given by:

$$\text{AIC}(k) = -2 \log \left(\frac{\prod_{i=k+1}^p \hat{\lambda}_i^{\frac{1}{p-k}}}{\frac{1}{p-k} \sum_{i=k+1}^p \hat{\lambda}_i} \right)^{(p-k)N} + 2k(2p - k) \quad (20)$$

$$\text{MDL}(k) = -\log \left(\frac{\prod_{i=k+1}^p \hat{\lambda}_i^{\frac{1}{p-k}}}{\frac{1}{p-k} \sum_{i=k+1}^p \hat{\lambda}_i} \right)^{(p-k)N} + \frac{k}{2}(2p - k) \log N \quad (21)$$

Distribution Analysis Detector (DAD)

- **Sub-bands Detection:** The proposed method is based on the sliding window technique.
- **PU Signal Detection:** The DAD detector can be formulated as a binary hypothesis test.

Theorem 1

The test statistic of the blind DAD algorithm is given by:

$$\Upsilon_{DAD}(\mathbf{x}) = \begin{cases} W_{Rice} - W_{Rayleigh} < \gamma_{DAD} & \text{noise} \\ W_{Rice} - W_{Rayleigh} > \gamma_{DAD} & \text{signal} \end{cases} \quad (22)$$

According to the system requirement on $P_{FA,DAD}$, we calculate a proper threshold γ_{DAD} . If $AIC_{Rice} - AIC_{Rayleigh} > \gamma_{DAD}$, we declare that the PU is present, otherwise, we declare the PU is absent.

DAD False Alarm Probability

The false alarm probability for DAD detector can be expressed as

$$P_{FA,DAD} = \Pr(W_{Rice} - W_{Rayleigh} > \gamma_{DAD} | H_0) \quad (23)$$

Theorem 2

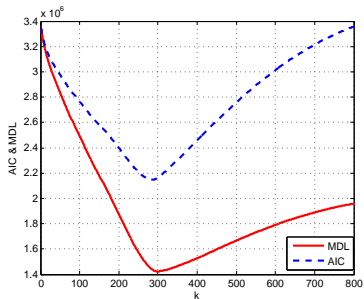
The probability of false alarm of the DAD algorithm can be approximated as

$$P_{FA,DAD} = F \left(\left(\frac{1 + \gamma_{DAD}}{1 - \gamma_{DAD}} \right)^2 (4\pi K)^{-p} \exp(p - 2) \right) \quad (24)$$

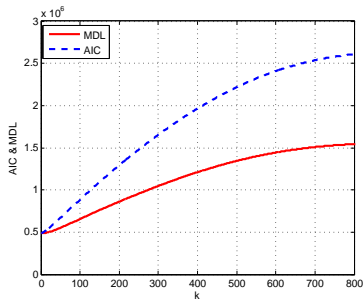
or, alternatively, the threshold can be expressed as

$$\gamma_{DAD} = \frac{\sqrt{(4\pi K)^p F^{-1}(P_{FA,DAD}) \exp(2 - p)} - 1}{\sqrt{(4\pi K)^p F^{-1}(P_{FA,DAD}) \exp(2 - p)} + 1} \quad (25)$$

Model Selection Akaike Information Criteria



(a) Data block



(b) Noise block

Figure 4: Akaike information criterion and minimum description length of captured noise block samples and data block samples using an UMTS signal.

Dimension Estimation Detector (DED)

- **PU Signal Detection:**

Theorem 3

The test statistic of the blind DED algorithm using AIC criteria is given by:

$$\Upsilon_{DED-AIC}(\mathbf{x}) = \begin{cases} AIC(0) - AIC(1) < \gamma_{DED-AIC} & \text{noise} \\ AIC(0) - AIC(1) > \gamma_{DED-AIC} & \text{signal} \end{cases} \quad (26)$$

and using MDL criteria:

$$\Upsilon_{DED-MDL}(\mathbf{x}) = \begin{cases} MDL(0) - MDL(1) < \gamma_{DED-MDL} & \text{noise} \\ MDL(0) - MDL(1) > \gamma_{DED-MDL} & \text{signal} \end{cases} \quad (27)$$

We define the two thresholds $\gamma_{DED-AIC}$ and $\gamma_{DED-MDL}$ in order to decide on the nature of the received signal.

DED-AIC False Alarm Probability

The false alarm probability for DAD detector can be expressed as

$$P_{FA,DED-AIC} \approx Pr\left(AIC(0) - AIC(1) > \gamma_{DED-AIC} | H_0\right) \quad (28)$$

Theorem 4

The probability of false alarm of the DED algorithm using AIC criteria can be approximated as

$$P_{FA,DED-AIC} = F_2\left(\frac{N \exp\left(\frac{2-4p-\gamma_{DED-AIC}}{2N}\right) - \mu}{\nu}\right) \quad (29)$$

and the threshold

$$\gamma_{DED-AIC} = 2 - 4p - 2N \ln\left(\frac{\nu F_2^{-1}(P_{FA,DED-AIC}) + \mu}{N}\right) \quad (30)$$

DED-MDL False Alarm Probability

The false alarm probability for DAD detector can be expressed as

$$P_{FA,DED-MDL} \approx Pr\left(\text{MDL}(0) - \text{MDL}(1) > \gamma_{DED-MDL} | H_0\right) \quad (31)$$

Theorem 5

The probability of false alarm of the DED algorithm using MDL criteria can be approximated as

$$P_{FA,DED-MDL} = F_2\left(\frac{N \exp\left(\frac{\gamma_{DED-MDL} + \left(p - \frac{1}{2}\right) \log N}{N}\right) - \mu}{\nu}\right) \quad (32)$$

and the threshold is given by

$$\gamma_{DED-MDL} = \left(p - \frac{1}{2}\right) \log N - N \ln\left(\frac{\nu F_2^{-1}(P_{FA,DED-MDL}) + \mu}{N}\right) \quad (33)$$

Evaluation and Simulation Framework

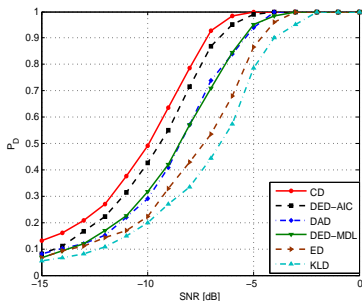
Three different scenarios with different properties have been chosen to evaluate the spectral detection performance using a DVB-T OFDM signal:

- ④ **Scenario 1:** OFDM signal in AWGN channel.
- ② **Scenario 2:** OFDM signal in Rayleigh multipath fading with shadowing.
- ③ **Scenario 3:** OFDM signal in Rician multipath fading with shadowing.

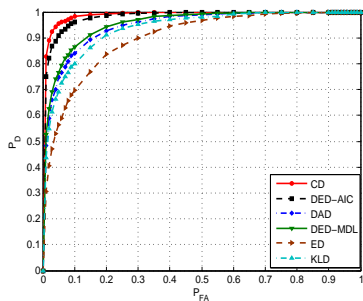
Bandwidth	8MHz
Mode	2K
Guard interval	1/4
Channel models	Rayleigh/Rician (K=1)
Maximum Doppler shift	100Hz
Frequency-flat	Single path
Sensing time	1.25ms
Location variability	10dB

Table 1: The transmitted DVB-T primary user signal parameters.

PU Signal Detection: Scenario 1



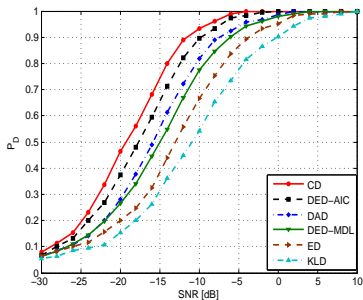
(a) P_D vs. SNR: Scenario 1



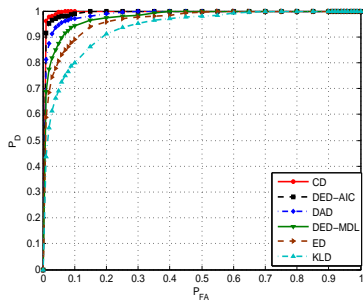
(b) ROC curves: Scenario 1

Figure 7: Monte Carlo simulation results assessing detection performance using an DVB-T OFDM primary user system in AWGN channel: Probability of detection versus SNR curves with $P_{FA} = 0.05$ and ROC curves with SNR = -7dB and sensing time = 1.12ms .

PU Signal Detection: Scenario 2



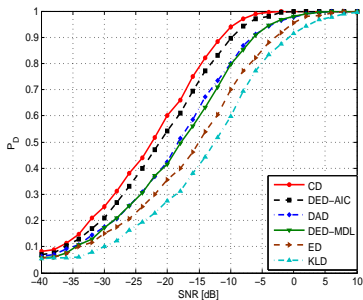
(a) P_D vs. SNR: Scenario 2



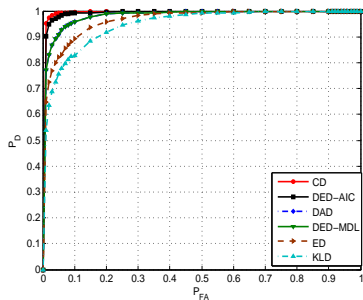
(b) ROC curves: Scenario 2

Figure 8: Monte Carlo simulation results assessing detection performance using an DVB-T OFDM primary user system in Rayleigh multipath fading with shadowing: Probability of detection versus SNR curves with $P_{FA} = 0.05$ and ROC curves with SNR = -7dB and sensing time = 1.12ms .

PU Signal Detection: Scenario 3



(a) P_D vs. SNR: Scenario 3



(b) ROC curves: Scenario 3

Figure 9: Monte Carlo simulation results assessing detection performance using an DVB-T OFDM primary user system in Rician multipath fading with shadowing: Probability of detection versus SNR curves with $P_{FA} = 0.05$ and ROC curves with SNR = -7dB and sensing time = 1.12ms .

Cooperative Sensing

- SU 1 is shown to be shadowed by a high building over the sensing channel
⇒ the CR cannot reliably sense the presence of the PU due to the very low SNR of the received signal (hidden node problem).
 - **Step 1:** Every SU performs local spectrum measurements independently and then makes a binary decision.
 - **Step 2:** All the SUs forward their binary decisions to a FC.
 - **Step 3:** The FC combines those binary decisions and makes a final decision to infer the absence or presence of the PU in the observed band.

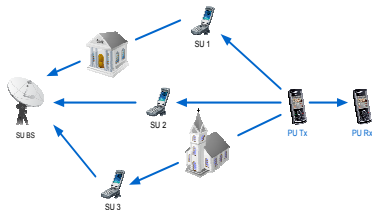
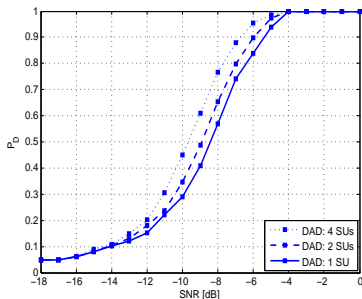
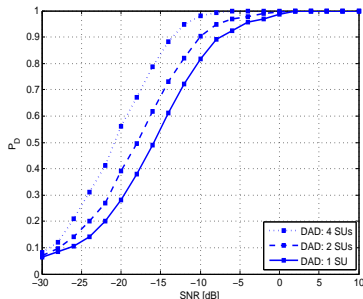


Figure 10: Cooperative spectrum sensing in cognitive radio networks.

DAD: Cooperative Sensing Evaluation



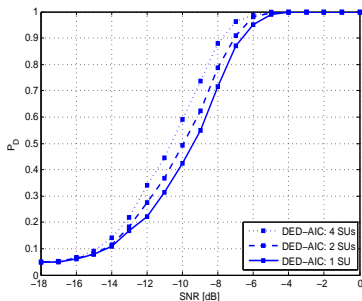
(a) P_D vs. SNR: Scenario 1



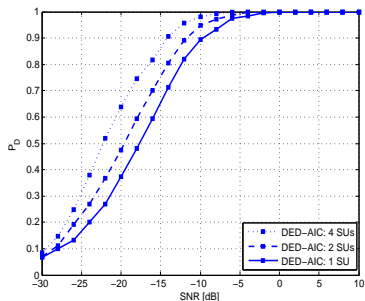
(b) P_D vs. SNR: Scenario 2

Figure 11: Performance evaluation of the DAD detector in terms of PU signal detection in cooperative way using an DVB-T OFDM primary user system: Probability of detection versus SNR curves with $P_{FA} = 0.05$ and the required SNR versus the number of collaborating users M .

DED: Cooperative Sensing Evaluation



(a) P_D vs. SNR: Scenario 1



(b) P_D vs. SNR: Scenario 2

Figure 12: Performance evaluation of the DED detector in terms of PU signal detection in cooperative way using an DVB-T OFDM primary user system: Probability of detection versus SNR curves with $P_{FA} = 0.05$ and the required SNR versus the number of collaborating users M .