Diversity-Multiplexing Tradeoff for the non-separated Two-way Relay DF Channel

Arun Singh, Petros Elia, Kiran T. Gowda and David Gesbert
Mobile Communications Department
EURECOM, Sophia Antipolis, France
{singhak, elia, gowda, gesbert}@eurecom.fr

***Abstract*—This work establishes the diversity-multiplexing tradeoff (DMT) of the four-phase decode-and-forward (DF) protocol in the half-duplex, non-separated two-way relay channel. We consider a fading channel model where the source relay links are Rayleigh distributed but the direct link between sources is left more general as Nakagami-m distributed and investigate for any possible gains in the achievable DMT using four-phase hybrid broadcast (HBC) protocol, as compared to three-phase time-division broadcast (TDBC) protocol.**

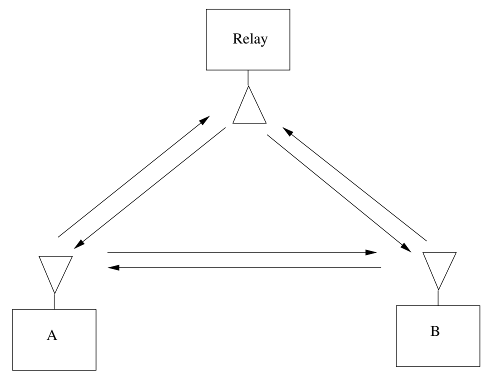
For the statistically symmetric case of Rayleigh fading and asymmetric fading with more stable direct link ($m > 1$), the optimal DMT of the HBC protocol is computed, and is shown to be achieved by a three-phase orthogonal variant, TDBC protocol. The operational meaning of this result is that the *multiple access channel* (MAC) phase of HBC protocol is not necessary to achieve optimal performance, this results in a simplification of the communication protocol. For less stable direct link ($\frac{1}{2} \leq m < 1$), the analysis establishes that the MAC phase of HBC protocol is necessary to achieve optimal DMT.

***Index Terms*—Diversity-multiplexing tradeoff, Nakagami-m fading, half duplex, decode-forward, two-way relay channels.**

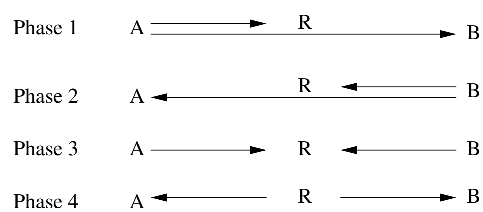
I. Introduction

Cooperative relays have found applications in many wireless networks to enhance network capacity, extend radio range, reduce terminal transmission power, provide spatial diversity etc. While several cooperation modes involving one-way relays have been proposed in the literature [1], [2], [3], in most practical communication scenarios data flows in both directions. Hence, recently these scenarios are revisited under the assumption that the two communication nodes exchange messages with cooperation of an intermediate relay node employing intelligent two-way relaying strategies. This approach has been modeled as Two-way Relay Channels (TRC) and has attracted significant interest [4], [5]. The fundamental advantage of TRC over classical one-way relay channels is that the duplexing loss due to half-duplex constraint (a node cannot transmit and receive simultaneously) can be avoided.

The presented work deals with a TRC with an asymmetric fading channel model, which corresponds to a very pertinent communication scenario where source-relay links are statistically different from the direct link between sources. In this setting, the source relay links are i.i.d. Rayleigh distributed but the direct link between sources is left more general as i.i.d. Nakagami-m (c.f. [6]) distributed (Nakagami-m includes Rayleigh for $m = 1$). This channel model is called a non-separated two-way relay channel (ns-TRC) because of existence of direct link between sources, and is more general than many TRC scenarios considered in previous works.



(a) System Model



(b) Four-phase DF Protocol

Fig. 1: non-separated Two-way Relay Channel model

The system model considered in this paper consists of two source nodes A and B who want to exchange information in the presence of an assisting relay node R as shown in Fig. 1(a). The relay employs decode-and-forward (DF) strategy and does not have any information of its own to transmit. Each node uses a single antenna and operates under half-duplex constraint. Note that this system model is statistically symmetrical with respect to the two source nodes.

Communication between nodes A and B takes place via a four-phase protocol as shown in Fig. 1(b) with the direction of arrow indicating the transmit/receive mode of each node. For instance, in the first phase node A transmits while all other nodes listen. We denote the fraction of time-slot allocated for the i^{th} phase with Δ_i, and hence $\sum \Delta_i = 1$. The third and the fourth phase are called *multiple access channel (MAC) phase* and *broadcast channel (BC) phase* respectively. Note that this is the most general scheduling possible, with the half-duplex constraint imposed.

The relative phase durations affect both the error performance as well as the achievable transmission rates. In this paper, we use the diversity-multiplexing tradeoff (DMT), introduced in [7], as a unified metric and compute individual phase durations that optimize this DMT.

Throughout this paper we assume that all the channels are frequency flat, quasi-static, and they are all independent of each other. We assume perfect channel state information (CSI) at the receiver (CSIR) of each link, but no CSI at the transmitters (CSIT).

A. Prior Work

The DMT analysis of TRC has attracted significant interest in the recent past. The work in [8], [9] considers a TRC model with no direct link between sources (the so called separated-TRC), and DMT analysis is presented for two different settings, with feedback [8] and without feedback [9]. The work in [10], [11] considers both separated and non-separated TRC, but the analysis is restricted to an orthogonal three-phase TDBC protocol. Optimal DMT achievable with TDBC protocol and Rayleigh fading can be found in [10], [12]. In this paper, we consider a more general setting viz., a non-separated TRC with statistically asymmetric fading channels and investigate for any possible gains in the achievable DMT using four-phase HBC protocol, as compared to three-phase TDBC protocol.

B. Contributions

Focusing on the high SNR regime and asymmetric fading channel model, we analyze diversity-multiplexing tradeoff (DMT) of a four-phase DF protocol for the ns-TRC system model. Specifically, the contributions are:

- For $m \geq 1$, which is a situation that can be interpreted as the direct link having similar or a more stable fading statistic compared to source-relay links, we establish the expression for optimal DMT of four-phase HBC protocol and show that an orthogonal three-phase TDBC protocol achieves this optimal DMT. The practical implication of this result is that the MAC phase of HBC protocol is not necessary to achieve optimal performance, this results in a simplification of the communication protocol.
- For $\frac{1}{2} \leq m < 1$, the analysis establishes that the MAC phase of HBC protocol is necessary to achieve optimal DMT.

C. Summary

The rest of the paper is organized as follows. In section II, we derive DMT of four-phase HBC protocol. In section III, we establish DMT optimality of TDBC and HBC protocols for specific cases of asymmetric fading model and finally conclusions are made in section IV.

D. Definitions and Notations

A coding scheme is a sequence of codes $\mathcal{C}(\rho)$, with $R(\rho)$ denoting the rate of code $\mathcal{C}(\rho)$. In high SNR regime, this coding scheme is said to achieve a spatial multiplexing gain r and diversity gain $d(r)$ if

$$\lim_{\rho\to\infty} \frac{R(\rho)}{\log\rho} = r, \qquad \text{and} \qquad -\lim_{\rho\to\infty} \frac{\log(P_e)}{\log\rho} = d(r)$$

where P_e denotes probability of codeword error and ρ denotes signal-to-noise ratio.

We use $\doteq$ to denote the *exponential equality*, i.e., we write $f(\rho) \doteq \rho^B$ to denote $\lim\limits_{\rho\to\infty} \frac{\log f(\rho)}{\log\rho} = B$ and $\dot{\leq}$, $\dot{\geq}$ are similarly defined. With this notation, we can write $P_e \doteq \rho^{-d(r)}$. We use $\overline{\{\bullet\}}$ to denote the complement of $\{\bullet\}$.

II. Diversity-Multiplexing Tradeoff

In this section, we derive the DMT $d_{4-ph}(\Delta_i, r)$ for a *static* four-phase protocol with fixed phase durations. The DMT for the optimal protocol, where the optimization is done over phase durations that can vary with multiplexing gain r, is presented in next section.

We are interested in the case where both source nodes transmit with the same rate R, and also, both demand the same DMT performance. Since the system model is symmetric with respect to the two source nodes, there is no loss of generality in making the two phase durations Δ_1 and Δ_2 equal.

To analyze the system performance for fixed phase durations, we define the average error probability $P_e = P[\{\mathcal{E}_{\mathrm{A}} \cup \mathcal{E}_{\mathrm{B}}\}]$, where $\mathcal{E}_{\mathrm{A}}$ and $\mathcal{E}_{\mathrm{B}}$, defined later, denote the error events at the receive node A and B respectively.

In order to analyze the information flow from source node A to B, we define the following error event $\mathcal{E}_{\mathrm{B}} \triangleq \overline{\{\overline{\mathcal{E}}_{\mathrm{B},1} \cup \{\overline{\mathcal{E}}_{\mathrm{R},3} \cap \overline{\mathcal{E}}_{\mathrm{B},4}\}\}}$, where

- $\overline{\mathcal{E}}_{\mathrm{B},1}$ - occurs if B is able to decode message from A at the end of phase 1.
- $\overline{\mathcal{E}}_{\mathrm{R},3}$ - occurs if R is able to decode message from A and B at the end of phase 3.
- $\overline{\mathcal{E}}_{\mathrm{B},4}$ - occurs if B is able to decode message from A at the end of phase 4, conditioned on the occurrence of event $\overline{\mathcal{E}}_{\mathrm{R},3}$.

Note that $\overline{\mathcal{E}}_{\mathrm{B},1} \subset \overline{\mathcal{E}}_{\mathrm{B},4}$. Error event corresponding to $\mathcal{E}_{\mathrm{A}}$ can be similarly defined by interchanging A and B in the above expressions. With these definitions, we have the upper bound

$$\begin{aligned} P[\mathcal{E}_{\mathrm{B}}] &= P[\overline{\overline{\mathcal{E}}_{\mathrm{B},1} \cup \{\overline{\mathcal{E}}_{R,3} \cap \overline{\mathcal{E}}_{\mathrm{B},4}\}}] \\ &= P[\mathcal{E}_{\mathrm{B},1} \cap \{\mathcal{E}_{R,3} \cup \mathcal{E}_{\mathrm{B},4}\}] \\ &= P[\{\mathcal{E}_{\mathrm{B},1} \cap \ \mathcal{E}_{R,3}\} \cup \{\mathcal{E}_{\mathrm{B},1} \cap \mathcal{E}_{\mathrm{B},4}\}] \\ &= P[\{\mathcal{E}_{\mathrm{B},1} \cap \ \mathcal{E}_{R,3}\} \cup \mathcal{E}_{\mathrm{B},4}] \\ &\overset{(f)}{\leq} P[\{\mathcal{E}_{\mathrm{B},1} \cap \ \mathcal{E}_{R,3}\}] + P[\mathcal{E}_{\mathrm{B},4}] \\ &= P[\mathcal{E}_{\mathrm{B},1}]P[\mathcal{E}_{R,3}] + P[\mathcal{E}_{\mathrm{B},4}] \end{aligned} \tag{1}$$

where (f) follows from union bound. We also have a lower bound,

$$\max\{P[\mathcal{E}_{\mathrm{B},1}]P[\mathcal{E}_{R,3}], P[\mathcal{E}_{\mathrm{B},4}]\} \leq P[\mathcal{E}_{\mathrm{B}}]. \tag{2}$$

Bounds corresponding to $\mathcal{E}_{\mathrm{A}}$ can be obtained similarly by replacing B with A in the expressions (1) and (2).

Now, since

$$\max\{P[\mathcal{E}_{\mathrm{A}}], P[\mathcal{E}_{\mathrm{B}}]\} \leq P_e \leq P[\mathcal{E}_{\mathrm{A}}] + P[\mathcal{E}_{\mathrm{B}}], \tag{3}$$

if we can derive the precise optimum SNR exponents for each of the $P[\mathcal{E}_{B,1}]$, $P[\mathcal{E}_{R,3}]$ and $P[\mathcal{E}_{B,4}]$ defined above and also show that $P[\mathcal{E}_{\mathrm{A}}] \doteq P[\mathcal{E}_{\mathrm{B}}]$, then we have the DMT expression for the static four phase protocol.

Toward this end, we define the outage events for information flow from A to B. Let $X_J^{(i)}$ denote the transmit signal, $Y_J^{(k)}$ denote the receive signal for any node $J \in \{\mathrm{A}, \mathrm{B}, \mathrm{R}\}$, in the i-th and the k-th phase, $i, k \in \{1, 2, 3, 4\}$. The outage event $\mathcal{O}_{J,k}$ for the respective error event $\mathcal{E}_{J,k}$ is defined as:

$$\mathcal{O}_{\mathrm{B},1} \triangleq \{h_{AB} : \Delta_1 I(X_{\mathrm{A}}^{(1)}; Y_{\mathrm{B}}^{(1)}|h_{AB}) < R\} \tag{4}$$

$$\mathcal{O}_{\mathrm{R},3} = \{\mathcal{O}_{\mathrm{R},3,1} \cup \mathcal{O}_{\mathrm{R},3,2}\}$$

$$\begin{aligned}\mathcal{O}_{\mathrm{R},3,1} \triangleq \Big\{&(h_{\mathrm{AR}}, h_{\mathrm{BR}}) : \Delta_1 I(X_{\mathrm{A}}^{(1)}; Y_{\mathrm{R}}^{(1)}|h_{\mathrm{AR}}) \\ &+ \Delta_3 I(X_{\mathrm{A}}^{(3)}; Y_{\mathrm{R}}^{(3)}|X_{\mathrm{B}}^{(3)}, h_{\mathrm{AR}}) < R\Big\}\end{aligned} \tag{5}$$

$$\begin{aligned}\mathcal{O}_{\mathrm{R},3,2} \triangleq \Big\{&(h_{\mathrm{AR}}, h_{\mathrm{BR}}) : \Delta_1 I(X_{\mathrm{A}}^{(1)}; Y_{\mathrm{R}}^{(1)}|h_{\mathrm{AR}}) \\ &+ \Delta_2 I(X_{\mathrm{B}}^{(2)}; Y_{\mathrm{R}}^{(2)}|h_{BR}) \\ &+ \Delta_3 I(X_{\mathrm{A}}^{(3)}, X_{\mathrm{B}}^{(3)}; Y_{\mathrm{R}}^{(3)}|h_{\mathrm{AR}}, h_{\mathrm{BR}}) < 2R\Big\}\end{aligned} \tag{6}$$

$$\begin{aligned}\mathcal{O}_{\mathrm{B},4} \triangleq \Big\{&(h_{AB}, h_{\mathrm{RB}}) : \Delta_1 I(X_{\mathrm{A}}^{(1)}; Y_{\mathrm{B}}^{(1)}|h_{AB}) \\ &+ \Delta_4 I(X_{\mathrm{R}}^{(4)}; Y_{\mathrm{B}}^{(4)}|h_{\mathrm{RB}}) < R\Big\}.\end{aligned} \tag{7}$$

Again, $\mathcal{O}_{\mathrm{B},4} \subset \mathcal{O}_{\mathrm{B},1}$, where h_{JK} denotes channel from node J to node K ($J, K \in \{\mathrm{A}, \mathrm{B}, \mathrm{R}\}$). The corresponding events for information flow from B to A are similarly defined by interchanging A and B in all the mutual information expressions.

We can now prove the following result for the asymmetric fading model:

Theorem 1: For the asymmetric fading model, the DMT for a static four-phase protocol with fixed phase-durations $\Delta_1, \Delta_3, \Delta_4$ satisfying $\Delta_1 = \Delta_2$ and $\Delta_1 \geq m\Delta_4$, is

$$d_{4-ph}(\Delta_i, r) = \min\{(d_1(r) + d_2(r)), d_3(r)\}. \tag{8}$$

where

$$d_1(r) = m\left(1 - \frac{r}{\Delta_1}\right) \quad \text{for} \quad 0 \leq r \leq \Delta_1, \tag{9}$$

$$d_2(r) = \begin{cases} 1 - \frac{r}{\Delta_1 + \Delta_3} & \text{for} \quad 0 \leq r \leq \Delta_1 \\ 1 + \frac{\Delta_1 - 2r}{\Delta_1 + \Delta_3} & \text{for} \quad \Delta_1 < r \leq \Delta_1 + \frac{\Delta_3}{2} \end{cases} \tag{10}$$

$$d_3(r) = \begin{cases} 1 + m - \frac{r}{\Delta_4} & \text{for } r \leq \Delta_4 \\ m\left(1 + \frac{\Delta_4 - r}{\Delta_1}\right) & \text{for } \Delta_4 < r \leq \Delta_1 + \Delta_4 \end{cases} \tag{11}$$

and

$$\begin{aligned} P[\mathcal{E}_{\mathrm{B},1}] &\doteq P[\mathcal{O}_{\mathrm{B},1}] \doteq \rho^{-d_1(r)}, \\ P[\mathcal{E}_{\mathrm{R},3}] &\doteq P[\mathcal{O}_{\mathrm{R},3}] \doteq \rho^{-d_2(r)}, \\ P[\mathcal{E}_{\mathrm{B},4}] &\doteq P[\mathcal{O}_{\mathrm{B},4}] \doteq \rho^{-d_3(r)} \end{aligned} \tag{12}$$

Proof: (sketch) To prove (12), one can follow similar analysis as in [7], [12], [13]. For each of the events $\mathcal{O}_{J,k}$ and $\mathcal{E}_{J,k}$ defined above, we have

$$P(\mathcal{O}_{J,k}) \leq P(\mathcal{E}_{J,k}) \leq P(\mathcal{O}_{J,k}) + P(\mathcal{E}_{J,k}|\overline{\mathcal{O}}_{J,k}). \tag{13}$$

Then, for sufficiently long, independent random Gaussian encoding at every transmitter, joint maximum-likelihood decode-and-forward relaying at the relay node, we show that

$$P[\mathcal{E}_{\mathrm{B},1}|\overline{\mathcal{O}}_{\mathrm{B},1}] \doteq P[\mathcal{O}_{\mathrm{B},1}] \doteq \rho^{-d_1(r)}, \tag{14}$$

$$P[\mathcal{E}_{\mathrm{R},3}|\overline{\mathcal{O}}_{\mathrm{R},3}] \doteq P[\mathcal{O}_{\mathrm{R},3}] \doteq \rho^{-d_2(r)}, \tag{15}$$

$$P[\mathcal{E}_{\mathrm{B},4}|\overline{\mathcal{O}}_{\mathrm{B},4}] \doteq P[\mathcal{O}_{\mathrm{B},4}] \doteq \rho^{-d_3(r)}. \tag{16}$$

In particular, (14) follows from DMT for single antenna point-to-point transmission [7], (15) follows from the DMT for MAC [13] and (16) follows from the DMT for parallel channels [12]. These along with (13) gives

$$P[\mathcal{E}_{\mathrm{B}}] \doteq \rho^{-\min\{(d_1(r)+d_2(r)), d_3(r)\}}, \tag{17}$$

where, (17) follows from (1), (2) and Varadhan's lemma [14]. Due to symmetry we have $P[\mathcal{E}_{\mathrm{A}}] \doteq P[\mathcal{E}_{\mathrm{B}}]$ and applying Varadhan's lemma to (3), we can write

$$P_e \doteq \rho^{-\min\{(d_1(r)+d_2(r)), d_3(r)\}}, \tag{18}$$

This completes the proof of Theorem 1. ■

Remark 1:
- For a static protocol with $\Delta_1 < m\Delta_4$, the DMT expression is similar as in Theorem 1, but with
$$d_3(r) = \begin{cases} 1 + m - \frac{mr}{\Delta_1} & \text{for } r \leq \Delta_1 \\ 1 + \frac{\Delta_1}{\Delta_4} - \frac{r}{\Delta_4} & \text{for } \Delta_1 < r \leq \Delta_1 + \Delta_4 \end{cases}$$
- For $m = 1$, we get the DMT for symmetric Rayleigh fading model.

Now, for the DMT optimization, we need to solve the following problem for any r:

$$\begin{aligned} d_{OPT}(r) = &\max_{\Delta_1, \Delta_3, \Delta_4} d_{4-ph}(\Delta_i, r) \\ \text{subject to} \quad &2\Delta_1 + \Delta_3 + \Delta_4 = 1, \qquad \Delta_i \geq 0. \end{aligned} \tag{19}$$

III. Optimal DMT

The optimal DMT achievable for $m \geq 1$, which is a situation that can be interpreted as the direct link having similar or a more stable fading statistic compared to source-relay links, is given by:

Theorem 2: For the given system settings with $m \geq 1$, optimal DMT is

$$d_{OPT}(r) = \begin{cases} 1 + m - (2m + 3)r & \text{for } r \leq \frac{1}{(2m+3)} \\ \frac{m(1+m)(1-2r)}{r+m} & \text{for } \frac{1}{(2m+3)} < r \leq \frac{1}{2} \end{cases}, \tag{20}$$

and it can be achieved using TDBC protocol with

$$\Delta_1 = \begin{cases} \frac{(m+1)}{(2m+3)} & \text{for } r \leq \frac{1}{(2m+3)} \\ \frac{r+m}{1+2m} & \text{for } \frac{1}{(2m+3)} < r \leq \frac{1}{2} \end{cases} \tag{21}$$

$\Delta_2 = \Delta_1$, $\Delta_3 = 0$ and $\Delta_4 = 1 - 2\Delta_1$.

Corollary 2a: For the Rayleigh fading case ($m = 1$), the optimal DMT $d_{OPT}(r)$ is attained by an orthogonal three-phase TDBC protocol.

Proof:

A. Proof for Theorem 2

From Theorem 1 we have:

$$d_1(r) + d_2(r) = \begin{cases} 1 + m - \frac{mr}{\Delta_1} - \frac{r}{\Delta_1+\Delta_3} & \text{for } r \leq \Delta_1 \\ 1 + \frac{\Delta_1}{\Delta_1+\Delta_3} - \frac{2r}{\Delta_1+\Delta_3} & \text{for } \Delta_1 < r \leq \Delta_1 + \frac{\Delta_3}{2} \end{cases} \tag{22}$$

We are looking at $d_{OPT} = \max_{\Delta_i} \min\{d_1(r) + d_2(r), d_3(r)\}$ under the sum-constraint and the positivity constraints in (19). First we prove that any optimal protocol must have $\Delta_3 = 0$, for $m \geq 1$.

This can be proved by contradiction - suppose (Δ_i^*) maximizes the DMT $(d_{OPT}(r))$ with $\Delta_3^* > 0$. Now consider the four phase protocol with new phase durations $\Delta_1 = \Delta_1^* + \frac{\Delta_3^*}{2}$, $\Delta_4 = \Delta_4^*$ and $\Delta_3 = 0$. These new phase durations are in the feasible set (19), and substituting these in (22) and (11) we get

$$d_1(\Delta_i, r) + d_2(\Delta_i, r) \geq d_1(\Delta_i^*, r) + d_2(\Delta_i^*, r) \quad \text{and}$$
$$d_3(\Delta_i, r) \geq d_3(\Delta_i^*, r)$$

for all values of r. This contradicts the optimality of (Δ_i^*) and show that for $m \geq 1$ any optimal protocol must have $\Delta_3 = 0$.

The optimization problem in (19) can now be written as

$$d_{OPT}(r) = \max_{\Delta_1, \Delta_4} d_{4-ph}(\Delta_1, \Delta_4, r)$$
$$\text{subject to} \quad 2\Delta_1 + \Delta_4 = 1, \quad \Delta_1, \Delta_4 \geq 0,$$

where we now have

$$d_1(r) + d_2(r) = (m+1)\left(1 - \frac{r}{\Delta_1}\right) \quad \text{for} \quad 0 \leq r \leq \Delta_1.$$

Since we are looking at a max-min problem, for any given multiplexing gain r, the optimal DMT is achieved when the two DMT curves $d_1(r) + d_2(r)$ and $d_3(r)$ meet at r. When $\Delta_1 < m\Delta_4$ it is easy to see that the curves meet only at $r = 0$. So we only need to consider the case $\Delta_1 \geq m\Delta_4$. Now we have

$$\mathrm{d}_3(r) = \begin{cases} 1 + m - \frac{r}{\Delta_4} & \text{for } r \leq \Delta_4 \\ m(1 + \frac{\Delta_4 - r}{\Delta_1}) & \text{for } \Delta_4 < r \leq \Delta_1 + \Delta_4 \end{cases}$$

The expression for $d_3(r)$ suggests that the optimization can be divided in to two exclusive cases: (i) $r \leq \Delta_4$ (ii) $\Delta_4 < r \leq \Delta_1$.

1) Case $r \leq \Delta_4$: With $d_1(r) + d_2(r) = d_3(r)$, and all other constraints, it is straight-forward to check that the optimum solution must satisfy $\Delta_1 = (m+1)\Delta_4$. Substituting this in the sum-constraint, we get

$$\Delta_4 = \frac{1}{(2m+3)}, \quad \Delta_1 = \frac{(m+1)}{(2m+3)},$$

and optimum DMT as $d_{OPT}(r) = 1 + m - (2m+3)r$ for any $r \leq \frac{1}{(2m+3)}$.

2) Case $\frac{1}{(2m+3)} < r \leq \frac{1}{2}$: Again with $d_1(r) + d_2(r) = d_3(r)$, we get

$$\Delta_1 = \frac{r+m}{1+2m}, \quad \Delta_3 = 0, \quad \text{and} \quad \Delta_4 = 1 - 2\Delta_1.$$

and optimum DMT as

$$d_{OPT}(r) = \frac{m(1+m)(1-2r)}{r+m} \text{ for } \frac{1}{(2m+3)} < r \leq \frac{1}{2}$$

This proves Theorem 2. Also, for all values of r and $m = 1$, we have proved that $\Delta_3 = 0$ is optimal, which proves Corollary 2a. ■

Now we consider the case $\frac{1}{2} \leq m < 1$, which corresponds to the case where direct link between sources is less stable compared to source-relay links. The optimal DMT achievable using four-phase HBC protocol is given by:

Theorem 3: For the given system settings with $\frac{1}{2} \leq m < 1$, optimal DMT is

$$d_{OPT}(r) = 1 + m - \frac{r}{\kappa} \quad \text{for} \quad r \leq \kappa, \tag{23}$$

where,

$$\kappa = \frac{(2+m) - (1-m)\Delta_3 - X}{3+2m}$$
$$X = \sqrt{(m^2+4)\Delta_3^2 + (2m^2+2m-4)\Delta_3 + (m+1)^2}$$

and it can be achieved using HBC protocol with

$$\Delta_3 = \frac{(1-m)(2+m-2\sqrt{m})}{4+m^2}, \quad \Delta_4 = \kappa,$$

and

$$\Delta_2 = \Delta_1 = \frac{1 - \Delta_3 - \Delta_4}{2}$$

For $\kappa < r \leq \frac{1}{2}$, we do not have a closed form expression for the optimal DMT, however, this optimization problem can be solved using linear programming.

Proof:

B. Proof for Theorem 3

Again we are looking at a max-min problem, for any given multiplexing gain r, the optimal DMT is achieved when the two DMT curves $d_1(r) + d_2(r)$ and $d_3(r)$ meet at r. Solving for $d_1(r) + d_2(r) = d_3(r)$ from (22) and (11) and sum constraint $2\Delta_1 + \Delta_3 + \Delta_4 = 1$, we get

$$\Delta_3 = \frac{(1-m)(2+m-2\sqrt{m})}{4+m^2}, \quad \Delta_4 = \kappa,$$

and

$$\Delta_1 = \frac{1 - \Delta_3 - \Delta_4}{2}$$

and optimum DMT as $d_{OPT}(r) = 1 + m - \frac{r}{\kappa}$ for $r \leq \kappa$. This proves Theorem 3. ∎

For $\kappa < r \leq \frac{1}{2}$, again, optimal DMT is computed by solving for $d_1(r) + d_2(r) = d_3(r)$ from (22) and (11) and sum constraint $2\Delta_1 + \Delta_3 + \Delta_4 = 1$. Now we have,

$$\Delta_1 = \frac{m + r - (3m+1)\Delta_3 + Y}{2(2m+1)}. \tag{24}$$

where,

$$Y = \sqrt{(m + r - (3m+1)\Delta_3)^2 + 4m(2m+1)\Delta_3(1 - \Delta_3)}.$$

Substituting for Δ_1 and $\Delta_4 = 1 - 2\Delta_1 - \Delta_3$, the optimization problem in (19) can now be written as,

$$\min_{0 \leq \Delta_3 \leq 1-r} f(\Delta_3, r),$$

where

$$f(\Delta_3, r) = \frac{m + r - (3m+1)\Delta_3 + Y}{1 - r - \Delta_3}.$$

This optimization problem can be solved using linear programing. Numerical results are shown in Fig. 2 for $m = \frac{1}{2}$.

Theorem 3 establishes DMT optimality of HBC protocol for the case where direct link between sources is less stable compared to source-relay links. However, numerical results show that the gap between the DMT achievable using three-phase TDBC protocol and optimal DMT for the four-phase HBC protocol is very small - indicating that the three-phase protocol is "nearly optimal" for $\frac{1}{2} \leq m < 1$. This is illustrated in Fig. 2 for $m = \frac{1}{2}$.

The practical implication of this result is that the MAC phase of HBC protocol is not necessary to achieve this "nearly optimal" performance, and this results in a simplification of the communication protocol.

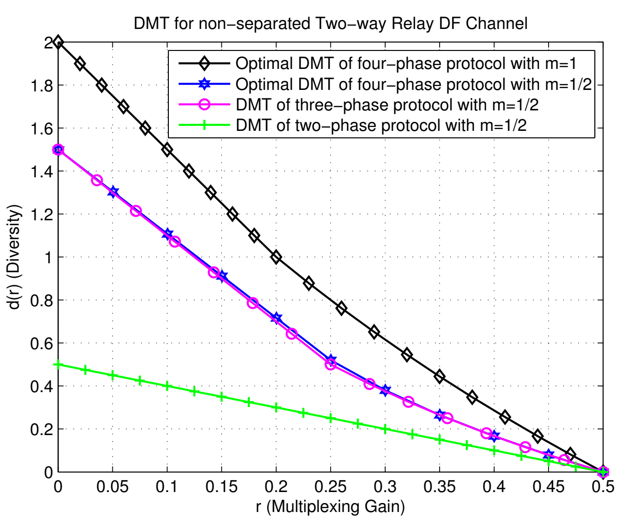


Fig. 2: DMT Comparison

IV. Conclusions

In this paper, optimal DMT of non-separated TRC for four-phase HBC protocol is established. The analysis indicates that for a DMT optimal protocol, duration of MAC phase decreases with improvement in direct link fading stability relative to source-relay links. For $m \geq 1$, which is a situation that can be interpreted as the direct link having similar or a more stable fading statistic compared to source-relay links, the optimal DMT is shown to be achieved by the orthogonal three-phase TDBC protocol. The operational meaning of this result is that the MAC phase of HBC protocol is not necessary to achieve optimal performance, this results in a simplification of the communication protocol. For less stable direct link ($\frac{1}{2} \leq m < 1$), the MAC phase of HBC protocol is necessary to achieve optimal DMT but a simpler TDBC protocol can achieve "nearly optimal" DMT performance.

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