A Performance Study of Kullback-Leibler Distance-based Spectrum Sensing Algorithm

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Abstract— In this paper, we will derive closed-form expressions of false alarm probabilities for a given threshold for the Kullback-Leibler distance-based spectrum sensing detector. This detector is based on the distribution analysis of the primary user received signal. A theoretical probability of false alarm will be derived for a fixed threshold using the Meijer G-function of the product of p independent Rayleigh random variables. The derived analytical decision threshold will be verified with Monte-Carlo simulations and a comparison between simulation and analytical results to confirm the theoretical results. These results confirm the very good match between simulation and theoretic results.

Keywords—**Spectrum sensing, Kullback-Leibler distance, distribution analysis, threshold calculation.**

I. INTRODUCTION

The discrepancy between current-day spectrum allocation and spectrum use suggests that radio spectrum shortage could be overcome by allowing a more flexible usage of the spectrum. Flexibility would mean that radios could find and adapt to any immediate local spectrum availability. A new class of radios that is able to reliably sense the spectral environment over a wide bandwidth detects the presence/absence of legacy users (*primary users*) and uses the spectrum only if the communication does not interfere with primary users (PUs). It is defined by the term *cognitive radio* [1]. Cognitive Radio (CR) technology has attracted worldwide interest and is believed to be a promising candidate for future wireless communications in heterogeneous wideband environments.

CR has been proposed as the means to promote efficient utilization of the spectrum by exploiting the existence of spectrum holes. The spectrum use is concentrated on certain portions of the spectrum while a significant amount of the spectrum remains unused. It is thus key for the development of CR to invent fast and highly robust ways of determining whether a frequency band is available or occupied. This is the area of spectrum sensing which will be considered in this paper. There are several spectrum sensing strategies that were proposed for CR. These strategies are categorized in two families: feature detection strategies and blind detection strategies. The feature detection approaches assume that a PU is transmitting information to a primary receiver when a secondary user (SU) is sensing the primary channel band. The elaboration of sensing techniques that use some prior information about the transmitted signal is interesting in terms of performance. In fact, feature detection algorithms employ knowledge of structural and statistical properties of PU signals when making the decision. The most known feature sensing technique is the cyclostationarity based detector (CD) [2]. Completely blind spectrum sensing techniques that do not consider any prior knowledge about the PU transmitted signal are more convenient to CR. A few methods that belong to this category have been proposed, but all of them suffer from the noise uncertainty and fading channels variations. One of the most popular blind detectors is the energy detector (ED) [3]. This detector is the most common method for spectrum sensing because of its non-coherency and low complexity. The CD and ED will serve as references when evaluating the performance of the dimension estimation-based detectors.

It is stated that current spectrum sensing techniques suffer from challenges in the low signal to noise range. The reasons for this have to be analyzed. It is suggested that Kullback-Leibler distance is possible area to look for a solution to overcome the problem. It is apparent that the problem at hand is wide and challenging [4] [5]. The initial attempt to apply Kullback-Leibler distance for spectrum sensing was presented in [4]. This work suggested to use Kullback-Leibler distance to conclude on the nature of the sensed band in a blind way. The proposed detector analysis the Kullback-Leibler distance between signal and noise distributions. Specifically, it compares the distribution of the received signal with the Gaussian distribution. The idea is to decide if the distribution of the observed signal fits the Gaussian model. The proposed algorithm, called the distribution analysis detector (DAD), exploits Akaike weights information derived using Akaike information criterion (AIC) as a reliability index in order to decide if the distribution of the received signal fits the noise distribution or not [6].

The work presented in [4] was a preliminary step for this idea. Indeed, no threshold expression was given. For this purpose, we will present in this paper the DAD detector as a binary hypothesis test and we will give then the exact threshold expressions of this detector for a given false alarm probability. We will use in this derivation the Meijer G-function of the product of p independent Rayleigh random variables [7]. The analytical results will be compared with simulation results.

The rest of this paper is organized as follows. In Section II we will present Kullback-Leibler distance formulation and in Section III we will analyze the Akaike weight information. The DAD algorithm will be presented in Section IV. We will derive in Section V closed-form expressions of false alarm probability for a given threshold. Performance evaluation and advantages will be described in Section VI and a comparison of the proposed detector with reference detectors will be given. The performance will be assessed under different conditions, using three simulation scenarios. Finally, Section VII presents the conclusions of this paper.

II. KULLBACK-LEIBLER DISTANCE

It is assumed that the samples of the received signal are distributed according to an original probability density function f, called the operating model. The operating model is usually unknown, since only a finite number of observations is available. Therefore, approximating probability model must be specified using the observed data, in order to estimate the operating model. The approximating model is denoted as g_θ, where the subscript θ indicates the U-dimensional parameter vector, which in turn specifies the probability density function. In information theory, the Kullback-Leibler distance describes the discrepancy between the two probability functions f and g_θ and is given by [6]:

$$D(f\|g_\theta) = -h(X) - \int f_X(x) \log g_\theta(x) dx \quad (1)$$

where the random variable X is distributed according to the original but unknown probability density function f, and $h(.)$ denotes differential entropy. This distance measure is not directly applicable, since the original probability density function f is not known. It is known, however, that the Kullback-Leibler distance is nonnegative, i.e., $D(f\|g_\theta) \geq 0$. This implies that the Kullback-Leibler discrepancy,

$$-\int f_X(x) \log g_\theta(x) dx = h(X) + D(f\|g_\theta) \quad (2)$$

approaches the differential entropy of X from above for increasing quality of the model g_θ. The differential entropy of X is reached if and only if $f = g_\theta$. Applying the weak law of large numbers, the second term in (1) can be approximated by averaging the log-likelihood values given the model over N independent observations $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ according to:

$$-\int f_X(x) \log g_\theta(x) dx \approx -\frac{1}{N} \sum_{n=1}^{N} \log g_\theta(\mathbf{x}_n) \quad (3)$$

The log-likelihood depends on the estimated vector θ, which itself is a function of the actual observations $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$. If another set of observations $\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, ..., \tilde{\mathbf{x}}_N$ is used, a different Kullback-Leibler discrepancy would be obtained. The expected Kullback-Leibler discrepancy is given by:

$$-E_\theta \left\{ \int f_X(x) \log g_\theta(x) dx \right\} \quad (4)$$

where the expectation is taken with respect to the distribution of the estimated parameter vector θ. This expression (4) cannot be computed, but estimated.

The information theoretic criteria was first introduced by Akaike in [6] for model selection. Assuming a candidate model, the idea is to decide if the distribution of the observed signal fits the candidate model. The AIC criterion is an approximately unbiased estimator for (4) and is given by:

$$\text{AIC} = -2 \sum_{n=1}^{N} \log g_{\hat{\theta}}(\mathbf{x}_n) + 2U \quad (5)$$

The parameter vector θ for each family should be estimated using the minimum discrepancy estimator $\hat{\theta}$, which minimizes the empirical discrepancy. This is the discrepancy between the approximating model and the model obtained by regarding the observations as the whole population. The maximum likelihood estimator is the minimum discrepancy estimator for the Kullback-Leibler discrepancy.

III. MODEL SELECTION USING AKAIKE WEIGHT

In this section, we analyze the Akaike weight information introduced by Akaike in [6] in order to decide if the distribution of the received signal fits the suitable distribution or not. Consider a probability distribution parameterized by an unknown parameter θ, associated with either a known probability density function or a known probability mass function, denoted as f_θ. As a function of θ with $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N$ fixed, the likelihood function is:

$$\begin{aligned} L(\theta) &= f_\theta(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N) \\ &= \prod_{n=1}^{N} f_\theta(\mathbf{x}_n) \end{aligned} \quad (6)$$

Commonly, one assumes that the data drawn from a particular distribution are i.i.d. with unknown parameters. This considerably simplifies the problem because the log-likelihood can then be written as follows:

$$L(\theta) = \sum_{n=1}^{N} \log f_\theta(\mathbf{x}_n) \quad (7)$$

The maximum of this expression can then be found numerically using various optimization algorithms. The method of maximum likelihood estimates θ by finding the value of θ that maximizes $L(\theta)$. Maximum likelihood estimator (MLE) is one of the most used methods to estimate functions parameters. This contrasts with seeking an unbiased estimator of θ, which may not necessarily yield the MLE but which will yield a value that (on average) will neither tend to over-estimate nor under-estimate the true value of θ. The maximum likelihood estimator may not be unique, or indeed may not even exist. The MLE of the parameters of θ is computed over a set of samples of length N. We assume that the samples are independent identically distributed (i.i.d.). The log-likelihood function $L^*(\theta)$ is given by:

$$L^*(\theta) = \sum_{n=1}^{N} \log g_\theta(\mathbf{x}_n) \quad (8)$$

Consequently, the MLE expression of θ in our case is:

$$\hat{\theta} = \arg_\theta \max \frac{1}{N} \sum_{n=1}^{N} \log g_\theta(\mathbf{x}_n) \quad (9)$$

The AIC is hence described by the following form:

$$\text{AIC} = -2L^*(\hat{\theta}) + 2U \qquad (10)$$

where U indicates the dimension of the parameter vector θ. We consider here that the envelop of a Gaussian noise can be modeled using Rayleigh distribution and the one of signal data can be modeled using Rician distribution [8]. In fact, recall that the distribution of a sum of independent random variables is the convolution of their distributions [8]. Hence, when the SNR is low, the noise distribution will dominate in the convolution and the resulting distribution will tend to become close to Gaussian even if the signal has an arbitrary non Gaussian distribution, and the envelope (norm) distribution of the signal is close to Rayleigh distribution [8]. Another important property is the contribution of the dominant propagation paths on the distribution of the communication signal. The envelope distribution of the received communication signal tend to become close to Rician even if the input has a non Rician distribution [8]. Hence, for the proposed DAD detector, we assume that the norm of the Gaussian noise can be modeled using Rayleigh distribution and the signal data can be modeled as a Rician distribution.

Therefore, Akaike weights can be interpreted as estimate of the probabilities that the corresponding candidate distribution show the best modeling fit. It provides another measure of the strength of evidence for this model, and is given by:

$$\text{W}_j = \frac{e^{-\frac{1}{2}\Phi_j}}{\sum_{i=1}^{N} e^{-\frac{1}{2}\Phi_i}} \qquad (11)$$

for a given distribution j, where Φ_j denotes the AIC difference defined by:

$$\Phi_j = \text{AIC}_j - \min_i \text{AIC}_i \qquad (12)$$

where $\min_i \text{AIC}_i$ denotes the minimum AIC value over all PU signals observations. In order to show the results of comparison between distributions in a clear manner, we introduce the Akaike weights W_{Rice} and $W_{Rayleigh}$ derived from AIC values. Akaike weights for Rice and Rayleigh can be expressed as:

$$W_{Rice} = \frac{exp\left(-\frac{1}{2}\Phi_{Rice}\right)}{exp\left(-\frac{1}{2}\Phi_{Rice}\right) + exp\left(-\frac{1}{2}\Phi_{Rayleigh}\right)} \qquad (13)$$

$$W_{Rayleigh} = \frac{exp\left(-\frac{1}{2}\Phi_{Rayleigh}\right)}{exp\left(-\frac{1}{2}\Phi_{Rayleigh}\right) + exp\left(-\frac{1}{2}\Phi_{Rice}\right)} \qquad (14)$$

where

$$\Phi_{Rice} = \text{AIC}_{Rice} - \min\left(\text{AIC}_{Rice}, \text{AIC}_{Rayleigh}\right) \qquad (15)$$

$$\Phi_{Rayleigh} = \text{AIC}_{Rayleigh} - \min\left(\text{AIC}_{Rayleigh}, \text{AIC}_{Rice}\right) \qquad (16)$$

and

$$\text{AIC}_{Rice} = -2L_{Rice} + 2U_{Rice} \qquad (17)$$

$$\text{AIC}_{Rayleigh} = -2L_{Rayleigh} + 2U_{Rayleigh} \qquad (18)$$

where $U_{Rayleigh} = 1$ and $U_{Rice} = 2$.

IV. DISTRIBUTION ANALYSIS DETECTOR

The goal of spectrum sensing is to decide between the following two hypothesizes [1]:

$$\mathbf{x} = \begin{cases} \mathbf{n} & \text{H}_0 \\ \mathbf{As} + \mathbf{n} & \text{H}_1 \end{cases} \qquad (19)$$

We decide that a spectrum band is unoccupied if there is only noise, as defined in H_0. On the other hand, once there exists a PU signal besides noise in a specific band, as defined in H_1, we say that the band is occupied. Thus the probability of false alarm can be expressed as

$$P_{FA} = Pr(\text{H}_1 \mid \text{H}_0) = Pr(\mathbf{x} \text{ is present} \mid \text{H}_0) \qquad (20)$$

The decision threshold is determined by using the required probability of false alarm P_{FA} given by (20). The threshold γ for a given false alarm probability is determined by solving the equation

$$P_{FA} = Pr(\Upsilon(\mathbf{x}) > \gamma | \text{H}_0) \qquad (21)$$

where $\Upsilon(\mathbf{x})$ denotes the test statistic for the given detector.

The DAD detector can be formulated as a binary hypothesis test. If PU is present, the Akaike weight of Rician distribution is higher than Akaike weight of Rayleigh distribution, and if PU is absent, we have the opposite. Therefore, the generalized blind DAD algorithm is given by:

$$\Upsilon_{DAD}(\mathbf{x}) = \begin{cases} W_{Rice} - W_{Rayleigh} < \gamma & \text{noise} \\ W_{Rice} - W_{Rayleigh} > \gamma & \text{signal} \end{cases} \qquad (22)$$

According to the system requirement on $P_{FA,DAD}$, we calculate a proper threshold γ. If $\text{AIC}_{Rice} - \text{AIC}_{Rayleigh} > \gamma$, we declare that the PU is present, otherwise, we declare the PU is absent. The threshold expression depends only on $P_{FA,DAD}$ and is given in the following section.

V. THRESHOLD DERIVATION

A theoretical probability of false alarm will be derived in this section. The analytical results will be compared with simulation results to confirm the theoretical expression of thresholds and probabilities of false alarm.

Since spectrum sensing is actually a binary hypothesis test, the performance we focus on is the probability for identifying the signal when the PU is absent. We will derive in this section a closed-form expression of $P_{FA,DAD}$. According to the sensing steps in (22), the false alarm occurs when the estimated decision $\Upsilon_{DAD}(\mathbf{x})$ is smaller than γ given that the PU is absent.

According to the presented sensing scheme, the false alarm probability for DAD detector can be expressed as

$$P_{FA,DAD} = Pr\left(W_{Rice} - W_{Rayleigh} > \gamma | \text{H}_0\right) \qquad (23)$$

According to AIC values for Rice and Rayleigh given in (17) and (18), we have

$$P_{FA,DAD} = Pr\left(\frac{\exp\left(L_{Rice}\right) - e\exp\left(L_{Rayleigh}\right)}{\exp\left(L_{Rice}\right) + e\exp\left(L_{Rayleigh}\right)} > \gamma \bigg| \text{H}_0\right) \qquad (24)$$

where $e = \exp(1)$.

$$P_{FA,DAD} = Pr\left(\left(\prod_{i=1}^{p} x_i\right)^{\frac{1}{2}} < \frac{1+\gamma}{e(1-\gamma)} \frac{\exp\left(\frac{p}{2} - \frac{pv^2}{\sigma^2} - \frac{pv^2}{2\sigma^2}\right)}{\left(\frac{2\pi v}{\sigma^2}\right)^{\frac{p}{2}}}\middle| H_0\right)$$

$$= Pr\left(\prod_{i=1}^{p} x_i < \left(\frac{1+\gamma}{1-\gamma}\right)^2 \left(\frac{2\pi v}{\sigma^2}\right)^{-p} \exp\left(p - \frac{3pv^2}{\sigma^2} - 2\right)\middle| H_0\right) \qquad (26)$$

Using the probability density function for the Rayleigh distribution and Rician distribution, we obtain

$$P_{FA,DAD} = Pr\left(\frac{e - \exp\left(-\frac{pv^2}{2\sigma^2}\right)\prod_{i=1}^{p} I_0\left(\frac{x_i v}{\sigma^2}\right)}{e + \exp\left(-\frac{pv^2}{2\sigma^2}\right)\prod_{i=1}^{p} I_0\left(\frac{x_i v}{\sigma^2}\right)} > \gamma\middle| H_0\right)$$
(25)

Using now I_0 expression $\left(I_0\left(\frac{x_i v}{\sigma^2}\right) = \frac{\exp\left(\frac{x_i v}{\sigma^2}\right)}{\sqrt{2\pi \frac{x_i v}{\sigma^2}}}\right)$, we obtain (26).

At hypothesis H_0, the distribution of the received signal is assumed as a Gaussian distribution. Therefore, the distribution of the envelope of this signal is Rayleigh. Therefore, we can find that $\frac{pv^2}{\sigma^2} \rightarrow 0$. If we introduce the Rician K-factor defined as the ratio of signal power in dominant component σ^2 over the (local-mean) scattered power v, the false alarm probability of the DAD detector can be approximated as

$$P_{FA,DAD} = Pr\left(\prod_{i=1}^{p} x_i < \left(\frac{1+\gamma}{1-\gamma}\right)^2 (4\pi K)^{-p} \exp\left(p - 2\right)\middle| H_0\right)$$
(27)

Applying now the distribution of the product of p independent Rayleigh random variables [7], the product $\prod_{i=1}^{p} x_i$ satisfies the distribution of p independent Rayleigh random variables represented by its CDF [7] given by:

$$F(t) = \left(2^p \sigma^{2p}\right)^{-\frac{1}{2}} t G_{1,p+1}^{p,1}\left(\left(2^p \sigma^{2p}\right)^{-1} t^2 \middle| \begin{matrix}\frac{1}{2}\\ \frac{1}{2},...,\frac{1}{2},-\frac{1}{2}\end{matrix}\right)(28)$$

where G denotes the Meijer G-function [7] defined by:

$$G_{1,p+1}^{p,1}\left(u\middle| \begin{matrix}\frac{1}{2}\\ \frac{1}{2},...,\frac{1}{2},-\frac{1}{2}\end{matrix}\right) = \frac{1}{j2\pi}\int_L \frac{\left(\Gamma\left(\frac{1}{2}-s\right)\right)^{p-1}\Gamma\left(\frac{1}{2}+s\right)}{\Gamma\left(\frac{3}{2}+s\right)}u^{-s}ds$$
(29)

The contour L is chosen so that it separates the poles of the gamma products in the numerator. The Meijer G-function has been implemented in some commercial mathematical software packages. Finally, the probability of false alarm of the DAD algorithm can be approximated as

$$P_{FA,DAD} = F\left(\left(\frac{1+\gamma}{1-\gamma}\right)^2 (4\pi K)^{-p} \exp\left(p-2\right)\right) \quad (30)$$

or, alternatively, the threshold can be expressed as

$$\gamma = \frac{\sqrt{(4\pi K)^p F^{-1}(P_{FA,DAD})\exp\left(2-p\right)} - 1}{\sqrt{(4\pi K)^p F^{-1}(P_{FA,DAD})\exp\left(2-p\right)} + 1} \quad (31)$$

Note that Meijer's G-function is a standard built-in function in most of the well known mathematical software packages, such as Matlab which used in this work.

From (30), it is clear that the probability of false alarm is independent of noise variances σ^2. Therefore, the proposed sensing algorithm based on distribution analysis is robust in practical applications. This remark will be verified in the following section.

Now, we will present a comparison between simulation and analytical results to confirm the theoretical results given in previously. For the proposed detector the threshold is computed based on p (the length of PU received signal in samples) and $P_{FA,DAD}$ value. Table I shows the comparison results for the thresholds γ for the DAD detector with $P_{FA} = 0.05$ and for $P_{FA,DAD}$ using different p values. In the presented results SNR $= -7$dB. One can find that, the simulation results are slightly lower than the analytical results. This is due to the approximation we have used during the derivation of $P_{FA,DAD}$ and γ for the presented detector. The presented table confirms the very good match between simulation and theoretic results.

		$p = 100$	$p = 150$	$p = 200$
Simulation results	$P_{FA,DAD}$	0.0571	0.0544	0.0502
	γ	0.9948	0.9814	0.9561
Analytical results	$P_{FA,DAD}$	0.0582	0.0563	0.0529
	γ	0.9965	0.9907	0.9614

TABLE I
SIMULATION AND ANALYTICAL RESULTS COMPARISON.

VI. PERFORMANCES EVALUATION

Actual sensing results and performance studies will be provided in this section. The evaluation framework for all simulations has been implemented in Matlab and all results are obtained as the average of a number of Monte Carlo simulations. For the Monte Carlo simulation, each signal block consists of one symbol which contains 2048 samples. 500 iterations are performed in the simulation. The primary system used is a Digital Television Broadcast-Terrestrial (DVB-T) system. The choice of the DVB-T PU system is justified by the fact that most of the PU systems utilize the OFDM modulation format. The channel models implemented are AWGN, Rician and Rayleigh channels. The latter two correspond to the two different types of propagation that have to be handled in practice, namely line-of-sight (LOS) and non-line-of-sight (NLOS). Slow fading is simulated by adding log-normal shadowing.

Three different scenarios with different properties have been chosen to evaluate the spectral detection performance, subject to provide different attributes so that the performance can be assessed under different conditions, aiming to provide fair conditions before making conclusions. OFDM is the modulation of choice for the three simulation scenarios to be used as

evaluation tools in this report. In OFDM, a wideband channel is divided into a set of narrowband orthogonal subchannels. OFDM modulation is implemented through digital signal processing via to the FFT algorithm. In scenario 1, we use a DVB-T OFDM signal in an AWGN channel. It is assumed that the detection performance in AWGN will provide a good impression of the performance, but it is necessary to extend the simulations to include signal distortion due to multipath and shadow fading. Scenario 2 utilizes the same DVB-T OFDM signal as scenario 1, but to make the simulations more realistic, the signal is subjected to Rayleigh multipath fading and shadowing following a log normal distribution in addition to the AWGN. The maximum Doppler shift of the channel is 100Hz and the standard deviation for the log normal shadowing is 10dB. Since the fading causes the channel to be time variant, it is necessary to apply longer averaging than in scenario 1 to obtain good simulation results. Thus the number of iterations in the Monte Carlo simulation is increased from 500 to 1000. The third simulation scenario utilizes also a DVB-T OFDM signal in Rician multipath fading with shadowing. The K-factor for the Rician fading is 10, which represents a very strong line of sight component. The maximum Doppler shift of the channel and the standard deviation for the log normal shadowing are the same as in the second scenario.

Now we will assess the performance of the proposed detector in terms of PU signal detection using the binary hypothesis test expressed in (22). The results from these simulations can be seen in the batch Fig. 1. The best performance is obtained from the CD detector. Subsequent to the CD detector is the proposed DAD detector, with approximately 2dB reduced performance compared to the CD, and ED detector, approximately 3dB behind CD. It is expected that if knowledge of signal parameters is provided, feature detectors are the optimal schemes for detecting the PU signal. From Fig. 1, we remark also that relative detection results for scenario 2 and scenario 3 are to a large extent aligned with the results for scenario 1. This is expected as the underlying used signals are the same. The main difference is in absolute performance which is caused by the addition of multipath and shadow fading.

VII. CONCLUSION

In this paper, we derived the exact threshold expressions of the distribution analysis based spectrum sensing using Kullback-Leibler distance. This is based on the Meijer G-function of the product of p independent Rayleigh random variables. Simulations using three different scenarios with different properties DVB-T PU systems were presented in order to verify the derived threshold value based on the probability of detection performance. It has been shown that analytical and empirical results are coincide with each other.

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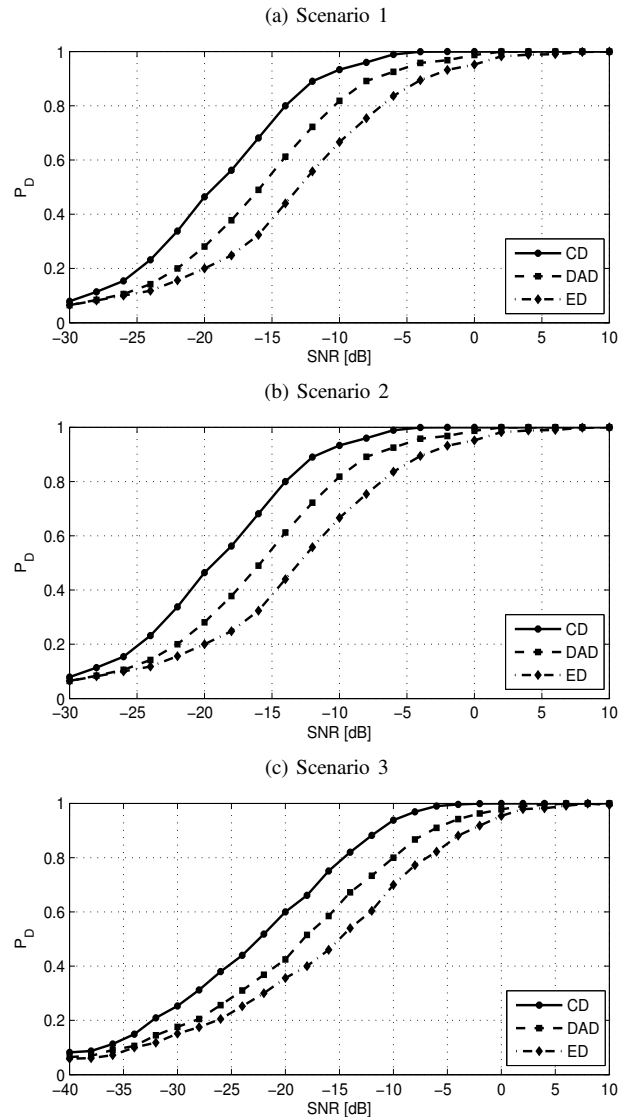(a) Scenario 1

(b) Scenario 2

(c) Scenario 3

Fig. 1. Performance evaluation of the DAD detector in terms of PU signal detection using an DVB-T OFDM PU system: Probability of detection versus SNR curves with $P_{FA} = 0.05$, sensing time = $1.12ms$ and $p = 2048$.

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