Maximum SINR Prefiltering for Reduced–State Trellis–Based Equalization

Uyen Ly Dang[1], Wolfgang H. Gerstacker[1], and Dirk T.M. Slock[2]
[1]Chair of Mobile Communications, University of Erlangen-Nürnberg, Cauerstrasse 7,
D-91058 Erlangen, Germany, {dang, gersta}@LNT.de
[2]EURECOM, Department of Mobile Comm., BP 193, 06904 Sophia Antipolis, France, dirk.slock@eurecom.fr

***Abstract*—We consider prefiltering for a single–carrier transmission over frequency–selective channels, where reduced–state trellis–based equalization is employed at the receiver, such as delayed decision–feedback sequence estimation (DDFSE) or reduced–state sequence estimation (RSSE). While previously proposed prefiltering schemes are based on the optimum filters of decision–feedback equalization (DFE), the prefiltering scheme introduced in this paper is designed according to a signal–to–interference–plus–noise ratio (SINR), whose definition takes into account explicitely the subsequent trellis–based equalizer and its complexity. In addition to the prefilter, a finite–length target impulse response for DDFSE/RSSE and an infinite–length feedback filter for state–dependent decision feedback in DDFSE/RSSE, respectively, is optimized. The developed solutions lend themselves to an interpretation of the tasks of the optimum filters. The presented numerical results show that noticeable gains can be achieved compared to state–of–the–art prefilters.**

I. Introduction

For a transmission with single–carrier modulation over frequency–selective channels producing intersymbol interference (ISI), optimum maximum–likelihood sequence estimation (MLSE) [1] is too complex for an implementation if higher–order modulation is employed and/or the length of the channel impulse response (CIR) is high. In such a case, suboptimum trellis–based equalization with a reduced number of states might be adopted because it offers a very good tradeoff between performance and complexity.

In order to limit the required number of states of the Viterbi algorithm (VA), the CIR might be shortened by linear prefiltering. Various approaches have been proposed in the literature for the design of the prefilter. In [2], the prefilter has been chosen for channel memory truncation, i.e., the overall impulse response of the cascade of channel and prefilter approximates a desired impulse response (DIR) with an order which is lower than that of the channel and taps which are given by the first few channel taps. In [3], Falconer and Magee have shown how to optimize the prefilter coefficients and the DIR jointly according to a minimum mean–squared error (MMSE) criterion. By doing so, the approximation error can be reduced compared to the approach of [2]. However, the schemes of [2], [3] do not take into account a possible correlation of the noise after prefiltering which may lead to a performance degradation of the VA due to a metric mismatch, i.e., the Euclidean branch metrics of the VA are no longer adjusted to the true noise characteristics. The aim of the scheme proposed in [4] is to control also the noise correlation when shortening the CIR via prefiltering. However, for all known linear prefiltering schemes for channel shortening a coloring and enhancement of the noise is inevitable if the prefilter has to be designed for a significant shortening of the CIR corresponding to a drastic reduction of the complexity of the VA.

In order to avoid the mentioned shortcomings of channel shortening by linear prefiltering, the principles of trellis–based equalization and decision–feedback equalization (DFE) might be combined [5]. However, such a scheme is prone to error propagation if the feedback of decisions is done outside of the VA, using tentative decisions. This problem can be circumvented if state–dependent feedback is performed within the VA, exploiting the symbols of the surviving paths of the VA assigned to the trellis states as has been proposed by Duel-Hallen and Heegard [6], [7]. The resulting *delayed decision–feedback sequence estimation (DDFSE)* algorithm is characterized by an excellent tradeoff between performance and complexity. A further refinement of DDFSE has been introduced by Eyuboğlu and Qureshi [8] which is referred to as *reduced–state sequence estimation (RSSE)*. Here, an additional reduction of the number of states of the VA is accomplished by defining the trellis states via the index numbers of the subsets of a set partitioning of the signal constellation.

In [9], it has been shown for the GSM / Enhanced Data Rates for GSM Evolution (EDGE) system which employs 8–ary phase–shift keying (8PSK) modulation that the performance of MLSE can be closely approached with a DDFSE or an RSSE operating in a trellis with only a few states. However, this holds only for a proper prefiltering of the received signal because only the front part of the overall CIR can be exploited by DDFSE/RSSE. It is well known that the minimum–phase equivalent CIR is a suitable overall impulse response for DDFSE/RSSE corresponding to an allpass prefilter transforming the CIR into its minimum–phase version while leaving the noise characteristics unaffected. Significant research effort has been devoted to the problem of computing a finite impulse response (FIR) approximation of the allpass prefilter with a low computational complexity. For example, in [10] it has been proposed to employ the cascade of a discrete–time matched filter and a prediction–error filter. Motivated by the fact that the feedforward filter of an MMSE–DFE with infinite–length filters tends to the desired allpass filter corresponding to the optimum feedforward filter of a zero–

forcing (ZF) DFE for high signal–to–noise ratios (SNRs), also the feedforward filter of an FIR MMSE–DFE might be chosen for prefiltering, cf. e.g. [11]. In [12], it has been shown that for DDFSE, noticeable gains can be achieved by the MMSE–DFE prefilter for certain channels compared to an allpass prefilter. Approaches for a fast computation of the MMSE–DFE prefilter coefficients have been proposed in [13]–[15].

The aforementioned prefilters seem to be not optimally adjusted to DDFSE/RSSE because they have been originally designed for ZF–DFE and MMSE–DFE, respectively, neglecting the fact that not only the first tap of the prefiltered CIR is relevant for the detection performance but several consecutive taps whose number depends on the trellis definition of DDFSE/RSSE. In this paper, we propose a prefiltering scheme which offers an improved performance compared to the DFE–based prefilters and is better matched to the characteristics of DDFSE/RSSE. Our scheme is related to optimum filter design for fixed delay tree search with decision feedback for data storage channels according to [16] and an algorithm for maximum SNR prefiltering for multiple–input multiple–output (MIMO) systems introduced in [17]. It should be noted that the filter optimization in [16] was done for magnetic recording systems with special run–length constraints and a different detection scheme as considered in this paper. In [17] the focus lies on prefiltering for joint DDFSE/RSSE for reception of cochannel EDGE user signals employing a single–antenna receiver, whereas here we consider a single–user transmission and, also unlike [17], optimize also the target impulse response of equalization and the feedback filter used in metric calculations of DDFSE/RSSE. In contrast to the schemes of [16], [17] which are based on FIR filters, the limit case of an infinite–length prefilter and feedback filter, respectively, is studied. The corresponding results lend themselves to a clear interpretation of the tasks of the filters and offer further insight.

This paper is organized as follows. In Section II, the underlying system model for a single–carrier transmission over an ISI channel with DDFSE/RSSE in the receiver is described, and a filter design criterion matched to the characteristics of DDFSE/RSSE is introduced. In Section III, the optimum infinite–length prefilter and feedback filter and the optimum finite–length target response of the scheme, respectively, are derived. Numerical results for the proposed scheme are presented in Section IV which demonstrate that it is able to outperform previously proposed schemes.

Notation: $\mathcal{E}\{\cdot\}$, $*$, $(\cdot)^T$ and $(\cdot)^H$ denote expectation, convolution, transposition and Hermitian transposition, respectively. Bold lower case letters and bold upper case letters stand for column vectors and matrices, respectively. $[\mathbf{A}]_{m,n}$ denotes the element in the mth row and nth column of $\mathbf{A}$; $\mathbf{I}_X$ is the $X \times X$ identity matrix. $P(z)$ stands for the z–transform of a sequence $p[k]$. The correlation sequence of signals $p[k]$ and $q[k]$ and its z–transform are denoted as $\varphi_{pq}[\kappa] = E\{p[k]\, q^*[k-\kappa]\}$ and $\Phi_{pq}(z)$, respectively.

II. System Model

We consider a single–carrier transmission with linear modulation over a frequency–selective channel producing ISI. In discrete–time equivalent complex baseband representation, the received signal is given by

$$r[k] = \sum_{\kappa=0}^{q_h} h[\kappa]\, a[k-\kappa] + n[k], \qquad (1)$$

where $a[k]$ denote the independent, identically distributed (i.i.d.) symbols of the transmit sequence of variance σ_a^2 which are taken from a signal constellation $\mathcal{A}$, e.g. an M–ary PSK or quadrature amplitude modulation (QAM) constellation. The discrete–time CIR $h[k]$ of order q_h comprises the effects of transmit filtering, channel, receiver input filtering, and symbol–spaced sampling. $n[k]$ stands for additive white Gaussian noise (AWGN) of variance σ_n^2. The received signal is prefiltered by a two–sided infinite–length filter with transfer function $F(z) = \sum_{k=-\infty}^{+\infty} f[k]\, z^{-k}$.

The prefiltered signal is processed by a DDFSE or RSSE algorithm. For simplicity, we discuss only DDFSE in more detail[1]. The states of the reduced–state trellis diagram of DDFSE are defined as

$$\tilde{\boldsymbol{s}}_r[k] = [\tilde{a}[k-1]\ \tilde{a}[k-2]\ \ldots\ \tilde{a}[k-q_d]]^T, \qquad (2)$$

where $\tilde{a}[\cdot] \in \mathcal{A}$ are equalizer trial symbols. The number of states per time step is $Z = M^{q_d}$, $0 \le q_d \le q_h$. Here, the extreme cases of $q_d = 0$ and $q_d = q_h$ correspond to DFE and a full–state VA, respectively. The metric of the DDFSE trellis branch emerging from state $\tilde{\boldsymbol{s}}_r[k]$ with trial symbol $\tilde{a}[k]$ is given by [7]

$$\begin{aligned}\lambda(\tilde{a}[k], \tilde{\boldsymbol{s}}_r[k]) \;=\; & \Big| u[k] - \sum_{\kappa=0}^{q_d} d[\kappa]\, \tilde{a}[k-\kappa] - \\ & \sum_{\kappa=0}^{q_b} b[\kappa]\, \hat{a}[k-(q_d+1)-\kappa, \tilde{\boldsymbol{s}}_r[k]] \Big|^2 .\end{aligned} \qquad (3)$$

Here, $u[k]$ denotes the prefiltered received signal, $u[k] = f[k] * r[k]$, and $d[k]$ refers to the target impulse response of prefiltering of order q_d which is used for trellis definition. $b[k]$ is the causal impulse response of the feedback filter employed in metric calculations by which postcursor taps with delays higher than q_d can be taken into account properly, exploiting the contents $\hat{a}[k-\kappa, \tilde{\boldsymbol{s}}_r[k]]$ of the registers of the survivor paths of states $\tilde{\boldsymbol{s}}_r[k]$ via state–dependent decision feedback.

Commonly, $f[k]$ is chosen as the impulse response of the feedforward filter of a ZF–DFE or MMSE–DFE, and $d[k]$ and $b[k]$ are selected as the leading and backmost portion of the causal part of the prefiltered CIR, respectively. However, in general this seems to be not the optimum choice.

As a novel criterion for filter optimization, we consider the signal–to–interference–plus–noise ratio (SINR) seen by the

[1] The resulting filters are also directly applicable to RSSE.

DDFSE algorithm,

$$\mathrm{SINR} = \frac{\sigma_a^2 \sum\limits_{\kappa=0}^{q_d} |d[\kappa]|^2}{\mathcal{E}\{|e[k]|^2\}}, \quad (4)$$

where the error signal $e[k]$ is defined as

$$\begin{aligned} e[k] \quad = \quad & \sum_{\kappa=-\infty}^{+\infty} f[\kappa]\, r[k-\kappa] - \sum_{\kappa=0}^{q_d} d[\kappa]\, a[k-\kappa] - \\ & \sum_{\kappa=0}^{q_b} b[\kappa]\, a[k-(q_d+1)-\kappa], \end{aligned} \quad (5)$$

representing the difference of the signal after feedforward and feedback filtering from the desired signal $d[k] * a[k]$ of trellis–based equalization. For (5), perfect feedback in branch metric computations has been assumed corresponding to error–free symbols in the survivor path registers, as usual in the design of systems with decision feedback.

It should be noted that for the criterion (4) a possible loss in minimum Euclidean distance for trellis–based equalization compared to an ISI–free channel has been ignored. However, for low–to–moderate q_d this distance loss is expected to be small. For instance, a distance loss does not occur for $q_d = 1$, and the distance loss is limited to 2.3 dB for $q_d = 2$ and binary PSK even for a worst–case channel [18]. Furthermore, the effect of a possible correlation of $e[k]$ on the performance of trellis–based equalization with Euclidean metric has been not taken into account. However, it can be assumed that the performance degradation due to a metric mismatch is only slight for low–to–moderate q_d because an excessive noise correlation can be avoided by allowing a feedback filter in addition to the feedforward filter, similar to a DFE, cf. also the numerical results of Section IV.

III. Filter Optimization

In the following, filter optimization is performed in three steps. First, $f[k]$ is optimized for given $d[k]$ and $b[k]$. In the next step, $b[k]$ is also optimized resulting in a cost function which depends solely on $d[k]$ and is finally maximized.

A. Optimization of $f[k]$

For given $d[k]$ and $b[k]$, the optimum transfer function $F(z)$ can be directly calculated via MMSE filter theory, cf. e.g. [19], resulting in

$$F(z) = F_{\mathrm{LE}}(z)\,(D(z) + z^{-k_0}\, B(z)) \quad (6)$$

where $D(z) = \sum_{k=0}^{q_d} d[k] z^{-k}$, $B(z) = \sum_{k=0}^{q_b} b[k]\, z^{-k}$, $k_0 = q_d + 1$, and $F_{\mathrm{LE}}(z)$ is the transfer function of the optimum infinite–length MMSE linear equalizer [18],

$$F_{\mathrm{LE}}(z) = \frac{H^*(1/z^*)}{H(z)\, H^*(1/z^*) + \zeta}, \quad (7)$$

with $H(z) = \sum_{k=0}^{q_h} h[k] z^{-k}$ and $\zeta = \sigma_n^2/\sigma_a^2$.

B. Optimization of $b[k]$

For optimization of the SINR w.r.t. $b[k]$ we insert the optimum $f[k]$ according to (6) in (4) and consider only the denominator further, which can be written as

$$\mathcal{E}\{|e[k]|^2\} = \mathcal{E}\{|e_{\mathrm{LE}}[k] * d[k] + e_{\mathrm{LE}}[k-k_0] * b[k]|^2\}, \quad (8)$$

where $e_{\mathrm{LE}}[k]$ denotes the error signal of MMSE linear equalization,

$$e_{\mathrm{LE}}[k] = f_{\mathrm{LE}}[k] * r[k] - a[k], \quad (9)$$

whose autocorrelation sequence $\varphi_{e_{\mathrm{LE}} e_{\mathrm{LE}}}[\kappa]$ has a z–transform

$$\Phi_{e_{\mathrm{LE}} e_{\mathrm{LE}}}(z) = \frac{\sigma_n^2}{H(z)\, H^*(1/z^*) + \zeta}. \quad (10)$$

The task is now to find the optimum causal MMSE filter with order $q_b \to \infty$ minimizing $\mathcal{E}\{|e[k]|^2\}$ for given $d[k]$.[2] Invoking causal infinite impulse response (IIR) MMSE filter theory, cf. e.g. [19], and denoting the input signal of filter $b[k]$ and its desired output signal as $x[k] = e_{\mathrm{LE}}[k-k_0]$ and $m[k] = -\sum_{\kappa=0}^{k_0-1} d[\kappa]\, a[k-\kappa]$, respectively, the optimum filter can be obtained as follows. First, a spectral factorization [20] of $\Phi_{xx}(z)$ has to be performed, yielding

$$\Phi_{xx}(z) = \Phi_{e_{\mathrm{LE}} e_{\mathrm{LE}}}(z) = \Phi_{\min}(z)\, \Phi^*_{\min}(1/z^*), \quad (11)$$

with a z–transform $\Phi_{\min}(z) = \sum_{k=0}^{+\infty} \varphi_{\min}[k]\, z^{-k}$ corresponding to a causal, stable and minimum–phase sequence $\varphi_{\min}[k]$. In addition, $\Phi_{mx}(z)$ is needed which can be calculated to

$$\Phi_{mx}(z) = -z^{k_0}\, D(z)\, \Phi_{\min}(z)\, \Phi^*_{\min}(1/z^*). \quad (12)$$

Next, a z–transform

$$\begin{aligned} G(z) = \frac{\Phi_{mx}(z)}{\Phi^*_{\min}(1/z^*)} = -z^{k_0}\, D(z)\, \Phi_{\min}(z) = \\ -\sum_{\kappa=0}^{k_0-1} d[\kappa]\, z^{k_0-\kappa}\, \Phi_{\min}(z) \end{aligned} \quad (13)$$

is calculated which has to be decomposed according to

$$G(z) = G_+(z) + G_-(z), \quad (14)$$

where $G_+(z)$ and $G_-(z)$ correspond to the causal part and the strictly anticausal part of sequence $g[k]$, respectively, resulting in

$$G_+(z) = -\sum_{\kappa=0}^{k_0-1} d[\kappa]\, \Phi_{k_0-\kappa,\min}(z) \quad (15)$$

with

$$\Phi_{\mu,\min}(z) = \sum_{\nu=\mu}^{+\infty} \varphi_{\min}[\nu]\, z^{-\nu}, \quad \mu \in \{1, 2, \ldots, k_0\}. \quad (16)$$

[2] Feedback filters for DDFSE with $q_b \to \infty$ have been also considered in the original paper [7].

Finally, the optimum feedback filter transfer function is obtained as

$$B(z) = \frac{G_+(z)}{\Phi_{\min}(z)} = -\sum_{\kappa=0}^{k_0-1} d[\kappa]\, P_{k_0-\kappa}(z), \tag{17}$$

where

$$P_\mu(z) = \frac{\Phi_{\mu,\min}(z)}{\Phi_{\min}(z)}, \quad \mu \in \{1,\, 2,\, \ldots,\, k_0\} \tag{18}$$

can be easily identified as the optimum causal IIR transfer function of a μ–step forward predictor [21] for $e_{\rm LE}[k]$. Hence, the optimum feedback filter can be decomposed into a linear combination of prediction filters for $e_{\rm LE}[\cdot]$ with different prediction steps.

C. Optimization of $d[k]$

Using the optimum coefficients $f[k]$ and $b[k]$ according to (6) and (17), the error signal of (5) can be expressed as

$$e[k] = \sum_{\kappa=0}^{k_0-1} d[\kappa]\, w_{e,k_0-\kappa}[k-\kappa] \tag{19}$$

where $w_{e,k_0-\kappa}[k-\kappa]$ is the prediction error which occurs when $e_{\rm LE}[k-\kappa]$ is predicted by a causal IIR $(k_0-\kappa)$–step forward predictor from the values $e_{\rm LE}[k-\mu]$, $\mu \in \{k_0,\, k_0+1,\, \ldots\}$,

$$w_{e,k_0-\kappa}[k-\kappa] = e_{\rm LE}[k-\kappa] - p_{k_0-\kappa}[k] * e_{\rm LE}[k-k_0]. \tag{20}$$

Hence, the error variance can be compactly written as

$$\mathcal{E}\{|e[k]|^2\} = \mathbf{d}^H\, \mathbf{\Phi}_{w_e w_e}\, \mathbf{d}, \tag{21}$$

with

$$\mathbf{d} = [d^*[0]\, d^*[1]\, \ldots\, d^*[k_0-1]]^T \tag{22}$$

and the autocorrelation matrix of the prediction error vector

$$\mathbf{w}_e[k] = [w_{e,k_0}[k]\, w_{e,k_0-1}[k-1]\, \ldots\, w_{e,1}[k-k_0+1]]^T, \tag{23}$$

$$\mathbf{\Phi}_{w_e w_e} = \mathcal{E}\{\mathbf{w}_e[k]\, \mathbf{w}_e^H[k]\}. \tag{24}$$

The elements of $\mathbf{\Phi}_{w_e w_e}$ are obtained via inverse z–transform as

$$[\mathbf{\Phi}_{w_e w_e}]_{m,n} = \frac{1}{2\pi j} \oint z^{n-m}\, (1 - z^{-(k_0-m)} P_{k_0-m}(z)) \cdot (1 - z^{(k_0-n)} P^*_{k_0-n}(1/z^*))\, \Phi_{e_{\rm LE}e_{\rm LE}}(z)\, \frac{1}{z}\, {\rm d}z, \quad m,n \in \{0,\, 1,\, \ldots, k_0-1\}, \tag{25}$$

with the prediction–error filter transfer functions

$$(1 - z^{-(k_0-\kappa)} P_{k_0-\kappa}(z)) = \frac{\sum_{\mu=0}^{k_0-1-\kappa} \varphi_{\min}[\mu]\, z^{-\mu}}{\Phi_{\min}(z)}. \tag{26}$$

Alternatively, $\mathbf{\Phi}_{w_e w_e}$ can be closely approximated by using the fact that the filter impulse responses $p_{k_0-\kappa}[k]$ are stable and its coefficients are negligible for $k > q_p$ with a suitably chosen q_p. Hence,

$$\mathbf{\Phi}_{w_e w_e} = [\mathbf{I}_{k_0} - \mathbf{P}^H]\, \mathbf{\Phi}_{e_{\rm LE}e_{\rm LE}}\, [\mathbf{I}_{k_0} - \mathbf{P}^H]^H \tag{27}$$

with

$$\mathbf{P} = [\mathbf{p}^*_{k_0}\, \mathbf{p}^*_{k_0-1}\, \ldots\, \mathbf{p}^*_1], \tag{28}$$

$$\mathbf{p}_{k_0-\kappa} = [p_{k_0-\kappa}[0]\, p_{k_0-\kappa}[1]\, \ldots\, p_{k_0-\kappa}[q_p]]^T \tag{29}$$

and the autocorrelation matrix $\mathbf{\Phi}_{e_{\rm LE}e_{\rm LE}}$ of $e_{\rm LE}[k]$ of size $(k_0+q_p+1) \times (k_0+q_p+1)$.

Finally, with (21) the SINR (4) is rewritten as

$${\rm SINR} = \frac{\sigma_a^2\, \mathbf{d}^H\, \mathbf{d}}{\mathbf{d}^H\, \mathbf{\Phi}_{w_e w_e}\, \mathbf{d}} \tag{30}$$

and can be maximized via the Rayleigh–Ritz Theorem [22], resulting in the eigenvalue problem

$$\mathbf{\Phi}^{-1}_{w_e w_e}\, \mathbf{d}_{\rm opt} = \frac{{\rm SINR}_{\max}}{\sigma_a^2}\, \mathbf{d}_{\rm opt}. \tag{31}$$

Hence, the optimum vector $\mathbf{d}_{\rm opt}$ is identical to the eigenvector of $\mathbf{\Phi}^{-1}_{w_e w_e}$ corresponding to the maximum eigenvalue which yields the (normalized) maximum SINR, ${\rm SINR}_{\max}/\sigma_a^2$.

D. Discussion and Special Cases

The computational complexity of the proposed prefiltering scheme is mainly governed by the spectral factorization according to (11), the determination of the elements of matrix $\mathbf{\Phi}_{w_e w_e}$ according to (25) or (27), and the solution of the eigenvalue problem (31).

Because the optimum feedforward filter can be decomposed into the cascade of a linear equalizer and a system $D(z) + z^{-k_0}\, B(z)$, it first eliminates the ISI and then reintroduces ISI in a controlled manner according to the second factor of its transfer function (6). This ISI is partially taken into account for state definition of trellis–based equalization and partially cancelled by per–survivor processing within the VA using the feedback filter coefficients. The total error signal can be expressed as a linear combination of prediction errors corresponding to prediction of the error signal of linear equalization with different steps, cf. (19), where the combining coefficients $d[\kappa]$ are determined for a maximum SINR. It should be noted that the error signal contains also residual ISI as typical for MMSE filtering.

For the case of a target impulse response of order zero, $q_d = 0$, $k_0 = 1$, (21) simplifies to

$$\mathcal{E}\{|e[k]|^2\} = |d[0]|^2\, \mathcal{E}\{|w_{e,1}[k]|^2\}, \tag{32}$$

and (26) yields

$$(1 - z^{-1} P_1(z)) = \frac{\varphi_{\min}[0]}{\Phi_{\min}(z)}, \tag{33}$$

i.e., only a one–step prediction–error filter is required for filter calculations. Furthermore, (33), (11), and (25) result in

$$\mathcal{E}\{|w_{e,1}[k]|^2\} = |\varphi_{\min}[0]|^2, \tag{34}$$

where $|\varphi_{\min}[0]|^2$ is obtained as [20]

$$|\varphi_{\min}[0]|^2 = \exp\left(\int_{-1/2}^{1/2} \ln\left(\frac{\sigma_n^2}{|H({\rm e}^{j2\pi f})|^2 + \zeta} \right) {\rm d}f \right). \tag{35}$$

Finally, we can calculate the SINR as

$$\begin{aligned}\text{SINR} &= \frac{\sigma_a^2\,|d[0]|^2}{|d[0]|^2\,|\varphi_{\min}[0]|^2} \\ &= \exp\left(\int_{-1/2}^{1/2} \ln\left(\frac{\sigma_a^2\,|H(\mathrm{e}^{j2\pi f})|^2}{\sigma_n^2}+1\right)\,\mathrm{d}f\right), \quad (36)\end{aligned}$$

which is the well–known SINR of MMSE–DFE [23], as was to be expected. Of course, the SINR is independent of $d[0]$ which causes only a signal scaling, and the feedforward and feedback filter are equivalent to the feedforward and feedback filter of MMSE–DFE, respectively.

The involved prediction errors for $q_d > 0$ are in general not white, in contrast to the white one–step prediction error of MMSE–DFE ($q_d = 0$). This holds because via (26) and (11), the power spectral density of a $(k_0 - \kappa)$–step prediction error can be written as $|\sum_{\mu=0}^{k_0-1-\kappa} \varphi_{\min}[\mu]\,\mathrm{e}^{-j2\pi f\mu}|^2$.

We note that for a scheme without feedback filtering, matrix $\mathbf{P}$ in (27) has to be replaced by an all–zero matrix and $\mathbf{\Phi}_{w_e w_e}$ reduces to the $k_0 \times k_0$ autocorrelation matrix of $e_{\mathrm{LE}}[k]$. Thus, the solution of Falconer and Magee [3] is obtained in this case.

IV. Numerical Results

For numerical results the SINR is evaluated for different selected channels. The proposed SINR maximizing prefiltering scheme (SINR–max) is compared to prefiltering with a ZF–DFE and MMSE–DFE feedforward filter, respectively, at a signal–to–noise ratio (SNR) of $\mathrm{SNR} = \frac{\sigma_a^2}{\sigma_n^2} = 30\,\mathrm{dB}$. In the following, theoretical results for the SINR according to (4) are presented for different prefiltering schemes. It should be noted that for SINR results, only the SNR is relevant in addition to the CIR, but not the adopted modulation scheme. Fig. 1 shows the SINR versus q_d for a normalized impulse response of unit energy of order $q_h = 9$ with linearly increasing channel taps ($h[0] = \mathrm{const.}\cdot 1$, $h[9] = \mathrm{const.}\cdot 10$). In Fig. 2 the SINR comparison is given for a random realization of a mobile communications channel of channel order $q_h = 9$ with constant power delay profile[3], and Fig. 3 shows the performance for a test channel with $q_h = 6$, as given in [24].

For $q_d = 0$ it has been shown in Section III-D that the bank of μ–step forward predictors of the proposed prefilter reduces to a simple single one–step predictor, which turns the SINR-max prefilter into the MMSE–DFE feedforward filter. For a higher q_d, the SINR–max prefilter includes $q_d + 1$ μ–step predictors, with $\mu = \{1, 2, .., q_d + 1\}$, which enhances the SINR significantly, compared to the MMSE–DFE and ZF–DFE prefilter, which are not designed for maximization of the SINR according to (4), in contrast to the novel prefilter. Thus, significant gains of the novel prefilter are expected for reduced–state equalization. However, it is also anticipated that only a part of the SINR gain can be realized by trellis–based equalization because of a certain noise coloring introduced by the prefilter, cf. Fig. 4, where $|F(e^{j2\pi f})|^2$ is shown for the channel of Fig. 1 and different q_d.

[3]For different realizations of this channel, similar results have been obtained.

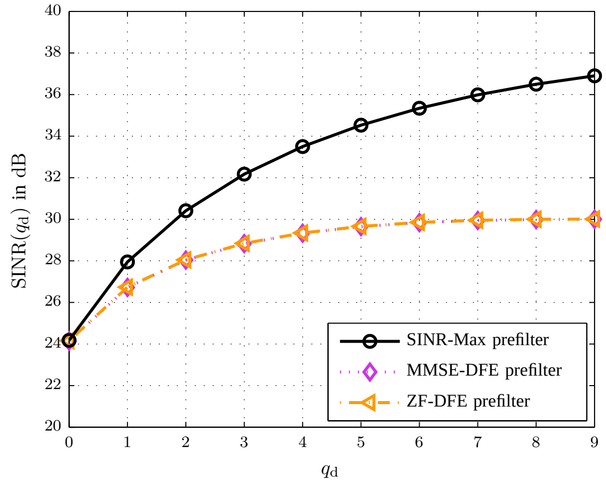

Fig. 1. SINR depending on q_d of the different prefiltering schemes for an impulse response with linearly increasing channel taps.

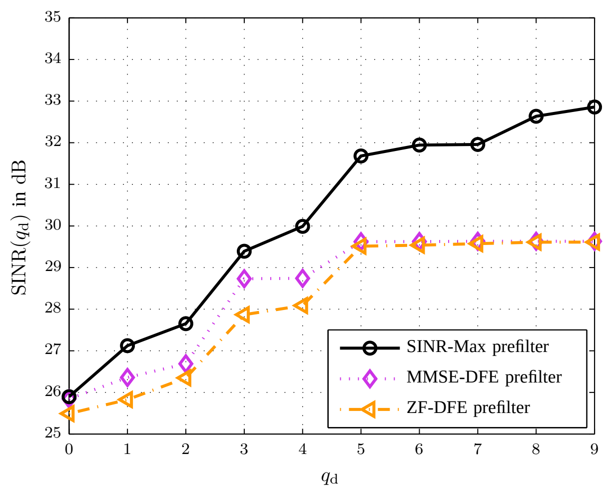

Fig. 2. SINR depending on q_d of the different prefiltering schemes for a random realization of an equalizer test channel.

V. Conclusions

We have introduced a novel prefiltering scheme for reduced–state trellis–based equalization which is better adjusted to the requirements of subsequent equalization than previous schemes. In particular, all filters are optimized according to a suitable SINR criterion. It has been shown that the prefilter can be represented as a cascade of a linear MMSE equalizer and a target system plus feedback filter, where the infinite–length causal feedback filter is a linear combination of multistep prediction filters with different steps and the coefficients of the target system result from an eigenvalue problem. In future work, the approach might be generalized to MIMO systems.

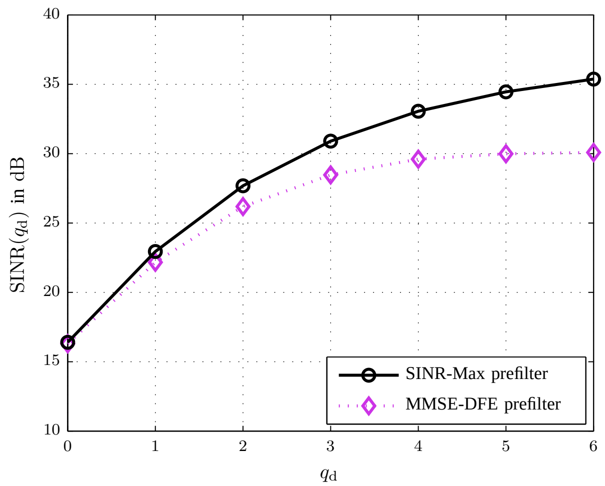

Fig. 3. SINR depending on q_d of the different prefiltering schemes for a test channel with $q_h = 6$ according to [24].

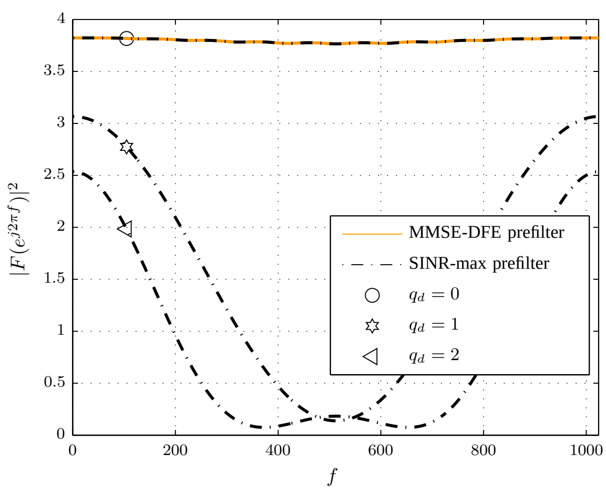

Fig. 4. $|F(e^{j2\pi f})|^2$ for the channel of Fig. 1 and different q_d.

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