Noname manuscript No.
(will be inserted by the editor)

Game Theory-based Resource Management Strategy for Cognitive Radio Networks

Bassem Zayen · Aawatif Hayar · Guevara Noubir

Received: date / Accepted: date

Abstract This paper investigates the application of game theory tools in the context of cognitive radio networks (CRN). Specifically, we propose a resource management strategy with the objective to maximize a defined utility function subject to minimize the mutual interference caused by secondary users (SUs) with protection for primary users (PUs). In fact, we formulate a utility function to reflect the needs of PUs by verifying the outage probability constraint, and the per-user capacity by satisfying the signal-to-noise and interference ratio (SNIR) constraint, as well as to limit interference to PUs. Furthermore, the existence of the Nash equilibrium of the proposed game is established, as well as its uniqueness under some sufficient conditions. Theoretical and simulation results based on a realistic network setting, and a comparison with a previously published resource management methods will be provided in this paper. The reported results demonstrate the efficiency of the proposed technique in terms of CRN deployment while maintaining quality-of-service (QoS) for the primary system.

Keywords Cognitive Radio · Resource Allocation · Game Theory · User Selection

Bassem Zayen
EURECOM, Mobile Communications Department
Sophia Antipolis, France
E-mail: zayen@eurecom.fr

Aawatif Hayar
GREENTIC-ENSEM University Hassan II
Casablanca, Morocco
E-mail: a.hayar@greentic.uh2c.ma

Guevara Noubir
College of Computer and Information Science
Boston, MA, USA
E-mail: noubir@ccs.neu.edu

1 Introduction

Cognitive radio (CR) is an emerging technology in wireless technology that uses software-defined radio to aim to the efficient use of the spectrum by exploiting the unused frequency bands at the time and space [1]. A look on the state of the art shows that CR research area is very open. A particularly problem in the context of CR, when we seek to optimize the secondary system capacity, is to guarantee a quality of service (QoS) to primary users (PUs) and a certain QoS to secondary users (SUs). There is a large number of proposals for all communication layers treating the increase of restrictions to spectrum utilization [2], but the QoS issue still has not been clearly defined. In addition, it is unclear how secondary system opportunism is compatible with the support of QoS for both, CR systems and primary systems. The U.S. Federal Communications Commission (FCC) proposed the concept of "*interference temperature*" as a way to have unlicensed transmitters sharing licensed bands without causing harmful interference [3] [4]. Rather than merely regulate transmitter power at fixed levels, as it has been done in the past, the scheme would have governed transmitter power on a variable basis calculated to limit the energy at victim receivers, where interference actually occurs. As a practical matter, however, the FCC abandoned the interference temperature concept recently [5] due to the fact that it is not a workable concept. While offering attractive promises, CRs face various challenges, starting from defining the fundamental performance limits of this radio technology, in order to achieve the capability of using the spectrum in an opportunistic manner. Specifically, CR is required to detect spectrum holes in the spectrum band and to determine if the spectrum allocation meets the QoS requirements of different users. This decision can be made by assessing the channel capacity, known as the most important factor for spectrum characterization.

The purpose of this paper is to present an analysis of the QoS problem along with a proposed solution, while maintaining a limited scope to provide coherency and depth. The QoS problem will be tackled in this work by proposing a resource management strategy based on game theory tools. The motivation behind doing so is that, in any case, the PU will not necessarily need all that multi-rate system. In fact, the PU will experience the SU's interference, and as long as all his target rate (depending on his QoS) to be achieved, he does not care about what he leaves more. In what follows, we adopt this setting and consider a CR network (CRN) in which primary and secondary users both attempt to communicate in a distributed way, subject to mutual interference. We propose a CR coordination that maximizes the CRN secondary rate while keeping the interference to the PU acceptable. Our goal is to realize PU-SU spectrum sharing by optimally allocating SU transmit powers, in order to maximize the total SU throughput under interference and noise impairments, and short term (minimum and peak) power constraints, while preserving the QoS of the primary system. In particular, it is of interest to determine the maximum number of SUs allowed to transmit threshold above which SUs can decide to transmit without affecting the PU's QoS. In such approaches, each user individually makes its decision on its transmit power so as to optimize its contribution to the system throughput. At the core of the concept lies the idea that the interference is more predictable when the network is dense, and consequently the resource allocation problem of a given user becomes more dependent to the average behavior, thus facilitating optimization.

Therefor, we propose in this paper a resource management strategy with the objective to maximize a defined utility function subject to minimize the mutual interference caused by SUs with protection for PUs. Specifically, we formulate a utility function to reflect the needs of PUs by verifying the outage probability constraint, and the per-user capacity by satisfying the signal-to-noise and interference ratio (SNIR) constraint, as well as to limit interference to PUs. Our contribution within this work is the investigation of the QoS issues from an outage point of view and using a game theory problem reformulation. Furthermore, the existence of the Nash equilibrium of the proposed game is established, as well as its uniqueness under some sufficient conditions.

The paper is organized as follows. In Section 2, we will provide a rather straightforward classification of resource allocation strategies attempting to show the diversity and advantages of these techniques. Two types of resource allocation strategies, centralized and distributed strategies, are discussed in this section. In Section 3, we will introduce the game theory approach and present the utility function to compute the transmitted power of each SU. Section 4 will introduce a number of theoretical concepts of importance. It will describe the CRN that will be used throughout this paper and present three resource management algorithms based on outage probability that will act as references when evaluating the proposed approach. In Section 5 the resource management algorithm based on game theory is presented. The existence of the Nash equilibrium of the proposed game is established in Section 6, as well as its uniqueness under some sufficient conditions. Simulation results and a comparison with methods presented in Section 4 are provided in Section 7, and Section 8 concludes the paper.

2 Resource Management Overview

In this paper we address the resource management problem in the context of CRN with special emphasis on QoS provisioning in a number of emerging broadband wireless networks. Specifically, depending on the choice of implementations, there are two approaches to allocate the spectrum resource. The first approach is based on a central controller that requires information about SUs and channel gains. This approach is referred as *centralized* solution. The second approach doesn't requires knowledge about the PU and SUs channels. This approach is so-called *distributed* solution. This section overviews the underlying standards and/or technologies and provides a literature review of related works on resource management and QoS provisioning in these broadband centralized and distributed systems.

The centralized resource allocation have been the main focus of some research efforts in CRNs. The authors in [6] for example derived a centralized power control method for the CRN to maximize the energy efficiency of the SUs and guarantee the QoS of both the PUs and the SUs. The feasibility condition was derived in [6] and a joint power control and admission control procedure was suggested such that the priority of the PUs is ensured all the time. However, in [6] only one CRN was considered. In [7], the authors considered spectrum sharing among a group of spread spectrum users with a constraint on the total interference temperature at a particular measurement point, and a QoS constraint for each secondary link. An optimization solution of this problem was proposed in [7] by using a game theory method. Specifically, the authors defined the secondary spectrum sharing problem as a potential game which takes different priority classes into consideration. Firstly, this game is solved through sequential play. Then a learning automata algorithm is introduced which only requires a feedback of the utility value. The same idea was proposed in [8], where the authors study a centralized auction mechanisms to allocate the received powers. They consider an objective function of maximizing utility which is a function of SINR. In [9] the authors tried to solve the centralized resource allocation problem by including a beamforming strategy. In this work, the primary systems are assumed to tolerate an amount of interference originating from secondary systems. This amount of interference is controlled by a pricing mechanism that penalizes the secondary systems in proportion to the interference they produce on the PUs. Two centralized optimization frameworks were proposed in [10] in order to solve for the optimal resource management strategies. In the first framework, authors determine the minimum transmit power that SUs should employ in order to maintain a certain SINR and use that result to calculate the optimal rate allocation strategy across channels. In the second framework, both transmit power and rate per channel are simultaneously optimized with the help of a bi-objective problem formulation. Though there have been ample research efforts on centralized resource management in CRNs, there is still a lack of a complete framework that considers QoS for SUs as well as resource management in a fair manner. One of the objective in this paper is to take a step towards such a solution.

In a realistic network, centralized system coordination is hard to implement, especially in fast fading environments and in particular if there is no fixed infrastructure for SUs. In fact, centralized channel state information for a dense network involves immense signaling overhead and will not allow the extraction of diversity gains in fast-fading channel components. To alleviate this problem, distributed methods were proposed in the literature where SUs can get rid of PU knowledge.

A number of distributed resource allocation strategies for CRNs have been proposed in literature. In addition to the two centralized frameworks presented in last section, the authors in [10] designed a distributed suboptimal joint coordination and power control mechanism to allocate transmit powers to SUs. A lower bound on SINR is used as a QoS constraint for SUs. In [11], the authors propose a game theoretic framework to analyze the behavior of CRs for distributed adaptive channel allocation. They define two different objective functions for the spectrum sharing games, which capture the utility of selfish users and cooperative users, respectively. The channel allocation problem is modeled in [11] to a potential game which converges to a deterministic Nash equilibrium channel allocation point. Game theory was applied in [12] to develop a distributed power allocation algorithm. In this work, each user maximizes its own utility function (which includes a pricing term) by performing a single-user price-based water-filling. However, in [12], coexistence of multiple SUs in a channel has not been considered. Also, the QoS requirement of SUs has been ignored. In [13], the authors studied the distributed multi channel power allocation for spectrum sharing CRNs with QoS guarantee. They formulate the problem as a noncooperative game with coupled strategy space to address both the co-channel interference among SUs and the interference temperature regulation imposed by primary systems. The authors in [14] presented a general analytical framework, in which SU's rate, frequency, and power resource can be jointly optimized under the interference temperature constraints. This framework was used to design an optimal distributed resource allocation algorithm with low polynomial time complexities in multiuser broadband CRNs. In [15], the authors focus on designing distributed resource allocation algorithms for cooperative networks. They proposed two share auction mechanisms, the SNR auction and the power auction, to distributively coordinate the relay power allocation among users. The authors in [15] demonstrate that the SNR auction achieves the fair allocation, while the power auction achieves the efficient allocation. In [16], the authors propose a distributed resource allocation scheme where SUs are penalized for interfering on the primary systems. The penalty is proportional to the interference rate produced from the secondary transmitter to each PU. This mechanism is referred to as pricing and is interpreted as introducing the effect of disturbance created from a user as a penalty measure in his utility function. In this means, the secondary transmitters can be controlled to choose their transmission strategies satisfying soft interference constraints on the PUs. In [17], this model of exogenous prices is used to analyze a noncooperative game between the SUs.

3 Game Theory Tools

In this section, we will provide the problem reformulation and will introduce the game model by defining the utility function to compute the transmitted power of each SU. Game theory was at first a mathematical tool used for economics, political and business studies. It helps understand situations in which decision-makers interact in a complex environment according to a set of rule [18]. Many different types of game exists which are used to reflect to analyzed situation for example potential games, repeated game, cooperative or noncooperative games. In the cognitive radio network (CRN), the formal game model for the power control can be defined as follows:

- Players: are the cognitive users (secondary users (SUs)).
- Actions: called also as the decisions, and are defined by the transmission power allocation strategy.
- Utility function: represents the value of the observed quality-of-service (QoS) for a player, and is defined later in this section.

The central idea in game theory is how the decision from one player will affects the decision-making process from all other players and how to reach a state of equilibrium that would satisfy most of the players. A well known contributor in the field is Nash for the Nash equilibrium [19]. The theory shows that you can reach a state equilibrium for your system where all decisions are set, unchanging and is the best possible situation for the players.

CR need to perform sophisticated adaptation and dynamically learn from the environment. This situation makes the learning process a very complicated one comparable to situation found in economics. Game theory is already used in other field of communication to better understand for example congestion control, routing, power control, topology control and trust management [20]. Our interests rest in its use for power control as it can be considered a game with fixed number of players where each tries to optimize their power levels. There are a number of properties that makes this problem appropriate for a cognitive radio game model:

- The player's payoff is a function of her own transmit power level and her signal-to-noise and interference ratio (SINR). The player's SINR is a function of her own transmit power and the transmit powers of the other players in the cell.
- When a player increases her power level, this will increase her own SINR, but will decrease the SINRs of all other players.
- For a fixed SINR, the players prefer lower power levels to higher ones. That is, players wish to conserve power and extend their battery life when possible.
- For a fixed power level, players prefer higher SINR to lower one. That is, players want the best possible channel conditions for a given expenditure of power.

There are many ways to cope with these issues such as to add restriction to the use of the power resource by charging it to users. This is done by adding a cost component to the *payoff function* to add fairness to the network. Another idea is to model the scenario as a repeated game [20].

In this paper we formulate the problem of resource allocation in the context of a CRN to reflect the needs of PUs and SUs. We consider the primary uplink of a single CRN, where cognitive transmitters transmit signals to a number of SUs, while the primary BS receives its desired signal from a primary transmitter and interference from all the cognitive transmitters.

To resolve the problem of resource allocation, we propose a *utility function* that meets the objective to maximize the SUs capacity, and the protection for PUs. Specifically, we define a *payoff function* that represents the SNIR constraint, and a *price function* specifies the outage probability constraint. The *utility function* is defined as:

$$utility\ function = payoff\ function - price\ function$$

We introduce a *payoff* to express the capacity need of SU m, and a *price function* to represent the protection for PUs by means of the outage probability. And each SU adjusts its transmitted power to maximize its *utility function*. Therefore, we will present in this paper a power allocation algorithm that maximize the defined *utility function* to compute the transmitted power of each SU.

4 Primary Outage-based Resource Allocation

In this section we will present the context of this study. We will start by the channel model and the primary/secondary performance metrics. Then, we will present three resource allocation algorithms based on outage probability.

Consider the uplink of a CRN that consists of a PU, a base station (BS), and M pairs of SUs randomly distributed over the system [1]. The channel gains are i.i.d random variable. Throughout this paper, we will use the following notation:

- the index of SUs m lies between 1 and M,
- $h_{l,m}$ denotes the channel gain from SU l to the desired user m,
- the data destined from SU m is transmitted with power p_m and a maximum power P_{max},
- $h_{pu,m}$ denotes the channel gain from the PU indexed by pu to the desired user m,
- the data destined from the primary system is transmitted with power p_{pu}.

In the coverage area of the primary system, there is an *interference boundary* within which no SUs can communicate in an ad-hoc manner. Thus, as can be seen in Fig. 1, for the impairment experienced by the primary system to be as small as possible, a SU must be able to detect very reliably whether it is far enough away from a primary base station, i.e., in the area of possible cognitive radio operation. The expression of the PU instantaneous capacity is

$$C_{pu} = \log_2 \left(1 + \frac{p_{pu}|h_{pu,pu}|^2}{\sum_{m=1}^{M} p_m |h_{m,pu}|^2 + \sigma^2} \right) \tag{1}$$

where σ^2 is the ambient noise variance. On the other hand, by making SUs access the primary system spectrum, the m^{th} SU experiences interference from the PU and all neighboring co-channel SU links that transmit on the same band. Accordingly, the m^{th} SU instantaneous capacity is given by:

$$C_m = \log_2 (1 + \mathrm{SINR}_m) \tag{2}$$

where

$$\mathrm{SINR}_m = \frac{p_m|h_{m,m}|^2}{\sum_{\substack{l=1 \\ l\neq m}}^{M} p_l|h_{l,m}|^2 + p_{pu}|h_{pu,m}|^2 + \sigma^2} \tag{3}$$

SUs need to recognize their communication environment and adapt the parameters of their communication scheme in order to maximize the per-user cognitive capacity, expressed as

$$C_{sum} = \sum_{m=1}^{M} C_m \tag{4}$$

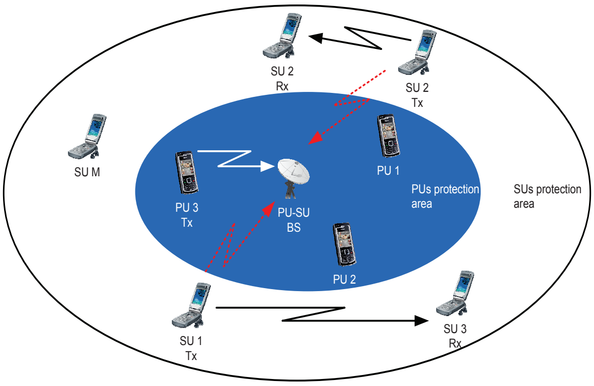

Fig. 1 The cognitive radio network with N primary users and M secondary users attempting to communicate with their respective pairs in an ad-hoc manner during a primary system transmission in uplink mode, subject to mutual interference.

while minimizing the interference to the PUs, in a *distributed* fashion. The sum here is made over the M SUs allowed to transmit [21] [22]. Moreover, we assume that the coherence time is sufficiently large so that the channel stays constant over each scheduling period length. We also assume that SUs know the channel state information (CSI) of their own links, but have no information on the channel conditions of other SUs. No interference cancelation capability is considered. Power control is used for SUs both in an effort to preserve power and to limit interference and fading effects. The interference power (Intf) is given by:

$$\text{Intf}_m = \sum_{\substack{l=1 \\ l \neq m}}^{M} p_l |h_{l,m}|^2 + p_{pu} |h_{pu,m}|^2 + \sigma^2 \tag{5}$$

Combining (3) and (5), we define the SINR as a function of Intf:

$$\text{SINR}_m = \frac{p_m |h_{m,m}|^2}{\text{Intf}_m} \tag{6}$$

and

$$p_m = \frac{\text{SINR}_m \text{Intf}_m}{|h_{m,m}|^2} \tag{7}$$

The protection for PU must be guaranteed in a CRN. This protection is guaranteed if the sum of all SUs transmitters' powers is not larger than the interference constraint P_T. Then, PU verifies his outage probability constraint. The interference constraint is given by:

$$\sum_{m=1}^{M} p_m |h_{pu,m}|^2 \leq P_T \tag{8}$$

and the notion of outage probability defined as the probability that the capacity of the user is below the transmitted code rate [23]. In the proposed framework, the outage probability can be expressed as [24]:

$$P_{out} \equiv Prob\{C_{pu} \leq R_{pu}\} \leq q, \tag{9}$$

where R_{pu} is the PU transmitted data rate and q is the maximum outage probability. The information about the outage failure can be carried out by a band manager that mediates between the primary and secondary users [2], or can be directly fed back from the PU to the secondary transmitters through collaboration and exchange of the CSI between the primary and secondary users as proposed in [25].

In this work, we will propose a resource management strategy based on outage probability. Specifically, we allow SUs to transmit simultaneously with the PU as long as the interference from the SUs to the PU that transmits on the same band remains within an acceptable range. We impose that SUs may transmit simultaneously with the PU as long as the PU in question does not have his QoS affected in terms of outage probability. We consider that PUs operate at a desired rate (depending on their respective QoS demands). Based on PU channel statistics, we determine the outage failure or in other words the probability that the PU of interest is actually under that rate. From a practical point of view the outage probability as well as the requested rate can be broadcasted before the start of the communication by the primary system, and it is used as a preamble for the PU to get informed which data rate is requested. This preamble can also be overheard by SUs who can then learn about these outage values. The proposed method guarantees also a certain QoS to SUs and ensures the continuity of service even when the detected spectrum holes become occupied by the PU, this is done by the outage probability control. Three resource management strategies were proposed in the literature using the outage probability control. A centralized algorithm was presented in [26,27]. The idea in [26] and [27] is to adopt a QoS guarantee to the PU by means of an outage constraint. This knowledge is obtained with a centralized mode where the resource allocation system would require information from a third party (i.e. central database maintained by regulator or another authorized entity) to schedule SUs coming. In fact, to compute the P_{out}, the CR system requires knowledge of the PU and SUs channels. In [21], the authors proposed a distributed manner to compute the outage probability without exchange of information between the primary and secondary users. In [28], the authors adopt the same framework as in [26] and [21] by using the outage probability as protection constraint for the PU. They proposed in [28] a centralized user selection strategy combined with an efficient transmit beamforming technique using a multiuser SU system. The proposed strategy tries to maximize the system throughput and to satisfy the SINR constraint, as well as to limit interference to the PU. The three algorithms presented in [26], [27] and [21] will serve as references when evaluating the performance of the game theory based resource management approach proposed in this paper.

5 Power Allocation Algorithm

We derive in this section the *utility function*: we define a *payoff function* specifies the SU capacity constraint and a *price function* that represents the interference constraint as a function of the outage probability constraint. Therefore, the *price function* is given by (2), and we will derive here the equation of the interference constraint P_T.

The margin of $P_T - \sum_{\substack{l=1 \\ l\neq m}}^{M} p_l |h_{pu,l}|^2$ is the maximum interference that SU m could generate under the description of (8). Divide $p_m|h_{pu,m}|^2$ by $P_T - \sum_{\substack{l=1 \\ l\neq m}}^{M} p_l |h_{pu,l}|^2$, we found the interference level expression:

$$\mathrm{L}_{\mathrm{Intf}m} = \frac{p_m |h_{pu,m}|^2}{P_T - \sum\limits_{\substack{l=1 \\ l\neq m}}^{M} p_l |h_{pu,l}|^2} \tag{10}$$

which is a normalized value. As long as this ratio $\in [0,1]$, the protection for PU is met. We compute now P_T as a function of the outage probability.

To proceed further with the analysis and for the sake of emphasis, we introduce the PU average channel gain estimate G_{pu} based on the following decomposition:

$$h_{pu,pu} \equiv G_{pu} * h'_{pu,pu} \tag{11}$$

where h'_{pupu} is the random component of channel gain and represents the *normalized* channel impulse response tap. This gives us the following PU outage probability expression in an interference-limited context:

$$\begin{aligned} P_{out} &= Pr\left\{ \log_2 \left(1 + \frac{p_{pu} G_{pu}^2 |h'_{pupu}|^2}{\sum\limits_{m=1}^{M} p_m |h_{m,pu}|^2} \right) \leq R_{pu} \right\} \\ &\simeq Pr\left\{ \frac{p_{pu} G_{pu}^2 |h'_{pu,pu}|^2}{\sum\limits_{m=1}^{M} p_m |h_{m,pu}|^2} \leq 2^{R_{pu}} - 1 \right\} \\ &\simeq Pr\left\{ |h'_{pu,pu}|^2 \leq \left(2^{R_{pu}} - 1\right) \left(\frac{\sum\limits_{m=1}^{M} p_m |h_{m,pu}|^2}{G_{pu}^2 p_{pu}} \right) \right\} \end{aligned} \tag{12}$$

From now on we assume for simplicity of analysis that the channel gains are i.i.d rayleigh distributed. However, the results can be immediately translated into results for any other channel model by replacing by the appropriate probability distribution function. Continuing from (12), we have:

$$P_{out} \simeq \int_0^{\left(2^{R_{pu}} - 1\right) \left(\frac{\sum\limits_{m=1}^{M} p_m |h_{m,pu}|^2}{G_{pu}^2 p_{pu}} \right)} \exp(-t) dt \tag{13}$$

Finally, we get the following outage constraint:

$$P_{out} \simeq 1 - \exp\left[-\left(2^{R_{pu}} - 1\right) \left(\frac{\sum\limits_{m=1}^{M} p_m |h_{m,pu}|^2}{G_{pu}^2 p_{pu}} \right) \right] \tag{14}$$

Replacing the interference constraint equation in (14), we can express the probability outage as:

$$P_{out} = 1 - \exp\left[-\left(2^{R_{pu}} - 1\right) \frac{P_T}{G_{pu}^2 p_{pu}} \right] \tag{15}$$

Then, the corresponding interference constraint is:

$$P_T = \frac{p_{pu} G_{pu}^2}{1 - 2^{R_{pu}}} \ln\left(1 - P_{out}\right) \tag{16}$$

We introduce now a *utility function* for which each SU adjusts its transmitted power in order to maximize it. It is composed of a *payoff function* expressed as the capacity C_m of the SU, and of a *price function* composed of the interference level to the PU and the power consumption.

Then, the *utility function* is expressed as follow:

$$U_m = C_m - \left(\frac{p_m |h_{pu,m}|^2}{P_T - \sum\limits_{\substack{l=1 \\ l\neq m}}^{M} p_l |h_{l,m}|^2} \right)^{a_m} \tag{17}$$

The parameter a_m is adjustable to have a comparable values, i.e. the *payoff function* value and the *price function* value. This parameter gives the flexibility needed to adjust the SU capacity over the interference to the PU. We choose $a_m < 0$. It could be easily obtained that the *price function* decreases as the ratio $\mathrm{L}_{\mathrm{Intf}m}$ increases. This fact is caused by the negative property of a_m.

Mathematically, the game G can be expressed as:

$$\text{Find } \; p_m|_{m=1,\ldots,M} = \arg\max_{p_m} U_m(p_m, \mathbf{P}_{-m}) \tag{18}$$

subject to:

$$\begin{cases} \sum_{m=1}^{M} p_m |h_{pu,m}|^2 \leq P_T \\ P_{out} \leq q \\ 0 \leq p_m \leq P_{max} \end{cases} \tag{19}$$

Recall that p_m denotes the strategy adopted by SU m and $\mathbf{P}_{-m} = (p_l)_{l \neq m, l \in \{1,...,M\}}$ denotes the strategy adopted by the other SUs. We replace the capacity by expression given by (2) and use (7) to obtain the following equation:

$$U_m = \log_2(1 + \text{SINR}_m) - \left(\frac{|h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^{M} p_l |h_{l,m}|^2} \right)^{a_m} \times \left(\frac{\text{SINR}_m \text{Intf}_m}{|h_{m,m}|^2} \right)^{a_m} \tag{20}$$

We are going to maximize the *utility function* in terms of the SINR, which is equivalent to the transmitted power. The solution of the system is found by calculating the derivatives of U_m with respect to the signal-to-noise and interference ratio parameters SINR_m:

$$\frac{\partial U_m}{\partial \text{SINR}_m} = \frac{1}{(1 + \text{SINR}_m) \ln 2} - \left(\frac{|h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^{M} p_l h_{l,m}} \right)^{a_m} \times a_m \left(\frac{\text{SINR}_m \text{Intf}_m}{|h_{m,m}|^2} \right)^{a_m - 1} \frac{\text{Intf}_m}{|h_{m,m}|^2} \tag{21}$$

We can express the solution of (21) as:

$$(1 + \text{SINR}_m) \, \text{SINR}_m^{a_m - 1} = \frac{1}{a_m \beta_m \ln 2} \tag{22}$$

where:

$$\beta_m = \left(\frac{|h_{pu,m}|^2}{P_T - \sum_{\substack{l=1 \\ l \neq m}}^{M} p_l |h_{l,m}|^2} \right)^{a_m} \left(\frac{\text{Intf}_m}{|h_{m,m}|^2} \right)^{a_m} \tag{23}$$

denoting the slope of the *price function*. Let $f(\text{SINR}_m) = (1 + \text{SINR}_m) \, \text{SINR}_m^{a_m - 1}$. Finally, we obtain the following set of equalities:

$$\text{SINR}_m = f^{-1} \left(\frac{1}{a_m \beta_m \ln 2} \right) \tag{24}$$

The maximization problem is dependent on a_m which is defined in the *utility function* as an adjustment parameter to the *price function*. For simulation results $a_m = -0.2$. It was chosen to stay with this value after different simulations to show its influence on the obtained results.

6 Existence and Uniqueness of the Nash Equilibrium

In the proposed game, each SU chooses an appropriate power to maximize its *utility function*. In this context, it is important to ensure the stability of the system. A concept which relates to this issue is the Nash equilibrium. As definition in [19], a pure strategy profile $\{p_l^*\}_{l \neq m, l \in \{1,...,M\}}$ is a Nash equilibrium of the proposed game if, for every player m (i.e. SU m):

$$U_m(p_m^*, \mathbf{P}_{-m}^*) \geq U_m(p_m, \mathbf{P}_{-m}^*), \quad \forall m \in \{1, ..., M\} \tag{25}$$

A Nash equilibrium can be regraded as a stable solution, at which none of the users has the incentive to change its power p_m.

6.1 Existence of the Nash Equilibrium

Theorem 1: Game *G* admits at least one Nash equilibrium.

proof: The conditions for the existence of Nash equilibrium in a strategic game are given in [29]:

1. The set P_m is a nonempty, convex, and compact subset of some Euclidean space for all m.
2. The *utility function* $U_m(p_m, \mathbf{P}_{-m})$ is continuous on *P* and quasi-concave on P_m.

According to the above description of the strategy space, it is straightforward to see that P_m is nonempty, convex and compact. Notice that $U_m(p_m, \mathbf{P}_{-m})$ is a linear function of either p_m, which means the second condition is satisfied. Hence, game *G* admits at least one Nash equilibrium.

6.2 Uniqueness of the Nash Equilibrium

Theorem 2: Game *G* always possesses a unique Nash equilibrium under the sufficient conditions.

proof: It's established in [30] that if the *utility function* $U_m(p_m) : (p_m)_{m \in \{1,...,M\}}$ is a standard function, then the Nash equilibrium in this game will be unique. A function $f(x)$ is said to be a standard function if it satisfies the following three properties [30]:

1. Positivity: $f(x) > 0$.
2. Monotonicity: If $x \geq x'$, then $f(x) \geq f(x')$.
3. Scalability: For all $\mu > 1$, $\mu f(x) \geq f(\mu x)$.

The positivity is obviously satisfied by adjusting parameter a_m.

Considering $p_m \geq p'_m$, we have

$$C_m(p_m) \geq C_m(p'_m) \tag{26}$$

Using the propriety that $a_m < 0$, we can obtain that

$$\left(\frac{p_m|h_{pu,m}|^2}{P_T - \sum\limits_{\substack{l=1\\l\neq m}}^{M} p_l|h_{l,m}|^2}\right)^{a_m} \leq \left(\frac{p'_m|h_{pu,m}|^2}{P_T - \sum\limits_{\substack{l=1\\l\neq m}}^{M} p_l|h_{l,m}|^2}\right)^{a_m} \quad (27)$$

According to (26) and (27), the monotonicity property is proved $\forall m \in \{1, ..., M\}$.

For all $\mu > 1$, it's got that:

$$\begin{aligned}\mu C_m(p_m) &= \mu \log_2 (1 + \text{SINR}_m) \\ &= \log_2 (1 + \text{SINR}_m)^{\mu} \\ &\geq \log_2 (1 + \mu\text{SINR}_m) = C_m(\mu p_m) \qquad (28)\end{aligned}$$

Since $a_m < 0$, we have also:

$$\begin{aligned}\left(\frac{\mu p_m|h_{pu,m}|^2}{P_T - \sum\limits_{\substack{l=1\\l\neq m}}^{M} p_l|h_{l,m}|^2}\right)^{a_m} &= \mu^{a_m} \left(\frac{p_m|h_{pu,m}|^2}{P_T - \sum\limits_{\substack{l=1\\l\neq m}}^{M} p_l|h_{l,m}|^2}\right)^{a_m} \\ &\leq \mu \left(\frac{p_m|h_{pu,m}|^2}{P_T - \sum\limits_{\substack{l=1\\l\neq m}}^{M} p_l|h_{l,m}|^2}\right)^{a_m} \qquad (29)\end{aligned}$$

Finally, according to (28) and (29) the scalability property is proved. Therefore, the proposed game *G* always possesses a unique Nash equilibrium.

7 Performance Evaluation

This section will provide a number of simulations aimed at assessing the performance of the proposed method in comparison with the reference methods presented in Section 4. To go further with the analysis, we resort to realistic network simulations. Specifically, we consider a CRN as described in Fig. 1 with one PU and M SUs attempting to communicate during a transmission, subject to mutual interference. A hexagonal cellular system functioning at 1.8 GHz with a primary cell of radius R meters and a primary protection area of radius R_p meters is considered. Secondary transmitters may communicate with their respective receivers of distances $d < R_p$ from the BS. Channel gains are based on the COST-231 path loss model including log-normal shadowing with standard deviation of 10 dB, plus fast-fading assumed to be i.i.d. circularly symmetric with distribution $\mathcal{CN}(0, 1)$ [31].

The performance of the proposed strategy is evaluated by Monte Carlo simulations ($IT_{max} = 10^4$). It is assumed that the maximum outage probability $q = 1\%$ for all outage probability-based algorithms. We considered also that the radius of the secondary cell $R = 1000$ meters and the radius of the primary protection area $R_p = 600$ meters. The derivation of the maximum number of SUs allowed to transmit using the game theory algorithm is based on the average estimation channel gain G_{pu}. From the locations of the users in the two-dimensional plane and the propagation characteristics of the environment, we can estimate this average channel gain. This value is estimated assuming a wireless ad hoc network affected by a large number of interferers.

7.1 Number of active SUs

In Fig. 2, the number of active SU links under the proposed algorithm as a function of the total number of users, for a target outage probability = 1%, tradeoff variable $a_m = -0.3$ and a rate = 0.3, is depicted. It can be seen from the figure that increasing the number of SUs yields improvements in the number of active users. Asymptotically, i.e., as the number of SUs goes large, the number of active SUs keeps constant due to the influence of interference impairments on the PU's QoS. We also compare the results obtained by the proposed method to those obtained using the distributed binary power allocation [21]. It can be observed that the proposed scheme allows almost 5 additional active SUs more than the binary power allocation scheme. As an example, we get 12 and 7 active SUs for 25 potential SUs for the proposed method and the one presented in [21], respectively.

7.2 QoS management

Our main contribution within this work is the QoS management of the CR system. The originality in the proposed method is that we guarantee a QoS to PU by maintaining the PU's outage probability unaffected in addition to a certain QoS to SUs and ensuring the continuity of service even when the spectrum sub-bands change from vacant to occupied. Thus by the outage probability control, if we have a vacant spectrum holes in the PU band, we set the outage probability $P_{out} = 1$ to exploit the available spectrum band by SUs, and if we have occupied sub-bands, the outage probability is set to $P_{out} = q$ depending on the PU's QoS.

In Table 1 we summarize the number of active SUs versus the maximum outage probability q (i.e. the absolute limit) ranging between 0.001 and 1. From this results, we remark that, increasing the maximum outage probability produces improvements in the number of active SUs: for a maximum outage probability equal to 3%, the number of allowed SUs

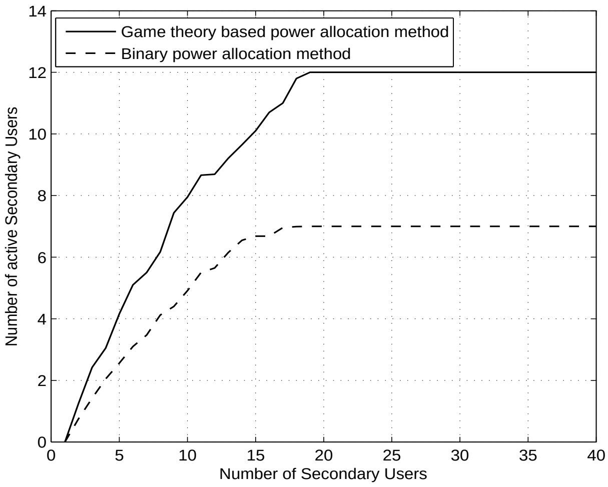

Fig. 2 Number of active SUs vs. number of SUs at rate = 0.3 bits/s/Hz, a tradeoff variable $a_m = -0.3$ and an outage probability = 1% in the uplink (the uplink distributed binary power allocation method and the proposed method).

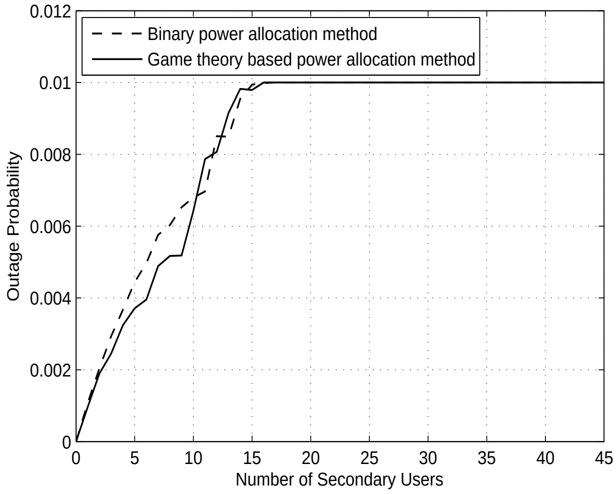

Fig. 3 The uplink outage probability as function of the number of SUs for a target outage probability = 1%, a tradeoff variable $a_m = -0.3$ and a rate = 0.3 bits/s/Hz (the uplink distributed binary power allocation method and the proposed method).

Table 1 Average of number of active SUs for different maximum outage probability values.

	$q = 1\%$	$q = 5\%$	$q = 10\%$	$q = 100\%$
Number of active SUs	1.5	6.02	6.9	7

Table 2 Average sum rate for different maximum outage probability values.

	$q = 1\%$	$q = 5\%$	$q = 10\%$	$q = 100\%$
Average sum rate (Mbit/s)	124.2	264.1	411.8	736.5

to transmit is equal to 5 and for $q = 100\%$, the number of active SUs is equal to the maximum number of SUs; as the number of SUs goes large, the number of active SUs keeps constant due to the influence of interference impairments on the PU's QoS. This tends to confirm the intuition from formula (15) where the number of active SUs is always upper-bounded by $\tilde{M}_{theory}$ in the distributed case, and PU outage probability protection given by the maximum outage q. From the presented results, we verified that we can maintain a QoS guarantee to the PU.

In order to validate our theoretical derivation, we also compare the outage probability defined in (15) for both the proposed method and the distributed binary power allocation method. As an example we carry out simulations at PU rate = 0.3 bits/s/Hz. First, it is clear from Fig. 3 that the outage probability using both schemes are similar. We also remark that, for the outage probability of interest, the number of allowed SUs to transmit is equal to 18 SUs. Now, how about the Nash equilibrium?

7.3 Power Control

The motivation behind the user selection technique is that, by opportunistically adapting their transmit power with the guide of the binary power allocation policy, SUs can maximize the achievable sum rate under the constraint of maintaining the outage probability of the PU not degraded. Therefore, we use a simple binary power allocation control: The power p_m of the m-th SU transmitter is selected from the binary set $\{0, P_{max}\}$. The proposed strategy tries in a first step to maximize the system throughput and to satisfy the signal-to-interference ratio (SIR) constraint. In the proposed user selection algorithm, SUs are first pre-selected to maximize the per-user sum capacity subject to minimize the mutual interference. Each SU verifies the SIR constraint and remains active or inactive during the next time slot: If the SU is active, he allowed to transmit with a power $p_m = P_{max}$, else $p_m = 0$.

7.4 The throughput

Table 2 presents the average sum rate computed for different values of maximum outage probability q. From Table 2 we remark that increasing maximum outage probability threshold increase in sum rate because the increase in degree of freedom more than compensates for the decrease in SINR due to interference. However, reaching a certain values of maximum outage probability threshold, the sum rate stabilizes. In addition, the current results claim that in CRN, when one attempts to maximize the number of active SUs (i.e. the outage probability threshold), the cognitive rate degrades asymptotically. Typically, there is a fundamental trade-off between sum rate maximization and number of active SUs maximization.

7.5 Nash equilibrium

In general, a Nash equilibrium is a profile of strategies such that each player's strategy is a best response to the other players' strategy. Thus, no player (i.e. SU) has the incentive to leave the Nash equilibrium, as a deviating action would imply a reduction of its own *utility function*. Therefore, the Nash equilibrium is a value for the game's stability. Hence, it can be seen as a lower limit for the QoS that can be guaranteed. As depicted in Fig. 3, depending on QoS to the PU, a unique Nash equilibrium is found. This is shown in the saturation state.

8 Conclusion

In this paper, we explored the idea of combining game theory with resource allocation in CRN to maximize the SU capacity while maintaining a QoS to the PU. Our contribution within this paper is to define a *utility/pricing* strategy that meets the objective to maximize the SUs capacity, and the protection for PUs by means of outage probability. Indeed, we discussed the existence of the Nash equilibrium of the proposed game, as well as its uniqueness. We demonstrated that the proposed game admits one and only one Nash equilibrium. Simulation results show that the proposed method exhibits a significant number of cognitive users able to transmit while minimizing interference to guarantee a QoS for the PU. We also compare the results obtained by the proposed method to those obtained using a binary power allocation method. The reported results demonstrate the efficiency of the proposed technique to maximize the SU rate while maintaining a QoS to PUs.

Acknowledgements

The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement SACRA (Spectrum and energy efficiency through multi-band cognitive radio) n°249060, and was partially supported by the Hassan II Foundation for the Moroccans residing abroad. Parts of this paper were presented at ICMCS 2011 [32].

References

1. J. Mitola. Cognitive radio for flexible mobile multimedia communications. In *Mobile Multimedia Communications (MoMUC)*, New York, Nov. 1999.
2. J. M. Peha. Approaches to spectrum sharing. *IEEE Communications Magazine*, 43(2):10–12, February 2005.
3. FCC. Cognitive radio technologies proceeding. Web Page, http://www.fcc.gov/oet/cognitiveradio/, 2003.
4. FCC. Spectrum access and the promise of cognitive radio technology. Web Page, ftp://ftp.fcc.gov/pub/Bureaus/Engineering_Technology/Documents/cognitive_radio/vanwazercognitive_radio_presentation.ppt, 2003.
5. Cognitive Radio Technologies Federal Communications Commission. http://hraunfoss.fcc.gov/edocs public/attachmatch/fcc−07−78a1.doc.
6. L. Qian, X. Li, J. Attia and Z. Gajic. Power control for cognitive radio ad hoc networks. In *IEEE Workshop on Local and Metropolitan Area Networks*, pages 07–12, Jun. 2007.
7. Y. Xing, C.N. Mathur, M.A. Haleem, R. Chandramouli and K.P. Subbalakshmi. Dynamic spectrum access with qos and interference temperature constraints. *IEEE Transactions on Mobile Computing*, 6:423–433, Apr. 2007.
8. J. Huang, R.A. Berry and M.L. Honig. Auction-based spectrum sharing. In *Mobile Networks and Applications*, volume 11, pages 405–418, 2006.
9. E. Jorswieck and R. Mochaourab. Beamforming in underlay cognitive radio: Null-shaping design for efficient nash equilibrium. In *International Workshop on Cognitive Information Processing*, Jun. 2010.
10. L. Akter and B. Natarajan. Qos constrained resource allocation to secondary users in cognitive radio networks. *Elsevier, Computer Communications*, Jun. 2009.
11. N. Nie and C. Comaniciu. Adaptive channel allocation spectrum etiquette for cognitive radio networks. In *New Frontiers in Dynamic Spectrum Access Networks*, 2005.
12. F. Wang, M. Krunz and S. Cui. Spectrum sharing in cognitive radio networks. In *IEEE Conference on Computer Communications*, pages 1885–1893, Jan. 2008.
13. Y. Wu and D.H.K. Tsang. Distributed power allocation algorithm for spectrum sharing cognitive radio networks with qos guarantee. In *IEEE INFOCOM*, 2009.
14. P. Cheng, Z. Zhang, H. Huang and P. Qiu. A distributed algorithm for optimal resource allocation in cognitive ofdma systems. In *IEEE International Conference on Communications*, May 2008.
15. J. Huang, Z. Han, M. Chiang and V. Poor. Auction-based distributed resource allocation for cooperation transmission in wireless networks. In *IEEE Global Communications Conference (GLOBECOM)*, Nov. 2007.
16. D. Schmidt, C. Shi, R. Berry, M. Honig and W. Utschick. Distributed resource allocation schemes. *IEEE Signal Process. Mag.*, 26(5):53–63, Sep. 2009.
17. J. S. Pang, G. Scutari, D. Palomar and F. Facchinei. Design of cognitive radio systems under temperature-interference constraints: A variational inequality approach. *IEEE Trans. Signal Process.*, 58(6):3251–3271, Jun. 2010.
18. M. J. Osborne. An introduction to game theory. *Oxford University Press*, Aug. 2003.
19. R. W. Rosenthal. A class of games possessing pure-strategy nash equilibria. *International Journal of Game Theory 2*, pages 65–67, 1973.
20. A. B. MacKenzie, L. Dasilva, and W. Tranter. Game theory for wireless engineers. *Morgan and Claypool Publishers*, 2006.
21. B. Zayen, M. Haddad, A. Hayar and G.E. Øien. Binary power allocation for cognitive radio networks with centralized and distributed user selection strategies. *Elsevier Physical Communication Journal*, 1(3):183–193, September 2008.

22. B. Zayen. Spectrum sensing and resource allocation strategies for cognitive radio. *Doctor of Electronic and Communications, TELECOM ParisTech*, Nov. 2010.
23. L. H. Ozarow, S. Shamai and A.D. Wyner. Information theoretic considerations for cellular mobile radio. *IEEE Trans. Veh. Technol.*, 43(5):359–378, May 1994.
24. Saad G. Kiani, Geir Egil Øien, and David Gesbert. Maximizing multi-cell capacity using distributed power allocation and scheduling. In *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, Hong Kong, China, March 2007.
25. A. Jovicic and P. Viswanath. Cognitive radio: An information-theoretic perspective. *IEEE Transactions on Information Theory*, 2006.
26. M. Haddad, A. Hayar and G.E. Øien. Uplink distributed binary power allocation for cognitive radio networks. In *CrownCom*, Singapore, Mai 2008.
27. M. Haddad, A. Hayar and G.E. Øien. Downlink distributed binary power allocation for cognitive radio networks. In *PIMRC*, Cannes, France, Sep. 2008.
28. B. Zayen, A. Hayar and G.E. Øien. Resource allocation for cognitive radio networks with a beamforming user selection strategy. *Asilomar*, Nov. 1-4 2009.
29. D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, Cambridge,MA, 1991.
30. Roy D. Yates. A framework for uplink power control in cellular radio systems. 13:1341–1347, 1995.
31. Urban transmission loss models for mobile radio in the 900 and 1800 MHz bands, 1991.
32. B. Zayen, A. Hayar and G. Noubir. Utility/pricing-based resource allocation strategy for cognitive radio systems. *ICMCS'11, 2nd International Conference on Multimedia Computing and Systems*, Apr. 7-9 2011.