On Enhancing Inter-user Spatial Separation for Downlink Multiuser MIMO Systems

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***Abstract*—In practical downlink multiuser MIMO channels, users' channels are non-orthogonal, which results in inter-user interference and in turn degrades system performance. In this paper, a new subspace-domain linear transmit preprocessing technique is proposed in enhancing users' spatial separation and suppressing inter-user interference. We show by simulations that sum rate is significantly improved and its saturation is avoided.**

I. Introduction

In downlink MU-MIMO channels, sum capacity can be significantly improved by allowing simultaneous transmission to multiple users and applying precoding to minimize inter-user interference. Aiming at maximizing the achievable data rate, a great number of linear precoding algorithms have been proposed. Block Diagonalization (BD) [1] is one popular alternative due to not only its low implementation complexity, but also its capability of approaching the sum capacity at high SNR. Regularized Block Diagonalization (RBD) [2], which is a generalized version of BD by relaxing the dimensionality constraint on the numbers of transmit and receive antennas, further improves the achievable sum rate at low and medium SNRs by taking noise suppression into account. At high SNR, however, both BD and RBD suffer from rate saturation when the number of transmit antennas is less than the total of receive antennas or the users are close to each other. Such a severe performance degradation is due to the incapability of linear precoding schemes in distinguishing user's transmission channel from other users' (i.e., interference) channels, which are overlapped without being spatially separable [3].

To counter the impact led by spatial inseparability, a natural solution is adopting user grouping, which divides users into several groups and serves each group per time slot. User grouping, however, will be degraded to TDMA when the numbers of transmit and per-user-receive antennas are comparable or any two users are spatially inseparable. Another alternative is Dominant Eigenmode Transmission (DET) but the spatial separation among the dominant eigenmodes is again not guaranteed. In this paper, we introduce a subspace reallocation method to enhance spatial separation among users' channels, which in turn improves their orthogonality and reduce inter-user interference. In particular, our algorithm identifies spatial overlapping among users in subspace domain and authentically suppresses inter-user interference by performing overlapped-subspace reallocation. Since our proposed approach redeploys transmission subspace without exerting any influence on other modules, it can be applied with various precoding algorithms and user scheduling algorithms for systems with limited number of users with spatial inseparability.

Notation: Matrices and vectors are represented as uppercase and lowercase letters, and transpose and conjugate transpose of a matrix are denoted as $(\cdot)^T$ and $(\cdot)^H$. We reserve $\mathcal{R}(\cdot)$[1] for range space and $\mathcal{N}(\cdot)$ for null space. We use $\langle\cdot\rangle$ to denote one representation of a matrix subspace. Further, $rank(\cdot)$ denotes the rank of a matrix, and $|\mathcal{O}|$ is the cardinality of the set $\mathcal{O}$.

II. Background

A. System Model

Consider the downlink of a K-user MIMO system with M_T transmit antennas at the Base Station (BS) and M_{R_i} receive antennas at the i-th user. We denote this MU-MIMO system as an $\{M_{R_1}, M_{R_2}, \cdots, M_{R_K}\} \times M_T$ system, where $\sum_{k=1}^{K} M_{R_k} = M_R$. Given an r_i-dimensional transmit vector $\mathbf{x}_i \in \mathbb{C}^{r_i \times 1}$, the received signal of user-i is expressed as

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{F}_i\mathbf{x}_i + \mathbf{H}_i \sum_{j=1,\ j\neq i}^{K} \mathbf{F}_j\mathbf{x}_j + \mathbf{n}_i, \quad i = 1, 2, \cdots, K, \quad (1)$$

where $\mathbf{H}_i$ is an $M_{R_i} \times M_T$ channel matrix between the BS and user-i, whose entries are complex Gaussian random variables with zero mean and unit variance, $\mathbf{F}_i \in \mathbb{C}^{M_T \times r_i}$ is the i-th user's precoding matrix, and $\mathbf{n}_i \in \mathbb{C}^{M_{R_i} \times 1}$ is the additive white Gaussian noise with zero mean and variance σ_n^2.

In general, users' channels are non-orthogonal (i.e., $\mathbf{H}_i\mathbf{H}_j^H \neq \mathbf{0}$) and therefore, inter-user interference inevitably exists for multiuser simultaneous transmission. Typical examples include: (1) $M_T < M_R$, which means that the degree-of-freedom cannot support interference-free transmission for all users; (2) users are located closely to one another, which renders their channels nearly the same. Under these situations, the desired signal and interference can hardly be separated from one another by linear precoding [4].

[1]For ease of notation, $\mathcal{R}(\mathbf{A})$ refers to for the column space of $\mathbf{A}$. Unless otherwise stated, the subspace we mentioned in the paper means column space.

B. Review on BD and RBD

We take BD and RBD as an illustrative example and review these precoding approaches from a subspace point of view.

Denote user-i's transmission and interference channels as $\mathbf{H}_i \in \mathbb{C}^{M_{R_i}\times M_T}$ and $\tilde{\mathbf{H}}_i = \left[\mathbf{H}_1^T \cdots \mathbf{H}_{i-1}^T \mathbf{H}_{i+1}^T \cdots \mathbf{H}_K^T\right]^T \in \mathbb{C}^{(M_R - M_{R_i})\times M_T}$, and further denote L_i and $\tilde{L}_i$ as their ranks. In principle, BD and RBD share a similar two-stage precoding mechanism in which for each user, inter-user interference is first suppressed and then its performance is optimized. However, while the former suppresses interference by nullifying the first $\tilde{L}_i$ spatial substreams and performing equal power allocation to the remaining $M_T - \tilde{L}_i$ ones, the latter minimizes both interference and noise by using an MMSE criterion such that the power allocated to each substream is inversely proportional to the interference level. From a subspace viewpoint, these precoding schemes can be interpreted by defining transmission and interference subspaces as the spaces occupied by the desired and interference channels, respectively. As for BD, transmission of the i-th user takes place in the intersection of the range space of its transmission channel $\mathbf{H}_i$ and the null space of its interference channel $\tilde{\mathbf{H}}_i$. For RBD, its interference subspace is also the range space of user-i's interference channel $\tilde{\mathbf{H}}_i$ but it takes all the available subspace as the transmission subspace to relax the dimensionality constraint.

In general, RBD performs as good as BD at high SNR and exhibits a superior performance at low and medium SNRs. When users' channels are spatial inseparable, however, it experiences the same degradation as BD because both precoding schemes cannot distinguish the transmission subspace from the interference one. A key observation behind BD and RBD using the subspace terminology is that all channel matrices are located in the whole space and their spanned subspaces (i.e., transmission and interference subspaces) are the subsets of the whole space. In the presence of non-orthogonality among users' channels, these subspaces are more or less overlapped. When the data transmits within the overlapped subspace, inter-user interference might heavily degrade the overall system performance. On the other hand, users can experience interference-free transmission if their data is transmitted in an non-overlapped subspace. Therefore, it would be interesting to take subspace overlapping as a measure of the level of spatial inseparability among users' channels and utilize the overlapped subspace in suppressing inter-user interference[2].

III. Subspace Reallocation Approach

We propose a new subspace reallocation approach that aims at enhancing spatial separability among users' channels. It consists of three stages, namely overlapped-subspace identification, subspace reassignment, and subspace reconstruction[3], which is outlined in Table I and described as follows.

[2]Distinctly, one can assign the overlapped subspace to a specific user or deploy TDMA. These alternatives, however, may respectively ignore the fairness consideration and suffer from a loss in spatial dimension.

[3]For the ease of description, we consider channel matrices of individual users to be with full rank $L_i = rank(\mathbf{H}_i) = \min\{M_T, M_{R_i}\}$, but it is easily generalized to rank deficient channels with $L_i < rank(\mathbf{H}_i)$.

Table I
Subspace Reallocation-based Precoding Algorithm

Initialization
for $i = 1 : K$
  $\mathbf{P}_i = \langle \mathbf{H}_i \rangle^{\perp} + \langle \tilde{\mathbf{H}}_i \rangle^{\perp} = \mathbf{U}_{\mathbf{P}_i}\mathbf{\Sigma}_{\mathbf{P}_i}\left[\mathbf{V}_{\mathbf{P}_i}^{(1)} \quad \mathbf{V}_{\mathbf{P}_i}^{(0)}\right]^H$;
  $\bar{\mathcal{O}}_i = \mathcal{N}(\mathbf{P}_i)$, $\mathcal{O}_i = \tilde{\mathcal{O}}_i = \varnothing$;
  for $j = 1 : |\bar{\mathcal{O}}_i|$
    choose $\mathbf{v}_{\mathbf{P}_i,j}^{(0)}$ from the overlapped set $\bar{\mathcal{O}}_i$;
    if $\|\mathbf{H}_i \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\| > \|\tilde{\mathbf{H}}_i \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\|$
      $\mathcal{O}_i \leftarrow \mathbf{v}_{\mathbf{P}_i,j}^{(0)}$;
    else
      $\tilde{\mathcal{O}}_i \leftarrow \mathbf{v}_{\mathbf{P}_i,j}^{(0)}$;
    end
  end
  $\mathbf{\Gamma}_i = \mathbf{V}_{\mathbf{P}_i}^{(1)}\mathbf{V}_{\mathbf{P}_i}^{(1)H} + \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \mathcal{O}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H}$;
  $\mathbf{\Upsilon}_i = \mathbf{V}_{\mathbf{P}_i}^{(1)}\mathbf{V}_{\mathbf{P}_i}^{(1)H} + \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \tilde{\mathcal{O}}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H}$
  $\mathbf{H}_i = \mathbf{H}_i\mathbf{\Gamma}_i$, $\tilde{\mathbf{H}}_i = \tilde{\mathbf{H}}_i\mathbf{\Upsilon}_i$;
  Perform MU-MIMO precoding using the updated $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$
end

A. Subspace Identification

Motivated by the fact that non-orthogonality between users' channels can be represented by subspace overlapping, our aim in this initial stage is to span the overlapped subspace, which corresponds to the intersection of the two range spaces $\mathcal{R}(\mathbf{H}_i)$ and $\mathcal{R}(\tilde{\mathbf{H}}_i)$. Apparently, it cannot be obtained directly from the two range spaces. One possible solution is to construct the overlapped subspaces by applying repeated projection among the two range spaces [5] but it suffers from a huge computational complexity becausethe exact equality is achieved only with a huge number of repeated projections[4]. Alternatively, we resort to construct the *non-overlapped subspace* by using the two sets of basis vectors. Then, the overlapped subspace can be easily extracted from the whole space because these two subspaces are *complement* to each other.

Proposition 1: For user-i, there exists a matrix

$$\mathbf{P}_i = \langle \mathbf{H}_i \rangle^{\perp} + \langle \tilde{\mathbf{H}}_i \rangle^{\perp} \in \mathbb{C}^{M_T \times M_T}, \tag{2}$$

where $(\cdot)^{\perp}$ denotes the orthogonal counterpart of a matrix, and $\langle \mathbf{H}_i \rangle$ and $\langle \tilde{\mathbf{H}}_i \rangle$ are ones of the representations of $\mathcal{R}(\mathbf{H}_i)$ and $\mathcal{R}(\tilde{\mathbf{H}}_i)$, respectively. For instance, we can make

$$\langle \mathbf{H}_i \rangle^{\perp} = \mathbf{I} - \mathbf{H}_i^H(\mathbf{H}_i\mathbf{H}_i^H)^{-1}\mathbf{H}_i, \tag{3}$$

$$\langle \tilde{\mathbf{H}}_i \rangle^{\perp} = \mathbf{I} - \tilde{\mathbf{H}}_i^H(\tilde{\mathbf{H}}_i\tilde{\mathbf{H}}_i^H)^{-1}\tilde{\mathbf{H}}_i \tag{4}$$

or

$$\langle \mathbf{H}_i \rangle^{\perp} = \mathbf{V}_i^{(0)}\mathbf{V}_i^{(0)H} \tag{5}$$

$$\langle \tilde{\mathbf{H}}_i \rangle^{\perp} = \tilde{\mathbf{V}}_i^{(0)}\tilde{\mathbf{V}}_i^{(0)H}, \tag{6}$$

where $\mathbf{V}_i^{(0)} \in \mathbb{C}^{M_T\times(M_T-L_i)}$ and $\tilde{\mathbf{V}}_i^{(0)} \in \mathbb{C}^{M_T\times(M_T-\tilde{L}_i)}$ are right singular vectors of $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$ associated with zero singular values, respectively. Apparently, the range space

[4]Although it is shown by simulations in [5] that an approximation can be achieved by 3 repeated projection in certain scenarios, our proposed approach is with a much lower computational complexity.

$\mathcal{R}(\mathbf{P}_i)$ is a non-overlapped component of the transmission and interference subspaces, while its null space $\mathcal{N}(\mathbf{P}_i)$ is the overlapped counterpart. □

Proof 1: Due to the page limit, here we only present the outline of the proof. Firstly, $\mathcal{S} = \mathcal{N}(\mathbf{H}_i) \cup \mathcal{N}(\tilde{\mathbf{H}}_i)$ is the non-overlapped subspace of $\langle \mathbf{H}_i \rangle$ and $\langle \tilde{\mathbf{H}}_i \rangle$. Then, we prove $\langle \mathbf{P}_i \rangle$ is equivalent to $\mathcal{N}(\mathbf{H}_i) \cup \mathcal{N}(\tilde{\mathbf{H}}_i)$ by justifying the following two conditions, namely $\mathbf{P}_i \in \mathcal{N}(\mathbf{H}_i) \cup \mathcal{N}(\tilde{\mathbf{H}}_i)$ and $rank(\mathbf{P}_i) = dim(\mathcal{N}(\mathbf{H}_i) \cup \mathcal{N}(\tilde{\mathbf{H}}_i))$. Lastly, since $\mathbf{P}_i \in \mathcal{N}(\mathbf{H}_i) \cup \mathcal{N}(\tilde{\mathbf{H}}_i)$ and its rank satisfies the condition that $rank(\mathbf{P}_i) = dim(\mathcal{N}(\mathbf{H}_i) \cup \mathcal{N}(\tilde{\mathbf{H}}_i))$, we can argue that the range space of $\mathbf{P}_i$ is identical to the combined subspace $\mathcal{N}(\mathbf{H}_i) \cup \mathcal{N}(\tilde{\mathbf{H}}_i)$ that represents the non-overlapped subspace, while the null space of $\mathbf{P}_i$ correspondingly stands for the overlapped counterpart. This completes the proof. □

It is observed from (2) that $\mathbf{P}_i$ is the summation of two projection matrices that project an arbitrary matrix subspace onto the null space of the transmission channel matrix and that of the interference channel matrix, respectively. In other words, $\mathbf{P}_i$ represents the subspace spanned by the two projectors. The reasons of using projection matrices as the components of the constructed matrix are threefold [5]: (1) the projection matrices are all M_T-dimensional and therefore, we can replace the union of the subspaces by the direct sum of projection matrices; (2) a projection matrix uniquely identifies the subspace it projects; and (3) since the eigenvalues of a projection matrix are either 0 or 1, we can take the projection matrix as the sum of tensors. Through the construction of $\mathbf{P}_i$, the overlapped and non-overlapped subspaces are detached[5] and represented by the null and range spaces, respectively. The overlapped subspace can now be extracted from the whole space.

If we perform SVD on $\mathbf{P}_i$, i.e.,

$$\mathbf{P}_i = \mathbf{U}_{\mathbf{P}_i} \begin{bmatrix} \mathbf{\Sigma}_{\mathbf{P}_i} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{\mathbf{P}_i}^{(1)} & \mathbf{V}_{\mathbf{P}_i}^{(0)} \end{bmatrix}^H, \quad (7)$$

then the overlapped and non-overlapped subspaces can further be separated into individual components, which are represented by the column vectors of $\mathbf{V}_{\mathbf{P}_i}^{(0)} \in \mathbb{C}^{M_T \times (L_i + \tilde{L}_i - L)}$ and the column vectors of $\mathbf{V}_{\mathbf{P}_i}^{(1)} \in \mathbb{C}^{M_T \times (M_T - L_i - \tilde{L}_i + L)}$, respectively, with L being the rank of the system channel matrix $\mathbf{H} \in \mathbb{C}^{M_R \times M_T}$ that aggregates $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$. It is clear that the number of overlapped components depends on the value of $L_i + \tilde{L}_i - L$. In general, there are two possible scenarios[6]: (1) $L_i + \tilde{L}_i = L$, i.e., there is no overlapping; and (2) $L_i + \tilde{L}_i > L$, i.e., there is an $(L_i + \tilde{L}_i - L)$-dimensional overlapped subspace. As a side remark, if there is no overlapped subspace but the channel matrix is ill-conditioned, we can consider a number of eigenvalues whose values are very small to be 0. Consequently, the corresponding subspaces would be considered as overlapped subspaces and they can be reallocated by using our proposed approach.

In summary, while Proposition 1 helps extract the overlapped subspace from the whole space, the SVD in (7) further divides the overlapped subspace into $L_i + \tilde{L}_i - L$ components that are used as an input for subspace reassignment.

B. Subspace Reassignment

We now present a subspace reassignment method that reallocates each subspace component to a user for different purposes. In particular, we adopt a low-complexity yet efficient norm-based comparison criterion such that the overlapped-subspace component is either used for transmission or treated as interference for user-i. Let $\bar{\mathcal{O}}_i$ be a set of vectors spanning the overlapped subspace. This set can be further divided into two subsets $\mathcal{O}_i$ and $\tilde{\mathcal{O}}_i$ that respectively collect the overlapped-subspace components to the transmission and interference subspaces of the user. These subsets are disjoined and satisfy $\mathcal{O}_i \cup \tilde{\mathcal{O}}_i = \bar{\mathcal{O}}_i$ and $\mathcal{O}_i \cap \tilde{\mathcal{O}}_i = \varnothing$. It is clear from (7) the column vectors of $\mathbf{V}_{\mathbf{P}_i}^{(0)}$ span the overlapped subspace and therefore, $\bar{\mathcal{O}}_i = \left\{ \mathbf{v}_{\mathbf{P}_i,1}^{(0)}, \mathbf{v}_{\mathbf{P}_i,2}^{(0)}, \cdots, \mathbf{v}_{\mathbf{P}_i,|\bar{\mathcal{O}}_i|}^{(0)} \right\}$, where $\mathbf{v}_{\mathbf{P}_i,j}^{(0)}$ is the j-th column vector of $\mathbf{V}_{\mathbf{P}_i}^{(0)}$, $|\bar{\mathcal{O}}_i| = M_T - L_p$ and L_p is the rank of $\mathbf{P}_i$. It is interesting to note that when there is no overlapped subspace, $L_p = M_T$ and the set $\bar{\mathcal{O}}_i$ is empty (i.e., $|\bar{\mathcal{O}}_i| = 0$). Since there is no overlapped component, the subsequent procedure of subspace assignment can be skipped. In this case, $\mathbf{V}_i^{(0)}$ and $\tilde{\mathbf{V}}_i^{(0)}$ are now all-zero vectors, which gives $\mathbf{P}_i = \mathbf{0}$ in (2). Therefore, the set $\bar{\mathcal{O}}_i$ can be the columns of an arbitrary unitary matrix.

By way of example, we introduce the following norm-based criterion for assigning appropriately each overlapped-subspace component $\mathbf{v}_{\mathbf{P}_i,j}^{(0)}$ to either $\mathcal{O}_i$ or $\tilde{\mathcal{O}}_i$, where $j = 1, 2, \ldots, |\bar{\mathcal{O}}_i|$. Particularly, the selection criterion is such that if the Frobenius norm of the transmission channel projected on a subspace component surpasses that of the interference channel projected on the same component, then this component is allocated for transmission. In other words, if $\|\mathbf{H}_i \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\|_F > \|\tilde{\mathbf{H}}_i \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\|_F$,, then $\mathcal{O}_i \leftarrow \mathbf{v}_{\mathbf{P}_i,j}^{(0)}$, or otherwise, the overlapped-subspace component is treated as interference, i.e., $\tilde{\mathcal{O}}_i \leftarrow \mathbf{v}_{\mathbf{P}_i,j}^{(0)}$.

C. Subspace Reconstruction

After the second stage, the overlapped subspace that is spanned by the elements of $\bar{\mathcal{O}}_i$ is separated into two disjoined sets $\mathcal{O}_i$ and $\tilde{\mathcal{O}}_i$, which are respectively affiliated with the transmission and interference subspaces of the i-th user. Consequently, the new transmission subspace $\mathbf{\Gamma}_i$ and interference subspace $\mathbf{\Upsilon}_i$ are constructed and represented by means of projection matrices as follows.

$$\mathbf{\Gamma}_i = \mathbf{V}_{\mathbf{P}_i}^{(1)} \mathbf{V}_{\mathbf{P}_i}^{(1)H} + \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \mathcal{O}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)} \mathbf{v}_{\mathbf{P}_i,j}^{(0)H} \in \mathbb{C}^{M_T \times M_T}, \quad (8)$$

$$\mathbf{\Upsilon}_i = \mathbf{V}_{\mathbf{P}_i}^{(1)} \mathbf{V}_{\mathbf{P}_i}^{(1)H} + \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \tilde{\mathcal{O}}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)} \mathbf{v}_{\mathbf{P}_i,j}^{(0)H} \in \mathbb{C}^{M_T \times M_T}. \quad (9)$$

Remarks:

[5]From Proposition 1, we know that $\mathbf{P}_i$ contains not only the components corresponding to non-overlapped null spaces of $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$, but also the common null space of $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$. Though this common null space is neither used for transmission nor treated as interference, it is included in our constructed subspace for simplification. This subspace does not affect any system performance as it is eliminated in the subsequent precoding procedure.

[6]Since $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$ are sub-matrices of $\mathbf{H}$, it is impossible that $L_i + \tilde{L}_i < L$.

1) Though the transmission and interference subspaces share $\mathbf{V}_{\mathbf{P}_i}^{(1)}\mathbf{V}_{\mathbf{P}_i}^{(1)H}$, no separation is required because $\mathbf{V}_{\mathbf{P}_i}^{(1)}\mathbf{V}_{\mathbf{P}_i}^{(1)H}$ is free from interference.
2) $\mathbf{\Gamma}_i$ is an orthogonal projection matrix because its components in (8) are not only orthogonal projection matrices but also mutually disjoined with each other. The same also applies for $\mathbf{\Upsilon}_i$.
3) Since the basis of transmission and interference subspaces differs among users, their reconstructed subspaces might be overlapped. In other words, though spatial separability among users' channels is enhanced, inter-user interference would not be perfectly eliminated.

With the newly constructed transmission and interference subspaces for user-i, the corresponding channel matrices are updated prior to precoding as $\mathbf{H}_i^U = \mathbf{H}_i\mathbf{\Gamma}_i$ and $\tilde{\mathbf{H}}_i^U = \tilde{\mathbf{H}}_i\mathbf{\Upsilon}_i$.

IV. Spatial Separation Enhancement

When two users' channels $\mathbf{H}_i$ and $\mathbf{H}_j$ are spatially overlapped, both BD and RBD cannot distinguish clearly between a user's transmission and interference channel matrices. In RBD, for example, the precoding matrix $\mathbf{F}_i$ suppresses multiuser interference induced by not only $\mathbf{H}_j$, but also portion of $\mathbf{H}_i$ that is spatially overlapped with $\mathbf{H}_j$. Since the transmission and interference subspaces are overlapped, portion of the user's effective transmission channel is also eliminated. This results in a reduction of the transmission channel norm.

With subspace reallocation, the transmission and interference subspaces of a user can be detached and it results in a significant enhancement in inter-user spatial separation. For justification, we follow [6] and consider matrix collinearity as a measure. Generally speaking, matrix collinearity reflects the similarity of the two compared matrix subspaces $\mathbf{A}$ and $\mathbf{B}$ [3]:

$$col(\mathbf{A},\mathbf{B}) = \frac{|tr(\mathbf{A}\mathbf{B}^H)|}{\|\mathbf{A}\|_F\|\mathbf{B}\|_F}, \tag{10}$$

where $0 \leq col(\mathbf{A},\mathbf{B}) \leq 1$. As discussed in [6], a low collinearity means a slightly-overlapped matrix subspace that in turn increases the received SNR and improves the achievable sum rate.

In order to show the reduction in collinearity of the transmission and interference subspaces by using the proposed subspace reallocation (SR) approach, we consider a simplified scenario in which both the transmission subspace and the interference subspace share the whole channel space, i.e., $\mathcal{R}(\mathbf{H}_i) = \mathcal{R}(\tilde{\mathbf{H}}_i)$. This simplification can be justified by the fact that SR-RBD follows RBD in employing the same two-stage precoding mechanism but with different transmission and interference subspaces. With neither precoding nor subspace reallocation, the transmission and interference subspaces share the whole channel space as their available subspaces and it is easy to see that the collinearity of these two subspaces is 1, i.e., $col(\mathcal{T}_i^{no-SR}, \mathcal{I}_i^{no-SR}) = 1$. By employing space reallocation, the transmission and interference subspaces are respectively the range spaces of $\mathbf{\Gamma}_i$ and $\mathbf{\Upsilon}_i$, i.e., $\mathcal{T}_i^{with-SR} = \mathcal{R}(\mathbf{\Gamma}_i)$ and $\mathcal{I}_i^{with-SR} = \mathcal{R}(\mathbf{\Upsilon}_i)$, where $\mathbf{\Gamma}_i$ and $\mathbf{\Upsilon}_i$ in (8) and (9) can be alternatively re-expressed as $\mathbf{\Gamma}_i = \mathbf{I} - \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \tilde{\mathcal{O}}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H}$ and $\mathbf{\Upsilon}_i = \mathbf{I} - \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \mathcal{O}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H}$, respectively. Further, it is important to note from the conditions $\mathcal{O}_i \cup \tilde{\mathcal{O}}_i = \bar{\mathcal{O}}_i$ and $\mathcal{O}_i \cap \tilde{\mathcal{O}}_i = \varnothing$ in Section III.B that

$$\begin{aligned} &\sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \mathcal{O}} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H} \\ &= \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \mathcal{O}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H} + \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \tilde{\mathcal{O}}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H}. \end{aligned} \tag{11}$$

For notational convenience, denote $\mathbf{O}_i = \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \mathcal{O}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H}$, $\tilde{\mathbf{O}}_i = \sum_{\mathbf{v}_{\mathbf{P}_i,j}^{(0)} \in \tilde{\mathcal{O}}_i} \mathbf{v}_{\mathbf{P}_i,j}^{(0)}\mathbf{v}_{\mathbf{P}_i,j}^{(0)H}$ and $\bar{\mathbf{O}}_i = \mathbf{O}_i + \tilde{\mathbf{O}}_i$. The collinearity of $\mathcal{T}_i^{with-SR}$ and $\mathcal{I}_i^{with-SR}$ can be derived as follows.

$$\begin{aligned} & col(\mathcal{T}_i^{with-SR}, \mathcal{I}_i^{with-SR}) & (12)\\ =\ & col(\mathcal{R}(\mathbf{\Gamma}_i), \mathcal{R}(\mathbf{\Upsilon}_i)) = col(\mathbf{\Gamma}_i, \mathbf{\Upsilon}_i) & (13)\\ =\ & \frac{|tr(\mathbf{\Gamma}_i\mathbf{\Upsilon}_i^H)|}{\|\mathbf{\Gamma}_i\|_F\|\mathbf{\Upsilon}_i\|_F} = \frac{|tr(\mathbf{I} - \bar{\mathbf{O}}_i)|}{\|(\mathbf{I} - \tilde{\mathbf{O}}_i)\|_F\|(\mathbf{I} - \mathbf{O}_i)\|_F} & (14)\\ =\ & \frac{M_T - |\bar{\mathbf{O}}_i|}{\sqrt{M_T - |\tilde{\mathbf{O}}_i|}\sqrt{M_T - |\mathbf{O}_i|}}. & (15) \end{aligned}$$

We can observe from (15) the following two remarks.

1) When $|\bar{\mathbf{O}}_i| = |\tilde{\mathbf{O}}_i| = |\mathbf{O}_i|$, then $col(\mathcal{T}_i^{with-SR}, \mathcal{I}_i^{with-SR}) = 1$. Since $\bar{\mathbf{O}}_i = \mathbf{O}_i + \tilde{\mathbf{O}}_i$, a unity collinearity is achieved when the users are totally spatially separable. This counter-intuitive observation is due to the fact that subspace overlapping does not exist and subspace reallocation is not required. Therefore, it does no harm in allowing both the transmission and the interference subspaces share the whole channel space.
2) When the users are inseparable, or equivalently when the transmission and interference subspaces are completely overlapped, we have M_T elements in the set $\bar{\mathcal{O}}_i$ (i.e., $|\bar{\mathcal{O}}_i| = M_T$) and $col(\mathcal{T}_i^{with-SR}, \mathcal{I}_i^{with-SR}) = 0$.

In summary, it can be observed from (15) that subspace reallocation significantly reduces the collinearity of the transmission and interference subspaces and hence enhances the inter-user spatial separation.

V. Numerical Results

The effectiveness of the proposed subspace reallocation (SR) approach is evaluated in terms of 10% outage capacity. As the first example, we consider BD and RBD with and without the proposed SR approach in a $\{3,3,3,3\} \times 5$ system over a spatial i.i.d. channel. Here, we employ water filling ("WF") and equal power allocation ("no PL") as the two power loading algorithms. We also consider TDMA as the degraded version of user grouping and DET for comparison purpose. Fig. 1 shows that our proposed scheme yields a significant capacity improvement. For example, while "RBD no PL" reaches a sum rate saturation of around 12 bps/Hz at a received SNR of 20 dB, a much higher capacity of around 23 bps/Hz is provided by "SR-RBD no PL". Further, no rate saturation is observed because our algorithm can better utilize spatial components of all users' channels. Lastly, we investigate into the importance of subspace reallocation by comparing the performance of "SR-RBD WF" with "TDMA" and "DET". We can see that the performance improvement due to subspace reallocation is more significant, e.g., at received SNR of 40 dB, the differences are as large as 19 bps/Hz and 9 bps/Hz. The slope of the curves also present their differences in achievable degree-of-freedom, which are 5, 3, and 4, respectively.

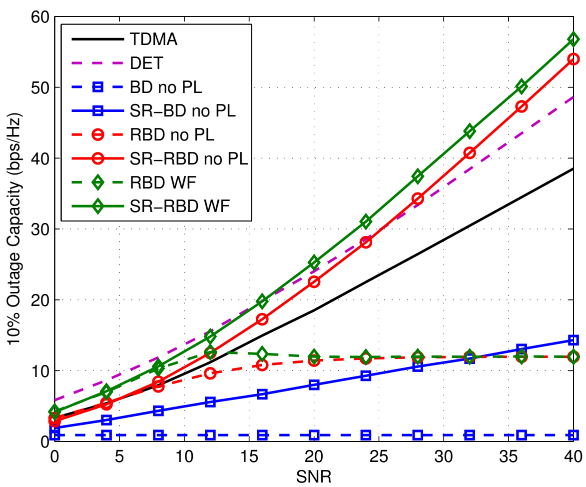

Fig. 1. 10% outage capacity performance of various precoding schemes with and without the proposed subspace reallocation approach. A $\{3,3,3,3\} \times 5$ system over an i.i.d. channel is considered.

Next, we compare the 10% outage capacity of RBD and SR-RBD in both a spatial i.i.d. channel and a spatial correlated channel [7] by considering a $\{4,4,4\} \times M_T$ system, where $M_T = 1, 2, \ldots, 14$. It can be seen from Fig. 2 that when $M_T \geq M_R$, our algorithm performs as well as the conventional RBD scheme because the users are spatially separable on the M_T-dimensional whole space that supports all the users for transmission. On the contrary, when $M_R > M_T$, SR-RBD surpasses RBD since our proposed SR approach reduces the overlap between the transmission and interference subspaces and therefore, enhance the spatial separation among users. There is also an interesting observation regarding the spatial correlated channel. When compared with the spatial i.i.d. channel, the sum rate of RBD over correlated channel is significantly worsened and it saturates earlier in terms of M_T. In addition, while SR-RBD and RBD converge with each other at high received SNR in the i.i.d. counterpart, no convergence is observed here and the performance gap between SR-RBD and RBD increases with the received SNR. These observations are due to the fact that while spatial inseparability among users' channels can be mitigated by increasing M_T in a spatial i.i.d. channel, it cannot be solved in the same way in the spatial correlated channel with rank deficiency because the problem is irrelative to the numbers of transmit and receive antennas. Nevertheless, thanks to the subspace reallocation technique that significantly enhances spatial separation, our proposed SR-RBD always surpasses RBD even when $M_T \geq M_R$.

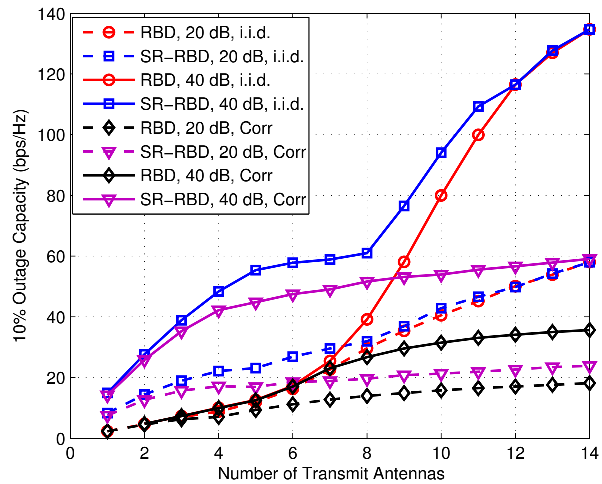

Fig. 2. 10% outage capacity performance of RBD and SR-RBD at received SNRs of 20 dB and 40 dB. A $\{4,4,4\} \times M_T$ system over an i.i.d. and a spatial correlated channels are considered, where $M_T = 1, 2, \ldots, 14$.

VI. Conclusion

We proposed a new transmit preprocessing technique in enhancing spatial separation among users' channels in downlink MU-MIMO systems. Our proposed algorithm suppresses interuser interference by exploiting the overlap between transmission and interference subspaces and allocating the overlapped-subspace components to different users according to a simple but efficient norm-based criterion. Since our algorithm redeploys transmission subspace for without exerting any influence on other modules, it can be directly applied with MU-MIMO precoding and user scheduling algorithms.

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