

# The throughput of Hybrid-ARQ protocols for the Gaussian collision channel

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January 22, 2001

## Abstract

In next generation wireless communication systems, packet-oriented data transmission will be implemented in addition to standard mobile telephony. We take an information-theoretic view of some simple protocols for reliable packet communication based on “Hybrid-ARQ”, over a slotted multiple access Gaussian channel with fading and study their throughput (total bit/s/Hz) and average delay under idealized but fairly general assumptions. As an application of the *renewal-reward* theorem, we obtain closed-form throughput formulas. Then, we consider asymptotic behaviors with respect to various system parameters. The throughput of ARQ protocols is compared to that of CDMA with conventional decoding. Interestingly, the ARQ systems are not interference-limited even if no multiuser detection or joint decoding is used, as opposed to conventional CDMA.

**Keywords.** Packet radio, information theory, multiuser systems, wireless communications.

## 1 Introduction

In order to support new services (e.g., wireless mobile access to the Internet), next generation wireless communication systems will implement packet-oriented data transmission in addition to standard mobile telephony [60]. This implies bursty sporadic communication from a large population of users, that may require instantaneous large data rates and very small error probabilities for a short time. Motivated by the above consideration, we take an information-theoretic

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view of some simple protocols for reliable packet communication based on “Hybrid-ARQ”, i.e., on combining *channel coding* and *Automatic Retransmission reQuest* (ARQ) [48, 6].

Several types of Hybrid-ARQ protocols have been proposed (see [48, 6] and references therein). The throughput of ARQ schemes can be improved by packet combining, i.e., by keeping the erroneous received packets and using them for detection. Packet combining can be based on hard decisions [36, 16, 43, 51, 50] or on soft channel outputs [8, 3, 45]. In the latter case, several noisy observations of the same packet obtained by retransmission are combined by a suitable diversity technique (e.g., maximal-ratio, equal-gain or selection combining [29]). Soft decoding of maximal-ratio combined packets can be seen as an elementary form of Hybrid-ARQ, based on soft-decoding of a repetition code of variable length. This idea can be extended to more general classes of codes. In [22, 45, 46], Rate Compatible Punctured Convolutional (RCPC) codes are used in an *incremental redundancy* ARQ scheme. Transmission starts with the highest rate code of the RCPC code family and additional redundancy bits are requested whenever necessary. More recently, Turbo-codes [5] have been suggested as candidates for packet combining, exploiting the fact that they are systematic and produce incremental redundancy by puncturing the parity bits [62, 25].

Analysis of Hybrid-ARQ protocols in terms of throughput, error rates and delay can be found in [34, 33, 31, 30, 47, 38, 37, 58, 55, 41, 20]. Most works carry out a “separated analysis”, i.e., consider a completely symmetric system with respect to any user, and study the behavior of the protocol for a particular reference user modeled as a Markov chain. In general, analysis depends on the type of codes and decoding/error detection technique employed. Modelling the system by a Markov chain might be complicated, since in each state one must convey all the information about the memory of the system. In [32], Zorzi proposes the use of *renewal theory* [42] in order to analyze ARQ protocols.

As remarkably well illustrated by Ephremides in [1], information theoretic techniques are not yet of widespread use in the domain of networking with random user activity. Steps in this direction are represented by the work of Shamai and Wyner on cellular systems [52, 53] and of Telatar and Gallager [13]. In [13], the multiple access Gaussian channel is assimilated to a processor-sharing system and is analyzed as a queue with single server and an infinite buffer. The required service for each user is defined in terms of random coding bound on the error probability. A code-independent analysis of the mean transmission duration is obtained as an

application of Little's Theorem [7].

This paper is mainly inspired by the work of [13, 32]. We assume that users transmit their signal *bursts* at high instantaneous power and in a completely uncoordinated way. The receiver is formed by a bank of single-user decoders, and does not implement joint decoding, i.e., each decoder treats the signals from other users as noise.<sup>1</sup> We refer to this model as the *Gaussian collision channel* [17]. The transmission of each user is governed by an Hybrid-ARQ protocol, designed to cope with background noise, fading and interference (or “collisions”) from other users.

We study the system performance in terms of throughput (total bit/s/Hz) and average delay for three simple idealized protocols: a coded version of Aloha, a repetition scheme with maximal-ratio packet combining and an incremental redundancy scheme with general coding. By applying the *renewal-reward* theorem [42], we obtain a closed-form throughput formula under a delay constraint (time-out) and code rate constraint. Since we consider random coding and typical set decoding, our results are independent of the particular coding/decoding technique and should be regarded as a limit in the information theoretic sense.

The system throughput is compared to that of CDMA with conventional single-user detection and decoding. Interestingly, the ARQ system is not interference-limited even if no multiuser detection or joint decoding is used, i.e., arbitrarily high throughput can be obtained simply by increasing the transmit power of all users, as opposed to conventional CDMA where the throughput tends to a finite limit as all users increase their transmit power [10, 56]. As a byproduct of this analysis, we provide a stronger operational meaning to the information outage probability of block-fading channels and we obtain the closed form probability distribution of signal-to-interference plus noise ratio (SINR) with Rayleigh fading and a Poisson-distributed number of interferers, extending the result of [53].

The paper is organized as follows: in Section II we describe the system model; in Section III we deal with typical set decoding and error detection; in Section IV we carry out system throughput analysis; in Section V we present some limiting behaviors and in Section VI we point out our conclusions. The proofs of the results are provided in the Appendices.

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<sup>1</sup>Even though any point in the capacity region of multiple access channels can be implemented with low complexity by successive “stripping” [4], this requires a good deal of coordination among the users which must allocate their rate and power in a proper way [9, 44, 27] and may not be suited to random user activity.

## 2 The slotted Gaussian collision channel with feedback

In the system under investigation,  $N_u$  users share a common radio channel with complex base-band equivalent bandwidth  $[-W/2, W/2]$  in order to transmit their information messages to a single receiver. Users are provided with a common time reference. The time axis is divided in *slots* of duration  $T$  and users transmit signal *bursts* of duration slightly less than  $T$ , aligned with the slots. Apart from the slotted transmission mode, users are completely uncoordinated. Each user can transmit about  $L = \lfloor WT \rfloor$  independent complex symbols over one slot (assuming  $WT \gg 1$  [59]).

A discrete-time signal model is adopted and slots are denoted by their index  $s$ . Let  $\mathbf{y}_s \triangleq (y_{s,1}, \dots, y_{s,L})$ ,  $\mathbf{x}_{k,s} \triangleq (x_{k,s,1}, \dots, x_{k,s,L})$  and  $\boldsymbol{\nu}_s \triangleq (\nu_{s,1}, \dots, \nu_{s,L})$  denote the received signal, the transmitted signal from user  $k$  and the background noise during slot  $s$ , respectively. Noise is assumed circularly-symmetric complex Gaussian, with i.i.d. components with variance  $N_0$ . User  $k$  transmits with constant average energy per symbol  $E_k \triangleq E[|x_{k,s,\ell}|^2]$ .

The propagation channel is assumed slowly time-varying and frequency-flat for each user. In particular, the complex channel gain  $c_{k,s}$  of user  $k$  over slot  $s$  is assumed to be constant on the whole slot (block-fading [26]). The received signal over slot  $s$  can be written as

$$\mathbf{y}_s = \sum_{k \in \mathcal{K}(s)} c_{k,s} \mathbf{x}_{k,s} + \boldsymbol{\nu}_s \quad (1)$$

where  $\mathcal{K}(s) \subseteq \{1, \dots, N_u\}$  denotes the set of *active* users over slot  $s$ .

User  $k$  encodes its information messages, of  $b$  bits each, independently of other users, by using a channel code with code book  $\mathcal{C}_k \subset \mathbb{C}^{LM}$  of length  $LM$  over the complex numbers, where  $M$  is a given integer. Code words are divided into  $M$  subblocks of length  $L$ , each of which is modulated into a signal burst and is transmitted over one slot. We let  $\mathcal{C}_{k,m}$ , for  $m = 1, \dots, M$ , denote the punctured code of length  $mL$  obtained from  $\mathcal{C}_k$  by deleting the last  $M - m$  subblocks.

Each user selects the slots for transmission according to its own *time-hopping* (pseudo-)random sequence, independently of the other users [24]. Time-hopping sequences can be seen as random “on-off” processes, where a user can transmit only when it is “on”. We assume that the receiver knows *a priori* the time-hopping rule of all users in the system [24].<sup>2</sup> Transmission is

<sup>2</sup>This assumption is not particularly restrictive, and is analogous to the standard assumption of CDMA with pseudo-random “long” spreading [2], where the receiver is assumed to know the spreading sequences of all users

governed by the following simple Hybrid-ARQ protocol, run in a decentralized way by each user  $k$ . When a new code word is ready for transmission, user  $k$  sends the first  $L$  symbols on the first allowed slot, say  $s_1$ , according to its time-hopping rule. The receiver decodes the code  $\mathcal{C}_{k,1}$  by processing the received signal  $\mathbf{y}_{s_1}$ . If decoding is successful, a positive acknowledgement (ACK) is sent back to user  $k$  over an error free and delay free feedback channel and the transmission of the current code word stops. On the contrary, if the receiver detects an error, a negative acknowledgement (NACK) is sent. In this case, user  $k$  sends the second block of  $L$  symbols of the same code word on the next allowed slot, say  $s_2$ . Now, the receiver decodes the code  $\mathcal{C}_{k,2}$  by processing the received signal blocks  $\{\mathbf{y}_{s_1}, \mathbf{y}_{s_2}\}$ . Again, if decoding is successful an ACK is sent and the transmission of the current code word stops. On the contrary, if a decoding error is detected, a NACK is sent back and user  $k$  transmits the third block of  $L$  symbols of the same code word on the next allowed slot. The process goes on this way: after the transmission of  $m$  bursts of the current code word, code  $\mathcal{C}_{k,m}$  is decoded by processing the received signal  $\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\}$ , where  $\mathcal{S}_{k,m} \triangleq \{s_1, \dots, s_m\}$  denotes the sequence of slots where transmission of user  $k$  took place. If successful decoding occurs at the  $m$ -th transmission, the effective coding rate for the current code word is  $R/m$  bits/s/Hz, where  $R \triangleq b/L$ .<sup>3</sup>

In general, the slots  $s \in \mathcal{S}_{k,m}$  are non-adjacent. We let  $n$  denote the delay (expressed in number of slots) between the instant where a code word is generated and the current time (time ticks at the slot rate). Obviously,  $m \leq n$  (see example in Fig. 1). In any practical application, an information message must be delivered to the receiver within a maximum delay of  $N$  slots, where for simplicity  $N$  is assumed common to all users and all messages. If successful decoding does not occur within delay  $N$ , the message becomes useless. Moreover, since the code words of  $\mathcal{C}_k$  have  $M$  subblocks, the same message can be transmitted in at most  $M$  signal bursts. If successful decoding does not occur within  $M$  transmitted bursts, the message is lost. We shall refer to  $N$  and  $M$  as the “delay” and “rate” constraints, respectively. The transmission of a code word can stop in three cases: i) Successful decoding occurs at the  $m$ -th transmitted burst it wishes to decode. In practice, we might think of an access mechanism, run at a much slower time-scale than packet transmission, that assigns to new users entering the system a time-hopping sequence.

<sup>3</sup>For large  $WT$ , a complex symbol (or dimension) can be transmitted approximately in one second and one Hz. More precisely, the spectral efficiency expressed in bit/s/Hz can be obtained by multiplying the coding rate (bit/complex symbol) by the modulation spectral efficiency (expressed in complex symbols/s/Hz), that depends on the modulation excess bandwidth [21].

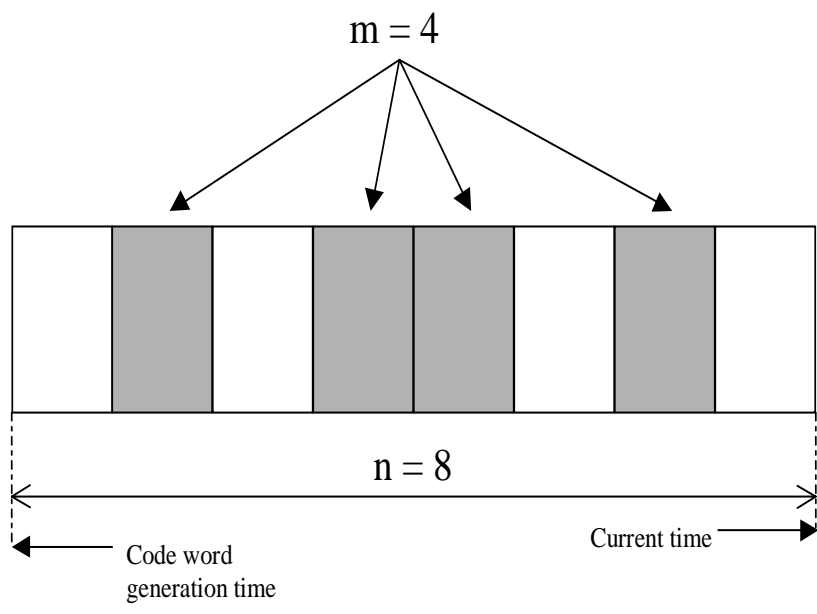


Figure 1: Example:  $m = 4$  transmitted bursts (shaded) over  $n = 8$  slots since the current code word generation.

and in  $n$  slots, with  $m \leq M$  and  $n \leq N$ ; ii) No successful decoding occurs after  $M$  transmitted bursts and  $n \leq N$  slots; ii) No successful decoding occurs after  $N$  slots and  $m \leq M$  transmitted bursts.

There are several ways to handle transmission failures (cases (ii) and (iii) above). For example, in the case of time-sensitive information, the current message is simply discarded. In other applications, delay is not a strict requirement ( $N$  is very large) and the current message may be kept in the transmission buffer for a later attempt. Several practical ARQ protocols have been proposed to handle transmission failures (see references in Section 1). The analysis carried out in the rest of this paper considers the simplified scenario where an infinite sequence of messages is available to all users and, in the case of transmission failure, the current message is discarded and the next message is encoded and transmitted in exactly the same way. The time-hopping sequence for slot selection is not modified by transmission failures (e.g., there is no idle state, waiting for better channel conditions). It is important to notice that each user runs its own ARQ protocol independently of the other users. The only way in which users influence each others is through mutual interference (collisions) that occurs when several users transmit their bursts over the same slot.

The single-user decoder for user  $k$  has perfect knowledge of the channel gain  $c_{k,s}$  and of the SINR

$$\beta_{k,s} \triangleq \frac{\alpha_{k,s} E_k}{N_0 + \sum_{j \in \mathcal{K}(s): j \neq k} \alpha_{j,s} E_j} \quad (2)$$

for all  $s$  such that  $k \in \mathcal{K}(s)$ , where  $\alpha_{k,s} \triangleq |c_{k,s}|^2$  denotes the channel power gain. Estimation of the channel gain and of the SINR can be achieved in practice by inserting training symbols into each signal burst, as currently done in most CDMA and TDMA cellular standards [61].

The Hybrid-ARQ protocol described above is a general incremental redundancy scheme (denoted by “INR” for brevity). We consider also the following particular cases.

**Generalized Slotted Aloha.** The slotted Aloha protocol [7] (denoted by “ALO” for brevity) is obtained by assuming for each user  $k$  a suboptimal decoder that considers only the last received signal block. In classical Aloha it is assumed that a decoding failure occurs (and is detected) whenever a collision occurs. In mobile systems, users might be received at very different power levels because of fading, shadowing and different distances from the receiver. In this case, a packet can be decoded successfully even if a collision occurs (capture effect) [20].

Here, we consider a *generalized* ALO where channel coding is used and messages may be decoded correctly even in the presence of collisions, depending on the SINR.

**Repetition time-diversity.** A simple time-diversity scheme (denoted by “RTD” for brevity) is obtained by repeating the same burst of  $L$  symbols [8, 3]. This is equivalent to construct the user code  $\mathcal{C}_k$  as a concatenated code, where  $\mathcal{C}_{k,1} \subset \mathbb{C}^L$  is the outer code and a simple repetition code of length  $M$  is the inner code. After the  $m$ -th transmission, the receiver performs Maximal-Ratio Combining (MRC) [8] of the signals  $\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\}$  and decodes the outer code  $\mathcal{C}_{k,1}$  based on the combined signal  $\mathbf{y} = \sum_{s \in \mathcal{S}_{k,m}} c_{k,s}^* \mathbf{y}_s$ . Because of the analogy with a rake receiver that performs MRC of the different multipath components of the received signal [21], this scheme is sometimes referred to as “repetition rake” [28].

### 3 Coding, decoding and error detection

We assume that all code books  $\mathcal{C}_k$  are generated randomly and independently, with i.i.d. components, according to a given pdf  $q(x)$  over  $\mathbb{C}$  with mean zero and variance  $E_k$ . For each user, an encoding function  $\psi_k : \{1, \dots, 2^{RL}\} \rightarrow \mathcal{C}_k$  is defined and revealed to the receiver.

A key point of the ARQ schemes described in Section 2 is that decoding errors should be detected. Any *complete decoding* function, based on a partition of the channel output space into  $2^{RL}$  regions (e.g., MAP decoding or ML decoding), is not suited to this purpose, unless an explicit error-detection stage after channel decoding is introduced (e.g., in many cellular systems a CRC is inserted into the information message [6, 61]). This however is undesirable since it decreases the throughput by adding extra redundancy. An alternative is the use of possibly suboptimal decoders in terms of error probability, but featuring a built-in error detection capability. Moreover, it is desirable to decode all punctured codes  $\mathcal{C}_{k,m}$ , for  $m = 1, \dots, M$ , by the same decoder.

In particular, we examine the following error correction/detection scheme. Consider decoding for user  $k$  after  $m$  received blocks, and let  $\mathbf{x}_k^{(w)} = \psi_k(w)$  be the transmitted code word corresponding to information message  $w$ . The decoder adds to the received signal  $\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\}$  other  $M - m$  dummy signal blocks  $\mathbf{z}_i$ , generated independently of the received signal,<sup>4</sup> to form

<sup>4</sup>In practice, in decoding of punctured convolutional codes dummy symbols are set to zero, but in the limiting case considered here it is sufficient that they are statistically independent of the channel input.



the observation  $\mathbf{Y} = (\mathbf{y}_{s_1}, \dots, \mathbf{y}_{s_m}, \mathbf{z}_1, \dots, \mathbf{z}_{M-m})$  of length  $LM$ , and then decodes the “mother code”  $\mathcal{C}_k$  according to the typical set rule  $\phi_k : \mathbb{C}^{LM} \rightarrow \{1, \dots, 2^{RL}, e\}$  (see [59] and Appendix A for details) defined as follows:

Let  $\mathcal{E}_w$  be the event that  $\mathbf{x}_k^{(w)}$  is the unique code word jointly typical with  $\mathbf{Y}$ . Then,

- $\phi_k(\mathbf{Y}) = \hat{w}$  if, for some  $\hat{w} \in \{1, \dots, 2^{RL}\}$ , the event  $\mathcal{E}_{\hat{w}}$  occurs.
- $\phi_k(\mathbf{Y}) = e$  in any other case.

Since decoder  $k$  treats all other user signals as additive noise, it “sees” a virtual additive noise channel given by

$$\mathbf{y}_s = c_{k,s} \mathbf{x}_{k,s} + \mathbf{v}_{k,s} \quad (3)$$

where  $\mathbf{v}_{k,s} = \boldsymbol{\nu}_s + \sum_{j \in \mathcal{K}(s): j \neq k} c_{j,s} \mathbf{x}_{j,s}$  is the interference plus noise vector. We let  $p_{k,s}(y|x)$  denote the single-letter transition pdf of the above channel, conditioned on the channel gains  $\{c_{j,s} : j \in \mathcal{K}(s)\}$  and on the set of active users  $\mathcal{K}(s)$ , and we define  $I(q(x), p_{k,s}(y|x))$  to be the mutual information (per letter) of channel (3), expressed as a functional of the pdfs  $q(x)$  and  $p_{k,s}(y|x)$ . Obviously,  $I(q(x), p_{k,s}(y|x))$  varies randomly from slot to slot, since it depends on the random set  $\mathcal{K}(s)$  and on the random channel gains  $\{c_{j,s} : j \in \mathcal{K}(s)\}$ .

We examine the behavior of codes  $\mathcal{C}_{k,m}$  with decoder  $\phi_k$  defined above, for a given sequence of channel transition pdfs  $\mathcal{P} \triangleq \{p_{k,s}(y|x) : s \in \mathcal{S}_{k,m}\}$ . The average error probability is defined by

$$\Pr(\text{error}|\mathcal{P}, \mathcal{C}_k) \triangleq 2^{-RL} \sum_{w=1}^{2^{RL}} \Pr(\overline{\mathcal{E}_w} | w, \mathcal{P}, \mathcal{C}_k) \quad (4)$$

A decoding error when message  $w$  is transmitted is not detected if, for some  $\hat{w} \neq w$ , the event  $\mathcal{E}_{\hat{w}}$  occurs. Then, the average probability of undetected error is defined by

$$\Pr(\text{undetected error}|\mathcal{P}, \mathcal{C}_k) \triangleq 2^{-RL} \sum_{w=1}^{2^{RL}} \Pr\left(\bigcup_{\hat{w} \neq w} \mathcal{E}_{\hat{w}} \middle| w, \mathcal{P}, \mathcal{C}_k\right) \quad (5)$$

The following results, proved in Appendix A, show that the typical set decoder defined above is asymptotically optimal for both error and undetected error probabilities, for large burst length  $L$ :

**Lemma 1 (achievability).** For all  $\epsilon > 0$  there exist  $L$  and codes  $\mathcal{C}_k \in \mathbb{C}^{LM}$  of size  $2^{RL}$  with  $\Pr(\text{error}|\mathcal{P}, \mathcal{C}_k) < \epsilon$  for all  $m = 1, \dots, M$  and channel sequences  $\mathcal{P}$  such that

$$\sum_{s \in \mathcal{S}_{k,m}} I(q(x), p_{k,s}(y|x)) > R. \quad \diamond$$

**Lemma 2 (converse).** For all channel sequences  $\mathcal{P}$  such that  $\sum_{s \in \mathcal{S}_{k,m}} I(q(x), p_{k,s}(y|x)) < R$ ,  $\Pr(\text{error}|\mathcal{P}, \mathcal{C}_{k,m}) \rightarrow 1$  for any code  $\mathcal{C}_{k,m} \in \mathbb{C}^{Lm}$  of size  $2^{RL}$  as  $L \rightarrow \infty$ .  $\diamond$

**Lemma 3 (error detection).** For all  $\epsilon > 0$  and channel sequences  $\mathcal{P}$  there exists  $L$  such that any code  $\mathcal{C}_k \in \mathbb{C}^{LM}$  of size  $2^{RL}$  satisfies  $\Pr(\text{undetected error}|\mathcal{P}, \mathcal{C}_k) < \epsilon$ .  $\diamond$

The optimal input distribution  $q(x)$  of the interference channel (3) is not known in general [59].

For the sake of mathematical tractability, we consider (somewhat arbitrarily) circularly-symmetric complex Gaussian inputs for all users. Then, the mutual information  $\sum_{s \in \mathcal{S}_{k,m}} I(q(x), p_{k,s}(y|x))$  takes the form

$$I_{k,m} \triangleq \sum_{s \in \mathcal{S}_{k,m}} \log_2(1 + \beta_{k,s}) \quad (6)$$

From the above results we have that, by using Gaussian codes and typical set decoding at each step  $m$  of the ARQ protocols of Section 2, the probability of decoding error is arbitrarily small if  $R < I_{k,m}$ , very large if  $R > I_{k,m}$  and decoding errors are detected with arbitrarily large probability, for sufficiently large  $L$ . Practical future system for mobile data transmission will be characterized by a very large value of the product  $WT$ , in order to support large instantaneous bit rates. This motivates a system analysis under the assumption of very large  $L$ .<sup>5</sup> In this regime, we shall assume that, for all  $k$  and  $m$ ,  $\Pr(\text{error}|R < I_{k,m}) = 0$ ,  $\Pr(\text{error}|R \geq I_{k,m}) = 1$  and  $\Pr(\text{undetected error}) = 0$ .

**Remark: Bounded distance and iterative decoding.** Obviously, the typical set decoder considered above is not suited for practical implementations. However, it is interesting to notice that some non-ML practical decoding schemes show a behavior similar to the typical-set decoder. For example, bounded-distance decoding [39] outputs the message  $w$  if the received signal falls inside a *sphere* centered on the code word corresponding to  $w$ , while if the received signal is not in any sphere, an error message  $e$  is declared. Another example is provided by the iterative decoding scheme [23] used to decode Turbo-codes. The component codes of the Turbo-code are individually decoded by symbol-by-symbol soft-in soft-out decoders sharing and updating some common information about the reliability of the symbol-wise decisions. Typically, if the code

<sup>5</sup>For example, in the 3rd generation UMTS standard a packet-radio random access scheme is supported with variable slot duration  $0.625 \leq T \leq 10$  ms, bandwidth  $W = 5$  MHz [49] and modulation spectral efficiency up to 0.2, obtained by using direct-sequence spread-spectrum modulation with raised-cosine pulses with roll-off 0.22 and spreading gain 4. This means that  $L = [0.2WT]$  is between 625 and 10000 complex symbols per slots.

word is correctly decoded all component decoders agree on the symbol-wise decisions, while in the presence of decoding errors the decoders keep on reversing the symbol decisions at each iteration [11]. This ill behavior, as well as the low reliability for some symbols, can be used as error indicators [14].

**Remark: analogy with the block-fading channel.** Under the assumption of Gaussian user code made here, the channel model (3) is totally analogous to the block-fading AWGN channel with perfect channel state information at the receiver introduced in [26]. In [26], decoding is always performed after  $M$  blocks and the probability of decoding failure for large  $L$  is given by  $\Pr(I_{k,M} \leq R)$ , and is referred to as *information outage probability*. Outage probability finds a very natural interpretation as the limiting error probability for large block length averaged over the random coding ensemble and over the fading states [18]. A question left open in [26] and in many subsequent works is whether it exists a code sequence (for increasing values of the block length  $L$ ) with error probability arbitrarily small *for all* fading states such that  $I_{k,M} > R$ . Notice that this is not a trivial question, since if the choice of the code sequence depends on the particular fading state, outage probability would not be achievable (it would require side information at the transmitter). The existence of codes *asymptotically good* for all fading states satisfying  $I_{k,M} > R$  is given by Lemma 1 (see the details of the proof in Appendix A). In this respect, information outage probability is not just an average probability of error over a code ensemble, but it can be approached by a given (deterministic) sequence of codes.

**ALO and RTD schemes.** In analogy to what done above for the INR scheme, we can define random coding and typical-set decoding for ALO and RTD. For the sake of brevity, we state without details that, as  $L \rightarrow \infty$ , also for these schemes there exist codes for which  $\Pr(\text{error}|R < I_{k,m})$  and  $\Pr(\text{undetected error})$  vanish and  $\Pr(\text{error}|R \geq I_{k,m})$  goes to 1, provided that the correct expression for the mutual information is used. ALO takes into account only the most recent received signal burst, therefore the corresponding  $I_{k,m}$  is given by

$$I_{k,m} = \log_2(1 + \beta_{k,s_m}) \quad (7)$$

In RTD, the SINR after MRC of  $m$  received bursts is given by the sum  $\sum_{s \in \mathcal{S}_{k,m}} \beta_{k,s}$  [21], therefore the corresponding  $I_{k,m}$  is given by

$$I_{k,m} = \log_2 \left( 1 + \sum_{s \in \mathcal{S}_{k,m}} \beta_{k,s} \right) \quad (8)$$

## 4 Throughput analysis

In this section we compute the throughput of the ARQ protocols of Section 2 with the coding and decoding scheme of Section 3, in the limit for large  $L$ . Our analysis is valid under the following idealized assumptions:

1. An infinite number of information messages is available for each users. As soon as a user stops the transmission of the current code word, it encodes the next packet and starts its transmission in the next selected slot. As explained in Section 2, transmission of a code word can stop either because successful decoding occurs, or because the delay or rate constraints  $N$  and  $M$  are violated (decoding failure).
2. The ACK/NACK feedback channel is delay-free and error-free.
3. Users select slots for transmission so that the number of slots between two consecutive transmissions of the same user is i.i.d., geometrically distributed with identical parameter  $p_t$  for all users. In order words, on each slot  $s$  each user transmits a signal burst with probability  $p_t$  and does not transmit with probability  $1 - p_t$ . The expected number of users transmitting over a slot is  $G = p_t N_u$  (average channel load).
4. The channel power gains  $\alpha_{k,s}$  are i.i.d. random variables (RVs), for all users and slots.
5. All users have the same *transmit* SNR  $\gamma \triangleq E/N_0$  (i.e.,  $E_k = E \forall k = 1, \dots, N_u$ ).

Let  $t$  count the number of slots,  $b_k(t)$  the number of information bits from user  $k$  successfully decoded up to slot  $t$  and  $R_k(t) \triangleq b_k(t)/L$  the corresponding number of bit/s/Hz. The overall throughput  $\eta$  measured in bit/s/Hz is given by

$$\begin{aligned} \eta &= \lim_{t \rightarrow \infty} \frac{1}{tL} \sum_{k=1}^{N_u} b_k(t) \\ &= N_u \lim_{t \rightarrow \infty} \frac{1}{t} R_1(t) \end{aligned} \tag{9}$$

where the second line follows from the symmetry of the system with respect to any user.

Consider user 1 transmission. Under the above assumptions, the event that user 1 stops transmitting the current code word is recognized to be a *recurrent event* [42]. A random *reward*  $\mathcal{R}$  is associated to the occurrence of the recurrent event. In particular,  $\mathcal{R} = R$  bit/s/Hz if

transmission stops because successful decoding, and  $\mathcal{R} = 0$  bit/s/Hz if transmission stops because delay/rate constraint violation. We can apply the renewal-reward theorem [42] and get

$$\lim_{t \rightarrow \infty} \frac{1}{t} R_1(t) = \frac{E[\mathcal{R}]}{E[\mathcal{T}]} \quad \text{with prob. 1} \quad (10)$$

where  $\mathcal{T}$  is the random time between two consecutive occurrences of the recurrent event (inter-renewal time). Thus, the desired throughput general expression is

$$\eta = N_u \frac{E[\mathcal{R}]}{E[\mathcal{T}]} \quad (11)$$

In order to evaluate  $E[\mathcal{R}]$ , the mean reward, and  $E[\mathcal{T}]$ , the mean inter-renewal time, we focus on the transmission of a given code word of user 1 and we define the auxiliary RV  $\mathcal{M}$  to be the number of transmitted bursts between the instant when the code word is generated and the instant when its transmission is stopped (i.e., between two consecutive occurrences of the recurrent event). We define the event  $\mathcal{A}_m \triangleq \{I_{1,m} > R\}$ , and the probability  $q(m)$  that the random sequence  $I_{1,1}, I_{1,2}, \dots, I_{1,m}, \dots$  of mutual information at the user 1 decoder crosses level  $R$  at the  $m$ -th step (and not before), or, in other words, the probability of having successful decoding with  $m$  transmitted bursts. This is given by

$$\begin{aligned} q(m) &= \Pr(\overline{\mathcal{A}}_1, \dots, \overline{\mathcal{A}}_{m-1}, \mathcal{A}_m) \\ &= \Pr(\overline{\mathcal{A}}_1, \dots, \overline{\mathcal{A}}_{m-1}) - \Pr(\overline{\mathcal{A}}_1, \dots, \overline{\mathcal{A}}_m) \\ &= p(m-1) - p(m) \end{aligned} \quad (12)$$

where

$$p(m) \triangleq \Pr(\overline{\mathcal{A}}_1, \dots, \overline{\mathcal{A}}_m) = 1 - \sum_{\ell=1}^m q(\ell) \quad (13)$$

The joint probability distribution of  $\mathcal{T}$  and  $\mathcal{M}$

$$f_{\mathcal{T}, \mathcal{M}}(n, m) \triangleq \Pr(\mathcal{T} = n, \mathcal{M} = m)$$

is obtained explicitly as follows (in the case  $M \leq N$  otherwise the rate constraint is meaningless):

$$f_{\mathcal{T}, \mathcal{M}}(n, m) = \begin{cases} (1 - p_t)^N & n = N, m = 0 \\ v(N, m) + \binom{N}{m} (1 - p_t)^{N-m} p_t^m p(m) & n = N, 1 \leq m \leq M - 1 \\ v(n, M) + \binom{n-1}{M-1} (1 - p_t)^{n-M} p_t^M p(M) & M \leq n \leq N, m = M \\ v(n, m) & m \leq n \leq N - 1, 1 \leq m \leq M - 1 \\ 0 & \text{elsewhere} \end{cases} \quad (14)$$

(we use the short-hand notation  $v(n, m)$  for  $\binom{n-1}{m-1} (1 - p_t)^{n-m} p_t^m q(m)$ ). In Appendix B we show that (14) is a well-defined probability distribution for all  $0 \leq p_t \leq 1$ ,  $N \geq M > 0$  and non-negative non-increasing sequence  $\{p(m)\}$  with  $p(0) = 1$ .

At this point, we are ready to compute  $E[\mathcal{R}]$  and  $E[\mathcal{T}]$ . A reward  $R$  is obtained for  $(\mathcal{T}, \mathcal{M}) = (n, m)$  if successful decoding occurs in  $n$  slots with  $m$  transmitted bursts. This corresponds to placing  $m - 1$  transmissions in the first  $n - 1$  slots without success, and the  $m$ -th transmission in the  $n$ -th slot with success, which occurs with probability  $v(n, m)$ . Therefore,

$$\begin{aligned} E[\mathcal{R}] &= R \sum_{m=1}^M \sum_{n=m}^N v(n, m) \\ &= R \left[ 1 - \sum_{\ell=0}^{M-1} \binom{N}{\ell} (1 - p_t)^{N-\ell} p_t^\ell p(\ell) - \sum_{\ell=M}^N \binom{N}{\ell} (1 - p_t)^{N-\ell} p_t^\ell p(M) \right] \end{aligned} \quad (15)$$

The average inter-renewal time is simply given by

$$\begin{aligned} E[\mathcal{T}] &= \sum_{m=0}^M \sum_{n=1}^N n f_{\mathcal{T}, \mathcal{M}}(n, m) \\ &= \sum_{m=0}^{M-1} \frac{1}{p_t} p(m) \left[ 1 - \sum_{\ell=0}^m \binom{N+1}{\ell} (1 - p_t)^{N+1-\ell} p_t^\ell - \binom{N}{m} (1 - p_t)^{N-m} p_t^{m+1} \right] \end{aligned} \quad (16)$$

Finally, the desired closed-form expression for the system throughput is given by

$$\eta_{N,M} = RG \frac{\left[ 1 - \sum_{\ell=0}^{M-1} \binom{N}{\ell} (1-p_t)^{N-\ell} p_t^\ell p(\ell) - \sum_{\ell=M}^N \binom{N}{\ell} (1-p_t)^{N-\ell} p_t^\ell p(M) \right]}{\sum_{m=0}^{M-1} p(m) \left[ 1 - \sum_{\ell=0}^m \binom{N+1}{\ell} (1-p_t)^{N+1-\ell} p_t^\ell - \binom{N}{m} (1-p_t)^{N-m} p_t^{m+1} \right]} \quad (17)$$

(we write  $\eta_{N,M}$  in order to stress the dependence on  $N$  and  $M$ ). Protocols INR, RTD and ALO described before, for given parameters  $R$ ,  $M$ ,  $N$ ,  $N_u$  and  $G$ , differ in the probabilities  $p(m)$ . Consider first INR and RTD. These schemes have memory, since the receiver accumulates mutual information, for INR, or SINR, for RTD, over the sequence of slots  $\{s \in \mathcal{S}_{1,m}\}$ . From (6) and (8), since  $\beta_{1,s}$  is non-negative, it is apparent that the random sequence  $\{I_{1,m}\}$  is non-decreasing with probability 1. Then,  $\overline{\mathcal{A}_\ell} \subseteq \overline{\mathcal{A}_m}$  for all  $\ell \leq m$  and we can write

$$p(m) = \Pr(\overline{\mathcal{A}_m}) = \Pr(I_{1,m} \leq R) \quad (18)$$

For ALO,  $I_{1,m}$  given by (7) has no particular monotone behavior. However, the receiver has no memory of past signal bursts and the events  $\mathcal{A}_m$  are i.i.d.. Then, we can write

$$\begin{aligned} p(m) &= \Pr(\overline{\mathcal{A}_1}, \dots, \overline{\mathcal{A}_m}) \\ &= \prod_{i=1}^m \Pr(\overline{\mathcal{A}_i}) = \Pr(\overline{\mathcal{A}_1})^m \end{aligned} \quad (19)$$

Finally, for all protocols examined we obtain a compact expression for  $p(m)$  as

$$p(m) = \begin{cases} \Pr\left(\sum_{s \in \mathcal{S}_{1,m}} \log_2(1 + \beta_{1,s}) \leq R\right) & \text{INR} \\ \Pr\left(\log_2(1 + \sum_{s \in \mathcal{S}_{1,m}} \beta_{1,s}) \leq R\right) & \text{RTD} \\ \Pr(\log_2(1 + \beta_{1,1}) \leq R)^m & \text{ALO} \end{cases} \quad (20)$$

Some interesting properties of  $\eta_{N,M}$  can be derived immediately from (17) and (20). It can be easily shown that  $\eta_{N,M}$  is a decreasing function of  $p(m)$  and that

$$\Pr\left(\sum_{s \in \mathcal{S}_{1,m}} \log_2(1 + \beta_{1,s}) \leq R\right) \leq \Pr\left(\log_2(1 + \sum_{s \in \mathcal{S}_{1,m}} \beta_{1,s}) \leq R\right) \leq \Pr(\log_2(1 + \beta_{1,1}) \leq R)^m$$

for all  $m = 1, \dots, M$ . Then, as expected, the three protocols are related by

$$\eta_{N,M}^{(\text{INR})} \geq \eta_{N,M}^{(\text{RTD})} \geq \eta_{N,M}^{(\text{ALO})} \quad (21)$$

For ALO, we have that

$$\eta_{N,M}^{(\text{ALO})} = RG(1 - p(1)) \quad (22)$$

independently of  $N$  and  $M$ . This result is expected, since ALO has no memory and both delay and rate constraints are irrelevant. It is easy to show that the sequence  $p(m)$  for both INR and RTD is “sub-geometric”, i.e., that  $p(m) \leq p(1)^m$  for  $m = 1, 2, \dots$ , with equality only for  $m = 1$ . From this observation, it is possible to show that for, both INR and RTD, the throughput is increasing in  $N$  and  $M$ , i.e., that

$$\eta_{N+\ell, M+r} \geq \eta_{N,M} \quad (23)$$

for all  $\ell, r \geq 0$ , with equality for  $\ell = 0, r = 0$  only. This result is intuitive, since it makes sense that the throughput is going to increase by relaxing the delay or the rate constraints. However, it is not completely trivial since both the numerator (average reward) and the denominator (average inter-renewal time) of (17) are increasing functions of  $N$  and  $M$ . As a matter of fact, both the INR and the RTD protocols have the nice feature that “the longer we wait the more we gain”.

In classical Aloha analysis, it is customary to let  $N_u \rightarrow \infty$  while keeping  $G$  fixed and finite (infinite population [7]). Since  $p_t = G/N_u$ , this is equivalent to let  $p_t \rightarrow 0$  in (17). Interestingly, for all finite  $N$ , we have

$$\lim_{p_t \rightarrow 0} \eta_{N,M} = RG(1 - p(1))$$

In other words, in the limit for infinite population, the INR and RTD protocols with finite delay constraint are equivalent to ALO. In fact, in this case a large number of users transmit with very small probability, and the probability that a user transmit more than once in any finite time  $N$  is negligible. Therefore, either the packet is successfully decoded at the first attempt, or it is discarded, like in ALO. On the contrary, for  $N \rightarrow \infty$  the limit for infinite population is different for the three protocols.

In the next section, we study some limiting behaviors of the throughput for an *unconstrained system*, i.e., for  $N, M \rightarrow \infty$ . We notice here that all three protocols without constraints yield zero packet loss probability: the transmission of a code word ends only when it is correctly decoded. The unconstrained throughput (denoted simply by  $\eta$  for the sake of brevity) is easily



obtained from (17) as

$$\eta = \frac{RG}{\sum_{m=0}^{\infty} p(m)} = \frac{RG}{E[\mathcal{M}]} \quad (24)$$

where we used the fact that  $\sum_{m=0}^{\infty} p(m) = \sum_{m=1}^{\infty} m q(m) = E[\mathcal{M}]$ , the average number of transmitted bursts needed for successful decoding. In passing, we notice that  $E[\mathcal{M}]/p_t$  is the mean delay (measured in slots) for the transmission of an information message (i.e., it is the average number of slots between the generation of a code word and its successful decoding).

For ALO,  $\eta$  can be computed in closed form since  $p(1) = \Pr(\log_2(1 + \beta_{1,1}) \leq R)$  can be obtained explicitly (see Appendix D). For the channel without fading we have

$$\eta^{(\text{ALO})} = RG \sum_{\ell=0}^{K(R,\gamma)-1} \binom{N_u-1}{\ell} \left(\frac{G}{N_u}\right)^\ell \left(1 - \frac{G}{N_u}\right)^{N_u-1-\ell} \quad (25)$$

where

$$K(R, \gamma) = \left\lfloor \frac{1}{2^R - 1} - \frac{1}{\gamma} \right\rfloor + 1 \quad (26)$$

is the maximum number of simultaneous users in a slot that can be correctly decoded (notice that, depending on  $R$  and  $\gamma$ , a collision does not correspond necessarily to an error, since  $K(R, \gamma)$  might be larger than 1). For  $N_u \rightarrow \infty$ , (25) yields

$$\eta^{(\text{ALO})} = RG \sum_{\ell=0}^{K(R,\gamma)-1} e^{-G} \frac{G^\ell}{\ell!} \quad (27)$$

that for  $K(R, \gamma) = 1$  reduces to the well-known result of classical slotted Aloha,  $\eta = RGe^{-G}$ .

For the channel with Rayleigh fading we have

$$\eta^{(\text{ALO})} = RG \sum_{\ell=0}^{N_u-1} \binom{N_u-1}{\ell} \left(\frac{G}{N_u}\right)^\ell \left(1 - \frac{G}{N_u}\right)^{N_u-1-\ell} e^{-(2^R-1)/\gamma} 2^{-\ell R} \quad (28)$$

which for  $N_u \rightarrow \infty$  yields

$$\eta^{(\text{ALO})} = RGe^{-(2^R-1)/\gamma - (1-2^{-R})G}$$

For the INR and RTD it is not possible to find closed-form expressions for the probabilities  $p(m)$ . However, these can be calculated easily for any  $m$  as follows. Let  $Z = \beta_{1,1}$  and  $I = \log_2(1 + \beta_{1,1})$ . Then, from definitions (20) we see that for INR,  $p(m)$  is the cdf of the sum of  $m$  i.i.d. RVs distributed as  $I$ , evaluated in  $R$ , and for RTD,  $p(m)$  is the cdf of the sum of  $m$  i.i.d. RVs

distributed as  $Z$ , evaluated in  $2^R - 1$ . For small  $m$ ,  $p(m)$  can be evaluated from the distribution of  $\beta_{1,1}$  (e.g., by using the characteristic function). Since this approach involves discrete Fourier transforms whose length increases with  $m$ , it cannot be applied for large  $m$ . In this case, from the central limit theorem [35] we have that  $\frac{1}{\sqrt{m}} \sum_{s \in \mathcal{S}_{1,m}} \beta_{1,s}$  and  $\frac{1}{\sqrt{m}} \sum_{s \in \mathcal{S}_{1,m}} \log_2(1 + \beta_{1,s})$  are close to Gaussian RVs, for large  $m$ . Therefore,  $p(m)$  can be easily evaluated from the Gaussian cdf. For the sake of brevity, we skip the details of numerical calculations. However, it is interesting to notice that none of the results of this paper are obtained by Monte Carlo simulation.

Figs. 2 and 3 show  $\eta$  vs.  $R$ , for the INR, RTD and ALO protocols, with  $\gamma = 10\text{dB}$ ,  $N_u = 50$ ,  $G = 1$  and  $N = M \rightarrow \infty$  in AWGN and Rayleigh fading, respectively. For ALO on AWGN channel,  $\eta$  is zero for  $R > \log_2(1 + \gamma) = 3.5$  since for higher rates the SINR is not enough even in the absence of interferers (the system becomes power-limited rather than interference-limited). In the case of Rayleigh fading,  $\eta$  decreases with  $R$  but it is positive even for  $R > \log_2(1 + \gamma)$ , since there is a non zero probability that the fading gain is larger than one.

Fig. 4 shows  $\eta$  vs.  $R$  for  $\gamma = 10\text{dB}$ ,  $N_u = 50$  and  $G = 1$ ,  $N = 100$  and increasing values of  $M$  for INR on AWGN channel. As already pointed out, the curve for  $M = 1$  coincides with ALO. The different curves overlap for small  $R$  since one transmitted burst is sufficient to decode. For  $M > 1$  the throughput is non zero also for  $R$  larger than  $\log_2(1 + \gamma)$ . For example, for  $M = 2$  the maximum mutual information that can be accumulated is  $2 \log_2(1 + \gamma) = 6.92$ .

## 5 Limiting behaviors

In this section we study the three protocols in terms of their unconstrained throughput  $\eta$  for large  $R$ ,  $G$  or  $\gamma$ .

### 5.1 Limits for large $R$

In Appendix C we show that

$$\lim_{R \rightarrow \infty} \eta = \begin{cases} GE[\log_2(1 + \beta_{1,1})] & \text{INR} \\ 0 & \text{RTD} \\ 0 & \text{ALO} \end{cases} \quad (29)$$

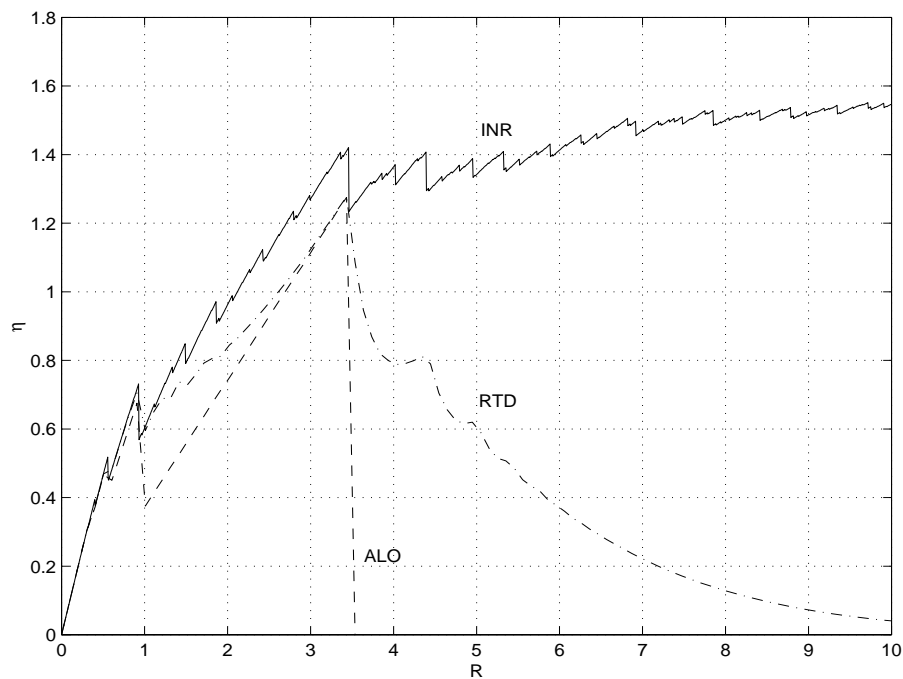


Figure 2:  $\eta$  vs.  $R$  for  $\gamma=10\text{dB}$ ,  $N_u=50$ ,  $G = 1$  and  $N, M \rightarrow \infty$  on AWGN channel.

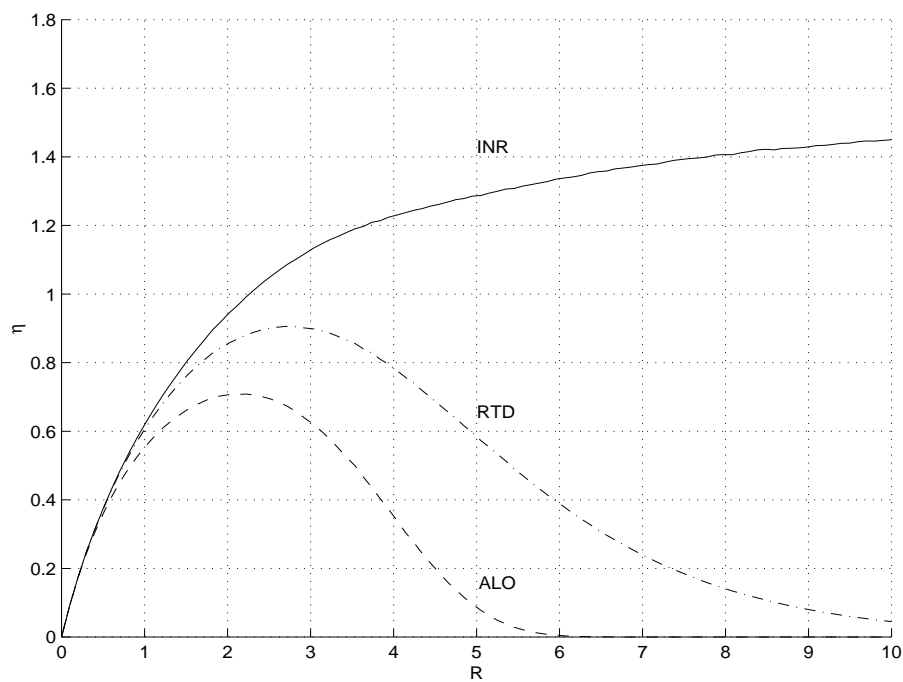


Figure 3:  $\eta$  vs.  $R$  for  $\gamma=10\text{dB}$ ,  $N_u=50$ ,  $G = 1$  and  $N, M \rightarrow \infty$  on Rayleigh fading channel.

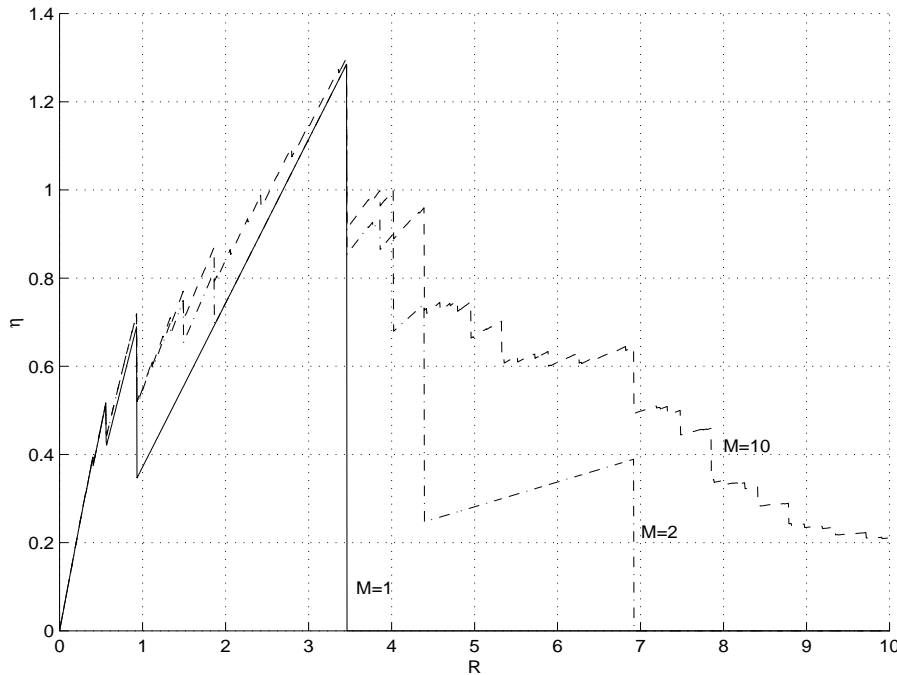


Figure 4:  $\eta$  vs.  $R$  for  $\gamma=10\text{dB}$ ,  $N_u=50$ ,  $G = 1$  and  $N = 100$  for INR on AWGN channel.

**ALO and RTD schemes.** ALO and RTD involve strongly suboptimal coding schemes, for which  $E[\mathcal{M}]$  grows faster than  $R$ . Thus, the limiting  $\eta$  is zero. Since  $\eta = 0$  for  $R = 0$  and goes to 0 for large  $R$ , for both protocols there exist an optimal choice of  $0 < R < \infty$  maximizing  $\eta$  (see Figs. 2 and 3). This optimal  $R$  depends, in general, on  $G$  and  $\gamma$ . From a practical system design point of view, this shows that the burst spectral efficiency  $R$  should be dimensioned according to the channel load  $G$  and the SNR  $\gamma$ .

**INR scheme.** For INR,  $\eta < GE[\log_2(1 + \beta_{1,1})]$  for all finite  $R$ . This fact is quite hard to show directly by using (24), since the probabilities  $p(m)$  depend on  $R$  but a closed form is not available. However, we can provide a simple indirect proof of the statement as follows. The quantity  $C \triangleq E[\log_2(1 + \beta_{1,1})]$  is the capacity of the memoryless  $L$ -block interference channel [40] given in (3), where the interference signal  $\mathbf{v}_{k,s}$  is circularly-symmetric complex Gaussian with i.i.d. components and where  $\beta_{k,s}$  is the SINR for block  $s$ . A well-known result states that feedback does not increase the capacity of memoryless channels [59]. Then, even if the encoder has available the sequence of past received vectors  $\mathbf{y}_1, \dots, \mathbf{y}_{s-1}$ , the maximum transmissible

rate for channel (3) is  $C$ .<sup>6</sup> Hence, we conclude that  $\eta = GC$  is actually the maximum achievable throughput on this channel, irrespectively of the feedback and for any choice of  $R$ . From a practical system design point of view, in the absence of rate and delay constraints it is convenient to work with a very high burst spectral efficiency  $R$ , irrespectively of the channel load  $G$  and the SNR  $\gamma$ .

The maximum throughput is achieved for infinite delay. It is interesting to notice that, with infinite delay, the same maximum throughput (with zero packet loss probability) can be achieved by a system without feedback (just forward error correcting codes) [17]. It is natural to ask why the ACK/NACK feedback channel should be implemented at all. The answer is provided by closer examination of the average delay: the system without feedback needs a very large (infinite) delay in order to transmit with arbitrarily small packet loss probability for all values of  $\eta$  [17]. On the contrary, the INR protocol achieves zero transmission failure probability with finite average delay for all  $\eta$  strictly less than  $GC$ .

Fig. 5 shows the average number of transmitted bursts  $E[\mathcal{M}]$  vs.  $\eta$  for the ALO and INR protocols in the case of Rayleigh fading, for  $\gamma = 10\text{dB}$ ,  $N_u = 50$  and  $G = 1$ ,  $N, M \rightarrow \infty$ . The corresponding average delay is given by  $N_u E[\mathcal{M}]/G$ .

## 5.2 Limits for large $G$

We consider an optimized system with respect to  $R$  and we let  $\bar{\eta} = \sup_R \eta$ . In Appendix C, we show that

$$\lim_{G \rightarrow \infty} \bar{\eta} = \begin{cases} \log_2(e) & \text{INR} \\ \log_2(e) & \text{RTD} \\ \frac{\log_2(e)}{\bar{\alpha}} \kappa & \text{ALO} \end{cases} \quad (30)$$

where  $\bar{\alpha} = E[\alpha_{1,1}]$ ,  $\kappa = \sup_{u \geq 0} u[1 - F_\alpha(u)]$  and  $F_\alpha(u) \triangleq \Pr(\alpha_{1,1} \leq u)$  is the cdf of  $\alpha_{1,1}$ . For AWGN,  $F_\alpha(u)$  is a step function with jump in  $u = 1$ , therefore  $\kappa = \bar{\alpha} = 1$ . For Rayleigh

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<sup>6</sup>It is important to notice that (3) is memoryless at the block level, but not at the symbol level. Feedback does not provide any capacity increase if the feedback channel works at the slot rate, i.e., it sends back the whole received vector  $\mathbf{y}_s$  at the end of each  $s$ -th slot. This is precisely the way the ACK/NACK feedback works. On the contrary, capacity would be clearly increased by a feedback working at faster rate, which sends back the components of  $\mathbf{y}_s$  as soon as they are received, during each  $s$ -th slot.

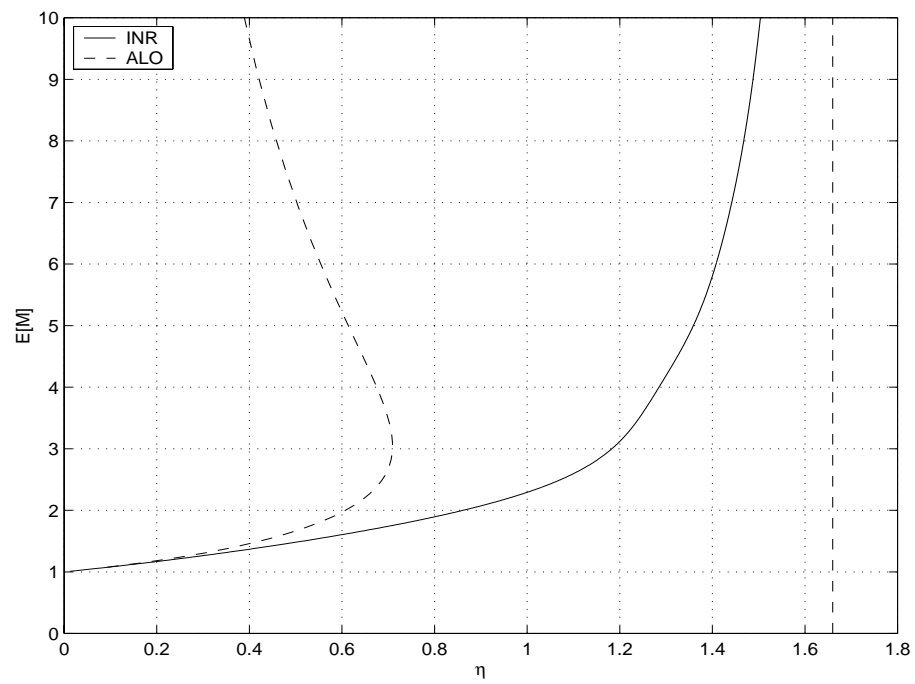


Figure 5:  $E[\mathcal{M}]$  vs.  $\eta$  for  $\gamma = 10\text{dB}$ ,  $N_u = 50$  and  $G = 1$  and  $N, M \rightarrow \infty$  for ALO and INR on Rayleigh fading channel.

fading,  $F_\alpha(u) = 1 - e^{-u/\bar{\alpha}}$ , therefore  $\kappa = \bar{\alpha}/e$  and  $\lim_{G \rightarrow \infty} \bar{\eta} = \log_2(e)/e$ . This shows that for large channel load  $G$  all schemes are equivalent in AWGN, while ALO performs worse than INR and RTD in Rayleigh fading. In fact, ALO considers only the most recent received block for decoding. Hence, there is no “averaging effect” with respect to the fading affecting the useful signal over a long sequence of slots. Fig. 6 shows  $\bar{\eta}$  vs.  $G$  for ALO,  $\gamma=10\text{dB}$  on AWGN and Rayleigh fading channel.

In order to gain insight in the behavior of the ALO protocol, it is useful to take a closer look at the throughput curve in the case of AWGN.<sup>7</sup> This curve is obtained by noticing that the supremum of  $\eta^{(\text{ALO})}$  given in (27) for fixed  $G$  and  $\gamma$  is always obtained when  $R = \log_2(1 + \gamma/(1 + K\gamma))$  for some integer  $K$ , where  $K + 1$  is the maximum number of users that can collide on the same slots without causing a decoding error. Therefore, maximizing with respect to  $R$  (for given  $G$ ) is equivalent to searching for the maximum of the expression  $\log_2(1 + \gamma/(1 + K\gamma))G \sum_{\ell=0}^K e^{-G} G^\ell / \ell!$  over the non-negative integers  $K$ . In particular, for small  $G$  the maximum is obtained by  $K = 0$ . In this case, the throughput is maximized by choosing the largest possible  $R$ , i.e.,  $R = \log_2(1 + \gamma)$ , and by letting the protocol alone to take care of collisions, like in conventional Aloha. As  $G$  increases, the maximum is obtained by larger and larger  $K$ . In this case, the throughput is maximized by choosing  $R$  in order to tolerate up to  $K$  interferers, i.e.,  $R = \log_2(1 + \gamma/(1 + K\gamma))$  (a decoding error occurs only when there are more than  $K$  interferers). In this way, the task of coping with collisions is shared by channel coding and by the retransmission protocol: channel coding yields no errors for up to  $K + 1$  active users in the slot, while if the number of active users is larger than  $K + 1$  retransmission is needed.

As  $G$  becomes large, a very large number of users transmit in every slot. In the limit, the system is equivalent to a CDMA system with an infinite number of users  $K$  and infinite spreading gain  $N$ , such that the ratio  $K/N$  is equal to  $G$  [10, 56]. In fact, the channel load  $G$  is precisely the (average) number of users per dimension (per chip). The throughput of such CDMA system is given by [54, 12]

$$\eta^{(\text{CDMA})} = GE \left[ \log_2 \left( 1 + \frac{\alpha_{1,1}\gamma}{1 + G\bar{\alpha}\gamma} \right) \right] \quad (31)$$

Its maximum is  $\log_2(e)$  bit/s/Hz, obtained for  $G \rightarrow \infty$ . Interestingly, the throughput of CDMA

<sup>7</sup>The throughput of ALO in AWGN converges to its limiting value  $\log_2(e)$  very slowly. Convergence can be seen on a much larger scale of  $G$ . We chose this scale in order to better illustrate the behavior in a practical range of  $G$ .

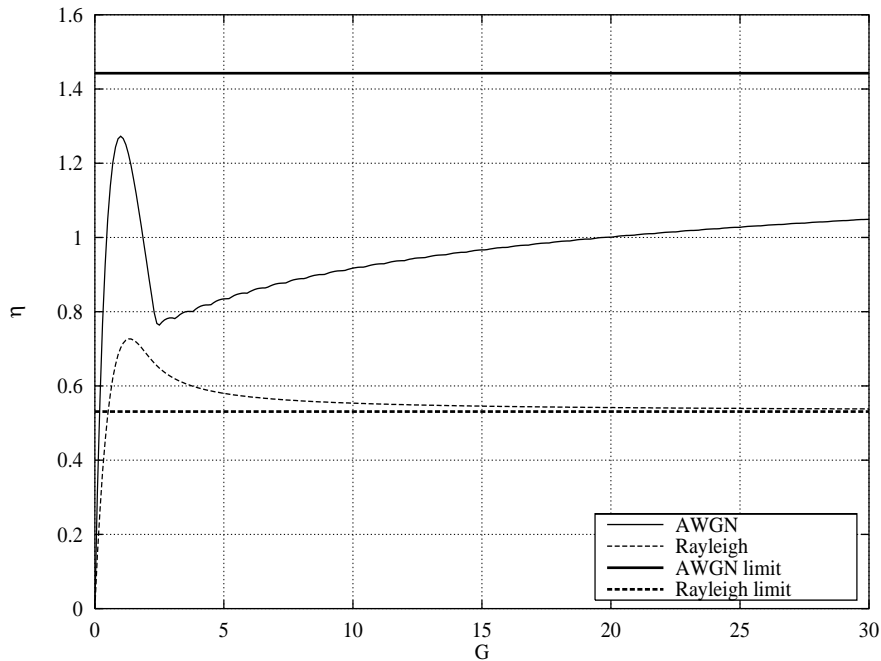


Figure 6:  $\bar{\eta}$  vs.  $G$  for ALO,  $\gamma=10\text{dB}$  in AWGN and Rayleigh fading.

is less than  $\log_2(e)$  for all finite  $G$ , while for the INR, RTD and ALO schemes there might exist a range of  $G$  for which  $\bar{\eta} > \log_2(e)$ .

In the case of INR,  $\bar{\eta}$  is the limiting value for large  $R$  given by (29). Fig. 7 shows  $\bar{\eta}$  vs.  $G$  for INR and CDMA,  $\gamma=10\text{dB}$  for AWGN and Rayleigh fading channel. Both the multiaccess schemes have the same limiting throughput, equal to  $\log_2(e)$ , for  $G \rightarrow \infty$ , however the INR scheme tends to this limit from above while CDMA from below. Also, notice that for large  $G$  the fading increases the throughput of the INR scheme, while it decreases the throughput of CDMA. This can be interpreted in terms of the *capture effect*. With the bursty discontinuous transmission of slotted INR, only a finite number of users are going to collide on every slot, for every finite  $G$ . Because of fading, some of the interferers are received with low power and, apparently, this provides a throughput increase for large  $G$ . On the contrary, in CDMA all users transmit all the time and interference converges quickly to a deterministic average. Because of the concavity of the logarithm, this provides a throughput decrease.



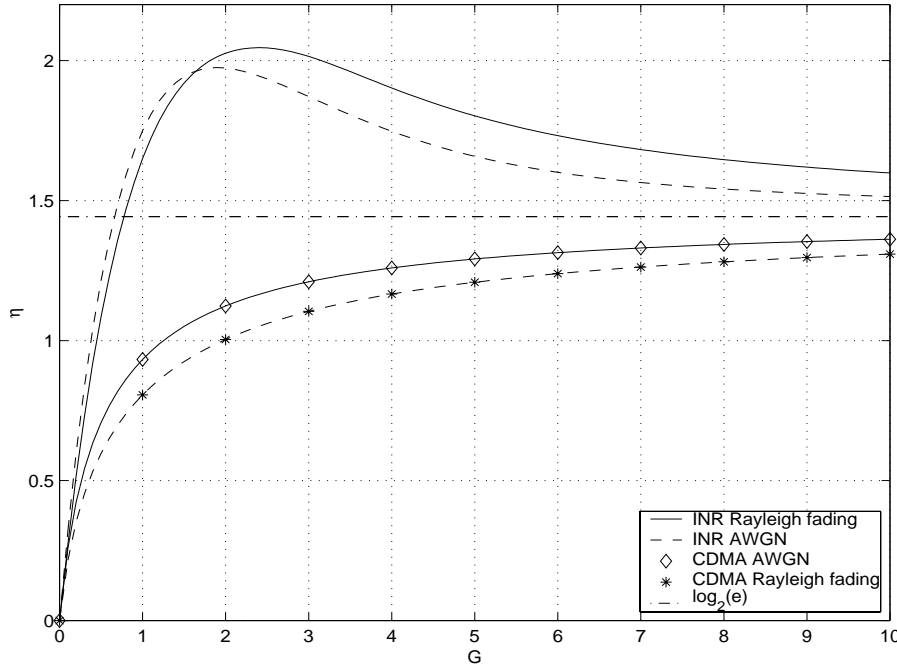


Figure 7:  $\bar{\eta}$  vs.  $G$  for INR and CDMA,  $\gamma=10$ dB in AWGN and Rayleigh fading.

### 5.3 Limits for large $\gamma$

In Appendix C we show that

$$\lim_{\gamma \rightarrow \infty} \bar{\eta} = \infty \quad (32)$$

for all protocols examined, as opposed to CDMA, for which the limit of (31) for large  $\gamma$  yields  $G \log_2(1 + 1/G)$  for AWGN and  $Ge^G \text{Ei}(1, G)$  in Rayleigh fading ( $\text{Ei}(1, z) \triangleq \int_1^\infty e^{-zt}/t dt$ ).

This means that the ARQ system is not interference limited, even if no joint decoding is implemented at the receiver: arbitrarily high throughput can be obtained by simply increasing transmit SNR of all users, irrespectively of power control, fading, etc ... Intuitively, this is due to the fact that there is a non-zero probability that only one user is active on any given slot, and can transmit at very high instantaneous rate.

## 6 Conclusions

Combined channel coding and retransmission protocols appear to be a viable and simple solution for reliable packet-radio communication requiring high instantaneous rates and very low error probability and characterized by bursty sporadic transmission and by mild delay constraints.

In this paper, we present an information-theoretic throughput analysis of some Hybrid-ARQ protocols under idealized but fairly general conditions. We showed that typical set decoding has very desirable properties for Hybrid-ARQ, in the limit for large slot dimension. From a renewal-reward theory approach, we obtained closed-form throughput formulas for three simple protocols: a generalization of slotted Aloha (ALO), a repetition time diversity scheme with maximal-ratio packet combining (RTD) and an incremental redundancy scheme based on progressively punctured codes (INR). We analyzed the effect of delay and rate constraints on the throughput, as well as the limiting behavior with respect to the slot spectral efficiency, the channel load and the transmit SNR. Interestingly, all three protocols are not interference-limited, and achieve arbitrarily large throughput by simply increasing the transmit power of all users.

We conclude by pointing out some future research directions inspired by this analysis.

- Practical coding and decoding schemes based on incremental redundancy and featuring built-in error detection capability should be used with Hybrid-ARQ. Turbo-codes (or other forms of concatenated coding) with iterative decoding appear to be a promising solution. However, the behavior of iterative decoders in the presence of decoding errors should be better characterized in order to exploit it for error detection.
- The assumption that the receiver knows exactly the time-hopping sequences of all users might not be realistic. If user activity is random and not known to the receiver, our results can be seen as an upperbound on the achievable throughput obtained by a *geine-aided* receiver which knows *a priori* the active users in each slot. True random access, where the receiver must also detect which users are active in order to make the appropriate packet combining, might be studied by inserting in our framework an *active user detection* scheme.
- In this work, we concentrated on a very simple receiver that does not attempt to decode the users jointly. A natural direction for future research is to consider joint decoding at

the receiver (e.g., implemented by stripping). A theoretical difficulty is represented by the user random activity [1]. In fact, because of random access, the capacity region varies from slot to slot and it is not known in advance, unless a complicated reservation/allocation scheme is implemented. Also, the set of interfering users might be different from slot to slot, and it is not clear how to carry out joint decoding across the slots. First steps in this direction are taken in [19, 27].

- In most practical applications, packet-radio networks must co-exist with other systems, as for example a connection-oriented CDMA system where a large number of low-power low-rate users transmit continuously. Quite a lot of work has been dedicated to the problem of power control for bursty transmission, where closed-loop schemes are not effective, in the fear that high-rate high-power bursty users might create too much interference to an underlying CDMA system. An appealing consequence of our study is the following: instead of trying to control bursty users, we can let them transmit at full-power. Thanks to the ARQ protocol, the signal from all bursty users can be eventually decoded correctly and subtracted from the received signal, so that the underlying CDMA system “sees” a clean channel, as if the bursty users were not there. In this way, the two quite different system could be layered one on top of the other. Obviously, in order to make this claim rigorous several issues must be addressed in the details: perhaps the most important of which is the delay. In fact, CDMA users can be decoded only after the signal from bursty users has been subtracted. Then, the variable decoding delay associated with the ARQ protocol imposes a variable decoding delay also on the CDMA system. If CDMA users have a strict delay constraint (e.g., due to real-time speech transmission, like in cellular telephony), outages due to the occurrence of large decoding delay events must be taken into account.

### Acknowledgement

The authors would like to acknowledge the kind and fruitful feedback from Prof. Sergio Verdú and Dr. Michael Mecking.

### Appendices

## A Proofs of Lemmas 1,2 and 3

Following standard continuity arguments [15], we consider a quantization of the input and a partition of the output of (3) and we work on the resulting discrete channel. The results for the continuous channel can be obtained by taking the supremum over all input quantizations and output partitions. Fix a sequence of channel transition probabilities  $\mathcal{P} = \{p_{k,s}(y|x) : s \in \mathcal{S}_{k,M}\}$ . Let  $P(\{\mathbf{x}_{k,s}, \mathbf{y}_s : s \in \mathcal{S}_{k,m}\})$ ,  $P(\{\mathbf{x}_{k,s} : s \in \mathcal{S}_{k,m}\})$  and  $P(\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\})$  be the joint and the marginal probability distributions induced by  $\mathcal{P}$  and by the input distribution  $q(x)$ . Since on every slot  $s \in \mathcal{S}_{k,m}$  the quantized version of channel (3) is a time-invariant DMC, for the weak law of large numbers [35] we have the following limits in probability:

$$\begin{aligned} \lim_{L \rightarrow \infty} \frac{1}{L} \log_2 P(\{\mathbf{x}_{k,s}, \mathbf{y}_s : s \in \mathcal{S}_{k,m}\}) &= - \sum_{s \in \mathcal{S}_{k,m}} H_{k,s}(X, Y) \\ \lim_{L \rightarrow \infty} \frac{1}{L} \log_2 P(\{\mathbf{x}_{k,s} : s \in \mathcal{S}_{k,m}\}) &= -mH_k(X) \\ \lim_{L \rightarrow \infty} \frac{1}{L} \log_2 P(\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\}) &= - \sum_{s \in \mathcal{S}_{k,m}} H_{k,s}(Y) \end{aligned} \quad (33)$$

where

$$\begin{aligned} H_{k,s}(X, Y) &\triangleq - \sum_{x,y} q(x)p_{k,s}(y|x) \log_2 q(x)p_{k,s}(y|x) \\ H_k(X) &\triangleq - \sum_x q(x) \log_2 q(x) \\ H_{k,s}(Y) &\triangleq - \sum_{x,y} q(x)p_{k,s}(y|x) \log_2 \sum_{x'} q(x')p_{k,s}(y|x') \end{aligned} \quad (34)$$

are the joint, input and output entropies per letter in slot  $s$ . The typical set  $\mathcal{A}_{k,m}^\epsilon$  is defined as the set of all sequences  $\{\mathbf{x}_{k,s}, \mathbf{y}_s : s \in \mathcal{S}_{k,m}\}$  satisfying

$$\begin{aligned} \left| \frac{1}{L} \log_2 P(\{\mathbf{x}_{k,s}, \mathbf{y}_s : s \in \mathcal{S}_{k,m}\}) + \sum_{s \in \mathcal{S}_{k,m}} H_{k,s}(X, Y) \right| &\leq \epsilon \\ \left| \frac{1}{L} \log_2 P(\{\mathbf{x}_{k,s} : s \in \mathcal{S}_{k,m}\}) + mH_k(X) \right| &\leq \epsilon \\ \left| \frac{1}{L} \log_2 P(\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\}) + \sum_{s \in \mathcal{S}_{k,m}} H_{k,s}(Y) \right| &\leq \epsilon \end{aligned} \quad (35)$$

By letting  $I(q(x), p_{k,s}(y|x)) \triangleq H_k(X) + H_{k,s}(Y) - H_{k,s}(X, Y)$ , and by following the same steps in [59, Th. 8.7.1], we get that any rate less than  $\frac{1}{m} \sum_{s \in \mathcal{S}_{k,m}} I(q(x), p_{k,s}(y|x))$  is  $\epsilon$ -achievable. In

particular, for  $m = M$  and given sequence of channels  $\mathcal{P}$ , for sufficiently large  $L$  there exists codes  $\mathcal{C}_k$  of length  $LM$  and rate  $R/M$  with error probability (with typical set decoding) less than  $\epsilon$  if

$$R < \sum_{s \in \mathcal{S}_{k,M}} I(q(x), p_{k,s}(y|x)) \quad (36)$$

In order to prove Lemma 1 we need to show that: i) there is a single code  $\mathcal{C}_k$  having error probability uniformly less than  $\epsilon$  over all sequences of channels  $\mathcal{P}$  satisfying (36); ii) for all  $1 \leq m \leq M$ , if  $R < \sum_{s \in \mathcal{S}_{k,m}} I(q(x), p_{k,s}(y|x))$ , then the punctured code  $\mathcal{C}_{k,m}$  obtained from  $\mathcal{C}_k$  by taking the first  $m$  subblocks of length  $L$  has also error probability less than  $\epsilon$ .

From the random coding achievability part and from the strong converse (it holds for every sequence of channels as shown by Lemma 2) we have that

$$E_{\mathcal{C}}[\Pr(\text{error}|\mathcal{P}, \mathcal{C})] \rightarrow 1_{\{\sum_s I(q(x), p_{k,s}(y|x)) \leq R\}}$$

as  $L \rightarrow \infty$ , where  $1_{\{A\}}$  denotes the indicator function of the event  $A$  and where  $E_{\mathcal{C}}$  denotes expectation over the ensemble of all codes of size  $2^{RL}$  and block length  $LM$  generated according to the input distribution  $q(x)$ . By averaging also with respect to the sequence of channels and exchanging expectations with respect to  $\mathcal{C}$  and with respect to  $\mathcal{P}$  (we can always do it, since the integrand is non-negative and bounded by 1) we obtain

$$E_{\mathcal{P}}[E_{\mathcal{C}}[\Pr(\text{error}|\mathcal{P}, \mathcal{C})]] \rightarrow \Pr\left(\sum_{s \in \mathcal{S}_{k,M}} I(q(x), p_{k,s}(y|x)) \leq R\right)$$

Then, there exists a family of codes  $\mathcal{C}^*$  for increasing  $L$  such that

$$E_{\mathcal{P}}[\Pr(\text{error}|\mathcal{P}, \mathcal{C}^*)] \leq \Pr\left(\sum_{s \in \mathcal{S}_{k,M}} I(q(x), p_{k,s}(y|x)) \leq R\right) \quad (37)$$

for  $L$  sufficiently large. Because of the strong converse,  $\Pr(\text{error}|\mathcal{P}, \mathcal{C}^*) \rightarrow 1$  for all  $\mathcal{P}$  such that  $\sum_{s \in \mathcal{S}_{k,M}} I(q(x), p_{k,s}(y|x)) \leq R$ . Then, in order to satisfy (37) it must be  $\Pr(\text{error}|\mathcal{P}, \mathcal{C}^*) \rightarrow 0$  for all channel sequences  $\mathcal{P}$  such that  $\sum_{s \in \mathcal{S}_{k,M}} I(q(x), p_{k,s}(y|x)) > R$ . This shows that, asymptotically, there exist codes  $\mathcal{C}^*$  such that

$$\Pr(\text{error}|\mathcal{P}, \mathcal{C}^*) \rightarrow 1_{\{\sum_s I(q(x), p_{k,s}(y|x)) \leq R\}}$$

for all channel sequences  $\mathcal{P}$ .

Now, let  $\mathcal{C}_{k,M} = \mathcal{C}^*$  and assume that for a given sequence of channels

$$R < \sum_{s \in \mathcal{S}_{k,m}} I(q(x), p_{k,s}(y|x)) \quad (38)$$

for some  $1 \leq m \leq M$ . Then, we can extend the sequence of channels by adding to  $\{p_{k,s}(y|x) : s \in \mathcal{S}_{k,m}\}$  other  $M - m$  dummy useless memoryless channels whose output is independent of the input. Since the mutual information on the last  $M - m$  blocks is zero and because of (38), the resulting sequence of  $M$  channels  $\mathcal{P}'$  satisfies  $\Pr(\text{error}|\mathcal{P}', \mathcal{C}_{k,M}) \rightarrow 0$ . Notice that extending the sequence of channels is equivalent to appending dummy output signal blocks  $\mathbf{z}_i$  independent of the channel input to the received signal  $\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\}$ , as described in Section 3. This concludes the proof of Lemma 1.

In order to prove Lemma 2 we use the limit in probability

$$\lim_{L \rightarrow \infty} \frac{1}{L} \log_2 \frac{P(\{\mathbf{x}_{k,s}, \mathbf{y}_s : s \in \mathcal{S}_{k,m}\})}{P(\{\mathbf{x}_{k,s} : s \in \mathcal{S}_{k,m}\})P(\{\mathbf{y}_s : s \in \mathcal{S}_{k,m}\})} = \sum_{s \in \mathcal{S}_{k,m}} I(q(x), p_{k,s}(y|x)) \quad (39)$$

where the LHS is the limiting normalized *information density* over the  $m$  slots and where, for a fixed sequence of channels, the RHS is a constant. Therefore, the inf-information rate and the sup-information rate (see definitions in [57]) coincide and, from [57, Th. 7], the strong converse holds, conditionally on the sequence  $\{p_{k,s}(y|x) : s \in \mathcal{S}_{k,m}\}$ .

In order to prove Lemma 3 we use the simple relation

$$\bigcup_{\hat{w} \neq w} \mathcal{E}_{\hat{w}} \subseteq \left\{ \{\mathbf{x}_{k,s}^{(w)}, \mathbf{y}_s : s \in \mathcal{S}_{k,m}\} \notin \mathcal{A}_{k,m}^\epsilon \right\}$$

$\forall w \in \{1, \dots, 2^{RL}\}$  and for all  $m = 1, \dots, M$ . This implies that

$$\begin{aligned} \Pr(\text{undetected error}|w, \mathcal{P}, \mathcal{C}_{k,m}) &\leq \Pr\left(\left\{\{\mathbf{x}_{k,s}^{(w)}, \mathbf{y}_s : s \in \mathcal{S}_{k,m}\} \notin \mathcal{A}_{k,m}^\epsilon\right\} \middle| w\right) \\ &< \epsilon \end{aligned} \quad (40)$$

where the second inequality holds for arbitrary  $\epsilon > 0$  and sufficiently large  $L$ , since the probability that the channel input and output sequences are not jointly typical vanishes as  $L \rightarrow \infty$  [59, Th. 8.6.1]. Then, Lemma 3 follows from averaging (40) over all transmitted messages.

## B Probability distribution of the inter-renewal time

The joint pdf of  $\mathcal{T}$  and  $\mathcal{M}$  can be expressed by

$$f_{\mathcal{T},\mathcal{M}}(n, m) = \begin{cases} (1 - p_t)^N & n = N, m = 0 \\ v(N, m) + g(N, m) & n = N, 1 \leq m \leq M - 1 \\ v(n, M) + r(n, M) & M \leq n \leq N, m = M \\ v(n, m) & m \leq n \leq N - 1, 1 \leq m \leq M - 1 \\ 0 & \text{elsewhere} \end{cases} \quad (41)$$

where we define

$$\begin{aligned} v(n, m) &= \binom{n-1}{m-1} (1 - p_t)^{n-m} p_t^m q(m) \\ r(n, M) &= \binom{n-1}{M-1} (1 - p_t)^{n-M} p_t^M p(M) \\ g(N, m) &= \binom{N}{m} (1 - p_t)^{N-m} p_t^m p(m) \end{aligned}$$

where  $q(m)$  is defined in (12),  $p(m)$  in (13) and they are related by  $q(m) = p(m-1) - p(m)$ .

We show that (41) is a well-defined probability distribution for any  $N \geq M > 0$ ,  $0 \leq p_t \leq 1$  and non-negative decreasing sequence  $\{p(m)\}$  with  $p(0) = 1$ . Since all terms in (41) are non-negative, it is sufficient to show that their sum is 1. We use the identity

$$\sum_{n=k}^{N-1} \binom{n}{k} a^{n-k} (1-a)^{k+1} = 1 - \sum_{\ell=0}^k \binom{N}{\ell} a^{N-\ell} (1-a)^\ell \quad (42)$$

(for  $0 \leq a \leq 1$ ) and write

$$\sum_{n,m} f_{\mathcal{T},\mathcal{M}}(n, m) = \sum_{m=1}^M \sum_{n=m}^N v(n, m) + \sum_{m=0}^{M-1} g(N, m) + \sum_{n=M}^N r(n, M) \quad (43)$$

For the sake of brevity, we let  $s(\ell) \triangleq \binom{N}{\ell} (1 - p_t)^{N-\ell} p_t^\ell$ . The first, second and third terms in the RHS of (43) are given by

$$\sum_{m=1}^M \sum_{n=m}^N v(n, m) = 1 - \sum_{\ell=0}^{M-1} s(\ell) p(\ell) - p(M) \left( 1 - \sum_{\ell=0}^{M-1} s(\ell) \right) \quad (44)$$

by

$$\sum_{m=0}^{M-1} g(N, m) = \sum_{m=0}^{M-1} s(m)p(m) \quad (45)$$

and by

$$\sum_{n=M}^N r(n, M) = p(M) \left( 1 - \sum_{\ell=0}^{M-1} s(\ell) \right) \quad (46)$$

where we used the fact that  $\sum_{k=\ell+1}^N q(k) = p(\ell) - p(N)$ . The result follows by noting that the second and third term in the RHS of (44) are the opposite of the terms given in (45) and in (46).

## C Limits

### C.1 Limits for large R

We want to establish the limiting behavior of the system throughput for large  $R$ , in the case of  $N, M \rightarrow \infty$ . To this purpose we consider  $\lim_{R \rightarrow \infty} 1/\eta$ , where  $\eta$  is given in (24).

We need the following lemmas:

**Lemma C.1.** Let  $X$  be a RV with cdf  $F_X(x)$ . Then,  $\forall y$ ,

$$1_{\{x \geq y\}} F_X(y) \leq F_X(x) \leq F_X(y) + 1_{\{x \geq y\}} (1 - F_X(y)) \quad (47)$$

◇

**Lemma C.2.** If  $a_n \rightarrow a$  as  $n \rightarrow \infty$ , then for any non-negative finite integer  $k$

$$b_n = \frac{1}{n+k} \sum_{i=1}^n a_i \rightarrow a \text{ for } n \rightarrow \infty \quad (48)$$

**Proof.** It follows immediately from the *Cesaro's mean* theorem [59].

◇

**Lemma C.3.** Let  $X_\ell$  be i.i.d. zero-mean RVs with variance  $\sigma_X^2$ . For all  $\epsilon > 0$ , we have

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr \left( \frac{1}{n} \sum_{\ell=1}^n X_\ell < \epsilon \right) &= 1 \\ \lim_{n \rightarrow \infty} \Pr \left( \frac{1}{n} \sum_{\ell=1}^n X_\ell < -\epsilon \right) &= 0 \end{aligned} \quad (49)$$

Moreover, for sufficiently large  $n$  we have

$$\Pr \left( \frac{1}{\sqrt{n}} \sum_{\ell=1}^n X_\ell < -\sqrt{n}\epsilon \right) \leq e^{-n\epsilon^2/(2\sigma_X^2)} \quad (50)$$



**Proof.** (49) follows immediately from the weak law of large numbers and (50) from the central limit theorem [35], by using the bound on the Gaussian tail function  $Q(x) \leq \exp(-x^2/2)$ .  $\diamond$

**INR protocol.** We let  $X_i \triangleq \log_2(1 + \beta_{1,s_i})$  for  $s_i \in \mathcal{S}_{1,m}$  and  $\mu_X \triangleq E[X_i]$ . Then, for  $\epsilon_1 > 0$  and  $b = \mu_X + \epsilon_1$  we can write

$$\begin{aligned}
\lim_{R \rightarrow \infty} \frac{1}{\eta} &= \lim_{R \rightarrow \infty} \frac{1 + \sum_{m=1}^{\infty} \Pr(\sum_{i=1}^m X_i < R)}{RG} \\
&\stackrel{(a)}{\geq} \lim_{R \rightarrow \infty} \frac{1}{RG} \sum_{m=1}^{\infty} 1_{\{R \geq mb\}} \Pr\left(\frac{1}{m} \sum_{i=1}^m X_i \geq b\right) \\
&\stackrel{(b)}{\geq} \frac{1}{Gb} \lim_{R \rightarrow \infty} \frac{1}{\lfloor R/b \rfloor + 1} \sum_{m=1}^{\lfloor R/b \rfloor} \Pr\left(\frac{1}{m} \sum_{i=1}^m X_i < b\right) \\
&\stackrel{(c)}{=} \frac{1}{G(\mu_X + \epsilon_1)}
\end{aligned} \tag{51}$$

where (a) follows by applying Lemma C.1 to the RV  $\sum_{i=1}^m X_i$  with  $x = R$  and  $y = mb$ ; (b) follows by noting that  $b/R \geq 1/(\lfloor R/b \rfloor + 1)$ ; (c) follows from Lemma C.2 and Lemma C.3, since  $\lim_{m \rightarrow \infty} \Pr(\frac{1}{m} \sum_{i=1}^m X_i - \mu_X < \epsilon) = 1$ . Similarly,  $\epsilon_2 > 0$  and  $b = \mu_X - \epsilon_2$  we can write

$$\begin{aligned}
\lim_{R \rightarrow \infty} \frac{1}{\eta} &= \lim_{R \rightarrow \infty} \frac{1 + \sum_{m=1}^{+\infty} \Pr(\sum_{i=1}^m X_i < R)}{RG} \\
&\leq \lim_{R \rightarrow \infty} \frac{1}{RG} \sum_{m=1}^{\infty} \left[ \Pr\left(\frac{1}{m} \sum_{i=1}^m X_i < b\right) + 1_{\{R \geq mb\}} \right] \\
&\leq \lim_{R \rightarrow \infty} \frac{1}{GR} \sum_{m=1}^{\infty} \Pr\left(\frac{1}{m} \sum_{i=1}^m X_i < b\right) + \frac{1}{Gb} \lim_{R \rightarrow \infty} \frac{1}{\lfloor R/b \rfloor} \sum_{m=1}^{\lfloor R/b \rfloor} 1 \\
&\stackrel{(a)}{=} \frac{1}{G(\mu_X - \epsilon_2)}
\end{aligned} \tag{52}$$

In order to get (a), we use the fact that, by Lemma C.3, there exists  $n$  finite and independent of  $R$  such that

$$\begin{aligned}
&\sum_{m=1}^{\infty} \Pr\left(\frac{1}{m} \sum_{i=1}^m (X_i - \mu_X) < -\epsilon_2\right) \\
&= \sum_{m=1}^n \Pr\left(\frac{1}{m} \sum_{i=1}^m (X_i - \mu_X) < -\epsilon_2\right) \\
&\quad + \sum_{m=n+1}^{\infty} \Pr\left(\frac{1}{\sqrt{m}} \sum_{i=1}^m (X_i - \mu_X) < -\sqrt{m}\epsilon_2\right) \\
&\leq \sum_{m=1}^n \Pr\left(\frac{1}{m} \sum_{i=1}^m (X_i - \mu_X) < -\epsilon_2\right) + \sum_{m=n+1}^{\infty} e^{-m\epsilon_2^2/(2\sigma_X^2)}
\end{aligned} \tag{53}$$

Since the first sum contains a finite number of terms and the second converges for all  $\epsilon_2 > 0$ , we have that

$$\lim_{R \rightarrow \infty} \frac{1}{R} \sum_{m=1}^{\infty} \Pr \left( \frac{1}{m} \sum_{i=1}^m X_i < b \right) = 0$$

and (a) follows.

Eventually, we get  $G(\mu_X - \epsilon_2) \leq \lim_{R \rightarrow \infty} \eta \leq G(\mu_X + \epsilon_1)$  and by letting  $\epsilon_i \rightarrow 0$  for  $i = 1, 2$  and recalling that, by definition,  $\mu_X = E[\log_2(1 + \beta_{1,1})]$  we obtain the desired result.

**ALO and RTD protocols.** We let  $X_i \triangleq \beta_{1,s_i}$  for all  $s_i \in \mathcal{S}_{1,m}$  and  $\mu_X \triangleq E[X_i]$ . Then, for  $\epsilon > 0$  and  $b = \mu_X + \epsilon$  and by following the same steps of (51) for RTD we can write

$$\begin{aligned} \lim_{R \rightarrow \infty} \frac{1}{\eta} &= \lim_{R \rightarrow \infty} \frac{1 + \sum_{m=1}^{\infty} \Pr \left( \sum_{i=1}^m X_i < 2^R - 1 \right)}{RG} \\ &\geq \lim_{R \rightarrow \infty} \frac{1}{Gb} \frac{2^R - 1}{R} \frac{1}{\lfloor (2^R - 1)/b \rfloor + 1} \sum_{m=1}^{\lfloor (2^R - 1)/b \rfloor} \Pr \left( \frac{1}{m} \sum_{i=1}^m X_i < b \right) \\ &= \frac{1}{Gb} \lim_{R \rightarrow \infty} \frac{2^R - 1}{R} \\ &= \infty \end{aligned} \tag{54}$$

This shows that  $\lim_{R \rightarrow \infty} \eta^{(\text{RTD})} = 0$  and since  $\eta^{(\text{ALO})} \leq \eta^{(\text{RTD})}$ , the same result holds for ALO.

## C.2 Limits for large G

We need to consider the limiting behavior of the RV  $\sum_{j=1}^K \alpha_j$  where  $K$  is the number of interfering users in a given slot, binomially distributed and  $\alpha_j$  is the channel gain of user  $j$ , assumed to be i.i.d. and independent of  $K$  with finite mean  $\bar{\alpha}$  and variance  $\sigma_\alpha^2$ . Since  $G = p_t N_u \leq N_u$ , as  $G \rightarrow \infty$  also  $N_u \rightarrow \infty$ . The mean and the variance of  $K$  are given by

$$\begin{aligned} \bar{K} &= p_t(N_u - 1) = G \frac{N_u - 1}{N_u} \leq G \\ \sigma_K^2 &= (1 - p_t)p_t(N_u - 1) \leq G/4 \end{aligned} \tag{55}$$

By iterating expectation, we obtain

$$\begin{aligned} E \left[ \sum_{j=1}^K \alpha_j \right] &= \bar{\alpha} \bar{K} \\ \text{Var} \left[ \sum_{j=1}^K \alpha_j \right] &= \sigma_K^2 \bar{\alpha}^2 + \bar{K} \sigma_\alpha^2 \end{aligned} \tag{56}$$

Putting together (55) and (56) we conclude that  $\frac{1}{G} \sum_{j=1}^K \alpha_j$  converges in probability to  $\bar{\alpha}$  as  $G \rightarrow \infty$ , in fact

$$\begin{aligned} \frac{\text{Var} \left[ \sum_{j=1}^K \alpha_j \right]}{G^2} &\leq \frac{E[\alpha^2]}{G} \rightarrow 0 \\ \frac{\bar{K}}{G} &= G \frac{N_u - 1}{N_u} \frac{1}{G} \rightarrow 1 \end{aligned} \quad (57)$$

From continuity of the functions  $1/(1+x)$  and  $\log_2(1+x)$  for  $x > 0$ , the following limits for  $G \rightarrow \infty$  hold in probability

$$\begin{aligned} G\beta_{1,1} &\rightarrow \frac{\alpha_{1,1}}{\bar{\alpha}} \\ G \log_2(1 + \beta_{1,1}) &\rightarrow \log_2(e) \frac{\alpha_{1,1}}{\bar{\alpha}} \end{aligned} \quad (58)$$

By using (58), and the fact that, in the case of INR,  $\bar{\eta} = GE[\log_2(1 + \beta_{1,1})]$  (obtained for  $R \rightarrow \infty$ ), we have

$$\begin{aligned} \lim_{G \rightarrow \infty} \bar{\eta} &= \lim_{G \rightarrow \infty} GE[\log_2(1 + \beta_{1,1})] \\ &= \log_2(e) E \left[ \frac{\alpha_{1,1}}{\bar{\alpha}} \right] \\ &= \log_2(e) \end{aligned} \quad (59)$$

Notice that (58) implies  $\sum_{s \in \mathcal{S}_{1,m}} \log_2(1 + \beta_{1,s}) \rightarrow \log_2(1 + \sum_{s \in \mathcal{S}_{1,m}} \beta_{1,s})$  in probability, as  $G \rightarrow \infty$ . Then, the probabilities  $p(m)$  given in (20) for INR and RTD are equal in the limit for large  $G$ . Since  $\eta$  depends on the particular protocol only through the probabilities  $p(m)$ , we conclude that limit (59) holds also for RTD.

For ALO we have

$$\begin{aligned} &\lim_{G \rightarrow \infty} \sup_R RG [1 - \Pr(\log_2(1 + \beta_{1,1}) \leq R)] \\ &= \lim_{G \rightarrow \infty} \sup_R RG [1 - \Pr(G \log_2(1 + \beta_{1,1}) \leq RG)] \\ &= \lim_{G \rightarrow \infty} \sup_R RG \left[ 1 - \Pr \left( \log_2(e) \frac{\alpha_{1,1}}{\bar{\alpha}} \leq RG \right) \right] \\ &\stackrel{(a)}{=} \lim_{G \rightarrow \infty} \frac{\log_2(e)}{\bar{\alpha}} \sup_{u \geq 0} u(1 - F_\alpha(u)) \\ &= \frac{\log_2(e)}{\bar{\alpha}} \kappa \end{aligned} \quad (60)$$

where we let  $\kappa \triangleq \sup_{u \geq 0} u(1 - F_\alpha(u))$ , and where (a) follows by letting  $u = RG\bar{\alpha}/\log_2(e)$  and by noticing that the expression that must be maximized depends on the product  $RG$  and not on  $G$  alone, therefore maximization with respect to  $R$  or with respect to  $u$  yields the same.

### C.3 Limits for large SNR

We want to show that the throughput can be made arbitrarily large by increasing the user transmit SNR  $\gamma$ . Since the throughput for ALO is a lower bound for the other two protocols, it is sufficient to prove this statement for ALO. Let  $K$  be the number of interfering users. We can write,

$$\begin{aligned}
\lim_{\gamma \rightarrow \infty} \eta &= \lim_{\gamma \rightarrow \infty} GR(1 - p(1)) \\
&= \lim_{\gamma \rightarrow \infty} GR \Pr(\log_2(1 + \beta_{1,1}) > R) \\
&= \lim_{\gamma \rightarrow \infty} GR \sum_{k=0}^{N_u-1} \Pr(\beta_{1,1} > 2^R - 1 | K = k) \Pr(K = k) \\
&\geq \lim_{\gamma \rightarrow \infty} GR \Pr\left(\alpha_{1,1} > \frac{2^R - 1}{\gamma}\right) (1 - p_t)^{N_u-1}
\end{aligned} \tag{61}$$

Now, we choose  $\epsilon > 0$  such that  $F_\alpha(\epsilon) < 1$ , and we let  $R = \log_2(1 + \gamma\epsilon)$ . Finally, we obtain

$$\lim_{\gamma \rightarrow \infty} \eta \geq \lim_{\gamma \rightarrow \infty} G(1 - p_t)^{N_u-1} (1 - F_\alpha(\epsilon)) \log_2(1 + \gamma\epsilon) = \infty$$

as desired.

## D Some useful cdf's

In order to simplify the notation of (2), we indicate the active users on slot  $s$  by  $k = 0, 1, \dots, |\mathcal{K}(s)| - 1$  (user 0 is the reference user) and we define the following RVs:

- User  $k$  instantaneous SNR,  $X_k \triangleq \alpha_{k,s}\gamma$ .
- The number of interfering users  $K \triangleq |\mathcal{K}(s)| - 1$ .
- The MAI instantaneous power-to-noise ratio  $Y \triangleq \sum_{k=1}^K X_k$ .
- The instantaneous SINR  $Z \triangleq \frac{X_0}{1+Y}$ .
- The instantaneous mutual information (IMI)  $I \triangleq \log_2(1 + Z)$ .

$K$  is binomially distributed as

$$\Pr(K = u) = \binom{N_u - 1}{u} \left(\frac{G}{N_u}\right)^u \left(1 - \frac{G}{N_u}\right)^{N_u - u}$$

for  $u = 0, \dots, N_u - 1$ . For  $N_u \rightarrow \infty$ , this converges to the Poisson distribution

$$\Pr(K = u) = e^{-G} \frac{G^u}{u!}$$

for  $u \geq 0$ .

Without fading,  $X_k$  is constant and equal to  $\gamma$ . Then,  $Y, Z$  and  $I$  takes on the values  $u\gamma$ ,  $\gamma/(1 + u\gamma)$  and  $\log_2(1 + \gamma/(1 + u\gamma))$  with probability  $\Pr(K = u)$  given above, for  $u = 0, 1, \dots$

In the case of (normalized) Rayleigh fading,  $X_k$  is exponentially distributed with mean  $\gamma$ ,

$$F_X(x) = 1 - e^{-x/\gamma} \quad (62)$$

The pdf of  $Y$  is readily obtained as a sum of  $u$ -fold convolutions of the pdf corresponding to (62), weighted by  $\Pr(K = u)$ . This yields the cdf

$$F_Y(x) = 1 - \sum_{u=0}^{N_u-1} \Pr(K = u) \sum_{k=0}^{u-1} e^{-x/\gamma} \frac{(x/\gamma)^k}{k!} \quad (63)$$

The derivation of the cdf for the SINR  $Z$  is more involved (the details are postponed to the end of this Appendix). We obtain

$$F_Z(x) = 1 - \sum_{u=0}^{N_u-1} \Pr(K = u) \frac{e^{-x/\gamma}}{(1+x)^u} \quad (64)$$

Finally, the cdf of the IMI  $I$  is obtained from (64) by a simple change of variable as

$$F_I(x) = 1 - \sum_{u=0}^{N_u-1} \Pr(K = u) e^{-(2^x-1)/\gamma} 2^{-xu} \quad (65)$$

(obviously, all the above cdfs are defined for  $x \geq 0$  and are zero for  $x < 0$ ).

Interestingly, in the limiting case of  $N_u \rightarrow \infty$  we can sum the series (63), (64) and (65) and obtain closed forms. The pdf corresponding to (63) was found in [52], and it is given by

$$f_Y(x) = e^{-G} \left[ \delta(x) + e^{-x/\gamma} \sqrt{\frac{G}{x\gamma}} I_1 \left( \sqrt{\frac{4xG}{\gamma}} \right) \right]$$

where  $\delta(x)$  is the Dirac delta function and  $I_1(x)$  is the first-order modified Bessel function of the first kind. The SINR cdf for infinite users is given by

$$\begin{aligned} F_Z(x) &= 1 - \sum_{u=0}^{\infty} e^{-G} \frac{G^u}{u!} \frac{e^{-x/\gamma}}{(1+x)^u} \\ &= 1 - e^{-G} e^{-x/\gamma} \sum_{u=0}^{\infty} \frac{[G/(1+x)]^u}{u!} \\ &= 1 - e^{-G} e^{-x/\gamma} e^{+G/(1+x)} \\ &= 1 - \exp \left( -\frac{x}{\gamma} - \frac{Gx}{1+x} \right) \end{aligned} \quad (66)$$

and the corresponding IMI cdf is obtained from (66) by a change of variable as

$$F_I(x) = 1 - \exp\left(-\frac{2^x - 1}{\gamma} - (1 - 2^{-x})G\right)$$

**Calculation of the SINR cdf conditioned on the number of interfering users.** Let  $X$  and  $Y$  be two independent RV's obtained as the sum of  $A$  and  $B$  i.i.d. exponentially distributed RVs with mean  $1/\lambda$ , respectively.  $X$  and  $Y$  follow the *Gamma* cdf

$$F(x) = 1 - \sum_{k=0}^{N-1} \frac{(\lambda x)^k}{k!} e^{-\lambda x} \quad (67)$$

for  $N = A$  and  $N = B$ , respectively.

For an arbitrary  $b \geq 0$ , consider the RV  $Z = \frac{X}{b+Y}$ . This reduces to the SINR considered above given  $K = u$  interferers by letting  $A = 1$ ,  $B = u$ ,  $b = 1$  and  $\lambda = 1/\gamma$ . The following derivation generalizes the result obtained in [53]. The cdf of  $Z$  is given by

$$\begin{aligned} F_Z(z) &= \Pr\{Z \leq z\} = \Pr\{X \leq (b+Y)z\} \\ &= \int_{-\infty}^{+\infty} dy \int_{-\infty}^{z(b+y)} dx f_X(x) f_Y(y) \\ &= \int_0^{+\infty} f_Y(y) F_X(z(b+y)) dy \\ &= \int_0^{+\infty} \frac{\lambda}{(B-1)!} (\lambda y)^{B-1} e^{-\lambda y} \left[ 1 - \sum_{k=0}^{A-1} \frac{(\lambda x)^k}{k!} e^{-\lambda x} \right]_{x=0}^{z(b+y)} dy \\ &= 1 - \sum_{k=0}^{A-1} \sum_{\ell=0}^k \int_0^{+\infty} \frac{\lambda (\lambda y)^{B-1}}{(B-1)!} \frac{(\lambda z b)^{k-\ell}}{(k-\ell)!} \frac{(\lambda z y)^\ell}{\ell!} e^{-\lambda y - \lambda z b - \lambda z y} dy \\ &= 1 - \frac{e^{-\lambda z b}}{(1+z)^B} \left[ \sum_{k=0}^{A-1} \sum_{\ell=0}^k \frac{(\lambda z b)^{k-\ell}}{(k-\ell)!} \left( \frac{z}{1+z} \right)^\ell \binom{B-1+\ell}{B-1} \right] \\ &\quad \cdot \left\{ \int_0^{+\infty} \frac{\lambda(1+z)}{(B-1+\ell)!} e^{-\lambda(1+z)y} [\lambda y(1+z)]^{B-1+\ell} dy \right\} \end{aligned}$$

where the integral in braces in the last line is equal to 1, since the integrand is a Gamma pdf.

For  $A = 1$ , the double summation in the last line of (68) reduces to a single term, and we obtain

$$F_Z(z) = 1 - \frac{e^{-\lambda z b}}{(1+z)^B} \quad (68)$$

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