CDMA with fading: Effective bandwidth and spreading-coding tradeoff

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Abstract — **We find the capacity regions of large CDMA systems with linear receivers and random spreading subject to slow fading (nonergodic channel) and fast fading (ergodic channel).**

We consider the *uplink* of a single-cell, synchronous DS-CDMA system with K users and random spreading sequences of L *chips*. The received signal L-chip column vector corresponding to one symbol interval is given by

$$\mathbf{y} = \sum_{k=1}^{K} \sqrt{z_k} x_k \mathbf{s}_k + \mathbf{n} \tag{1}$$

where $\mathbf{n} \sim \mathcal{N}_{\mathcal{C}}(\mathbf{0}, N_0\mathbf{I})$, x_k is the complex modulation symbol of user k, $\mathbf{s}_k$ is the spreading sequence of user k, made of binary antipodal chips $\pm 1/\sqrt{L}$ generated at random with uniform probability and where z_k is the flat fading power gain. We assume that the base station receiver has perfect knowledge of all fading gains and phases, and without loss of generality, we include the phase rotation of the k-th channel into the modulation symbol x_k. User k is received with signal-to-noise ratio (SNR) $\gamma_k = z_k \Gamma_k$, where Γ_k is the *transmit* SNR. As in [3, 4], we consider an asymptotically large system with $K \to \infty$ and $K/L \to \alpha$. The receiver for user 1 (our reference user) is defined by $y_1 = \mathbf{h}_1^H \mathbf{y}$ followed by a single-user decoder operating on the sequence of filter outputs y_1. The filter $\mathbf{h}_1$ can be either a single-user matched filter (SUMF) or a linear MMSE filter [1]. Under the above assumptions, the output SINR β_1 of receiver 1 satisfies [3]:

$$\beta_1 = \begin{cases} \frac{\gamma_1}{1+\alpha \int_0^\infty x\, dF_\gamma(x)} & \text{SUMF} \\ \frac{\gamma_1}{1+\alpha \int_0^\infty \frac{x\gamma_1}{\gamma_1 + x\beta_1} dF_\gamma(x)} & \text{MMSE} \end{cases} \tag{2}$$

Where $F_\gamma(x)$ is the limiting cdf of the received user SNRs. In the following, we assume that users are partitioned into J classes. Each class j is characterized by a transmit SNR Γ_j. Each class has $p_j K$ users, where $\sum_{j=1}^J p_j = 1$, and the z_k are i.i.d. and normalized, so that $\int_0^\infty x dF_z(x) = 1$. Then, $F_\gamma(x) = \sum_{j=1}^J p_j F_z(x/\Gamma_j)$ where $F_z(x)$ is the fading cdf. Let user 1 belong to class i. Because of the uncompensated fading, user 1 SINR is a random variable $\beta_{i,1}$. However, the ratio $\xi = \beta_{i,1}/(\Gamma_i z_1)$ is non-random and independent of i, and can be calculated from (2).

Non-ergodic fading. In this case, we assume that the fading time-variations are very slow so that the output SINR is random but constant over one code word. Outage probability for users of class i is given by $P_{\mathrm{out},i} = P(\beta_{i,k} \le \bar{\beta}_i) = F_z\left(\frac{\bar{\beta}_i}{\xi \Gamma_i}\right)$ where $\bar{\beta}_i$ is a SINR threshold that depends on the coding scheme of class i. Assuming Gaussian codes and minimum distance decoding at the output of the receiving filter, we let $\bar{\beta}_i = 2^{R_i} - 1$.

Let $\bar{\mathbf{\Gamma}} = (\bar{\Gamma}_1, \ldots, \bar{\Gamma}_J)$ be a vector of input SNR constraints, $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_J)$ be a vector of target outage probabilities, and $\mathbf{R} = (R_1, \ldots, R_J)$ be a vector of coding rates. We find the outage capacity, i.e., the set $\mathcal{R} \subseteq \mathbb{R}_+^J$ of rate vectors $\mathbf{R}$ that can be assigned to the J classes such that, for all $i = 1, \ldots, J$, $P_{\mathrm{out},i} \le \epsilon_i$ and $\Gamma_i \le \bar{\Gamma}_i$. By letting

$$\mu_i = \frac{2^{R_i} - 1}{\sup\{x \in \mathbb{R}_+ : F_z(x) = \epsilon_i\}} \tag{3}$$

we rewrite the outage constraint as $\Gamma_i \xi \ge \mu_i$. For maximum R_i this must hold with equality, which implies that $\Gamma_i/\mu_i = \kappa$ is a constant independent of i. Solving for κ and imposing the input constraints, we obtain the capacity inequality

$$\alpha \sum_{j=1}^{J} p_j B_j \le \min_{1 \le i \le J} \left\{ 1 - \frac{\mu_i}{\bar{\Gamma}_i} \right\} \tag{4}$$

where the *effective bandwidth* B_j is given by

$$B_j = \begin{cases} \mu_j & \text{SUMF} \\ \int_0^\infty \frac{x\mu_j}{1+x\mu_j} dF_z(x) & \text{MMSE} \end{cases} \tag{5}$$

Ergodic fading. In this section we assume that the fading is sufficiently fast the channel can be considered *information stable* [2]. Assuming that all users generate their code book according to a complex circularly-symmetric Gaussian pdf, users in class i can communicate reliably at rate

$$R_i = \int_0^\infty \log_2(1 + x\xi\Gamma_i) dF_z(x)$$

We find the set of rates $\mathbf{R} = (R_1, \ldots, R_J)$ achievable with input constraints $\mathbf{\Gamma} \le \bar{\mathbf{\Gamma}}$. Since the function $f(y) = \int_0^\infty \log_2(1 + xy) dF_z(x)$ is monotonically increasing, we define $\nu_i = f^{-1}(R_i)$, then, $\Gamma_i \xi \ge \nu_i$ and from an argument similar to above we obtain a capacity inequality of the same form of (4), with the substitution $\nu_j \to \mu_j$. It follows that the effective bandwidth B_j in the ergodic case has the same form of (5), with $\nu_j \to \mu_j$.

References

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