PSP-based space-time schemes for large spectral efficiency

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Abstract — **We propose a low-complexity space-time architecture based on diagonal interleaving of a single trellis code and on a PSP receiver that makes use of predecisions on the surviving paths of the Viterbi algorithm in a decision-feedback scheme. Forward and backward filters of the decision-feedback front-end are designed either according to the ZF or to the MMSE criterion. Then, we develope an approximated union-bound semi-analythic performance analysis technique based on multivariate weight enumerators, by assuming perfect decisions in the feedback. The FER evaluated in this way is in very good agreement with computer simulations of the true PSP scheme.**

I. Introduction

Multiple antenna transmission/reception has been recognized as one of the most promising ways of enhancing the spectral efficiency of wireless links. Information theoretic results can be found in a very large number of recent papers (see for example [14, 5, 9, 1, 10], and the references therein). These works study the capacity of multiple antenna channels under different assumptions on the availability of channel state information (CSI). Capacity with no transmitter CSI has been addressed in [14, 5]. Capacity with perfect CSI both at the transmitter and at the receiver has been studied in [1] and capacity with no CSI available both at the transmitter and at the receiver is the object of [9]. The design of codes for multiple antenna transmission reception (known generally as space-time codes), is addressed, for example, in [13].

In this paper, we consider quasi-stationary frequency-flat Rayleigh fading representative of a narrowband wireless local loop or of an indoor wireless link. In this case, the channel is assumed to be random but constant during the transmission of each code word, perfectly known at the receiver and unknown at the transmitter. For very high spectral efficiency, a very large number of antennas is needed and Maximum-Likelihood decoding as considered in [13] becomes too complex. In [4], a *layered* space-time architecture is proposed in order to allow a very large number of antennas with moderate complexity. In this way, the information stream is demultiplexed into t substreams, which are independently encoded by t encoders. The t code words are interleaved by a *diagonal* interleaving scheme and send in parallel to t transmitting antennas. The interleaver is designed so that the symbols of a given code word are cyclically sent over all the t antennas, in order to guarantee the necessary diversity order. At the receiver side, the output of $r \geq t$ antennas is processed by a reduced-complexity suboptimal detector which mimics zero-forcing (ZF) decision feedback equalization. The output of the decision-feedback "equalizer" is deinterleaved and sent to a bank of t decoders.

The layered space-time architecture of [4] is very attractive but presents some problems. First, it requires several independent encoder/decoder pairs, running in parallel. Then, if used with trellis component encoders, not all symbols are decoded with the same decoding delay. This might pose a problem for the underying Viterbi algorithm. The interference cancellation via decision feedback is prone to error propagation due to unreliable pre-decisions, as in standard decision-feedback equalization.

In order to solve these problems at once, while still keeping the nice reduced-complexity receiver, we propose a modification of the scheme of [4] where a single trellis encoder is "wrapped" along the transmitting antennas by a diagonal interleaver, and where the decision-feedback scheme is integrated into a per-survivor processing (PSP) receiver. Thanks to the PSP, the impact of unreliable decisions in the feedback loop is greatly reduced. Our scheme is general and applies to any trellis code and number of antennas (provided that $r \geq t$). In order to demonstrate our ideas, we develop an efficient semi-analytic performance analysis technique based on modeling the channel with decision feedback as a block fading channel with cyclic interleaving of the code word over the fading blocks [16, 8]. Multivariate Euclidean weight enumerators are used to compute an upper bound on the frame error rate. Strictly speaking, our method does not provide true upper bounds, because of the optimistic assumption of perfect decisions in the feedback loop. However, simulations show that the approximations obtained are very accurate. Implicitly, this shows that the impact of wrong decisions on the performance of the PSP receiver is minimal.

II. System model

We consider a multiple-antenna system with t transmitting (Tx) and $r \geq t$ receiving (Rx) antennas. A *multidimensional* TCM encoder [2], with rate $R = k/d$ bit/complex symbol, produces d modulation symbols $x[i] \in \mathcal{X} \subset \mathbb{C}$ every k input information bits ($\mathcal{X}$ denotes the modulator signal set, e.g., QAM or PSK). The sequence of coded symbols is interleaved and grouped into blocks of t symbols, that are sent in parallel over the t Tx antennas. The resulting spectral efficiency is $\eta = tR$ bit/channel use.

The encoder is connected to the Tx antennas through a diagonal interleaver of depth t, as shown in Fig. 1 for $t = 8$. Diagonals in the interleaving array are written from top to bottom, and the numbers $i = 1, 2, \ldots$ in each array entry denote the corresponding code symbol $x[i]$. The n-th column of the interleaver array forms the block $\mathbf{a}[n] = (a_1[n], a_2[n], \ldots, a_t[n])^T$, which is transmitted in parallel from the t antennas We use indexes i and n to indicate the time ordering of code symbols at the decoder output (i.e., before interleaving) and the time ordering of blocks at the Tx antennas (i.e., after interleaving). Symbol $x[i]$ corresponds to the (ℓ, n)-th element $a_\ell[n]$ of the interleaving array, i.e., it is transmitted at time n from the ℓ-th antenna, where i is related to ℓ and n by

$$\begin{aligned} \ell &= i - \left\lfloor \frac{i-1}{t} \right\rfloor t \\ n &= i - \left\lfloor \frac{i-1}{t} \right\rfloor (t-1) \end{aligned} \quad (1)$$

The channel, during the time span of a code word, is described by

$$\mathbf{y}[n] = \mathbf{C}\mathbf{a}[n] + \boldsymbol{\nu}[n] \quad (2)$$

where $\boldsymbol{\nu}[n] \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$ is a circularly symmetric complex Gaussian noise vector. The channel matrix $\mathbf{C}$ is random but constant with time, and it is assumed to be known to the receiver and unknown to the transmitter. Its (i,j)-th element $c_{i,j}$ is the complex gain from the j-th Tx to the i-th Rx antenna. The channels are normalized in order to have average power gain $E[|c_{i,j}|^2] = \gamma/t$, and the symbols have average energy $E[|x[i]|^2] = 1$. In this way, $\gamma = \eta E_b/N_0$ is the total *transmit* SNR.

Decoding is done by processing the vectors $\mathbf{y}[n]$ according to the diagonal deinterleaving ordering. The branch metrics for the Viterbi Algorithm are obtained by the decision-feedback scheme proposed in [4] combined with a per-survivor processing (PSP) approach. The i-th code symbol is located in position ℓ of column n of the interleaver array, where ℓ and n are given by (1). Symbols located *below*, i.e., in positions $\ell+1, \dots, t$ of column n, have been already processed in the past decoding steps $i-(t-1), i-2(t-1), \dots, i-(t-\ell)(t-1)$. If t is sufficiently large with respect to the decoding delay, symbols processed more than t steps before are already reliably detected and can be treated as known. On the contrary, all symbols *above*, i.e., in positions $1, \dots, \ell-1$, belong to future decoding steps and are completely unknown.

The n-th column can be partitioned into *future*, *present* and *past* symbols, as

$$\mathbf{a}[n] = (\mathbf{a}_\ell^+[n]^T, a_\ell[n], \mathbf{a}_\ell^-[n]^T)^T$$

where $\mathbf{a}_\ell^+[n] = (a_1[n], \dots, a_{\ell-1}[n])^T$ and $\mathbf{a}_\ell^-[n] = (a_{\ell+1}[n], \dots, a_t[n])^T$. Assuming $\mathbf{a}_\ell^-[n]$ perfectly known, the branch metric for the i-th code symbol is obtained from the output $z_\ell[n]$ of the decision-feedback equalizer

$$z_\ell[n] = \mathbf{f}_\ell^H \mathbf{y}[n] - \mathbf{b}_\ell^H \mathbf{a}_\ell^-[n] \quad (3)$$

where $\mathbf{f}_\ell$ and $\mathbf{b}_\ell$ are the forward and the feedback filter vectors, of length r and $t-\ell$, respectively, which depend on the channel matrix and on the position ℓ of the i-th code symbol in the vector $\mathbf{a}[n]$.

By partitioning the channel matrix as $\mathbf{C} = [\mathbf{C}_\ell^+, \mathbf{c}_\ell, \mathbf{C}_\ell^-]$, where $\mathbf{C}_\ell^+ \in \mathbb{C}^{r\times(\ell-1)}$ and $\mathbf{C}_\ell^- \in \mathbb{C}^{r\times(t-\ell)}$, (3) can be written as

$$\begin{aligned} z_\ell[n] &= \mathbf{f}_\ell^H \mathbf{c}_\ell a_\ell[n] + \mathbf{f}_\ell^H \mathbf{C}_\ell^+ \mathbf{a}_\ell^+[n] + \\ &\quad + (\mathbf{f}_\ell^H \mathbf{C}_\ell^- - \mathbf{b}_\ell^H)\mathbf{a}_\ell^-[n] + \mathbf{f}_\ell^H \boldsymbol{\nu}[n] \end{aligned} \quad (4)$$

The feedback vector is designed to satisfy the perfect cancellation condition $\mathbf{b}_\ell^H = \mathbf{f}_\ell^H \mathbf{C}_\ell^-$. The forward filter is designed either according to the minimum mean-square error (MMSE) or according to the zero-forcing (ZF) criterion, and normalized in order to make the variance of the residual noise+interference equal to 1. Explicitly, we have

$$\mathbf{f}_\ell = \begin{cases} \frac{1}{\sqrt{\mathbf{c}_\ell^H (\boldsymbol{\Sigma}_\ell^+)^{-1}\mathbf{c}_\ell}} (\boldsymbol{\Sigma}_\ell^+)^{-1}\mathbf{c}_\ell & \text{(MMSE)} \\ \mathbf{q}_\ell & \text{(ZF)} \end{cases} \quad (5)$$

where $\boldsymbol{\Sigma}_\ell^+ = \mathbf{C}_\ell^+(\mathbf{C}_\ell^+)^H + \mathbf{I}$ and where $\mathbf{q}_\ell$ is the ℓ-th column of the unitary matrix $\mathbf{Q}$ in the "QR" factorization [6] $\mathbf{C} = \mathbf{QR}$ of the channel matrix $\mathbf{C}$.

With the above filters, $z_\ell[n]$ can be written as

$$z_\ell[n] = \mu_\ell a_\ell[n] + \nu_\ell[n] \quad (6)$$

where $\nu_\ell[n]$ is the residual noise+interference term and $\beta_\ell = |\mu_\ell|^2$ is the signal-to-interference plus noise ratio (SINR) at the output of the decision-feedback front-end, given by

$$\beta_\ell = \begin{cases} \mathbf{c}_\ell^H (\boldsymbol{\Sigma}_\ell^+)^{-1}\mathbf{c}_\ell & \text{(MMSE)} \\ |[\mathbf{R}]_{\ell,\ell}|^2 & \text{(ZF)} \end{cases} \quad (7)$$

Assuming $\nu_\ell[n] \sim \mathcal{N}_{\mathbb{C}}(0,1)$ (this is true for ZF, and approximately true for MMSE [11]), the set of branch metrics for the i-th code symbol is $\{|z_\ell[n] - \mu_\ell x|^2 : x \in \mathcal{X}\}$.

In reality, the symbols in $\mathbf{a}_\ell^-[n]$ are not perfectly known. Thus, we propose to use PSP. Consider state s in the code trellis, and let $\widehat{\mathbf{a}}_\ell^-(s)$ be the vector of decisions corresponding to symbols in $\mathbf{a}_\ell^-[n]$, obtained from the survivor terminating in s. Let $\mathcal{M}_i(s)$ denote the path metric of the path terminating in state s at decoding step i. Then, the path metric update of the PSP decision-feedback scheme is given by

$$\begin{aligned} \mathcal{M}_i(s) &= \min_{s' \in \mathcal{P}(s)} \{\mathcal{M}_{i-1}(s') + \\ &\quad + \left| \mathbf{f}_\ell^H \mathbf{y}[n] - \mathbf{b}_\ell^H \widehat{\mathbf{a}}_\ell^-(s') - \mu_\ell x(s', s) \right|^2 \} \end{aligned} \quad (8)$$

where $\mathcal{P}(s)$ denotes the set of parent states of s and $x(s', s)$ denotes the symbol corresponding to the trellis transition $s' \to s$.[1]

Remark 1. If t is larger than the decoding delay of the Viterbi algorithm (typically, 6 times the code constraint length), the probability that all survivors merge is very high, therefore storing a single survivor is enough. Therefore, the complexity of standard PSP is greatly reduced.

Remark 2. For quasi-stationary fading channels, the filter pairs $(\mathbf{f}_\ell, \mathbf{b}_\ell)$ for $\ell = 1, \dots, t$ can be computed once and used for the whole code word (i.e., until the channel changes significantly).

III. Performance analysis

In this section we provide a semi-analytic method for efficient evaluation of the frame error rate of the proposed space-time scheme. Our method is based on multivariate transfer functions of trellis codes [16], on the modified union bound of [8] for block-fading channels, on an alternative integral representation of the Gaussian tail function (see [12]) and on the relation between frame error probability and error event probability for trellis-terminated trellis codes of [3]. For the sake of simplicity, we assume that the underlying TCM code is geometrically uniform [2]. Then, any sequence can be taken as the

[1] For simplicity, the metric updating rule is stated here in the case $d = 1$ (one symbol per branch) and no parallel transitions. The generalization to $d > 1$ symbols per branch and parallel transitions is straightforward.

reference sequence for calculating error probabilities. Generalizations to non-uniform codes is conceptually straightforward, even if computation might be considerably more complicated.

We consider the transmission of a code word $\mathbf{x}$ of length N symbols, obtained by trellis termination. For simplicity, we assume $d|t$ and $t|N$. Assuming perfect decision feedback, the system can be modeled as a set of t parallel Gaussian channels, with SNR $\beta_1, \ldots, \beta_t$. The symbols of $\mathbf{x}$ are cyclically sent to the channels $1, 2, \ldots, t$. All symbols sent to channel ℓ are grouped in the code subsequence $\mathbf{x}_\ell$, of length N/t. Then, the transmission of $\mathbf{x}$ can be compactly written as

$$\mathbf{y}_\ell = \mu_\ell \mathbf{x}_\ell + \boldsymbol{\nu}_\ell \qquad \text{for } \ell = 1, \ldots, t$$

where $\boldsymbol{\nu}_\ell = \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$. Consider the pairwise error event $\{\mathbf{x} \to \mathbf{x}'\}$ that the decoder chooses the sequence $\mathbf{x}' \neq \mathbf{x}$, given that $\mathbf{x}$ was transmitted, as if $\mathbf{x}$ and $\mathbf{x}'$ were the only two possible decoder outcomes. Straightforward calculation yields the conditional pairwise-error probability (PEP) in the form

$$P(\mathbf{x} \to \mathbf{x}'|\beta_1, \ldots, \beta_t) = Q\left(\sqrt{\frac{1}{2}\sum_{\ell=1}^{t} \beta_\ell |\mathbf{d}_\ell|^2}\right) \qquad (9)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy$ is the Gaussian tail function, and where $\mathbf{d}_\ell = \mathbf{x}_\ell - \mathbf{x}'_\ell$ is the vector of the componentwise differences for symbols sent to channel ℓ.

By using the integral form

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/(2\sin^2\phi)} d\phi \qquad x \geq 0 \qquad (10)$$

the conditional PEP can be written as

$$P(\mathbf{x} \to \mathbf{x}'|\beta_1, \ldots, \beta_t) = \frac{1}{\pi}\int_0^{\pi/2} \prod_{\ell=1}^{t} \exp\left(-\frac{\beta_\ell w_\ell}{\sin^2\phi}\right) d\phi \quad (11)$$

where we define the normalized squared Euclidean weights $w_\ell = |\mathbf{d}_\ell|^2/4$.

Next, we define an equivalent super-trellis by considering $L = t/d$ steps of the original trellis at once. Every branch of the super-trellis carries t symbols. We define $P(e|\beta_1, \ldots, \beta_t)$ to be the conditional error event probability of a Viterbi algorithm working on the super-trellis, i.e., the probability that the chosen path diverges form the correct one at a given step of the super-trellis. We can upperbound the conditional error event probability by the union bound

$$\begin{aligned} P(e|\beta_1, \ldots, \beta_t) &\leq \sum_{\mathbf{x}' \neq \mathbf{x}} P(\mathbf{x} \to \mathbf{x}'|\beta_1, \ldots, \beta_t) \\ &= \frac{1}{\pi}\int_0^{\pi/2} f(\phi, \beta_1, \ldots, \beta_t)\, d\phi \quad (12) \end{aligned}$$

where we define the multivariate weight enumerator

$$T(W_1, \ldots, W_t) = \sum_{w_1} \cdots \sum_{w_t} A(w_1, \ldots, w_t) \prod_{\ell=1}^{t} W_\ell^{w_\ell} \qquad (13)$$

and where we define

$$f(\phi, \beta_1, \ldots, \beta_t) = T(e^{-\beta_1/\sin^2\phi}, \ldots, e^{-\beta_t/\sin^2\phi}) \qquad (14)$$

$A(w_1, \ldots, w_t)$ in(13) is the number of code words $\mathbf{x}'$ diverging from $\mathbf{x}$ at a given step of the super-trellis and remerging after some step, having normalized squared Euclidean weights $w_1, \ldots, w_t$.

The conditional frame error rate (FER) $P_w(e|\beta_1, \ldots, \beta_t)$ can be upperbounded by [3] $P_w(e|\beta_1, \beta_t) \leq (N/t)P(e|\beta_1, \ldots, \beta_t)$. Finally, by following the approach of [8], an upper bound on the FER averaged over the joint statistics of the SINRs $\beta_1, \ldots, \beta_t$, is obtained as

$$P_w(e) \leq E\left[\min\left\{1, \frac{N}{\pi t}\int_0^{\pi/2} f(\phi, \beta_1, \ldots, \beta_t)\, d\phi\right\}\right] \qquad (15)$$

Remark 3. Strictly speaking, (15) is an approximation rather than a true upper bound, since we made the assumption of perfect decision feedback. Moreover, in the case of MMSE forward filter, we approximated the residual interference+noise term as Gaussian, even if it is not.

Remark 4. In (15), averaging with respect to the joint statistics of the β_ℓ's is normally done by Monte Carlo integration. The integral with respect to ϕ can be calculated as a finite Riemann sum and converges very quickly.

A Multivariate weight enumerators

We consider a TCM code based on a linear binary convolutional encoder followed by a signal mapper [2]. The linear binary convolutional encoder is defined by the difference equations over $\mathbb{F}_2$

$$\begin{bmatrix} \mathbf{c}[i] \\ \boldsymbol{\sigma}[i] \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{F} \\ \mathbf{G} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{b}[i] \\ \boldsymbol{\sigma}[i-1] \end{bmatrix}$$

where $\mathbf{c}[i], \mathbf{b}[i], \boldsymbol{\sigma}[i]$ are the binary vectors defining output, input and state of the encoder, and $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$ are binary matrices of the appropriate dimension, defining the output and the state equations.

A very simple technique for the computation of the multivariate weight enumerator of the super-trellis code, suited for automated computer implementation, is the following. First, we obtain the L-step super-trellis difference equations in the same form as above, with matrices

$$\begin{aligned} \mathbf{E}_L &= \begin{bmatrix} \mathbf{E} & \mathbf{0} & \cdots & & \mathbf{0} \\ \mathbf{FG} & \mathbf{E} & \mathbf{0} & \cdots & \vdots \\ \mathbf{FHG} & \mathbf{FG} & \mathbf{E} & & \\ \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{FH}^{L-2}\mathbf{G} & \cdots & \mathbf{FHG} & \mathbf{FG} & \mathbf{E} \end{bmatrix} \\ \mathbf{F}_L &= \begin{bmatrix} \mathbf{F} \\ \mathbf{FH} \\ \vdots \\ \mathbf{FH}^{L-1} \end{bmatrix} \\ \mathbf{G}_L &= \begin{bmatrix} \mathbf{H}^{L-1}\mathbf{G} & \cdots & \mathbf{HG} & \mathbf{G} \end{bmatrix} \\ \mathbf{H}_L &= \mathbf{H}^L \end{aligned}$$

Notice that even if the original TCM code has no parallel transitions, the L-step trellis might, depending on the rank of the matrix $[\mathbf{G}_L, \mathbf{H}_L]$. However, the number of states of the super-trellis is the same of the original trellis.

Next, from the super-encoder equations defined by the above matrices and from the signal mapper we derive the modified state diagram of the encoder, whose edges are labeled by monomials. Since each edge (or trellis branch) of the super-encoder trellis carries t symbols, the corresponding labeled is a monomial in the indeterminates $W_1, \ldots, W_t$, with exponents given by the normalized squared Euclidean weights of the symbols on the edge. Edges corresponding to parallel transitions can be merged into a single edge with polynomial label given by the sum of all monomials of the parallel transitions.

Finally, we split state 1 (the reference state, corresponding to the all-zero sequence) into two states, 1 and $1'$. We define the vector of state variables $\mathbf{V} = (V_2, \ldots, V_S)^T$, where S is the number of states, we let X and Y be the state variables for state 1 and $1'$, respectively, and we write the node equations

$$\begin{aligned} \mathbf{V} &= \mathbf{AV} + \mathbf{B}X \\ Y &= \mathbf{CV} + DX \end{aligned}$$

where $\mathbf{A}$ is the $(S-1) \times (S-1)$ polynomial matrix with elements $[\mathbf{A}]_{i,j}$ equal to the label of edge from state j to state i, for $i, j = 2, \ldots, S$, $\mathbf{B}$ is the polynomial $(S-1) \times 1$ vector with elements $[\mathbf{B}]_i$ equal to the label between state 1 to state $i > 1$, $\mathbf{C}$ is the polynomial $1 \times (S-1)$ vector with elements $[\mathbf{C}]_j$ equal to the label between state $j > 1$ to state $1'$, and D is the polynomial label of the transition between state 1 and state $1'$. The desired weight enumerator is obtained as the formal graph transfer function

$$T(W_1, \ldots, W_t) = \frac{Y}{X} - 1 = \mathbf{C}[\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B} + D - 1 \qquad (16)$$

where the term -1 eliminates the contribution of the correct path, with Euclidean weight zero.

Remark 5. Unless t and S are very small, closed form computation of $T(W_1, \ldots, W_t)$ via a symbolic manipulator is too complex, and (16) is more efficiently evaluated by first substituting the values of the indeterminates W_ℓ and then evaluating the resulting numerical matrix expression.

Example. The multivariate weight enumerator of the binary convolutional code with generators $(5, 7)$ (octal notation) interleaved over $t = 2$ channels is given by

$$T(W_1, W_2) = \frac{W_1^2 W_2^3 + W_1^4 W_2^2 - W_1^2 W_2^4}{1 - 2W_2 + W_2^2 - W_1^2}$$

$\diamond$

IV. Results

With space-time coding large spectral efficiencies can be achieved by increasing the number of antennas, rather than by increasing the size of the modulation alphabet $\mathcal{X}$. Therefore, coded BPSK is an attractive solution because of its simplicity, especially for PSP implementation.

We considered the performance of the proposed scheme with binary linear convolutional codes with rate $R = 1/2$ bit/symbol and BPSK modulation. Fig. 2 shows the approximated union bound (15), denoted by "AUB", and some simulation points for the $(5, 7)$ code with $t = r = 2, 4$ and 8 antennas, with the ZF and MMSE front-end. Simulations are in good agreement with the AUB. This shows that the effect of wrong decisions on the PSP receiver is minimal, and that (15) provides an effective method for evaluating the FER. In all examples, the FER is calculated for block length $N = 256$.

In the case of ZF it is not necessary to compute the SINRs β_ℓ via the QR factorization of $\mathbf{C}$. In fact, the joint statistics of the β_ℓ's is given by the following:

Proposition 1. If $\mathbf{C}$ has i.i.d. complex circularly-symmetric Gaussian elements, the SINRs $\{\beta_\ell : \ell = 1, \ldots, t\}$ resulting from the ZF front-end are statistically independent and β_ℓ is central Chi-squared with $2(r - \ell + 1)$ degress of freedom. $\square$

The fact that each β_ℓ is individually Chi-squared distributed is well-known (see [15] and references therein). The novelty is that the β_ℓ's are also statistically independent, even though they are all functions of the same channel matrix $\mathbf{C}$.

Figs. 3, 4 and 5 show the AUB for the optimal rate 1/2 binary convolutional codes with generators $(5, 7), (15, 17), (23, 35), (53, 75)$ and $(171, 133)$, with $t = r = 2, 4$ and 8 antennas. We notice that the code performance is not directly related to its minimum Hamming distance for small t, while as t grows, Hamming distance becomes more and more important. In fact, as observed in [7], the *diversity order* D of a code of rate R bit/symbol, based on the signal set $\mathcal{X}$ and interleaved over t parallel channels must satisfy

$$D \leq 1 + \left\lfloor t \left(1 - \frac{R}{\log_2 |\mathcal{X}|} \right) \right\rfloor$$

The code diversity order for a given t can be calculated from its multivariate weight enumerator. In fact, the code diversity is the maximum integer D for which $T(W_1, \ldots, W_t) = 0$ in all $\binom{t}{D-1}$ points $(W_1, \ldots, W_t) \in \{0, \alpha\}^t$ with Hamming weight $D - 1$, where $0 < \alpha < 1$ is a sufficiently small constant chosen such that $T(W_1, \ldots, W_t)$ is always defined in the points $\{0, \alpha\}^t$. Again, the weight enumerator can be evaluated numerically via (16), and there is no need for time-consuming symbolic manipulation.

Example. By the above method it is immediate to check that for $t = 8$ the 4- and 8-state codes achieve diversity $D = 4$, while the 16-,32- and 64-states codes achieve the maximum diversity $D = 5$. The large performance gap between these codes suggests that when the number of antennas is not too small, codes optimized for the Gaussian channel are good also for the proposed space-time scheme. Then, it is meaningful to consider powerful codes. On the other hand, for $t = 2$ the maximum diversity $D = 2$ is achieved by all codes considered, and the simple 4-state code performs roughly as well as the others. $\diamond$

Remark 6. Very small FER is achieved for low E_b/N_0, and for constant E_b/N_0 the FER decreases for increasing t and r. For given transmit power $\mathcal{P}$, constant E_b/N_0 implies constant bit-rate $R_b = \mathcal{P}/E_b$. Then, the system bandwidth decreases as $1/t$. In other papers, different systems are compared for constant γ, i.e., for constant symbol-rate. Then, the system bandwidth is constant and the bit-rate increases linearly with t.

Remark 7. Our results show that the MMSE front-end outperform the ZF front-end by some dB. This is a quite counterintuitive behavior, since it can be expected that, for a fixed matrix $\mathbf{C}$, the performance of the two front-ends are asymptotically equivalent for large E_b/N_0. We interpret this fact as an effect of averaging over the ensemble of random channel matrices. The FER is dominated by the occurrence of very "bad" channel realizations (outages). In these cases, the MMSE front-end provides a considerable advantage over the ZF front-end, which incurs in the noise-enhancement effect typical of ZF equalizers.

V. Conclusions

We proposed a low-complexity space-time architecture based on diagonal interleaving of a single trellis code and on a PSP receiver that makes use of predecisions on the surviving paths of the Viterbi algorithm in a decision-feedback scheme. Forward and backward filters of the decision-feedback front-end are designed either according to the ZF or to the MMSE criterion. We developed an approximated union-bound semi-analythic performance analysis technique based on multivariate weight enumerators, by assuming perfect decisions in the feedback. The FER evaluated in this way is in very good agreement with computer simulations of the true PSP scheme, thus showing that the degradation due to non-perfect decision feedback is negligible with the PSP approach. Results show that binary codes of rate 1/2 with $t = r = 8$ antennas, yielding spectral efficiency $\eta = 4$ bit/s/Hz, can achieve FER between $\approx 10^{-4}$ and $\approx 10^{-6}$ at $E_b/N_0 = 0$ dB (block length $N = 256$ coded bits) on a block-fading Rayleigh channel. Interestingly, powerful codes should be considered when the number of antennas is large, while simple codes are good enough when the number of antennas is small. Finally, contrarily to what stated in most papers on multiple antennas (e.g., [5, 4]), the MMSE front-end provides an advantage of some dB over the ZF front-end in terms of E_b/N_0.

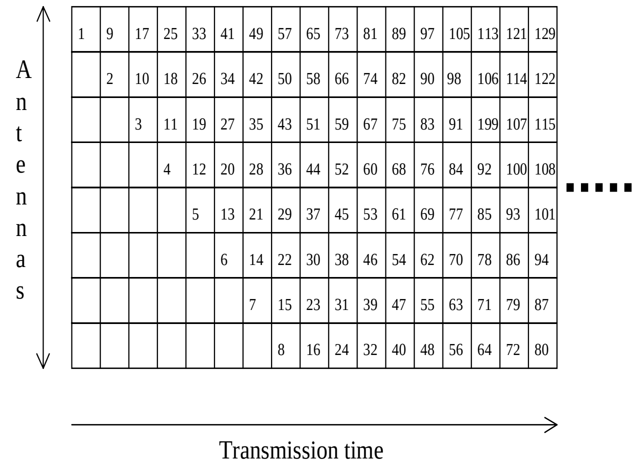

Figure 1: An example of diagonal interleaving with $t = 8$.

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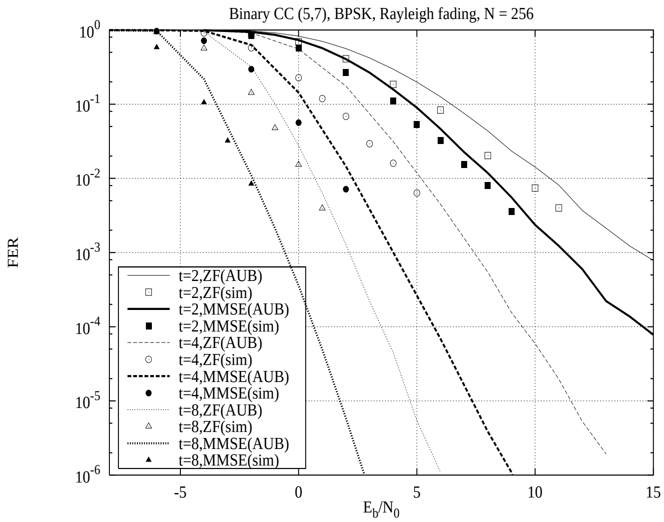

Figure 2: FER vs. E_b/N_0 for the convolutional code $(5,7)$ with 2, 4 and 8 antennas, ZF and MMSE PSP receivers.

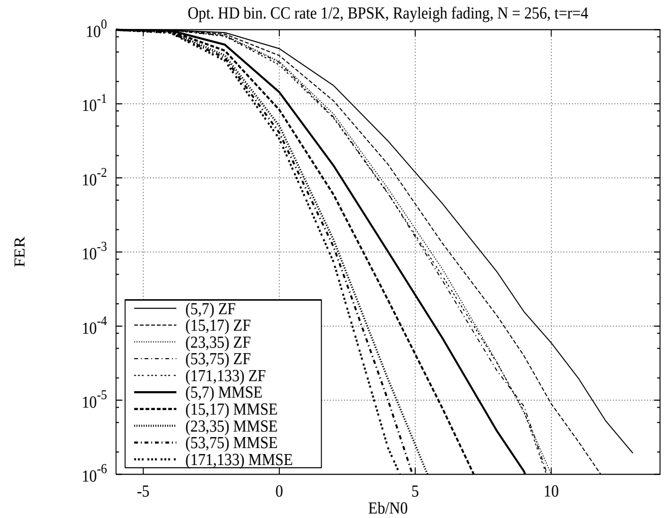

Figure 4: FER vs. E_b/N_0 for binary convolutional codes of rate 1/2 with $t = r = 4$ antennas.

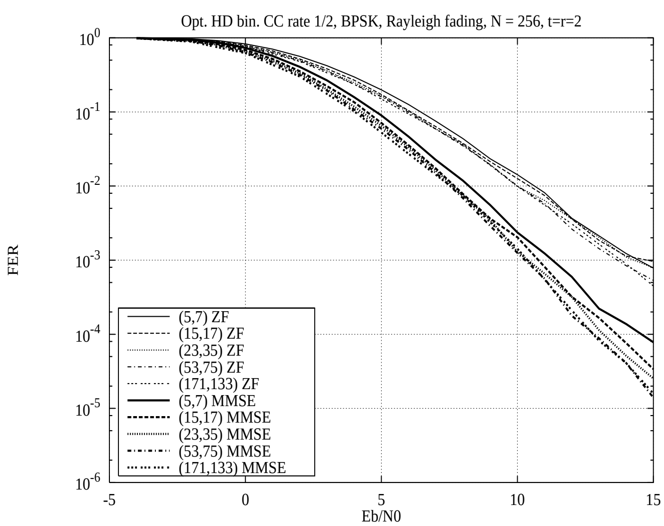

Figure 3: FER vs. E_b/N_0 for binary convolutional codes of rate 1/2 with $t = r = 2$ antennas.

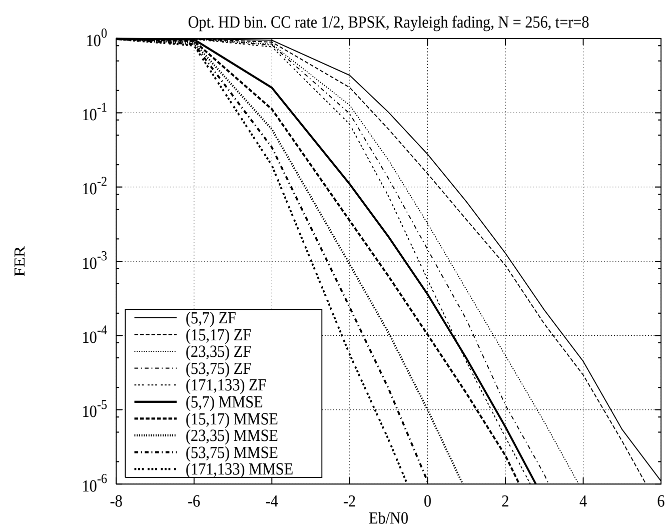

Figure 5: FER vs. E_b/N_0 for binary convolutional codes of rate 1/2 with $t = r = 8$ antennas.