Achieving Full Sum DoF in the SISO Interference Channel with Feedback Delay

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***Abstract*—Channel state information at the transmitter (CSIT) is of utmost importance to manage interference in multi-user wireless networks, in which transmission rates at high SNR are characterized by (sum) Degrees of Freedom (DoF). Ingenious techniques have been proposed to deal with delayed CSIT, notably a scheme by Maddah-Ali and Tse providing significant DoF with outdated CSIT. However, with most techniques, any delay in the CSIT feedback still results in a DoF loss. A notable exception is the work by Lee and Heath, allowing to preserve the optimal DoF in the underdetermined MISO BC and IC, but for feedback delays T_{fb} decreasing with the number of users K, up to a fraction $\frac{1}{K}$ of the channel coherence time T_c. In this work, an ergodic interference alignment scheme that preserves the $K/2$ sum DoF of the K-user IC is proposed for $T_{fb} \leq T_c/2$. It is also proven that $T_{fb} = T_c/2$ is the longest for which the optimal $K/2$ DoF can still be obtained for all K.**

***Index Terms*—SISO IC, delayed CSIT, interference alignment**

I. Introduction

Interference is a major limitation in wireless networks and the search for efficient ways of transmitting in this context has been productive and diversified [1]–[3]. If numerous techniques allow the increase of the degrees of freedom (DoF), most of them rely on accurate and timely channel state information at the transmitter (CSIT) and at the receiver (CSIR). Especially CSIT is problematic since it requires feedback which inevitably causes a delay, that may be substantial. It came as a surprise that with totally outdated CSIT, the MAT scheme [4] is still able to produce significant DoF gains in the K-user MISO broadcast channel (BC). Using a sophisticated variation of the MAT scheme, [5] and [6] independently proposed improved schemes for the the 2-user MISO BC with time-correlated channels. This result was extended to the 2-user MISO interference channel (IC) in [7].

Even with such promising results, it was still generally believed that any delay in the feedback necessarily causes a DoF loss. However, Lee and Heath, in [8], proposed a scheme that achieves N_t (sum) DoF in the block fading underdetermined MISO BC with N_t transmit antennas and $K = N_t + 1$ users if the feedback delay is small enough ($T_{fb} \leq T_c/K$). This scheme is also valid in the MISO IC with N_t antennas per transmitter and $K = N_t + 1$ transmitter-receiver pairs [9].

This possibility of achieving the full sum DoF of the MISO BC and IC with small feedback delays comes at the expense of a slight increase of the feedback overhead [10]. It was then demonstrated in [11] that the minimum fraction of time of perfect current CSIT required per user in order to achieve the optimal DoF of $\min(N_t, K)$ is given by $\min(N_t, K)/K$. Therefore, the lack of timeliness of CSIT can be compensated by having the CSIT of more users. Indeed, the achievability result in [11] relies on always having perfect current CSIT of N_t users at any time but not always of the same N_t users. In a classic block fading model, this would require an increase of the feedback overhead. In [12], the feedback versus performance trade-off is characterized extensively. For the square case, i.e., when $K = N_t$, the authors confirm that with a block fading model any feedback delay causes a DoF loss and that the basic combination of using MAT, when only delayed CSIT is available, and performing zero-forcing (ZF), when current CSIT is available, is optimal in terms of DoF, as was mentioned in [13].

Block fading and stationary bandlimited fading models are shown to be both special cases of the more general finite rate of innovation (FRoI) model in [14]. Furthemore, the authors demonstrate that, with adequate training and foresighted feedback, the CSI can be acquired at any time and be valid for the coherence time of the channel T_c. Thereby allowing for the permanent availability of CSIT and for the possibility of performing ZF without any dead time, at the cost of an increased rate of training and feedback.

For the 3-user SISO IC, [15] introduces *retrospective interference alignment* (IA) and reaches a DoF greater than one with outdated CSIT. Then, in [16], a general scheme for the K-user SISO IC with outdated CSIT was shown to reach a sum DoF that is greater than one and increases with K. However, these DoF are upper bounded by $\frac{4}{6\ln 2-1} \approx 1.266$. In [17], a scheme based on ergodic interference alignment is shown to yield a DoF that increases with K and approaches 2 in the K-user SISO IC with outdated CSIT. There is no proof of optimality of these DoF, but it is conjectured that the DoF of the SISO IC with completely outdated CSIT is upper bounded by a constant in [16]. This is in sharp contrast with the optimal sum DoF of $\frac{K}{2}$ in the SISO IC with current CSIT [3].

We shall here demonstrate that the optimal sum DoF of the K-user SISO IC is still $\frac{K}{2}$ for feedback delay $T_{fb} \leq T_c/2$ and propose a scheme which achieves this optimal sum DoF.

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Moreover, this feedback delay supported here will be proven to be the longest for which the optimal $K/2$ DoF can still be obtained for all K.

The approach is based on a variation of the *ergodic interference alignment* scheme proposed in [18], where the authors show that not only $\frac{K}{2}$ DoF are attainable but also that each user can get half of its interference-free rate at any signal to noise ratio (SNR). Our variation will conserve this property.

II. System Model

We consider a K-user SISO IC, i.e., there are K transmitter-receiver pairs, all equipped with a single antenna. Let $\mathbf{H}[t] = [h_{ji}(t)] \in \mathbb{C}^{K\times K}$ denote the channel matrix at time t where $h_{ji}(t)$ is the frequency flat time-varying channel coefficient between transmitter i and receiver j. We assume a block fading model, the channel coefficients are constant over blocks of length T_c and change independently between blocks. Furthermore, channel coefficients are drawn from a continuous distribution, their phases are uniformly distributed and are independent from their magnitude. It is assumed that, due to feedback delay, the transceivers possess the CSI only after a delay of T_{fb} channel uses.

The channel output observed at receiver $j \in [1, K]$ is a noisy linear combination of the inputs

$$Y_j[t] = \sum_{i=1}^{K} h_{ji}[t]X_i[t] + Z_j[t] \tag{1}$$

where $X_i[t]$ is the transmitted symbol of transmitter i, $Z_j[t]$ is the additive white Gaussian noise at receiver j.

The performance metric is the sum DoF, it is the prelog of the sum rate. Let $R_j(P)$ denote the achievable rate for user j with transmit power P, then the achievable DoF for user j is

$$d_j = \lim_{P\to\infty} \frac{R_j(P)}{\log_2(P)}$$

and the sum DoF is $\text{DoF}(K) = \sum_{j=1}^{K} d_j$.

Our starting point is the work on ergodic IA [18]; we use a similar system and channel model. Most of the remarks and improvements made to the original scheme could be applied also to our delayed CSIT version. For instance, the channel coefficient distribution does not need to be symmetric [19], the sum of channel matrices below can be relaxed to be an arbitrary diagonal matrix and not only the identity matrix [19], and simple strategies can be deployed to reduce latency [20].

III. Main Result

Our main result is the following theorem assessing the resilience of the IC sum DoF against the lack of timeliness of the CSIT.

Theorem 1: In the K-user SISO IC, as long as feedback delay $T_{fb} \le \frac{T_c}{2}$,

$$\text{DoF}(K) = \frac{K}{2} \quad . \tag{2}$$

To prove achievability, in section III-B, we introduce a variant of the ergodic IA scheme [18] that works in the block fading IC and does not require any CSIT before the second half of each block. The converse is trivial since $\frac{K}{2}$ is the DoF of the K-user SISO IC with instantaneous CSIT.

A. Ergodic IA [18]

The main idea behind ergodic IA is to transmit the data a first time during channel realization $\mathbf{H}[t_1]$, then to wait for the complementary channel realization $\mathbf{H}[t_2]$ such that $\mathbf{H}[t_1]+\mathbf{H}[t_2] = I$, the $K \times K$ identity matrix. We shall denote this relation by $\mathbf{H}[t_2] = \overline{\mathbf{H}[t_1]}$. It allows each receiver to cancel all interference by simply adding the signals received at times t_1 and t_2.

$\mathbf{H}[t_2] = \overline{\mathbf{H}[t_1]}$ will never happen when channel coefficients are drawn from a continuous distribution. However, it is still possible to match up channel matrices to an approximation error small enough to allow decoding [18]. This can be done through appropriately precise quantization. The authors of [18] prove that, by considering sequences of channel realizations that are long enough, it is possible to be sure, with a sufficient probability, that it will be possible to match up enough channel realizations to yield $\text{DoF}_{\text{ErgoIA}}$ that approaches $\frac{K}{2}$.

[18] assumes $T_c = 1$ and $T_{fb} = 0$, but the extension to larger T_c with $T_{fb} = 0$ is straightforward and is illustrated in Fig. 1 for $T_c = 2$. We will now prove that a similar strategy can be set up for any T_c and T_{fb} such that $T_{fb} \le \frac{T_c}{2}$.

B. Ergodic IA with delayed CSIT

Assuming $T_{fb} = \frac{T_c}{2}$, we can divide each block in two parts of *equal* length: the beginning of the block, when there is no current CSIT and the end of the block, when current CSI is available at the transmitter. In this configuration, the original ergodic IA cannot be performed anymore, or at least not on the first part of each block. However, it is possible not to associate whole complementary blocks but only half blocks. For example, consider the case $T_c = 2$ and $T_{fb} = 1$. For all t: $\mathbf{H}[2t] = \mathbf{H}[2t+1]$ and current CSIT is only available on odd time indices. Then, if $\mathbf{H}[2t_1] = \overline{\mathbf{H}[2t_2]}$, and thus $\mathbf{H}[2t_1 + 1] = \overline{\mathbf{H}[2t_2+1]}$, we can match the second channel use of the complementary channel realization, $\mathbf{H}[2t_2+1]$, with the first channel use of the first channel realization, $\mathbf{H}[2t_1]$, because, at time index $2t_2+1$, there is CSIT on both $\mathbf{H}[2t_1]$ and $\mathbf{H}[2t_2+1]$. This new pairing method is illustrated in Fig. 2.

In more detail, *new* data signals are sent during the first half of each block. As explained in [18], we quantize all possible channel realizations and can take blocks of channel realizations of length T coherence blocks such that the sequence of channel realizations will be γ-typical with probability $(1-\epsilon)$. We refer to these blocks as meta-blocks here to indicate that, in our case, they are blocks of coherence blocks. This means that the number of occurrences $\#H_i$ of any channel realization H_i in a given meta-block is bounded as follows

$$T_cT\left(p(H_i)-\gamma\right) \le \#H_i \le T_cT\left(p(H_i)+\gamma\right)$$

with probability $(1-\epsilon)$. Since the coefficients are drawn i.i.d. from a distribution with uniform phase, the probability that the complement of a channel matrix occurs in a given time frame is the same as the probability that the original matrix occurs. However, this requirement on the distribution can be relaxed [19]. We can ensure it will be possible to pair enough channel realizations between two consecutive meta-blocks to approach $\frac{K}{2}$ DoFs in *steady state* by defining γ and ϵ as in

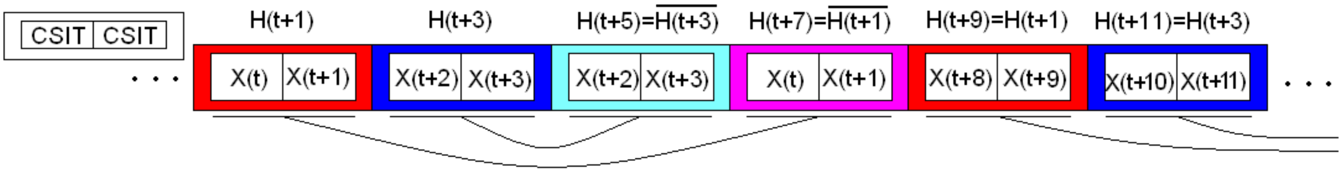

Fig. 1. Example of ergodic IA with $T_c = 2$ and $T_{fb} = 0$.

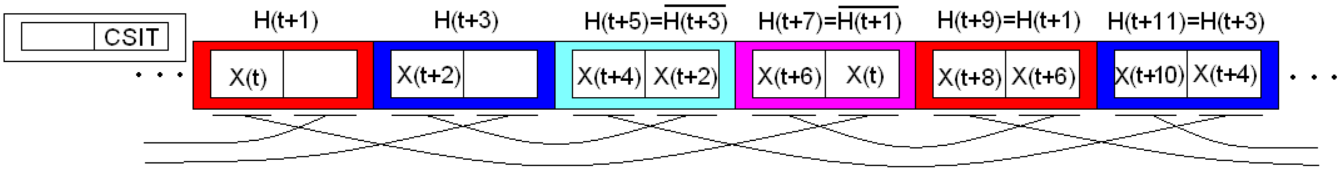

Fig. 2. Example of ergodic IA variant for delayed CSIT with $T_c = 2$ and $T_{fb} = 1$.

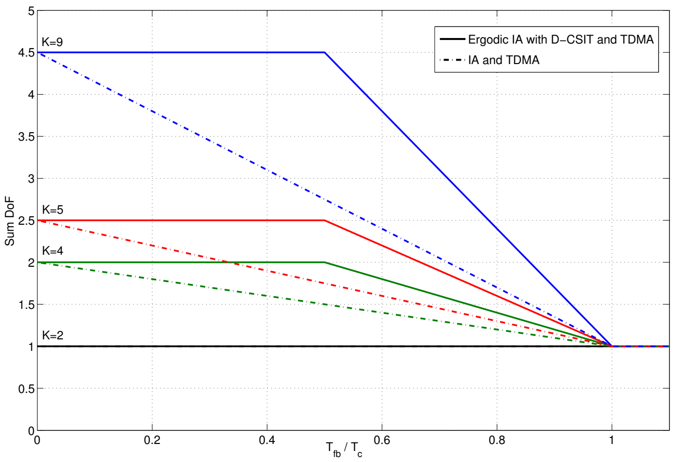

Fig. 3. Feedback delay-DoF tradeoff in the SISO IC.

[18]. Indeed, we notice that the second part of each block in the first meta-block will be wasted as well as the first part of each block of the last meta-block. However, by performing this scheme over a sequence of N meta-blocks, it is possible to transmit $(N-1)$ information blocks with the multiplexing gain of ergodic IA, thereby reaching DoF $= \frac{N-1}{N}\text{DoF}_{\text{ErgoIA}}$, which approaches $\text{DoF}_{\text{ErgoIA}}$ as N increases. It proves that $\text{DoF}_{\text{ErgoIA}}$ can be made arbitrarily close to $\frac{K}{2}$, which proves the achievability in Theorem 1 for $T_{fb} = \frac{T_c}{2}$. The case of smaller T_{fb} is trivially settled by still dividing each block in two parts of equal length, this time a fraction of the first part will correspond to the transmitter having current CSI instead of delayed CSI. However, what can be done with delayed CSIT can also be done with current CSIT and the proposed scheme is still applicable.

IV. Feedback delay-DoF tradeoff

A. *Time Sharing*

The variant of ergodic IA described above works for $T_{fb} = \frac{T_c}{2}$ but also for any shorter feedback delay without any modification. The case of longer feedback delay can be dealt with by doing time sharing between the variant we propose and time division multiple access (TDMA) transmission or any technique designed for the SISO IC with completely delayed CSIT. To the best of our knowledge, in the IC, only two other schemes deal with non trivial feedback delays that remain smaller than the channel coherence time. In [9], for $K = 2$, the authors also obtain the full sum DoF of the SISO IC with feedback delays up to half the channel coherence time but this could also be obtained by simple TDMA transmission. Their scheme is generalized for larger K but requires more transmit antennas N_t and preserves the optimal sum DoF of $N_t = K-1$ up to $T_{fb} = \frac{T_c}{K}$, which decreases with K. On the contrary, the scheme proposed here preserves the full sum DoF of the K-user SISO IC for $T_{fb} \leq \frac{T_c}{2}$ for any K. In [14], the full sum DoF can be preserved at the cost of an increase of the training and feedback frequency whereas the scheme proposed here does not. In Fig. 3, we plot the sum DoF that can be achieved (solid lines) in the K-user SISO IC as a function of $\frac{T_{fb}}{T_c}$. The dashed lines corresponds to time sharing between IA (when current CSIT is available) and TDMA (otherwise). We observe that the proposed ergodic IA variant significantly improves the sum DoF.

B. *Partial Optimality*

The scheme described presents a partial optimality in the sense that the feedback delay of $T_{fb} = T_c/2$ supported here is the longest for which the optimal $K/2$ DoF can still be obtained for all K. The idea for proving this converse result is to use the DoF of the K-user MISO BC with K transmit antennas as an upper bound since splitting the transmit antennas can only decrease the capacity. In [12], the authors show that time sharing between MAT and ZF is optimal when dealing with delayed CSIT in the square MISO BC. This means that the DoF of the SISO IC with delayed CSIT, as a function of $\frac{T_{fb}}{T_c}$, is upperbounded by a line going from $(0, K)$ to $(1, \frac{K}{\sum_{k=1}^{K}\frac{1}{k}})$. For large K, this results in the DoF having to be less than $\frac{K}{2}$ at an abscissa increasingly closer to $\frac{1}{2}$ as illustrated in Fig. 4 for $K = 20$ and $K = 30$. Formally, we have the following theorem,

Theorem 2: If there is a scheme such that

$$\forall K, \text{DoF} = \frac{K}{2} \text{ for } T_{fb} = \alpha T_c$$

then

$$\alpha \leq \frac{1}{2}.$$

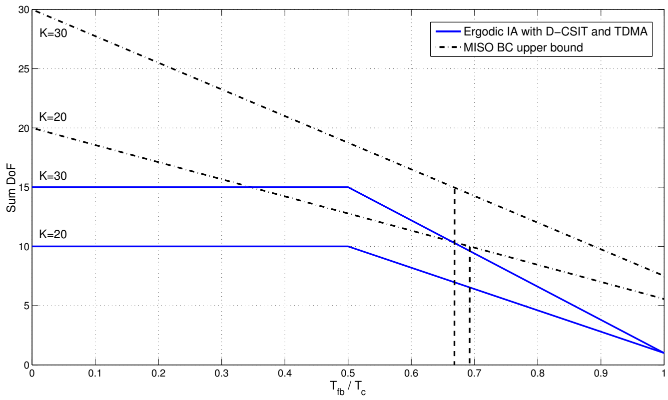

Fig. 4. MISO BC upper bound.

Proof: According to [12], the DoF of the K-user MISO BC with $T_{fb} = \alpha T_c$ is reached by time sharing between MAT and ZF and is

$$(1-\alpha)K + \alpha \frac{K}{\sum_{k=1}^{K} \frac{1}{k}}$$

which becomes less than $K/2$ for

$$\alpha > \frac{1}{2(1 - \frac{1}{\sum_{k=1}^{K} \frac{1}{k}})} \approx \frac{0.5}{1 - \frac{1}{\ln(K)}}$$

which approaches $\frac{1}{2}$ when K grows and makes it impossible to have a scheme that preserves the $K/2$ DoF for all K for any α strictly greater than $\frac{1}{2}$. $\square$

C. Square MIMO Configurations

The exact same (SISO) scheme is applicable to the square MIMO IC, i.e., when there are $N_t = N_r$ transmit and receive antennas. Indeed, in this configuration, one can do at least as well as in the KN_t-user SISO IC and achieve $\frac{KN_t}{2}$ DoF with $T_{fb} \leq \frac{T_c}{2}$, which is the optimal sum DoF of the square MIMO IC according to [3].

D. MISO Configurations

The scheme proposed in [18] for "recovering more messages" can be used to reach the decomposition bound in MISO cases and in MIMO cases, using the decomposability, if N_t/N_r is an integer. In such cases, the CSI is not needed for the first transmission but only for the last $R = N_t/N_r$ transmissions. Therefore, by doing the pairing in a similar fashion, it is possible to achieve the $N_r KR/(R+1)$ DoF with feedback delay up to $\frac{T_c}{R+1}$.

V. Concluding Remarks

The ergodic IA variant proposed here provides a strong theoretical result: the full sum DoF of the K-user SISO IC can be preserved with feedback delays up to $\frac{T_c}{2}$. Moreover, it is proven that this feedback delay supported here is the longest for which the optimal $K/2$ DoF can still be obtained for all K.

Most improvements applicable to ergodic IA, for instance to reduce latency, can also be applied to the proposed variant. The scheme also assures that half the interference free rate can be reached at finite SNR [18]. A different pairing rule could be chosen to reach an optimal SNR offset over the original scheme as proposed in [19].

We observe that, contrary to the MISO BC, in the SISO IC with delayed CSIT the full sum DoF can be preserved without requiring extra receivers [8], [11] or extra overhead [14].

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